PART III TI-89

1. Home Screen Topics

1.1 Built-in Functions and Constants

At this point you should be familiar with the basic operations of addition, subtraction, multiplication and division on your TI-89 calculator. Note: the HOME key takes you to the Home Screen. In addition to the basic operations, the TI-89 has several built-in functions that are used extensively in calculus. These include *square root* (2nd [√]), the trigonometric functions *sine* (2nd [SIN]), *cosine* (2nd [COS]), *tangent* (2nd [TAN]) and their inverses *arcsine* (◆ [SIN⁻¹]), *arccosine* (◆ [COS⁻¹]), *arctangent* (◆ [TAN⁻¹]), the *natural logarithm* (2nd [LN]), and the *natural exponential* (◆ [eˣ]). The *multiplicative inverse* or *reciprocal* of a number *x*, *1/x*, is obtained by 1 ÷ x ENTER. *Powers* of numbers, including *negative* and *fractional* powers, are computed using a sequence such as x ^ (2 ÷ 3) ENTER, which is the computation of $x^{2/3}$. Notice the use of the parenthesis around the entire exponent. The absolute value function |*x*| is listed in the CATALOG. Press CATALOG, then scroll down until you find the function denoted abs(.

Two mathematical constants that are used frequently in calculus are the numbers *e* and π. These are also built into the TI-89. The sequence ◆ [eˣ] 1) ENTER gives you e in symbolic form. If you want a numerical approximation for e, then press ◆ before you press ENTER. The key sequence 2nd [π] ENTER gives you π, also symbolically.

1.2 Expressions

After you enter a mathematical expression directly into the TI-89, press ENTER to evaluate it. When entering an expression, use the arrow keys to move the cursor within the expression, then use the delete (2nd [DEL]) and insert (2nd [INS]) keys to edit the expression as needed. The calculator automatically saves the answer into the system variable Ans. The TI-89 also allows you to save a value into a named variable. For example, if you want to compute $\sqrt{2}$ and save it under the name r, execute the sequence 2nd [√] 2) STO▸ ALPHA [R] ENTER (Figure 1).

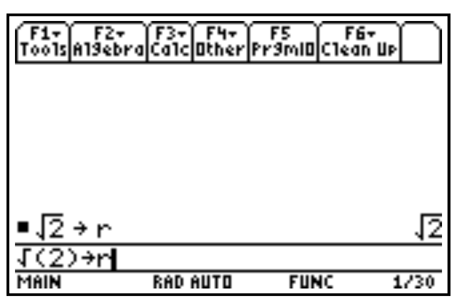


Figure 1: Storing a value to a variable.

1.3 Recalling an Entry

To retrieve the last entry from your previous calculation, press the (2nd [ENTRY]) keys, then position the cursor where you want to edit the expression. This feature is particularly useful if you are evaluating similar expressions repeatedly. The key sequence 2nd [ANS] will retrieve the last computed value.

1.4 Decimal to Fraction

The TI-89 can be set to operate in exact or approximate arithmetic. The best way to operate a TI-89 is to set it to AUTO mode. To do this, press the MODE key, move to Page 2 of the mode settings (F2), and scroll down to Exact/Approx. Use the right arrow to select AUTO, then press ENTER twice. See Figure 2.

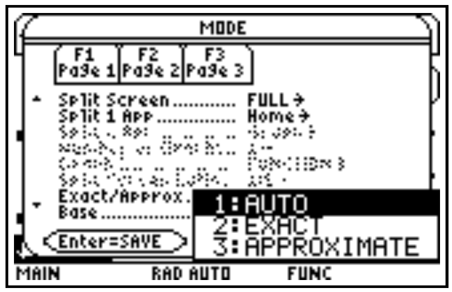


Figure 2: The AUTO mode.

With the TI-89, you have the advantage of using the History Area, which is displayed above the Entry Line on the Home Screen. You can use the arrow keys to scroll within this area. If you press ENTER, the item that is highlighted in the History Area will be entered into the Entry Line. This is helpful if you are computing similar operations with minor editing.

2. Mode Settings

To access the mode dialog box on the TI-89 press the MODE key. The calculator will display the first page of the mode dialog box (Figure 3) showing the current settings. You can use the F1, F2, and F3 keys to move through the Mode Settings pages.

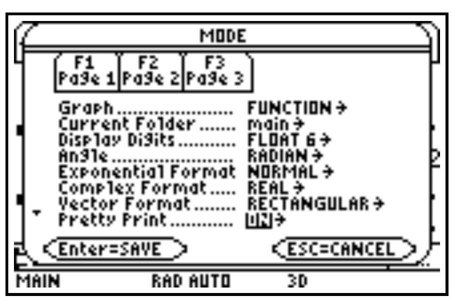


Figure 3: Mode Settings on the TI-89.

To change settings use the ▲, and ▼ keys to scroll up and down and the ▶ key to view specific options. Press ENTER to change a setting, and press ENTER one more time to return to the Home Screen. When you press ESC to exit the mode settings, any changes made will be canceled.

For more detail on Mode Settings refer to page 33 of the Guidebook that came with your TI-89 calculator. Specific settings may be required for certain calculus topics. We will describe these as needed. Your instructor may request that you change mode settings as needed. For now, make sure that your calculator has the same settings as shown in Figure 3.

3. Solving Equations

To solve an equation means that you want to find values of x for which $y = f(x) = 0$, where $f(x)$ is the given equation. That is, you want to find the points of intersection of the graph of the function with the x-axis.

3.1 Solve(

This feature is available in both the Algebra menu and the CATALOG. To access this feature, select solve(from the Algebra menu. The command requires that you input an equation and the variable. Type the equation and the variable directly in to the command line, then close the parenthesis and press ENTER. The solution will be displayed in the history area. See Figure 4.

When an equation has complex solutions the complete answer is not be displayed. In this case, it is best to use the cSolve(command instead of Solve(. This command requires the same inputs, and produces all real and complex solutions. See Figure 5. Of course, the TI-89 will automatically perform symbolic calculations if an entirely symbolic equation is entered.

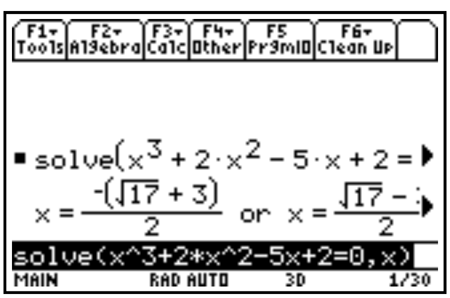


Figure 4: solve(.

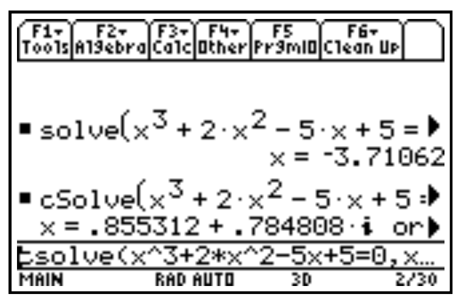


Figure 5: cSolve(.

Your TI-89 contains other features with which you can compute the solution to an equation. These will be described below in Section 4.

4. Functions

Calculus is an area of mathematics in which you study functions of one or more real variables in a variety of ways. The topics below will help you to enter functions into your calculator and to analyze their values and graphs. First, make sure that your calculator is set to Function Mode, (that is, FUNCTION should be displayed for the Graph mode). (See the Section 2 in Part III of this manual.)

4.1 Entering Functions

The TI-89 allows you to store functions into its memory. Press ◆ [Y =] to access the Y = Editor (Figure 6). Use the arrow keys to scroll up and down to select a function or to scroll left and right if you are editing a function. If you need to erase an entire line, press the CLEAR key. In function mode, the X key produces *x*, which is used as the independent variable. When a function is selected a check mark will appear to the left of the definition of the function. If you wish to deselect a function, highlight the definition of the function and press F4. One nice thing about the TI-89 is that you can use numbers, variables, matrices, lists, and other functions to define new functions. These features can be particularly useful when studying calculus.

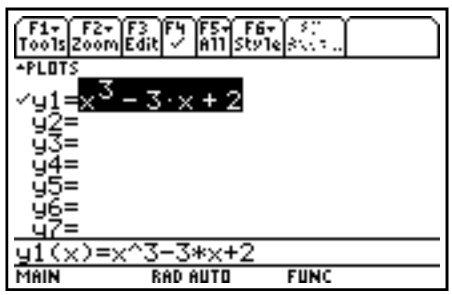


Figure 6: The Y = Editor.

4.2 Graph Style

Functions can be graphed in several different styles. Two such styles and the necessary keystrokes to display them are described in this section. For additional information see the Guidebook that came with your calculator.

The standard style for drawing graphs is called Line. This is the default style setting for you TI-89 calculator. This means that the calculator will plot certain points of the graph, then join them with tiny line segments creating a continuous-looking graph. In Dot style, the calculator simply plots certain points on the graph of the function. To change the style of the graph you must be in the Y = Editor, and must have a function highlighted in order to see the Style menu. Press the 2nd [F6] keys, to see the different styles that are available. Line and Dot styles are the first two. Use the arrow keys to scroll down to the desired style, press ENTER, and the new style will be selected (Figure 7).

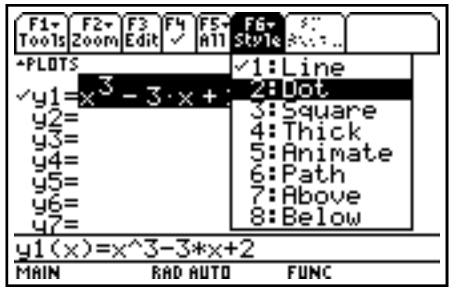


Figure 7: The Dot style selected.

4.3 Viewing Window

The viewing window represents a portion of the Cartesian plane. The standard viewing window is within the bounds $-10 \le x \le 10$, and $-10 \le y \le 10$. In many cases you will need to draw graphs of functions that are outside this range, but this is not a problem if you are using a TI-89, since you can set the viewing window as needed. Press ◆ [WINDOW] to access the window settings (Figure 8).

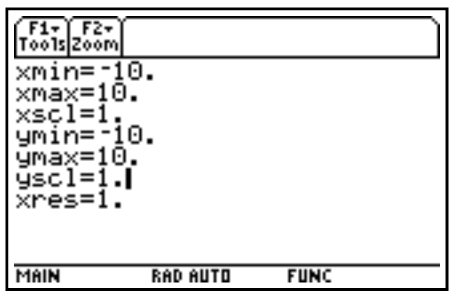


Figure 8: WINDOW.

The values of xmin, xmax, ymin, and ymax determine the portion of the Cartesian plane that will be shown. You must enter values that satisfy xmin < xmax, and ymin < ymax. The numbers xscl and yscl determine the distance between tick marks. Setting these numbers equal to zero will result in no tick marks. The number xres sets pixel resolution. For our purposes we want to set xres = 1.

4.4 Graphing a Function

Press ◆ GRAPH to display the graphs of the functions that are selected. Your calculator allows you to analyze the graphs in a variety of ways. The remainder of the section contains descriptions of several of the features connected to functions and their graphs. See Section 5 for additional topics.

4.5 Zoom

The ZOOM item in the menu (F2) allows you to change the viewing window in specific ways. Select the first item (ZoomBox) by entering 1. Move the cursor to a position that will become one corner of the viewing window, then press ENTER. Next, move the cursor to determine the opposite corner of the window, then press ENTER. The graph will be redrawn within the boundaries of the window. The ZoomIn and ZoomOut features allow you to look at the graph from closer or further away, respectively. Select one of these items by pressing 1 and 2, or by highlighting a specific feature and pressing ENTER. A cursor will appear on the graph, which will determine the center of the new viewing window. Move the cursor to the desired position and press ENTER. The graph will be redrawn. ZoomStd is the default set at the factory. It gives you the viewing window [–10, 10, –10, 10]. ZoomSqr sets the dimensions of the viewing window so that a circle will look like a circle. ZoomData is convenient when plotting statistical data points; it sets the viewing window so that all data points are visible. ZoomFit resizes the window by changing the Y values in such a way that the graph is displayed within the prespecified values of X. The other items in the Zoom menu are discussed in Chapter 6 of your Calculator Guidebook.

4.6 Trace

The Trace feature allows you to move the cursor along the graph of a function as the calculator displays the values of the coordinates of the points on the graph. Select Trace from the menu (F3), and you will see your graph displayed and the trace cursor appear on the graph. Use the left and right arrow keys to move the cursor along the graph. You can also move the cursor to a specific point by entering the *x*-value of the point and pressing the ENTER key. The cursor will immediately move to that point and display the coordinates of the point. Use the up and down arrows to move from function to function (Figure 9).

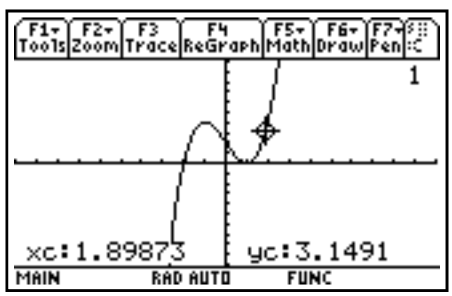


Figure 9: Trace.

4.7 Table

If you have entered a function into y1 (or any other dependent variable) the table feature will allow you to compute values for this function for many values of the independent variable. First, press ◆ [TblSet] to set the starting value of x (tblStart) and the increment of x (Δtbl). Set Independent to AUTO, and press ENTER to save the values (Figure 10). Press ◆ [TABLE] to view a table in which the values for y1 are computed automatically (Figure 11). Scroll through the table of values using the up and down arrow keys. The other commonly used selection is to set Independent to ASK. Press ENTER to save these options, then press ◆ [TABLE]. Enter a value for x, press ENTER and the corresponding value for y1 will be computed. For more information on tables, see Chapter 13 of the Guidebook that came with your calculator.

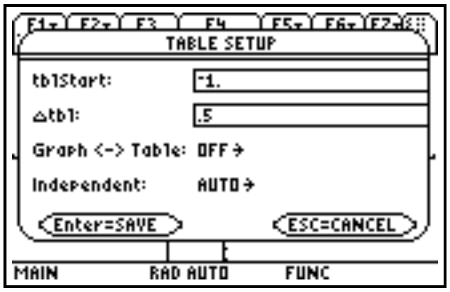


Figure 10: TblSet.

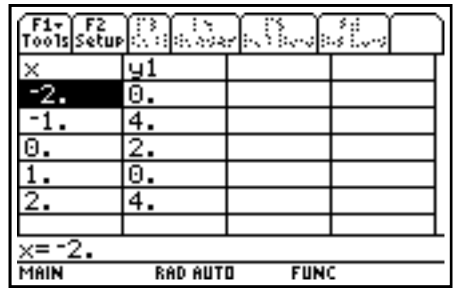


Figure 11: TABLE.

4.8 Solving Equations

You can solve an equation by using the trace and table features of a function, although the solution you get may be a very rough approximation, depending on the settings of your calculator. Your calculator has built-in algorithms for solving equations that make use of the graph feature. (See Section 3 in Part III of this manual for other methods on solving equations.)

Trace. Enter and graph a function. Select Trace from the menu and use the arrow keys to move the cursor to the point where the graph meets the *x*-axis. Once you establish an *x* -value that gives you a *y* -value close to zero, you can experiment by zooming in to find other *x*-values that may give a *y*-value of exactly zero (Figure 12). Often you will not arrive at an *x*-value that lies exactly on the *x*-axis.

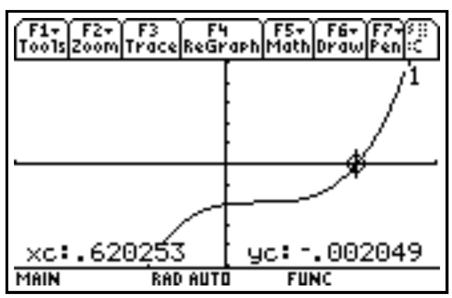


Figure 12: Solving an equation with Trace.

Table. Enter the function and construct a table of values for the function. Look at the graph to see if there is a solution between 0 and 1. If this is the case, it's a good idea to set tblStart = 0 and Δtbl = 0.01, and Independent to AUTO. Scroll through the values in the table to find values of the dependent variable close to zero. Once you establish an *x* -value that gives you a *y* -value close to zero, you can experiment with other values of tblStart and Δtbl to see if you can achieve a *y*-value of exactly zero (Figure 13). Often you will not arrive at an *x*-value that yields exactly zero.

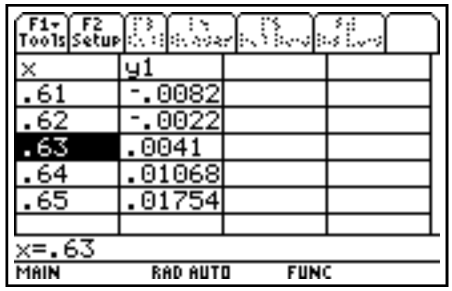


Figure 13: Using TABLE to solve an equation.

Zero. Enter a function into memory and select it. Press ◆[GRAPH] to graph the function, select Zero from the Math menu, and press ENTER. Use the arrows to move the cursor to select the lower bound and upper bound, as prompted by the calculator. Press ENTER to save each of your selections. The cursor will move to the zero of the function and the calculator will display the values of x and y (Figures 14–16).

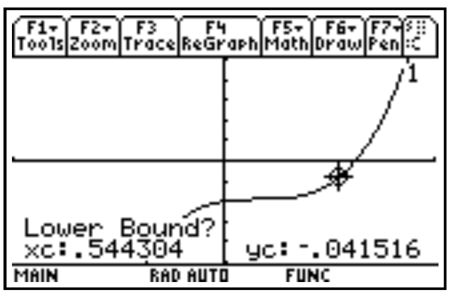


Figure 14: Lower bound.

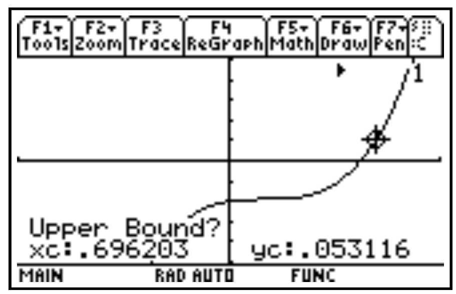


Figure 15: Upper bound.

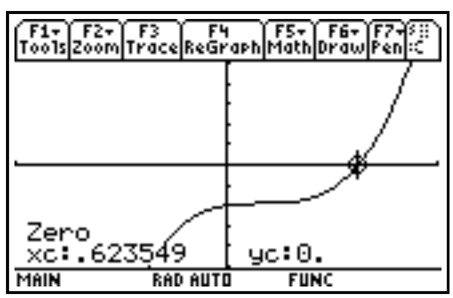


Figure 16: The zero.

Intersection. Suppose you want to solve the following: $e^{3x} - 5x - 7 = 0$. This problem is equivalent to finding the x-value at the point where the graphs of $y1 = e^{3x}$ and $y2 = 5x + 7$ meet. Enter both functions into your calculator's memory and select them, set your viewing window to [–5, 5, –3, 15]. Press ◆[GRAPH] to display the graphs, select Intersection from the Math menu, and press ENTER. Use the arrows to move the cursor to select the first curve, the second curve, a lower bound and an upper bound, as prompted by the calculator. Press ENTER to save each of your selections. The cursor will move to the point of intersection of the curves and the calculator will display the values of x and y (Figures 17–19).

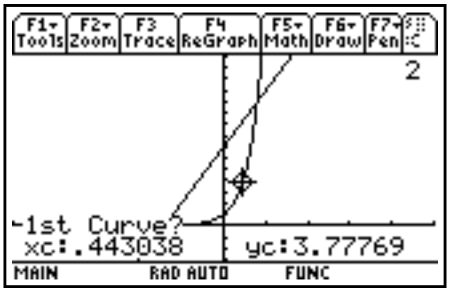


Figure 17: First curve.

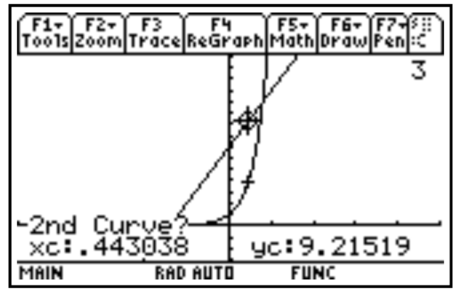
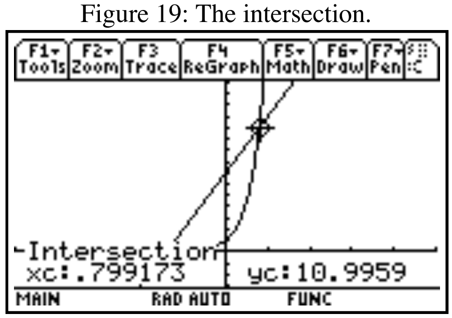


Figure 18: Second curve.

Figure 19: The intersection.



4.9 Composition of Functions

Functions defined in the TI-89 can be combined to form new functions. One such combination is the composition of two functions. Enter the functions $y1 = 1 - x$ and $y2 = e^x$ into your calculator. Both functions have domain equal to the set of real numbers, therefore the compositions $y1(y2(x))$, and $y2(y1(x))$ can both be formed without restrictions. Enter $y3 = y1(y2(x))$. This is the function $y3 = 1 - e^x$, its graph is shown in Figure 20. Enter $y4 = y2(y1(x))$. This is the function $y4 = e^{1-x}$, its graph is shown in Figure 21.

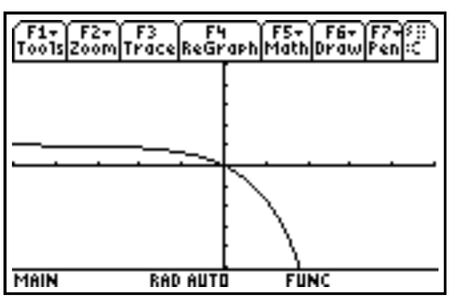


Figure 20: The graph of $y3 = 1 - e^x$.

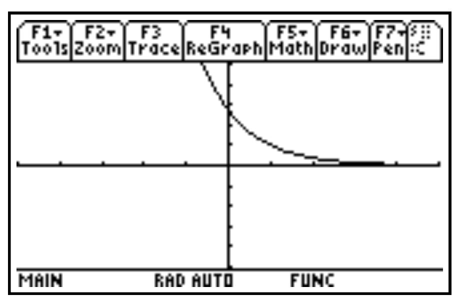


Figure 21: The graph of $y4 = e^{1-x}$.

4.10 Piecewise-defined functions

In many applications, functions cannot be given by one unique formula. Instead, functions related to applications are often given in parts. Such functions are called *piecewise-defined functions*. The TI-89 allows you to enter and graph piecewise-defined functions. Consider the function $f(x) = \begin{cases} e^x + 1 & -2 \le x \le 0 \\ x^2 - 2x + 2 & 0 < x \le \frac{3}{2} \end{cases}$. In order to avoid any vertical lines, you must first change the Graph Style to Dot, (see Section 4.2) then enter the function as

$$y1 = \text{when}\left(-2 \le x \text{ and } x \le 0, e^x + 1, \text{when}\left(x > 0 \text{ and } x \le \frac{3}{2}, x^2 - 2x + 2, \{\}\right)\right)$$

The format for the when command is: *when(condition, trueExpression, falseExpression)*. The definition of the function above requires *two* nested commands. Also notice that *falseExpression* for the second command is empty (Figure 22). The symbols "≤" and "≥" are in the CATALOG. Set the viewing window to [–2.5, 2, –1, 4] and press ◆ GRAPH. The graph of the piecewise-defined function is shown in Figure 23. Notice that the graph is limited to the interval [–2, $\frac{3}{2}$].

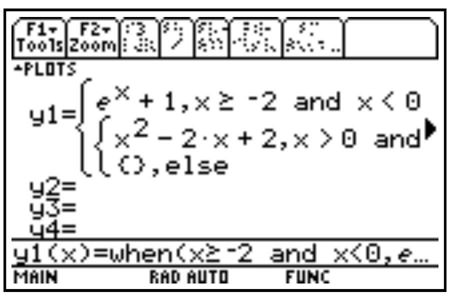


Figure 22: Entering a piecewise-defined function.

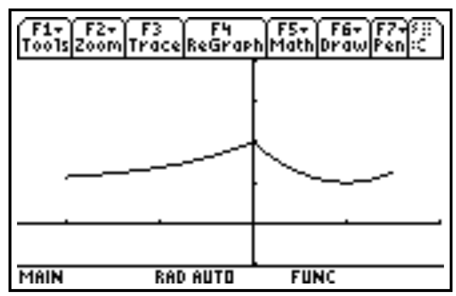


Figure 23: The graph of a piecewise-defined function.

4.11 Polar Graphing

Polar graphing is illustrated below with the equation $r = 2\cos(\theta) - 1$. To use polar graphing on the TI-89 you will need to change mode settings. Press the MODE key, then press F1, to move to Page 1 of the mode dialog window. Use the ▸ and ▾ keys to select POLAR as shown in Figure 24, and press ENTER twice. Press ◆ [Y =] to access the Y = Editor and enter the equation (Figure 25). To enter the variable θ press ◆ [θ].

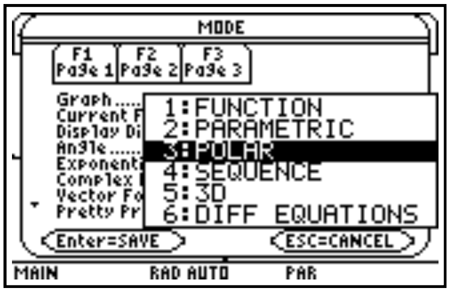


Figure 24: Mode Settings on the `TI-89`.

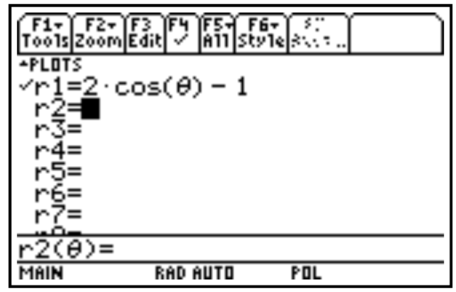


Figure 25: `Y = Editor` in polar mode.

Press ◆ `[WINDOW]` to change the window settings. In polar mode you need to specify values for `θ`. Let $0 \le \theta \le 2\pi$, then set `θstep` = $\pi / 24$, `xmin` = –3, `xmax` = 6, `xscl` = 1, `ymin` = –2, `ymax` = 2, `yscl` = 1 (Figure 26). Press ◆ `GRAPH` to view the graph determined by the equation and for the specified values of `θ` (Figure 27). You can also use the `TRACE` and `ZOOM` features when graphing in polar mode.

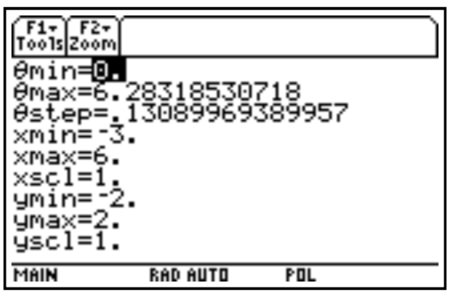


Figure 26: Window in polar mode.

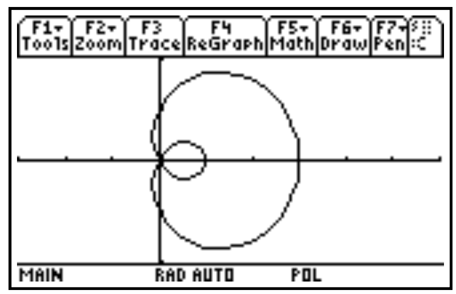


Figure 27: Polar graph.

4.12 Parametric Graphing

Parametric graphing is illustrated below with the equations $x = \cos(t - 1)$ and $y = \sin(t)$. For parametric graphing on the `TI-89` you will need to change mode settings. Press the `MODE` key, then press `F1`, to move to `Page 1` of the mode dialog window. Use the ▸ and ▾ keys to select `PARAMETRIC` as shown in Figure 28, and press `ENTER` twice. Press ◆ `[Y =]` to access the `Y = Editor` and enter the equations (Figure 29). To enter the variable `t` press `T`.

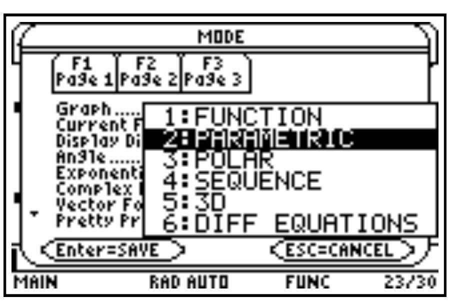


Figure 28: Mode Settings on the `TI-89`.

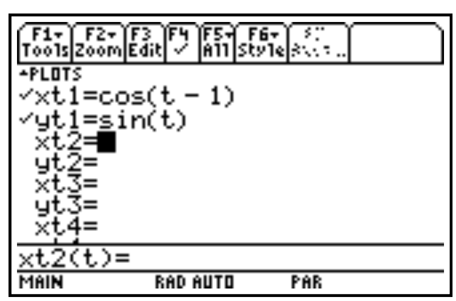


Figure 29: `Y = Editor` in parametric mode.

Press ◆ [WINDOW] to change window settings. In parametric mode you will also need to specify values for t. In this case you have trigonometric functions, so let $0 \le t \le 2\pi$, and set tstep $= \pi / 24$, xmin $= -2$, xmax $= 2$, xscl $= 1$, ymin $= -2$, ymax $= 2$, yscl $= 1$ (Figure 30). Press ◆ [GRAPH] to view the graph determined by the equation, and for the specified values of t (Figure 31). You can use the TRACE and ZOOM features when graphing in parametric mode.

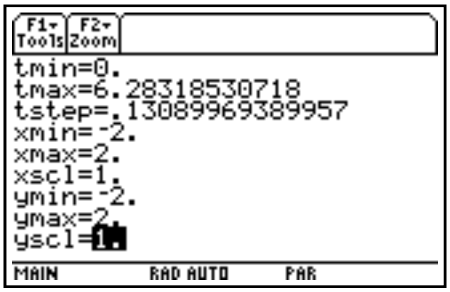


Figure 30: Window in parametric mode.

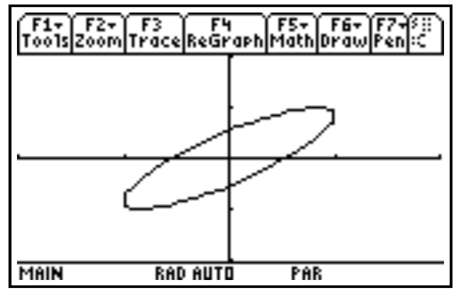


Figure 31: Parametric graph.

4.14 Split Screen

The TI-89 allows you to view two screens at a time. For example, you can look at the graph of a function, while computing its values in an adjacent table. To set up a split screen, press the MODE F2 keys, move the cursor to the top line, press the right arrow key to view the options as in Figure 32 press 3 followed by ENTER, to select the LEFT-RIGHT option. You can display any of the applications in the APPS menu on either side of the screen. Figure 33 shows a graph on the left hand side of the screen and a table on the right hand side. The key sequence 2nd APPS deactivates the current half of the screen and activates the other half. Split screens can be used when graphing in polar and parametric mode.

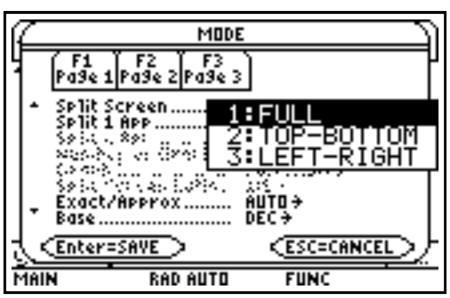


Figure 32: Selecting a spilt screen

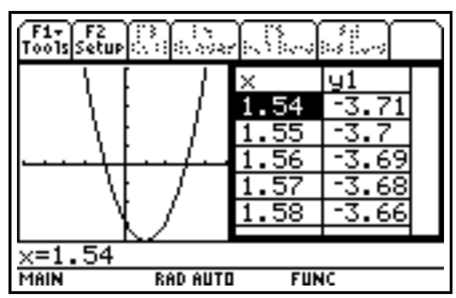


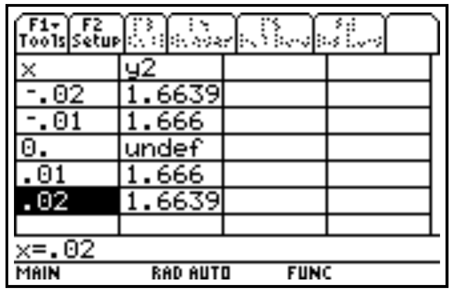
Figure 33: Split screen.

5 Limits, Derivatives, and Integrals

5.1 Limits

The study of limits of functions is a fundamental activity in calculus. Finding the limit of a function, *f*, as *x* approaches *a*, means that we analyze the values of $f(x)$ for values of *x* near *a*. If the values of $f(x)$ appear to be close to a value *L*, for values of *x* near *a*, then it is possible that the limit of the function is *L*. This analysis of a function can be carried out by examining the graph and values of a function, as illustrated in the example below.

Suppose we have the function $f(x) = \dfrac{\sin(5x)}{3x}$. We know that when $x = 0$ both the numerator and denominator are equal to zero. Does f have a limit as x goes to zero? How the values of this function behave when x is near zero? Construct a table of values for the function, selecting `tblStart` = –0.03 and Δ`tbl` = 0.01, and `Independent` to `AUTO`. The calculator will produce the table given in Figure 34.



x	y2
⁻.02	1.6639
⁻.01	1.666
0.	undef
.01	1.666
.02	1.6639

Figure 34: Values of the function near zero.

As suspected, the calculator cannot compute the value of the function at $x = 0$, but it can compute values for x near zero. As the values of x approach zero, either from the negative numbers or the positive numbers, the values of the function are getting closer to $\frac{5}{3} \approx 1.66666$. Now let's look at the graph of the function in Figures 35 and 36. Note that the viewing window is set at [–2.5, 2.5, –2.5, 2.5], with `xscl` = 1, and `yscl` = 1.

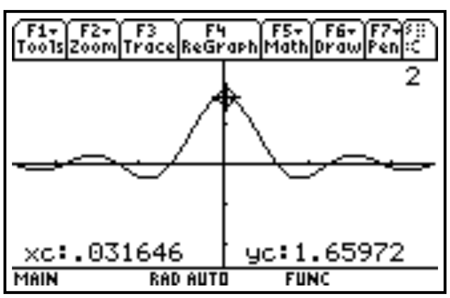


Figure 35: Graph of the function near zero.

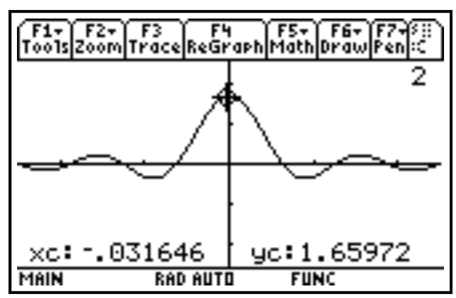


Figure 36: Graph of the function near zero.

The graph also indicates that the limit of $f(x)$ as x approaches zero is $\frac{5}{3}$. This is definitely the correct answer and can be verified analytically. The table of values and the graph of a function help you to understand limits. As this example shows, the calculator may only compute an approximate value of a limit (1.66666). Use algebraic methods to compute the exact value of a limit ($\frac{5}{3}$).

We can also study the limit of a function, f, as x goes to infinity. That is, as the values of x become very large. We can use the same techniques described above to examine the function for large values of x. Let $f(x) = \dfrac{5x-1}{x+2}$, and let your calculator construct a table for large values of x. Set `tblStart` = 999, Δ`tbl`

= 100, and `Independent` to `AUTO`. The calculator will produce the table given in Figure 37. As you continue to scroll down, observe that the values of the function approach 5.

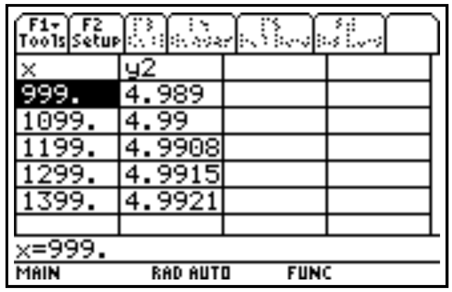


Figure 37: Values of the function for large *x*.

To look at the graph for large values of *x* select the viewing window [999, 1999, 0, 10]. Set `xscl` = 100 and `yscl` = 2. Graph the function and trace it. Notice how the value of the function is close to 5 (Figure 38). The value can be verified using analytical methods.

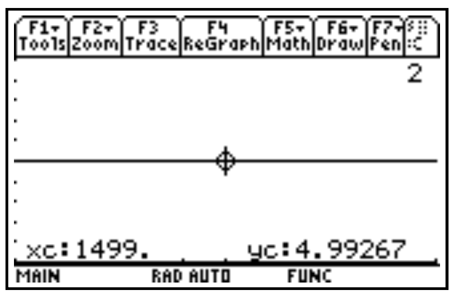


Figure 38: The function for large *x*.

We can use similar techniques to examine functions for extreme negative values of *x*. Further details and examples will appear in the exercise section.

5.2 Maximum and Minimum

To find the maximum of a function on your calculator, you can approach the subject from a geometric or numeric point of view. For the examples below use the function $f(x) = 2x^3 - 5x^2 + x - 3$. To find the max and min geometrically, graph the function, then use the `Trace` feature to move the cursor to the peaks and valleys of the graph and determine the *x*- and *y*-values (Figures 39 and 40).

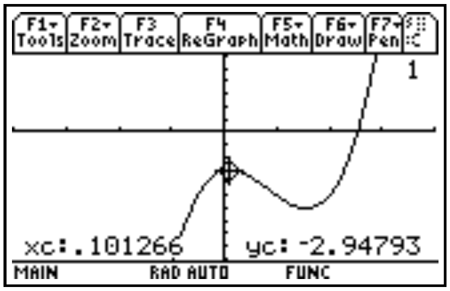


Figure 39: The maximum.

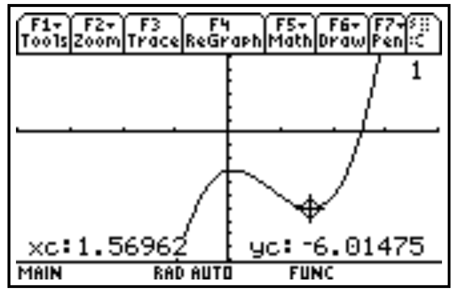
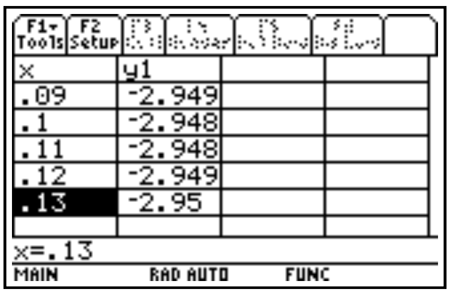


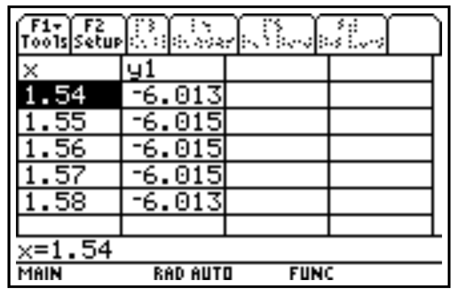
Figure 40: The minimum.

To find the max and min numerically, compute a table of values for the function and analyze the outputs of the function (Figures 41 and 42).



x	y1
.09	-2.949
.1	-2.948
.11	-2.948
.12	-2.949
.13	-2.95

Figure 41: The maximum.



x	y1
1.54	-6.013
1.55	-6.015
1.56	-6.015
1.57	-6.015
1.58	-6.013

Figure 42: The minimum.

Notice that answers can be different. Geometrically, we find the maximum of –2.94793 to occur at $x =$ .101266, and numerically, we find that the maximum of –2.948 occurs at $x = 0.11$. Although both values are good approximations, the exact value is only obtained analytically. (The exact value of x is $\frac{5-\sqrt{19}}{6}$, the exact value of y is $\frac{19\sqrt{19}}{54} - \frac{121}{27}$.)

The `TI-89` also has built-in functions that allow you to find the maximum and minimum of a function. The `Minimum` and `Maximum` features are in the `GRAPH Math` menu. The `fMin(` and `fMax(` features are in the `Calc` menu. We will describe these below.

To use the `Minimum` and `Maximum` features in the `GRAPH Math` menu, you must first graph the function. Press `Math`, and select `Minimum` by pressing [3]. Use the arrows to move the cursor to select the lower bound and the upper bound, as prompted by the calculator. Press [ENTER] to save each of your selections. The cursor will move to the lowest point of the function within the bounds selected and the calculator will display the values of x and y (Figures 43–45).

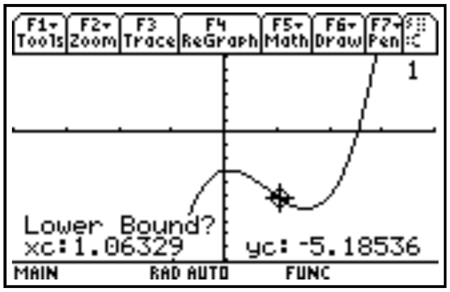


Figure 43: Lower bound.

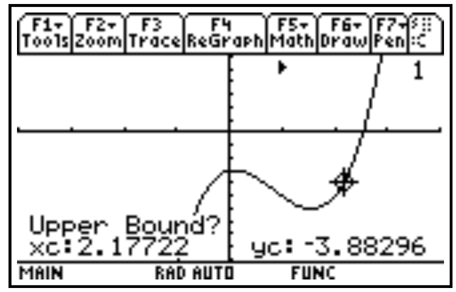


Figure 44: Upper bound.

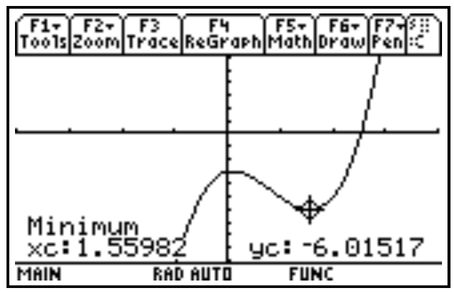


Figure 45: The minimum.

The commands for finding the maximum of the function are similar and will not be displayed here.

For fMin(and fMax(press [HOME] Calc and select fMax(by pressing [7]. This copies the command fMax(in the entry line. Enter the equation (if you have the function entered in y1, press [Y] [1] [(] [X] [)]), the variable (x), and [)]. Since the command fMax(does not accept a lower bound and an upper, you must enter these in the following way: |x>-1 and x<1. (The symbols > and < are in the bottom row of the calculator and require the [2nd] key) The TI-89 returns the value of *x* at which the maximum value of the function occurs. In this case, it is the exact value. Once the calculator gives you the *x*-value you can compute the *y*-value by computing the value of the function at the given *x*-value (Figures 46 and 47). The computations for the minimum are similar and will not be displayed here.

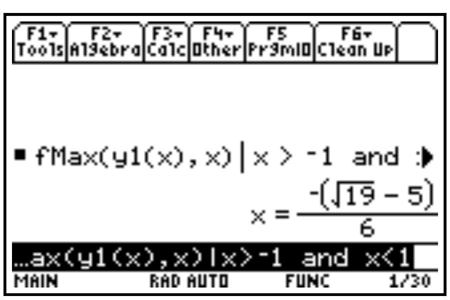


Figure 46: The *x*-value.

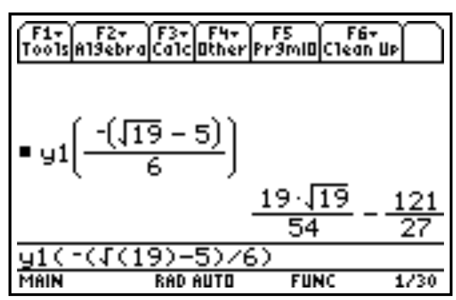


Figure 47: The Maximum.

5.3 Derivative

The TI-89 has built-in functions that allow you to find either the numerical derivative or the exact derivative of a function. The d(differentiate feature gives the exact value (even symbolically) and is found in the Calc and MATH Calculus menus. The dy/dx feature is in the GRAPH Math Derivatives menu and will return an approximation for the derivative. We illustrate these features below with the function $f(x) = x^4 + 2x^3 - x^2 + 1$ and a viewing window [–4, 4, –6, 6].

To use the dy/dx feature, first graph the function. Press Math Derivatives dy/dx. (F5 6 1). Use the arrows to move the cursor to select the point at which the derivative will be computed or enter a value for x, and press ENTER. The cursor will move to the point and the calculator will display the value of dy/dx (Figure 48).

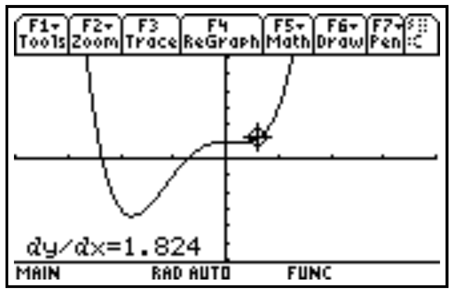


Figure 48: dy/dx computed at x = 0.6.

To use the d(differentiate, go to the Home Screen, and select Calc d(differentiate (F3 1). The command d(will be copied onto the screen. Complete the command by entering the equation (if you have the function entered press y 1 (x) , x) ENTER) and the variable (x). The calculator will return the derivative of the function. You can use this feature to obtain a numerical value for the derivative at a given value of x. To achieve this attach | x = 0.6 to the command and the derivative of the function will be calculated for $x = 0.6$ (Figure 49).

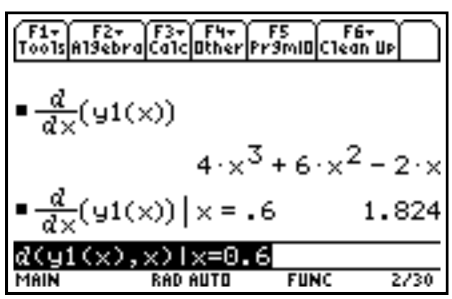


Figure 49: dy/dx computed at x = 0.6.

The derivative of a function is a function itself and therefore can be entered into the calculator, but you do not need to actually compute the derivative. Enter the function $y1 = x^3 - 5x^2 + 6x - 4$ into the calculator, then enter the derivative as shown in Figure 50. The graph of the original function and its derivative are shown in Figure 51.

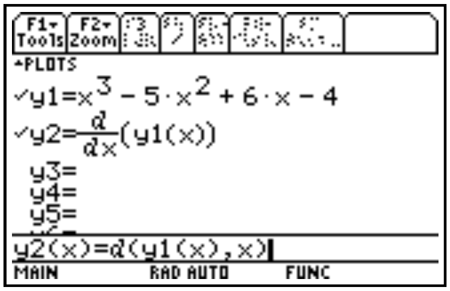


Figure 50: The derivative entered.

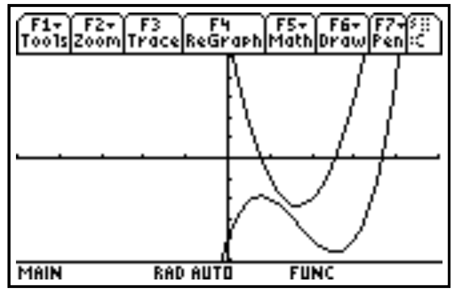


Figure 51: The graph of the function and its derivative.

5.4 Integrals

The TI-89 has built-in functions that allow you to find either the numerical or the exact integral—definite or indefinite—of a function. The ∫f(x)dx feature is in the GRAPH Math menu, the ∫(integrate and nInt(features are in the MATH Calculus menu. We illustrate these features below with the function $f(x) = 5e^{-(x-3)^2}$ and a viewing window [–2, 6, –2, 6].

To use the ∫f(x)dx feature, enter and graph the function. Press Math ∫f(x)dx (F5 7). Use the arrows to move the cursor to select the lower and upper limits of integration as prompted by the calculator or enter values for these limits, and press ENTER. The calculator will shade the area represented by the definite integral and display the value of ∫f(x)dx (Figures 52–54).

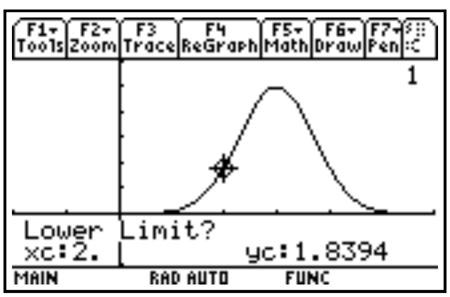


Figure 52: Lower limit.

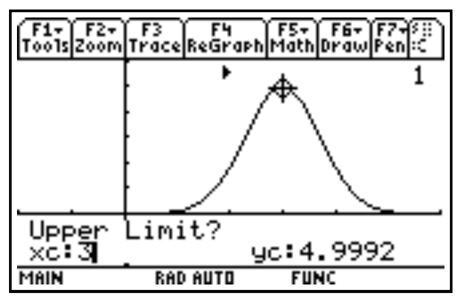


Figure 53: Upper limit.

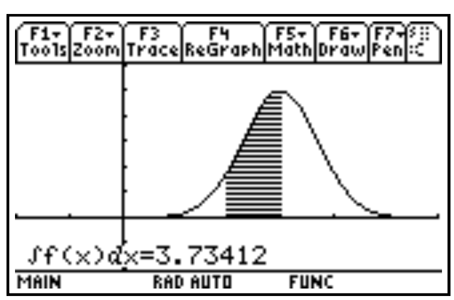


Figure 54: The integral.

To use the ∫f(integrate or the nInt(features, go to the Home Screen and select Calc ∫f(integrate (F3 2). The command ∫(will be copied onto the screen. Complete it by entering the equation (if you have the function entered press ENTER), the variable (x), and limits of integration for a definite integral, (separated by commas), close the parenthesis, and press ENTER. The calculator will return a value for the integral of the function. If you do not supply limits of integration, the calculator will attempt to find an antiderivative for the function. The steps are similar when applying `nInt(`, only this feature always will always return a numerical value (Figure 55).

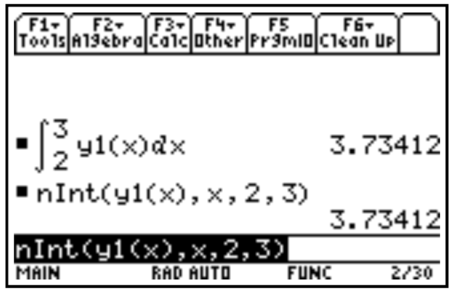


Figure 55: `∫f(` and `nInt(` computed in [2, 3].

The integral $\int_0^x f(t)dt$ of a function f is a function itself and therefore can be entered into the calculator, but you do not need to actually compute the integral. Enter the function $y1 = \dfrac{1}{1+x^2}$ into the calculator, then enter the integral as shown in Figure 56. The graph of the original function and its integral are shown in Figure 57.

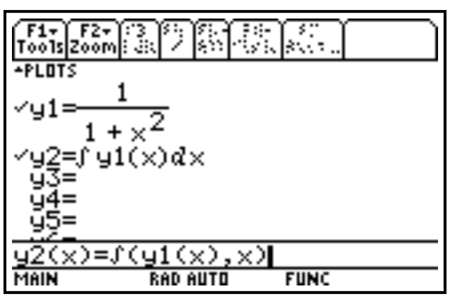


Figure 56: The integral entered.

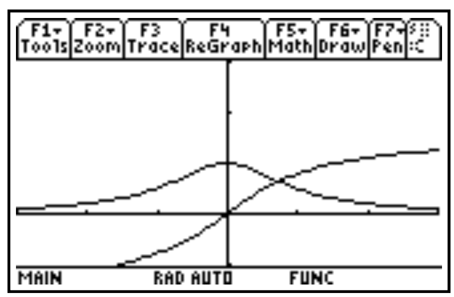


Figure 57: The graph of the function and its integral.

5.5 `GRAPH Draw` menu

The `TI-89` has a feature in the `GRAPH Draw` menu that is useful in calculus applications. First, press ◆ `[GRAPH] Draw` (F6 = 2nd F1) `ClrDraw` (1). This will erase any drawings in the window and graph only selected functions and plots. For additional topics in the `GRAPH Draw` menu, consult the guidebook that came with your calculator.

Inverse. To draw the graph of the inverse of $f(x) = \ln(2x-1)$, enter the function. Make sure this function is the only one selected, and set the viewing window to [–8, 8, –8, 8]. While you are viewing the graph press 2nd F1 (3) to select `DrawInv`. The command will appear on the home screen. Complete it with the name of the function, as in Figure 58, then press ENTER. The graph will reappear and the inverse function will be drawn on the screen (Figure 59).

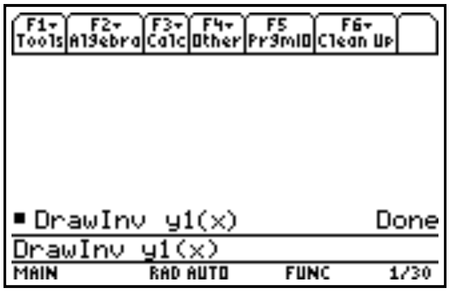


Figure 58: The DrawInv command.

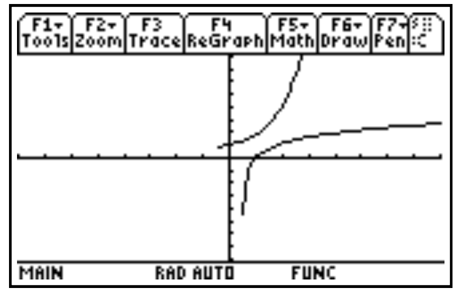


Figure 59: A function and its inverse.

5.6 GRAPH Math menu

The TI-89 has four features in the GRAPH Math menu that are useful in calculus applications. First, press ◆ [GRAPH] Draw (F6 = 2nd F1) ClrDraw (1). This will erase any drawings in the window and the calculator will graph only selected functions and plots. For additional topics in the GRAPH Math menu, consult the guidebook that came with your calculator.

Inflection Point. To find an inflection point for the function $f(x) = x^4 - 3x^3 + 2x^2 - x - 3$, enter the function. Make sure this function is the only one selected, and set the viewing window to [–4, 4, –8, 6]. While you are viewing the graph press F5 8 to select Infection. Enter a lower bound and an upper bound as prompted by the calculator. The cursor will position itself at a possible point of inflection of the graph, and the coordinates of the point will be displayed (Figure 60).

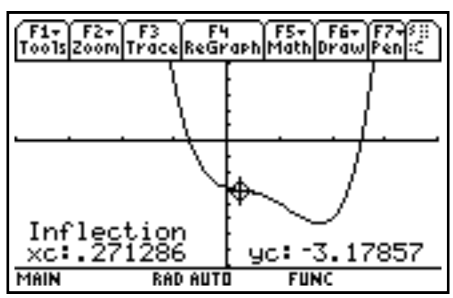


Figure 60: A point of inflection of the graph.

Tangent Line. To draw the tangent line to the graph of $f(x) = 2x^3 - 5x^2 + x - 3$ at the point (2, –5), enter the function. Make sure this function is the only one selected, and set the viewing window to [–4, 4, –10, 6]. While you are viewing the graph press F5, then scroll down and select Tangent. Enter 2 for a value of x and press ENTER. The tangent line will appear and the equation of the line will be displayed, as in Figure 61.

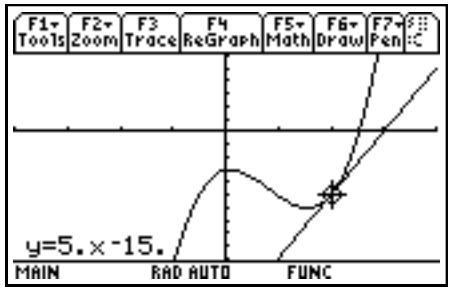


Figure 61: The tangent line.

Arc Length. To compute the length of the graph of the function $f(x) = \ln(9 + x^3)$ in the interval [–1, 1], enter the function. Make sure it is the only one selected, and set the viewing window to [–3, 5, –4, 4]. While you are viewing the graph press F5, then scroll down and select Arc. Input – 1 for the first point, press ENTER, and input 1 for the second point. The length of the curve will be displayed (Figure 62).

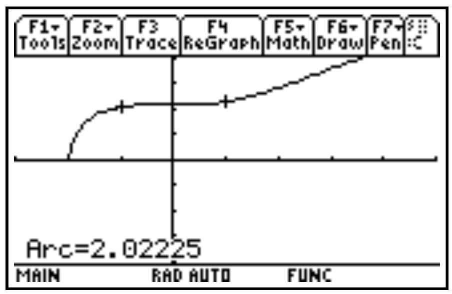


Figure 62: Arc length.

Shade. Use the Shade command to view the area between the graphs of two functions within a given interval. Enter and graph $y1 = -2\sin(x)$, and $y2 = -2\cos x$. Note that in the interval [–3 π/4, π/4], y2(x) ≤ y1(x). While you are viewing the graph press F5, then scroll down and select Shade. The calculator will prompt you to select the function to shade above. Use the arrow keys to select y2, and press ENTER. (This is the lower function as in Figure 63). The calculator will then prompt you to select the function to shade below. Use the arrow keys to select y1, and press ENTER. (This is the upper function as in Figure 64.) Next the calculator will prompt you to select the left endpoint. Input a value, and press ENTER (Figure 65). The next prompt will be for the right endpoint. Input the value, and press ENTER (Figure 66). The graphs will be shown again, and the shaded area will be above y2, below y1, and within the interval [–3 π/4, π/4] (Figure 67).

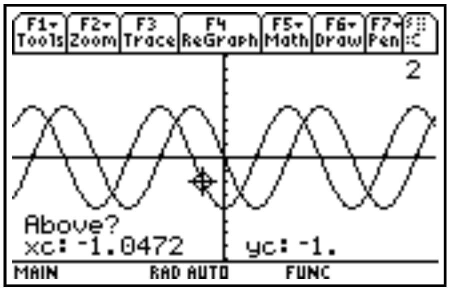


Figure 63: The lower function.

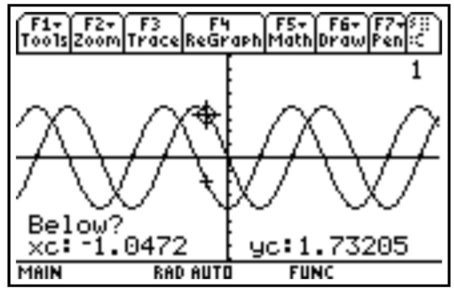


Figure 64: The upper function.

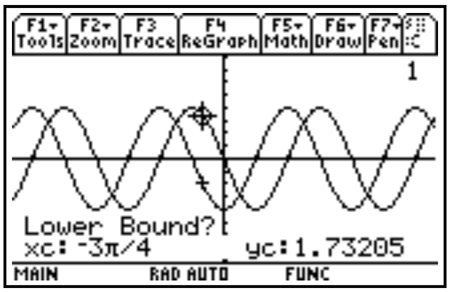


Figure 65: The lower bound.

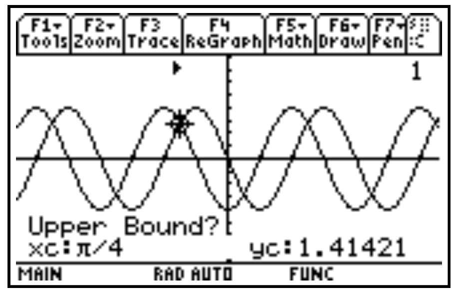


Figure 66: The upper bound.

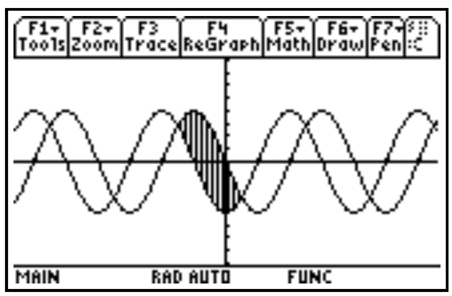


Figure 67: The shaded area.

6. Regression and Forecasting

Regression is a method used to analyze data obtained from real life phenomena with the purpose of developing mathematical models that can aid in forecasting. The topics below will show you how to enter data into your calculator, and analyze its behavior. You will plot data points and determine if a certain function describes the data points well. You will also be able to make predictions and forecast certain results based on your data. First, make sure that your calculator is set to Function Mode. (See the Section 2 in Part III of this manual.)

6.1 Entering Data

The data entered will reside in data variables. The TI-89 has a Data Editor that allows you to store, edit and view data. Press the APPS 6 3 keys, enter the information as shown in Figure 68, and press ENTER. For illustration purposes, we have chosen to enter the data from Example 9, on page 6, of your text (Figure 69). To begin entering data, position the cursor in the first entry of the table, r1c1, enter a value, and press ENTER. You can use the arrow keys to scroll through the entries of the table. To replace an entry, position the cursor at a desired entry, enter the replacement value, then press ENTER. Some of the features described here can be performed in a variety of ways. We have chosen to describe the one that is simplest to understand and use. There are many other features in the STAT menu that we have not covered (see Chapter 16 of the Calculator Guidebook).

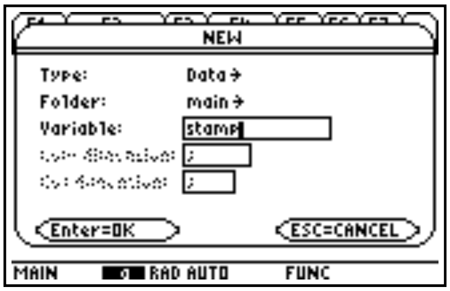


Figure 68: Naming the data.

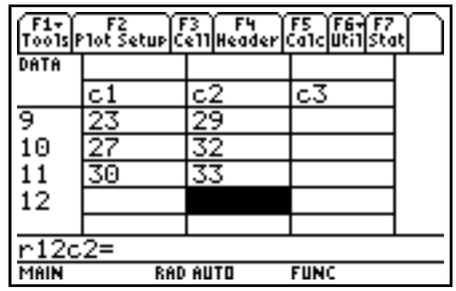


Figure 69: The Data Editor.

6.2 Plotting Data

Once you have entered the data into the calculator, you are ready to plot points. That is, you need to define a plot, turn it on, select the viewing window, display the plot, and explore. To define the plot and turn it on select Plot Setup (F2) from the menu, and highlight Plot1 (Figure 70). Press F1 to define the plot and enter the information as in Figure 71. Use the right arrow to select Scatter (the calculator will display the data as coordinate points). For Mark select Box. To enter c1, press ALPHA c 1 ENTER. Scroll down to the next box and enter c2. Make sure Free and Categories is set to NO. Press ENTER twice.

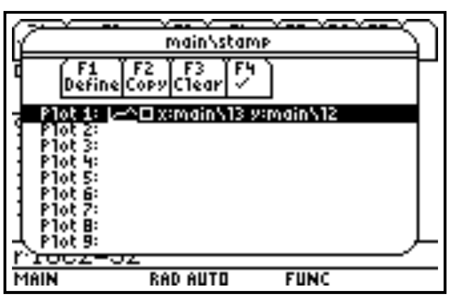


Figure 70: Selecting the plot.

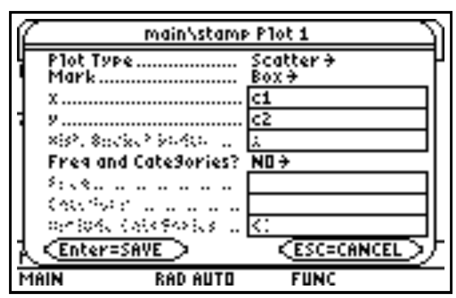


Figure 71: Defining the plot.

To view your data points you may want to deselect any functions as needed, so that their graphs aren't displayed. Press ◆ GRAPH. If data points do not display correctly, select ZoomData from the ZOOM menu (F2 9). The data will then be displayed as a plot (Figure 72).

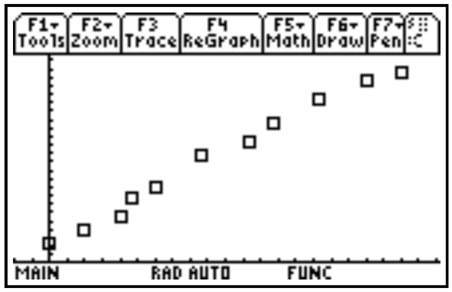


Figure 72: The Plot.

6.3 Regressions

Now you are ready to explore! First you will perform a Linear Regression. This means that you will find a linear function that is as close as possible (in a statistical sense) to the data points. Go back to the data you entered by using the sequence APPS 6 1. Select `Calc` from the menu (F5) and enter the information as in Figure 73. This tells the calculator to perform a Linear Regression, and to use `c1` for the *x*-values, and `c2` for the *y*-values. We have also indicated that we want the regression equation to be stored in `y1`. Press ENTER.

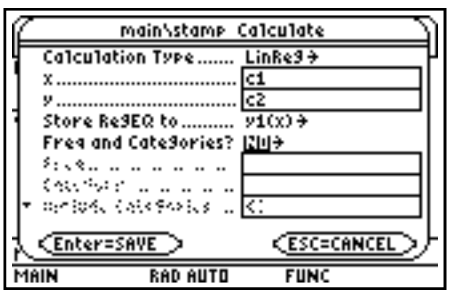


Figure 73: Setting the regression.

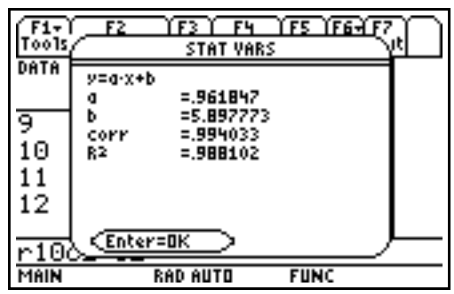


Figure 74: The regression data.

You have now computed a linear regression. The calculator will give you the values of *a* and *b* in the regression equation, and it will store the equation in `y1` (Figure 74). Press the ◆ GRAPH keys to see the original data points and the regression line together (Figure 75).

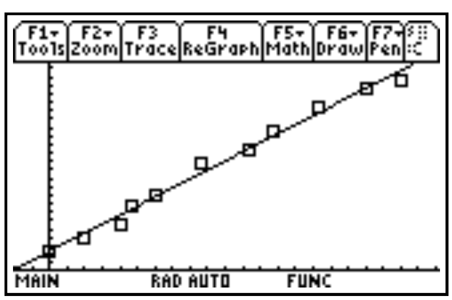


Figure 75: Data points and Regression Equation.

You can now make predictions. If $x = 42$, what is y? The calculator computes `y1(42)` as 46.2954.

6.4 Residuals

As indicated above, regressions provide a means of obtaining a mathematical model to describe real world phenomena. The residuals, defined by

residuals = observations – predictions

allow you to verify the model. In the equation above, observations are the numbers in column 1 of the data variable `stamp` and predictions are the values obtained by applying the regression equation to the numbers in column 2 of the data variable `stamp`. You can extract column 1, for example, from the data variable `stamp` with the command `stamp[1]`. With the `TI-89` compute the residual list as shown in Figure 76. The residuals are saved into the list `res`. It is a common practice to plot the residual points versus the independent variable to verify a model.

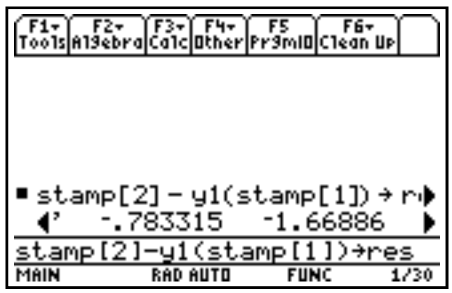


Figure 76: The Residuals.

7. Sequences and Series

7.1 Sequences

The `TI-89` has many tools in the `MATH List` menu that are useful in analyzing sequences and series. The `seq(` command allows you to generate the terms of a sequence. Let $a_n = \frac{(-1)^n}{n}$. Press [2nd] [MATH] [3] [1] to select `seq(` . Complete the command as shown in Figure 77 and press [ENTER]. (The command requires that you enter a formula for the sequence, the variable of the sequence, a starting value for the variable, and a stopping value for the variable.) The first five terms of the sequence will be displayed on the screen.

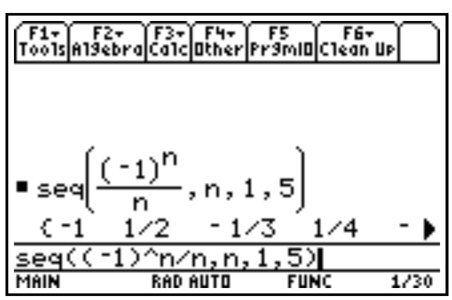


Figure 77: Generating the terms of a sequence.

7.2 Series

The sum(command adds the terms of a sequence. For example, to add the terms of the sequence in Section 7.1 above, go to the Home Screen, and move the cursor to the extreme left position. Press 2nd MATH 3 6 to select sum(. Move the cursor to the extreme right position and input the parenthesis, then press ENTER. The sum of the first five terms of the sequence will be displayed on the screen (Figure 78). For the sum of other sequences, select sum(then seq(, and complete the command with the new formula for the sequence, the variable of the sequence, a starting value for the variable, and a stopping value for the variable. Then press ENTER.

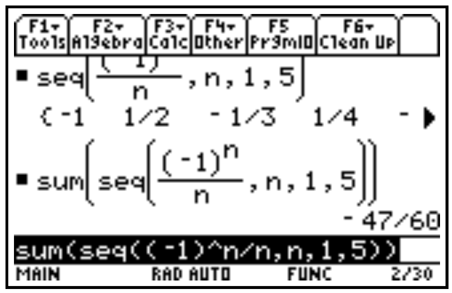


Figure 78: The sum of the terms of a sequence.