CHAPTER 23

The Electric Field II: Continuous Charge Distributions

1* • A uniform line charge of linear charge density λ = 3.5 nC/m extends from x = 0 to x = 5 m. (*a*) What is the total charge? Find the electric field on the x axis at (*b*) x = 6 m, (*c*) x = 9 m, and (*d*) x = 250 m. (*e*) Find the field at x = 250 m, using the approximation that the charge is a point charge at the origin, and compare your result with that for the exact calculation in part (*d*).

(*a*) $Q = \lambda L$ — $Q = (3.5\times10^{-9} \times 5)$ C = 17.5 nC

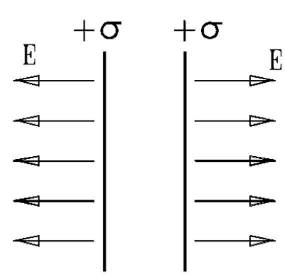
(*b*), (*c*), (*d*) $E_x(x_0) = kQ/[x_0(x_0 - L)]$, Equ. 23-5 — $E_x(6)$ = 26.2 N/C; $E_x(9)$ = 4.37 N/C; $E_x(250) = 2.57\times10^{-3}$ N/C

(*e*) $E_x \cong kQ/x^2$ — $E_x(250) = 2.52\times10^{-3}$ N/C, within 2% of (*d*)

2 • Two infinite vertical planes of charge are parallel to each other and are separated by a distance d = 4 m. Find the electric field to the left of the planes, to the right of the planes, and between the planes (*a*) when each plane has a uniform surface charge density σ = +3 μC/m^2 and (*b*) when the left plane has a uniform surface charge density σ = +3 μC/m^2 and that of the right plane is σ = –3 μC/m^2. Draw the electric field lines for each case.

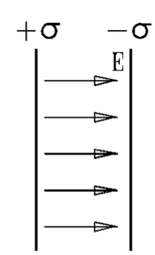
(*a*) $E = 4\pi k\sigma = 3.39\times10^5$ N/C

The field pattern is shown in the adjacent figure.
The field between the plates is zero.



(*b*) Again, $E = 3.39\times10^5$ N/C.

The field pattern is shown in the adjacent figure.
The field is confined to the region between the two plates and is zero elsewhere.



3 • A 2.75-μC charge is uniformly distributed on a ring of radius 8.5 cm. Find the electric field on the axis at (*a*) 1.2 cm, (*b*) 3.6 cm, and (*c*) 4.0 m from the center of the ring. (*d*) Find the field at 4.0 m using the approximation that the ring is a point charge at the origin, and compare your results with that for part (*c*).

(*a*) Use Equ. 23-10

$E_x = \dfrac{8.99\_10^9 \_2.75\_10^{•6} \_ 0.012}{((0.012)^2 + (0.085)^2)^{3/2}}$ N/C= 4.69_10^5 N/C

(*b*), (*c*) Proceed as in (*a*)

$E_x(0.036) = 1.13\times10^6$ N/C; $E_x(4) = 1.54\times10^3$ N/C

(*d*) $E_x \simeq kQ/x^2$

$E_x = 1.55\times10^3$ N/C; this is slightly greater than (*c*) because the point charge is nearer $x = 4$ m than the ring.

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4 • A disk of radius 2.5 cm carries a uniform surface charge density of 3.6 μC/m^2. Using reasonable approximations, find the electric field on the axis at distances of (*a*) 0.01 cm, (*b*) 0.04 cm, (*c*) 5 m, and (*d*) 5 cm.

For $x << r$, the disk appears like an infinite plane. For $x >> r$, the ring charge may be approximated by a point charge.

(*a*), (*b*) Use Equ. 23-12*a* — $E_x = 2.03\times10^5$ N/C

(*c*) $E_x = kQ/x^2 = k\pi r^2\sigma/x^2$ — $E_x = 2.54$ N/C

(*d*) $E_x = k\pi r^2\sigma/x^2$; this is not a good approximation since $x = 2r$ is not much greater than r. — $E_x = 2.54\times10^4$ N/C

5* • For the disk charge of Problem 4, calculate exactly the electric field on the axis at distances of (*a*) 0.04 cm and (*b*) 5 m, and compare your results with those for parts (*b*) and (*c*) of Problem 4.

(*a*) Use Equ. 23-11; $r = 2.5$ cm, $\sigma = 3.6$ μC/m^2 — $E_x = 2.00\times10^5$ N/C

(*b*) Proceed as in (*a*) — $E_x = 2.54$ N/C

For $x = 0.04$ cm, the exact value of E_x is only 1.5% smaller than the approximate value obtained in the preceding problem. For $x = 5$ m, the exact and approximate values agree within less than 1%.

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6 • A uniform line charge extends from $x = –2.5$ cm to $x = +2.5$ cm and has a linear charge density of λ = 6.0 nC/m. (*a*) Find the total charge. Find the electric field on the *y* axis at (*b*) $y = 4$ cm, (*c*) $y = 12$ cm, and (*d*) $y = 4.5$ m. (*e*) Find the field at $y = 4.5$ m, assuming the charge to be a point charge, and compare your result with that for part (*d*).

(*a*) $Q = \lambda L$ — $Q = 0.3$ nC

(*b*), (*c*), (*d*) $E_y = \dfrac{kQ}{y\sqrt{(L/2)^2 + y^2}}$, Equ. 23-8 — (*b*) $E_y = 1.43$ kN/C; (*c*) $E_y = 184$ N/C; (*d*) $E_y = 0.1332$ N/C

(*e*) $E_y = kQ/y^2$ — $E_y = 0.1332$ N/C, in good agreement with (*c*)

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7 • A disk of radius *a* lies in the *yz* plane with its axis along the *x* axis and carries a uniform surface charge density σ. Find the value of *x* for which $E_x = \frac{1}{2}\sigma/2\varepsilon_0$.

From Equ. 23-11, $E_x = \dfrac{\sigma}{2\varepsilon_0}\left(1 - \dfrac{x}{\sqrt{(x^2+a^2)}}\right)$. $E_x = \sigma/4\varepsilon_0$ when $\dfrac{x}{\sqrt{(x^2+a^2)}} = 1/2$. Solving for x gives

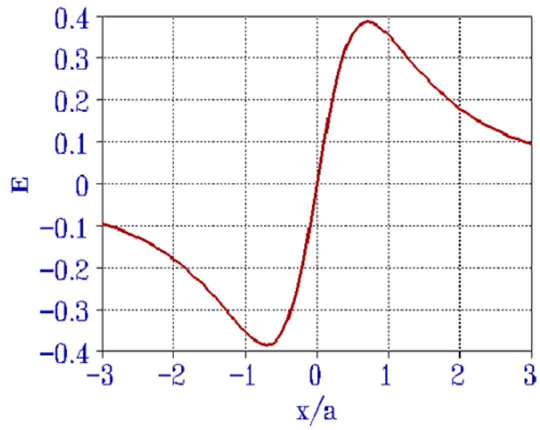
$x = a/\sqrt{3}$.

8 • A ring of radius a with its center at the origin and its axis along the x axis carries a total charge Q. Find E_x at (*a*) $x = 0.2a$, (*b*) $x = 0.5a$, (*c*) $x = 0.7a$, (*d*) $x = a$, and (*e*) $x = 2a$. (*f*) Use your results to plot E_x versus x for both positive and negative values of x.

a), (*b*), (*c*), (*d*), (*e*) Use Equ. 23-10; the results are given in units of kQ/a^2

a) $E_x = 0.189$ (*b*) $E_x = 0.358$ (*c*) $E_x = 0.385$ (*d*) $E_x = 0.354$ (*e*) $E_x = 0.179$

(*f*) The field along the x axis is plotted in the adjoining figure. The x coordinates are in units of x/a and E is in units of kQ/a^2.

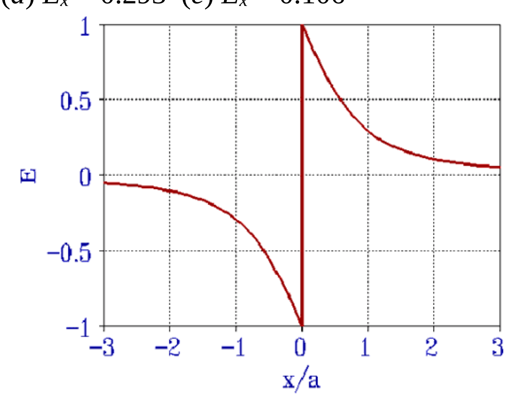


9* • Repeat Problem 8 for a disk of uniform surface charge density σ.

a), (*b*), (*c*), (*d*), (*e*) Use Equ. 23-11; the results are given in units of $\sigma/2\varepsilon_0$.

(*a*) $E_x = 0.804$ (*b*) $E_x = 0.553$ (*c*) $E_x = 0.427$ (*d*) $E_x = 0.293$ (*e*) $E_x = 0.106$

(*f*) The field along the x axis is plotted in the adjoining figure. The x coordinates are in units of x/a and E is in units of $\sigma/2\varepsilon_0$.



10 •• A disk of radius 30 cm carries a uniform charge density σ. (*a*) Compare the approximation $E = 2\pi k\sigma$ with the exact expression (Equation 23-11) for the electric field on the axis of the disk by computing the fractional difference $\Delta E/E \approx x/\sqrt{x^2+R^2}$ for the distances $x = 0.1$, $x = 0.2$, and $x = 3$ cm. (*b*) At what distance is the neglected term 1% of $2\pi k\sigma$?

(a) Evaluate $\Delta E/E = x/\sqrt{x^2 + R^2}$ for $x = 0.1$, 0.2, and 3 cm with $R = 30$ cm

$x = 0.1$ cm, $\Delta E/E = 0.00333$; $x = 0.2$ cm, $\Delta E/E = 0.00667$; $x = 3$ cm, $\Delta E/E = 0.0995$

(b) Set $\Delta E/E = 0.01$ and solve for x

$x = R/100 = 0.3$ cm

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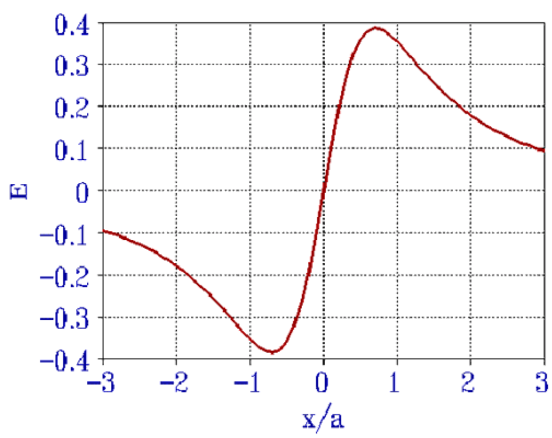
11 ·· Show that E_x on the axis of a ring charge of radius a has its maximum and minimum values at $x = +a/\sqrt{2}$ and $x = -a/\sqrt{2}$. Sketch E_x versus x for both positive and negative values of x.

Take the derivative of E_x given in Equ. 23-10 and set it equal to 0.

$$\frac{dE_x}{dx} = \frac{kQ}{(x^2 + a^2)^{3/2}}\left(1 - \frac{3x^2}{x^2 + a^2}\right) = 0$$. This gives $x = \pm a/\sqrt{2}$ as the values of x where $|E_x|$ is a maximum.

A plot of E versus x/a is shown in units of kQ/a^2.



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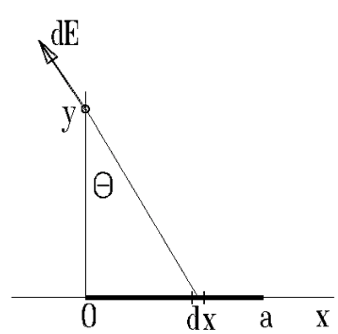
12 ·· A line charge of uniform linear charge density λ lies along the x axis from $x = 0$ to $x = a$. (a) Show that the x component of the electric field at a point on the y axis is given by

$$E_x = -\frac{k\lambda}{y} + \frac{k\lambda}{\sqrt{y^2 + a^2}}$$

(b) Show that if the line charge extends from $x = -b$ to $x = a$, the x component of the electric field at a point on the y axis is given by

$$E_x = \frac{k\lambda}{\sqrt{y^2 + a^2}} - \frac{k\lambda}{\sqrt{y^2 + b^2}}$$



(a) The line charge and point (0, y) are shown in the drawing.

Also shown is the line element dx and the corresponding field $d\mathbf{E}$. The x component of $d\mathbf{E}$ is then

$$dE_x = -\frac{k\lambda}{x^2 + y^2}\sin\theta\, dx = -\frac{k\lambda x}{(x^2 + y^2)^{3/2}}dx.$$ Integrating

dE_x from $x = 0$ to $x = a$ gives the result stated in the problem.

(*b*) Proceed as in part (*a*) but integrate from $x = -b$ to $x = a$. One obtains the expression given in the problem.

13* ·· (*a*) A finite line charge of uniform linear charge density λ lies on the x axis from $x = 0$ to $x = a$. Show that the y component of the electric field at a point on the y axis is given by

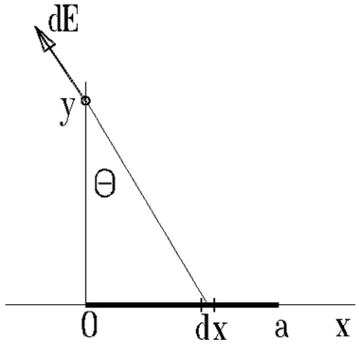
$$E = \frac{k\lambda}{y}\sin\theta_1 = \frac{k\lambda}{y}\frac{a}{\sqrt{y^2 + a^2}}$$

where θ_1 is the angle subtended by the line charge at the field point. (*b*) Show that if the line charge extends from $x = -b$ to $x = a$, the y component of the electric field at a point on the y axis is given by

$$E_y = \frac{k\lambda}{y}(\sin\theta_1 + \sin\theta_2)$$

where $\sin\theta_2 = b/\sqrt{y^2 + b^2}$.

(*a*) The line charge and the point (0, *y*) are shown in the drawing. Also shown is the line element dx and the corresponding field $d\mathbf{E}$. The y component of $d\mathbf{E}$ is then



$$dE_y = \frac{k\lambda}{x^2 + y^2}\cos\theta\, dx = \frac{k\lambda y}{(x^2 + y^2)^{3/2}}dx.$$

Integrating dE_y from $x = 0$ to $x = a$ one obtains

$$E_y = \frac{k\lambda a}{y\sqrt{y^2 + a^2}} = \frac{k\lambda}{y}\sin\theta_1,$$

where here $\theta_1 = \theta$ shown in the drawing.

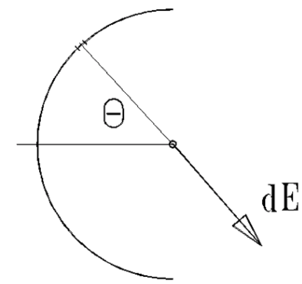
(*b*) Proceed as in part (*a*) but integrate dE_y from $x = -b$ to $x = a$. The result is

$$E_y = \frac{k\lambda}{y}\left(\frac{a}{\sqrt{y^2 + a^2}} + \frac{b}{\sqrt{y^2 + b^2}}\right) = \frac{k\lambda}{y}(\sin\theta_1 + \sin\theta_2),$$ where θ_2 is the angle subtended by the line segment b at the point y.

14 ·· A semicircular ring of radius R carries a uniform line charge of λ. Find the electric field at the center of the semicircle.

The semicircular ring is shown in the drawing. From symmetry it is evident that $E_y = 0$. The field $d\mathbf{E} = \lambda r\, d\theta/r^2 = \lambda\, d\theta/r$ and $dE_x = [(\lambda \cos \theta)/r]\, d\theta$.

Integrating from $\theta = -\pi/2$ to $\pi/2$ one obtains $E = E_x = 2\lambda/r$



15 ··· A hemispherical thin shell of radius R carries a uniform surface charge σ. Find the electric field at the center of the hemispherical shell ($r = 0$).

Consider a ring with its axis along the z direction of radius $r \sin \theta$ and width $r\, d\theta$. The field dE is given by Equ. 23-10, where the distance to the point of interest is $z = r \cos \theta$. The charge on the ring is $\sigma 2\pi r^2 \sin \theta\, d\theta$ and the field $dE = 2\pi k\sigma(r \cos \theta)r^2 \sin \theta\, d\theta/(r^2 \sin^2 \theta + r^2 \cos^2 \theta)^{3/2} = 2\pi k\sigma \cos \theta \sin \theta\, d\theta$. Integrating from $\theta = 0$ to $\theta = \pi/2$, one obtains $E = \pi k\sigma$.

16 ··· A line charge of linear charge density λ with the shape of a square of side L lies in the yz plane with its center at the origin. Find the electric field on the x axis at an arbitrary distance x, and compare your result to that for the field on the axis of a charged ring of radius $r = \frac{1}{2}L$ with its center at the origin and carrying the same total charge. (*Hint:* Use Equation 23-8 for the field due to each segment of the square.)

Note that any point on the x axis is on the perpendicular bisector of each of the four sides of the square. From symmetry, the total field has no z or y components. The x component of the field due to one of the four sides is

$E_x = Ex/\sqrt{2a^2 + x^2}$, where $a = L/2$. The field due to one side is $E = 2k\lambda a/(a^2 + x^2)$. There are four line charges, so the total field in the x direction is $E = 8k\lambda ax/[(a^2 + x^2)\sqrt{2a^2 + x^2}] = 4k\lambda Lx/[(x^2 + L^2/4)\sqrt{x^2 + L^2/2}]$.

For a ring of radius $r = L/2 = a$ the field is $E = 2\pi k\lambda ax/(a^2 + x^2)^{3/2}$.

17* ·· True or false:

(*a*) Gauss's law holds only for symmetric charge distributions.

(*b*) The result that $E = 0$ inside a conductor can be derived from Gauss's law.

(*a*) False (*b*) False

18 ·· What information in addition to the total charge inside a surface is needed to use Gauss's law to find the electric field?

To use Gauss’s law the system must display some symmetry.

19 ··· Is the electric field E in Gauss's law only that part of the electric field due to the charge inside a surface, or is it the total electric field due to all charges both inside and outside the surface?

The electric field is that due to all the charges, inside and outside the surface.

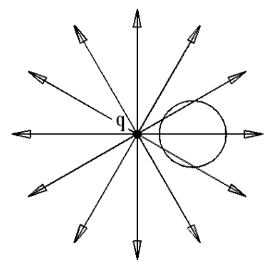
20 · Consider a uniform electric field $\mathbf{E} = 2$ kN/C $\mathbf{i}$. (*a*) What is the flux of this field through a square of side 10 cm in a plane parallel to the yz plane? (*b*) What is the flux through the same square if the normal to its plane makes a 30° angle with the x axis?

(*a*) Use Equ. 23-14 • $= (2\times10^3 \times 10^{-2})$ N·m²/C = 20 N·m²/C

(*b*) Here $\mathbf{i}\cdot\mathbf{n} = \cos 30^\circ$ • = 20 cos 30°N·m²/C = 17.3 N·m²/C

21* · A single point charge $q = +2\ \mu$C is at the origin. A spherical surface of radius 3.0 m has its center on the x axis at $x = 5$ m. (*a*) Sketch electric field lines for the point charge. Do any lines enter the spherical surface? (*b*) What is the net number of lines that cross the spherical surface, counting those that enter as negative? (*c*) What is the net flux of the electric field due to the point charge through the spherical surface?

(*a*) A sketch of the field lines and of the sphere is shown. Three lines enter the sphere.
(*b*) The net number of lines crossing the surface is zero.
(*c*) The net flux is zero.



22 · An electric field is $\boldsymbol{E} = 300$ N/C $\boldsymbol{i}$ for $x > 0$ and $\boldsymbol{E} = -300$ N/C $\boldsymbol{i}$ for $x < 0$. A cylinder of length 20 cm and radius 4 cm has its center at the origin and its axis along the x axis such that one end is at $x = +10$ cm and the other is at $x = -10$ cm. (*a*) What is the flux through each end? (*b*) What is the flux through the curved surface of the cylinder? (*c*) What is the net outward flux through the entire cylindrical surface? (*d*) What is the net charge inside the cylinder?

This problem is identical to Example 23-4 except for a change in the data. Following the procedure of that Example, one obtains the following results. (*a*) $\phi_{\text{right}} = \phi_{\text{left}} = 1.51\ \text{N·m}^2/\text{C}$, (*b*) $\phi_{\text{curved}} = 0$, (*c*) $\phi_{\text{net}} = 3.02\ \text{N·m}^2/\text{C}$, and (*d*) $Q = \varepsilon_0\ \ _{\text{net}} = 2.67\times10^{-11}$ C.

23 · A positive point charge q is at the center of a cube of side L. A large number N of electric field lines are drawn from the point charge. (*a*) How many of the field lines pass through the surface of the cube? (*b*) How many lines pass through each face, assuming that none pass through the edges or corners? (*c*) What is the net outward flux of the electric field through the cubic surface? (*d*) Use symmetry arguments to find the flux of the electric field through one face of the cube. (*e*) Which, if any, of your answers would change if the charge were inside the cube but not at its center?

(*a*) All N lines pass through the surface of the cube.
(*b*) By symmetry, N/6 lines pass through each face of the cube.
(*c*) $\phi = q/\varepsilon_0$ (see Equ. 23-20)
(*d*) Through one face, $\phi = q/6\varepsilon_0$
(*e*) Parts (*b*) and (*d*) would change if the charge is not centered.

24 · Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 6.0 kN·m^2/C. (*a*) What is the net charge inside the box? (*b*) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or why not?

(*a*) $Q = \varepsilon_0$ $\qquad Q = 5.31\times10^{-8}$ C
(*b*) One can only conclude that the net charge is zero. There may be an equal number of positive and negative charges present inside the box.

25* · A point charge $q = +2\ \mu$C is at the center of a sphere of radius 0.5 m. (*a*) Find the surface area of the sphere. (*b*) Find the magnitude of the electric field at points on the surface of the sphere. (*c*) What is the flux of the electric field due to the point charge through the surface of the sphere? (*d*) Would your answer to part (*c*) change if the point charge were moved so that it was inside the sphere but not at its center? (*e*) What is the net flux through a cube of side 1 m that encloses the sphere?

(*a*) $A = 4\pi r^2$	$A = 3.14\ \mathrm{m}^2$
(*b*) Use Equ. 23-19	$E = 7.19\times10^4$ N/C
(*c*) $\boldsymbol{E\cdot n} = E$, so $\phi = EA$	$\phi = 2.26\times10^5\ \mathrm{N\cdot m^2/C}$
(*d*) No change in ϕ if q is inside sphere	
(*e*) Apply Gauss's law; ϕ unchanged	$\phi = 2.26\times10^5\ \mathrm{N\cdot m^2/C}$

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26 • Since Newton's law of gravity and Coulomb's law have the same inverse-square dependence on distance, an expression analogous in form to Gauss's law can be found for gravity. The gravitational field ***g*** is the force per unit mass on a test mass m_0. Then for a point mass m at the origin, the gravitational field g at some position r is

$$g = -\frac{Gm}{r^2} r$$

Compute the flux of the gravitational field through a spherical surface of radius r centered at the origin, and show that the gravitational analog of Gauss's law is $\phi_{\mathrm{net}} = -4\pi G m_{\mathrm{inside}}$.

Define the gravitational flux as $\phi_g = \int_S 0\boldsymbol{g\cdot n}\, dA$. Then $\phi_g = -(Gm/r^2)(4\pi r^2) = -4\pi Gm$.

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27 •• A charge of 2 μC is 20 cm above the center of a square of side length 40 cm. Find the flux through the square. (*Hint:* Don't integrate.)

Assume the square is one face of a cube of side length 40 cm. Then the charge is at the center of the cube and $\phi_{\mathrm{total}} = q/\varepsilon_0$. So for the face, $\phi = q/6\varepsilon_0 = 3.77\times10^4\ \mathrm{N\cdot m^2/C}$.

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28 •• In a particular region of the earth's atmosphere, the electric field above the earth's surface has been measured to be 150 N/C downward at an altitude of 250 m and 170 N/C downward at an altitude of 400 m. Calculate the volume charge density of the atmosphere assuming it to be uniform between 250 and 400 m. (You may neglect the curvature of the earth. Why?)

1. Calculate the charge inside a cylinder of base area	$Q = \rho AH$; we take our zero at 250 m above the earth
2. Use Gauss's law to find Q; take up as positive	surface and can consider the earth as flat.
3. Evaluate ρ	$Q = -(E_H A - E_0 A)\varepsilon_0 = \rho AH$; $\rho = (E_0 - E_H)/\varepsilon_0 H$
	$\rho = -1.18\times10^{-12}\ \mathrm{C/m^3}$

29* •• Explain why the electric field increases with r rather than decreasing as $1/r^2$ as one moves out from the center inside a spherical charge distribution of constant volume charge density.

The charge inside a sphere of radius r is proportional to r^3. The area of the sphere is proportional to r^2. Using Gauss's law, one sees that the field must be proportional to $r^3/r^2 = r$.

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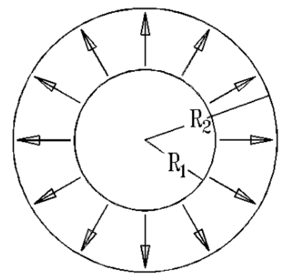
30 • A spherical shell of radius R_1 carries a total charge q_1 that is uniformly distributed on its surface. A second, larger spherical shell of radius R_2 that is concentric with the first carries a charge q_2 that is uniformly distributed on its surface. (*a*) Use Gauss's law to find the electric field in the regions $r < R_1$, $R_1 < r < R_2$, and $r > R_2$. (*b*) What should the ratio of the charges q_1/q_2 and their relative signs be for the electric field to be zero for $r > R_2$? (*c*) Sketch the electric field lines for the situation in part (*b*) when q_1 is positive.

(*a*) $r < R_1$, $Q = 0$; $R_1 < r < R_2$, $Q = q_1$; $r > R_2$, $Q = q_1 + q_2$

(*b*) Set $E = 0$ for $r > R_2$

$E = 0$ for $r < R_1$; $E = kq_1/r^2$ for $R_1 < r < R_2$
$E = k(q_1 + q_2)/r^2$ for $r > R_2$
$q_1 = -q_2$; $q_1/q_2 = -1$

c) The electric field lines for the case (*b*) are shown in the figure.



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31 · A spherical shell of radius 6 cm carries a uniform surface charge density $\sigma = 9\ \mathrm{nC/m^2}$. (*a*) What is the total charge on the shell? Find the electric field at (*b*) $r = 2$ cm, (*c*) $r = 5.9$ cm, (*d*) $r = 6.1$ cm, and (*e*) $r = 10$ cm.

(*a*) $Q = 4\pi r^2\sigma$ — $Q = 0.407$ nC
(*b*), (*c*) $Q_{in} = 0$ for $r < 6$ cm — $E(2\text{ cm}) = E(5.9\text{ cm}) = 0$
(*d*), (*e*) $Q_{in} = Q$; $E = kQ/r^2$ — $E(6.1\text{ cm}) = 983$ N/C; $E(10\text{ cm}) = 366$ N/C

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32 ·· A sphere of radius 6 cm carries a uniform volume charge density $\rho = 450\ \mathrm{nC/m^3}$. (*a*) What is the total charge of the sphere? Find the electric field at (*b*) $r = 2$ cm, (*c*) $r = 5.9$ cm, (*d*) $r = 6.1$ cm, and (*e*) $r = 10$ cm. Compare your answers with Problem 31.

(*a*) $Q = (4/3)\pi r^3\rho$ — $Q = 0.407$ nC
(*b*), (*c*) For $r < R$, $E = kQr/R^3$ — $E(2\text{ cm}) = 339$ N/C; $E(5.9\text{ cm}) = 1000$ N/C
(*d*), (*e*) For $r > R$, $E = kQ/r^2$ — $E(6.1\text{ cm}) = 984$ N/C; $E(10\text{ cm}) = 366$ N/C

33* ·· Consider two concentric conducting spheres (Figure 23-34). The outer sphere is hollow and initially has a charge $-7Q$ deposited on it. The inner sphere is solid and has a charge $+2Q$ on it. (*a*) How is the charge distributed on the outer sphere? That is, how much charge is on the outer surface and how much charge is on the inner surface? (*b*) Suppose a wire is connected between the inner and outer spheres. After electrostatic equilibrium is established, how much total charge is on the outside sphere? How much charge is on the outer surface of the outside sphere and how much is on the inner surface? Does the electric field at the surface of the inside sphere change when the wire is connected? If so, how? (*c*) Suppose we return to the original conditions in (*a*), with $+2Q$ on the inner sphere and $-7Q$ on the outer. We now connect the outer sphere to ground with a wire and then disconnect it. How much total charge will be on the outer sphere? How much charge will be on the inner surface of the outer sphere and how much will be on the outer surface?

(*a*) Since the outer sphere is conducting, the field in the thin shell must vanish. Therefore, $-2Q$, uniformly distributed, resides on the inner surface, and $-5Q$, uniformly distributed, resides on the outer surface.

(*b*) Now there is no charge on the inner surface and $-5Q$ on the outer surface of the spherical shell. The electric field just outside the surface of the inner sphere changes from a finite value to zero.

(*c*) In this case, the $-5Q$ is drained off, leaving no charge on the outer surface and $-2Q$ on the inner surface. The total charge on the outer sphere is then $-2Q$.

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34 ·· A nonconducting sphere of radius $R = 0.1$ m carries a uniform volume charge of charge density $\rho = 2.0$ nC/m^3. The magnitude of the electric field at $r = 2R$ is 1883 N/C. Find the magnitude of the electric field at $r = 0.5R$.

1. Write the expression for $E(r < R)$ — $E(r < R) = (4/3)\pi r k\rho$
2. Evaluate E at $r = 0.5R = 0.05$ m and $\rho = 2\ \mu$C/m^3 — $E = 3.77$ kN/C

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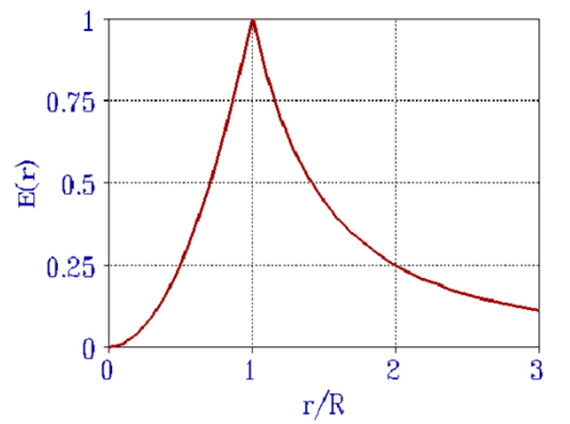
35 ·· A nonconducting sphere of radius R carries a volume charge density that is proportional to the distance from the center: $\rho = Ar$ for $r \le R$, where A is a constant; $\rho = 0$ for $r > R$. (*a*) Find the total charge on the sphere by summing the charges on shells of thickness dr and volume $4\pi r^2\, dr$. (*b*) Find the electric field E_r both inside and outside the charge distribution, and sketch E_r versus r.

(*a*) The charge in a shell of thickness dr is $dq = 4\pi r^2\rho dr = 4\pi A r^3 dr$. $Q = 4\pi A\int_0^R r^3\, dr = \pi A R^4$ 0.

(*b*) For $r < R$, $Q_{in} = \pi A r^4$ and by Gauss's law $E(r) = Ar^2/4\varepsilon_0$.
For $r > R$, $E(r) = AR^4/4\varepsilon_0 r^2$.
For $r > R$, $Q_{in} = \pi AR^4$ and $E(r > R) = AR^4/4\varepsilon_0 r^2$
A plot of $E(r)$ versus r/R is shown. Here $E(r)$ is in units of $A/4\varepsilon_0$.



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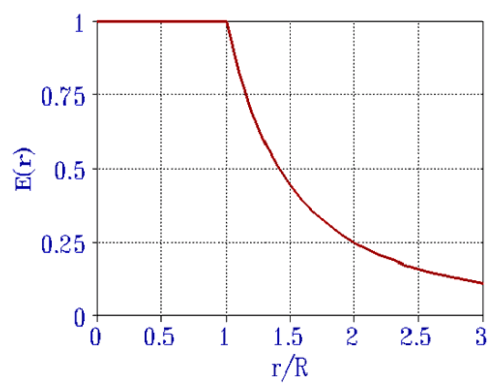
36 ·· Repeat Problem 35 for a sphere with volume charge density $\rho = B/r$ for $r < R$; $\rho = 0$ for $r > R$.

(*a*) Following the procedure of the preceding problem, one finds $Q = 2\pi BR^2$

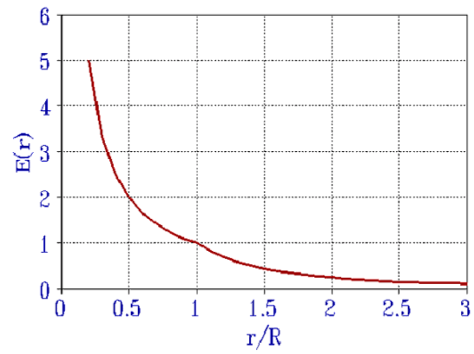
(b) Following the procedure of the preceeding problem, one finds $E(r < R) = B/2\varepsilon_0$, $E(r > R) = BR^2/2\varepsilon_0 r^2$. A plot of $E(r)$ versus r/R is shown. Here $E(r)$ is in units of $B/2\varepsilon_0$.



37* ·· Repeat Problem 35 for a sphere with volume charge density $\rho = C/r^2$ for $r < R$; $\rho = 0$ for $r > R$.

(*a*) The charge in a shell of thickness dr is $dq = 4\pi r^2\rho dr = 4\pi C dr$. $Q = 4\pi C\int_0^R dr = 4\pi C R$.

(*b*) For $r < R$, $Q_{\rm in} = 4\pi Cr$ and by Gauss's law $E(r < R) = C/\varepsilon_0 r$.
For $r > R$, $Q_{\rm in} = 4\pi CR$ and $E(r > R) = CR/\varepsilon_0 r^2$
A plot of $E(r)$ versus r/R is shown. Here $E(r)$ is in units of $C/\varepsilon_0 R$.



38 ··· The charge density in a region of space is spherically symmetric and is given by $\rho(r) = Ce^{-r/a}$ when $r < R$ and $\rho = 0$ when $r > R$. Find the electric field as a function of r.

The element of charge in a shell of thickness dr is $dq = 4\pi r^2\rho\, dr = 4\pi Cr^2e^{-r/a}\, dr$. The charge within a sphere of radius $r < R$ is obtained by integrating dq.

$$Q(r<R) = 4\pi C\int_0^r r^2e^{-r/a}dr = 4\pi C(-a^3e^{-r/a})(r^2/a^2 + 2r/a + 2)\Big|_0^r \;= 4\pi C\left[2a^3 - a^3e^{-r/a}\left(r^2/a^2 + 2r/a + 2\right)\right].$$

For $r > R$, Q, the charge within the sphere of radius r is given by the above expression where r is replaced by R. The electric field as a function of r is obtained from Gauss's law. That is,
$E(r < R) = (Ca^3/\varepsilon_0 r^2)[2 - e^{-r/a}(r^2/a^2 + 2r/a + 2)]$, and $E(r > R)$ is given by the same expression with r replaced by R inside the square brackets. Note that as in previous problems, E is continuous at $r = R$.

39 ··· A thick, nonconducting spherical shell of inner radius a and outer radius b has a uniform volume charge density ρ. Find (*a*) the total charge and (*b*) the electric field everywhere.

(*a*), (*b*) 1. For $r < a$, $Q_{\rm in} = 0$, and $E(r < a) = 0$. 2. For $a < r < b$, $Q_{\rm in} = (4\pi/3)\rho\,(r^3 - a^3)$; using Gauss's law, $E(a < r < b) = \rho\,(r^3 - a^3)/3\varepsilon_0 r^2$. 3. For $r > b$, $Q_{\rm in} = (4\pi/3)\rho\,(b^3 - a^3) = Q_{\rm tot}$ and $E(r > b) = \rho\,(b^3 - a^3)/3\varepsilon_0 r^2$.

40 ··· A point charge of +5 nC is located at the origin. This charge is surrounded by a spherically symmetric negative charge distribution with volume density $\rho(r) = Ce^{-r/a}$. (*a*) Find the constant C in terms of a if the total charge of the system is zero. (*b*) What is the electric field at $r = a$?

(*a*)	See Problem 38; set integration limits 0 to ∞	$Q_{\rm tot} = 5\ \mu{\rm C} + 8\pi Ca^3 = 0$; $C = -5/8\pi a^3\ \mu{\rm C}$
(*b*)	Use the expression in Problem 38 for $r = a$ to find $Q_{\rm in}$ and E	$Q_{\rm in} = [5 + 4\pi Ca^3(2 - 5/e)]\ \mu{\rm C} = 4.6\ \mu{\rm C}$ $E = kQ_{\rm in}/a^2 = 4.14\times10^4/a^2$ N/C

41* ··· A nonconducting solid sphere of radius a with its center at the origin has a spherical cavity of radius b with its center at the point $x = b$, $y = 0$ as shown in Figure 23-35. The sphere has a uniform volume charge density ρ. Show that the electric field in the cavity is uniform and is given by $E_y = 0$, $E_x = \rho b/3\varepsilon_0$. (*Hint:* Replace the cavity with spheres of equal positive and negative charge densities.)

Using the *Hint* we shall find the x and y components of the field due to the uniform positive charge distribution of the solid sphere, and then the x and y components of the field due to a uniform negative charge distribution centered at $x = b$. We denote the field due to the solid positively charged sphere as $\boldsymbol{E}_+$ and that due to the negatively charged sphere at $x = b$ by $\boldsymbol{E}_-$. The field E_+ is $(4\pi/3)k\rho\, r$ and its x and y components are E_{+x} =

$(4\pi/3)k\rho\, x$ and $E_{+y} = (4\pi/3)k\rho\, y$. For the negatively charged sphere, $E_- = -(4\pi/3)k\rho\, r'$, where r' is the radial distance from $x = b$, $y = 0$. We can again find the x and y components of E_- at a point (x, y); they are $E_{-x} = -(4\pi/3)k\rho\,(x - b)$ and $E_{-y} = -(4\pi/3)k\rho\, y$. Thus $E_x = E_{+x} + E_{-x} = (4\pi/3)k\rho b = \rho b/3\varepsilon_0$ and $E_y = E_{+y} + E_{-y} = 0$. Since the system is symmetric for rotation about the x axis, $E_z = E_y = 0$.

42 ·· Show that the electric field due to an infinitely long, uniformly charged cylindrical shell of radius R carrying a surface charge density σ is given by

$$E_r = 0{,}0 \qquad r < R \qquad \text{23-28}a$$

$$E_r = \frac{\sigma R}{\varepsilon_0 \mathrm{r}} = \frac{\lambda}{2\pi\,\varepsilon_0 \mathrm{r}}, \quad r > R \qquad \text{23-28}b$$

where $\lambda = 2\pi R\sigma$ is the charge per unit length on the shell.

From symmetry, the field in the tangential direction must vanish. Construct a Gaussian surface in the shape of a cylinder of radius r and length L. If $r < R$, $Q_{\rm in} = 0$ and $E_{\rm r} = 0$. If $r > R$, $Q_{\rm in} = \lambda L$. The area of the Gaussian surface surrounding $Q_{\rm in}$ is $2\pi RL$, neglecting the end areas since no flux crosses those. The charge $Q_{\rm in} = 2\pi RL\sigma$, and using Gauss's law, one obtains $E_{\rm r} = R\sigma/\varepsilon_0 r$.

43 ·· A cylindrical shell of length 200 m and radius 6 cm carries a uniform surface charge density of $\sigma = 9$ nC/m^2. (*a*) What is the total charge on the shell? Find the electric field at (*b*) $r = 2$ cm, (*c*) $r = 5.9$ cm, (*d*) $r = 6.1$ cm, and (*e*) $r = 10$ cm. (Use the results of Problem 42.)

(*a*) $Q = 2\pi RL\sigma$	$Q = 679$ nC
(*b*), (*c*) See Equ. 23-28*a*	$E(2\text{ cm}) = E(5.9\text{ cm}) = 0$
(*d*), (*e*) Use Equ. 23-28*b*	$E(6.1\text{ cm}) = 1$ kN/C; $E(10\text{ cm}) = 610$ N/C

44 ·· An infinitely long nonconducting cylinder of radius R carries a uniform volume charge density of $\rho(r) = \rho_0$. Show that the electric field is given by

$$E_r = \frac{\rho R^2}{2\varepsilon_0 \mathrm{r}} = \frac{1}{2\pi\,\varepsilon_0}\frac{\lambda}{r},0 \qquad r > R \qquad \text{23-29}a$$

$$E_r = \frac{\rho}{2\varepsilon_0}\mathrm{r} = \frac{\lambda}{2\pi\,\varepsilon_0 \mathrm{R}^2}\mathrm{r},0 \qquad r < R \qquad \text{23-29}b$$

where $\lambda = \rho\pi R^2 0$ is the charge per unit length.

Proceed as in Problem 42, constructing cylindrical Gaussian surfaces. For $r < R$, the charge within the Gaussian surface is $Q_{\rm in} = \pi r^2 L\rho$. The area of the Gaussian surface, neglecting the end areas, is $2\pi rL$. Using Gauss's law one obtains $E_{\rm r} = r\rho/2\varepsilon_0 = \lambda/2\pi r\varepsilon_0$. For $r > R$, the charge within the Gaussian surface is $\pi R^2 L\rho$ and the area of the Gaussian surface is $2\pi rL$. From Gauss's law one obtains $E_{\rm r} = \rho R^2/2r\varepsilon_0 = \lambda/2\pi r\varepsilon_0$, since $\lambda = \pi R^2\rho$.

45* ·· A cylinder of length 200 m and radius 6 cm carries a uniform volume charge density of $\rho = 300$ nC/m^3. (*a*) What is the total charge of the cylinder? Use the formulas given in Problem 44 to calculate the electric field at a point equidistant from the ends at (*b*) $r = 2$ cm, (*c*) $r = 5.9$ cm, (*d*) $r = 6.1$ cm, and (*e*) $r = 10$ cm. Compare your results with those in Problem 43.

(*a*) $Q = \pi R^2 L\rho$ — $Q = 679$ nC

(*b*), (*c*) Use Equ. 23-29*b* — $E_r(2\text{ cm}) = 339$ N/C; $E_r(5.9\text{ cm}) = 1.00$ kN/C

(*d*), (*e*) Use Equ. 23-29*a* — $E_r(6.1\text{ cm}) = 1.00$ kN/C; $E_r(10\text{ cm}) = 610$ N/C

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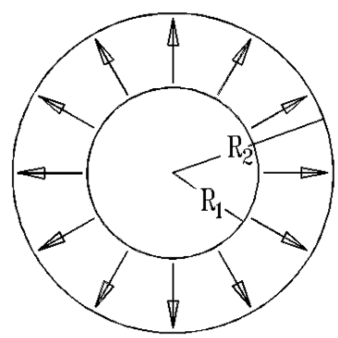
46 ·· Consider two infinitely long, concentric cylindrical shells. The inner shell has a radius R_1 and carries a uniform surface charge density of σ_1, and the outer shell has a radius R_2 and carries a uniform surface charge density of σ_2. (*a*) Use Gauss's law to find the electric field in the regions $r < R_1$, $R_1 < r < R_2$, and $r > R_2$. (*b*) What is the ratio of the surface charge densities σ_2/σ_1 and their relative signs if the electric field is zero at $r > R_2$? What would the electric field between the shells be in this case? (*c*) Sketch the electric field lines for the situation in (*b*) if σ_1 is positive.

From symmetry, the field must be radial.

(*a*) Apply Equs. 23-28*a* and 23-28*b* — $E = 0$ for $r < R_1$; $E = \sigma_1 R_1/\varepsilon_0 r$ for $R_1 < r < R_2$;
$E = E_1 + E_2 = (\sigma_1 R_1 + \sigma_2 R_2)/\varepsilon_0 r$ for $r > R_2$

(*b*) Set E for $r > R_2 = 0$ — $\sigma_1 R_1 + \sigma_2 R_2 = 0$, $\sigma_1/\sigma_2 = -R_2/R_1$

(*c*) The field lines for case (*b*) are shown.
Here we assume that σ_1 is positive.



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47 ·· Figure 23-36 shows a portion of an infinitely long, concentric cable in cross section. The inner conductor carries a charge of 6 nC/m; the outer conductor is uncharged. (*a*) Find the electric field for all values of *r*, where *r* is the distance from the axis of the cylindrical system. (*b*) What are the surface charge densities on the inside and the outside surfaces of the outer conductor?

(*a*) The cylinders are conductors; $E = 0$ inside them. — $E = 0$ for $r < 1.5$ cm and for 4.5 cm $< r <$ 6.5 cm

For 1.5 cm$< r <$ 4.5 cm, $r >$ 6.5 cm use Equ. 23-28*b*. — $E = 108/r$ N/C

(b) $E = 0$ inside the outer cylinder; see Problem 46(*b*). $Q_{2,\text{net}} = 0$; therefore, $\sigma_{2,\text{in}} R_{2,\text{inner}} = \sigma_{2,\text{out}} R_{2,\text{outer}}$ — $\sigma_1 = (6/2\pi\times 0.015)$ nC/m^2; $\sigma_{2,\text{in}} = \sigma_1 R_1/R_2 = 21.2$ nC/m^2
$\sigma_{2,\text{out}} = (21.2\times 4.5/6.5)$ nC/m^2 = 14.7 nC/m^2

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48 ·· Repeat Problem 44 for a cylinder with volume charge density (*a*) $\rho(r) = ar$ and (*b*) $\rho = Cr^2$

(a) Find λ_{in} within a radius r for $r < R$ — $\lambda_{in} = \int_0^r 2\pi r(ar^2)\,dr = 2\pi a r^3/3$

Use Gauss's law to determine E — $E = (2\pi ar^3/3)/2\pi\varepsilon_0 r = ar^2/3\varepsilon_0$ for $r < R$

(b) Find λ_{in} for $r > R$ — $\lambda_{in} = \int_0^R 2\pi r(ar^2)\,dr = 2\pi a R^3/3$

Find E using Gauss's law — $E = aR^3/3\varepsilon_0 r$

49* ·· Repeat Problem 44 with $\rho = C/r$.

(a) Find λ_{in} within a radius r for $r < R$ — $\lambda_{in} = \int_0^r 2\pi r(C/r)\,dr = 2\pi C r$

Use Gauss's law to determine E — $E = 2\pi Cr/2\pi\varepsilon_0 r = C/\varepsilon_0$ for $r < R$

(b) Find λ_{in} for $r > R$ — $\lambda_{in} = \int_0^R 2\pi r(C/r)\,dr = 2\pi C R$

Find E using Gauss's law — $E = CR/\varepsilon_0 r$

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50 ··· An infinitely long, thick, nonconducting cylindrical shell of inner radius a and outer radius b has a uniform volume charge density ρ. Find the electric field everywhere.

1. For $r < a$, $\lambda_{in} = 0$. Use Gauss's law — $E = 0$ for $r < a$
2. For a < r < b, find $\lambda_{in}(r)$ — $\lambda_{in}(r) = \int_a^r 2\pi r\rho\,dr = \pi\rho(r^2 - a^2)$

   Use Gauss's law to find E — $E = \rho(r^2 - a^2)/2\varepsilon_0 r$ for $a < r < b$
3. For $r > b$, $\lambda_{in} = \pi\rho(b^2 - a^2)$. Use Gauss's law — $E = \rho(b^2 - a^2)/2\varepsilon_0 r$

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51 ··· Suppose that the inner cylinder of Figure 23-36 is made of nonconducting material and carries a volume charge distribution given by $\rho(r) = C/r$, where C = 200 nC/m^2. The outer cylinder is metallic. (a) Find the charge per meter carried by the inner cylinder. (b) Calculate the electric field for all values of r.

(a) See Problem 49(b); $\lambda_{inner} = 2\pi CR$ — λ_{inner} = 18.85 nC/m

(b) 1. For r < 1.5 cm, see Problem 49(a) — $E = C/\varepsilon_0$ = 22.6 kN/C

2. For 1.5 cm < r < 4.5 cm, see Problem 49(b) — $E = 339/r$ N/C

3. For 4.5 cm < r < 6.5 cm, conductor — $E = 0$

4. For r > 6.5 cm, see part 2 — $E = 339/r$ N/C

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52 · A penny is in an external electric field of magnitude 1.6 kN/C directed perpendicular to its faces. (a) Find the charge density on each face of the penny, assuming the faces are planes. (b) If the radius of the penny is 1 cm, find the total charge on one face.

(a) $E = \sigma/\varepsilon_0$; $\sigma = E\varepsilon_0$ — σ = 14.2 nC/m^2

(b) $Q = A\sigma = \pi r^2\sigma$ — Q = 4.45 pC

53* · An uncharged metal slab has square faces with 12-cm sides. It is placed in an external electric field that is perpendicular to its faces. The total charge induced on one of the faces is 1.2 nC. What is the magnitude of the electric field?

$\sigma = Q/L^2$; $E = Q/L^2\varepsilon_0$ $E = 9.41$ kN/C

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54 • A charge of 6 nC is placed uniformly on a square sheet of nonconducting material of side 20 cm in the *yz* plane. (*a*) What is the surface charge density σ? (*b*) What is the magnitude of the electric field just to the right and just to the left of the sheet? (*c*) The same charge is placed on a square conducting slab of side 20 cm and thickness 1 mm. What is the surface charge density σ? (Assume that the charge distributes itself uniformly on the large square surfaces.) (*d*) What is the magnitude of the electric field just to the right and just to the left of each face of the slab?

(*a*) $\sigma = Q/A$ $\sigma = 150$ nC/m^2

(*b*) Use Equ. 23-12; $E = \sigma/2\varepsilon_0$ $E = 8.47$ kN/C

(*c*) Now Q on each face is 3 nC $\sigma = 75$ nC/m^2

(*d*) Use Equ. 23-25 $E = 8.47$ kN/C

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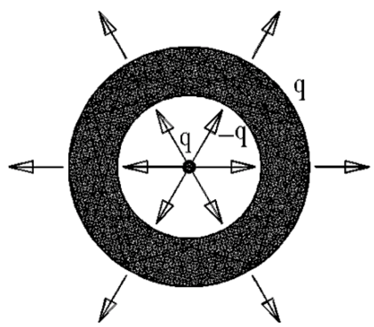
55 • A spherical conducting shell with zero net charge has an inner radius *a* and an outer radius *b*. A point charge *q* is placed at the center of the shell. (*a*) Use Gauss's law and the properties of conductors in equilibrium to find the electric field in the regions $r < a$, $a < r < b$, and $b < r$. (*b*) Draw the electric field lines for this situation. (*c*) Find the charge density on the inner surface ($r = a$) and on the outer surface ($r = b$) of the shell.

(*a*) For $r < a$, $E = kq/r^2$. For $a < r < b$, $E = 0$. For $r > b$, $E = kq/r^2$

(*b*) See the figure

(*c*) At $r = a$, $\sigma = -q/4\pi a^2$; at $r = b$, $\sigma = q/4\pi b^2$



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56 •• The electric field just above the surface of the earth has been measured to be 150 N/C downward. What total charge on the earth is implied by this measurement?

The earth is a sphere, so $Q = ER_E^2/k$ $Q = 150\times(6.37\times10^6)^2/8.99\times10^9 = 6.77\times10^5$ C

57* •• A positive point charge of magnitude 2.5 μC is at the center of an uncharged spherical conducting shell of inner radius 60 cm and outer radius 90 cm. (*a*) Find the charge densities on the inner and outer surfaces of the shell and the total charge on each surface. (*b*) Find the electric field everywhere. (*c*) Repeat (*a*) and (*b*) with a net charge of +3.5 μC placed on the shell.

(*a*) For 60 cm $< r <$ 90 cm, $E = 0$ $q_{in} = -2.5$ μC; $\sigma_{in} = -2.5\times10^{-6}/4\pi\times0.6^2 = -0.553$ μC/m^2

$q_{inner} + q_{outer} = 0$ $q_{out} = 2.5$ μC; $\sigma_{out} = 2.5\times10^{-6}/4\pi\times0.9^2 = 0.246$ μC/m^2

(*b*) For $r < 0.6$ m, $E = kq/r^2$ $E = 2.25\times10^4/r^2$ N/C

For 0.6 m $< r <$ 0.9 m, conductor $E = 0$

For $r > 0.9$ m, $E = kq/r^2$ $E = 2.25\times10^4/r^2$ N/C

(c) Since $E = 0$ in the conductor, q_{inner} is again $-2.5\ \mu$C and $\sigma_{\text{inner}} = -0.553\ \mu\text{C/m}^2$. Now $q_{\text{inner}} + q_{\text{outer}} = 3.5\ \mu$C; consequently, $q_{\text{outer}} = 6.0\ \mu$C and $\sigma_{\text{outer}} = 0.59\ \mu\text{C/m}^2$. The fields are $2.25\times10^4/r^2$ N/C for $r < 0.6$ m, zero within the shell, and $5.4\times10^4/r^2$ N/C for $r > 0.9$ m.

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58 ·· If the magnitude of an electric field in air is as great as 3×10^6 N/C, the air becomes ionized and begins to conduct electricity. This phenomenon is called dieletric breakdown. A charge of 18 μC is to be placed on a conducting sphere. What is the minimum radius of a sphere that can hold this charge without breakdown?

$R_{\text{min}}^2 = kq/E_{\text{max}}$ $\qquad R_{\text{min}} = (8.99\times10^9 \times 18\times10^{-6}/3\times10^6)^{1/2}$ m = 23.2 cm

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59 ·· A square conducting slab with 5-m sides carries a net charge of 80 μC. (a) Find the charge density on each face of the slab and the electric field just outside one face of the slab. (b) The slab is placed to the right of an infinite charged nonconducting plane with charge density 2.0 $\mu\text{C/m}^2$ so that the faces of the slab are parallel to the plane. Find the electric field on each side of the slab far from its edges and the charge density on each face.

(a) σ per face $= q/2L^2$; $E = \sigma/\varepsilon_0$ $\qquad \sigma/\text{face} = 1.6\ \mu\text{C/m}^2$; $E = 1.81\times10^5$ N/C

(b) $E_{\text{slab}} = \sigma_{\text{slab}}/2\varepsilon_0$; $E_{\text{total}} = E_{\text{face}} + E_{\text{slab}}$ on face away from the plane. On face near the plane, fields oppose. $\qquad E_{\text{total}}$(away from plane) $= 2.94\times10^5$ N/C; E_{total}(near plane) $= 0.68\times10^5$ N/C

σ(near plane) $= \varepsilon_0 E$(near plane) $\qquad \sigma$(near plane) $= 0.6\ \mu\text{C/m}^2$;

σ(away from plane) $= \varepsilon_0 E$(away from plane) $\qquad \sigma$(away from plane) $= 2.6\ \mu\text{C/m}^2$

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60 ·· Imagine that a small hole has been punched through the wall of a thin, uniformly charged spherical shell whose surface charge density is σ. Find the electric field near the center of the hole.

We can view the system as a spherical shell of surface charge density $\sigma = Q/4\pi r^2$ with a small disk with a surface charge density $-\sigma$. From Coulomb's law, the field just outside the shell is $kQ/r^2 = 4\pi k\sigma = \sigma/\varepsilon_0$ and is zero inside. The field due to the disk is $\sigma/2\varepsilon_0$ inside the shell and immediately adjacent to the disk, pointing radially outward; just outside the disk the field is $\sigma/2\varepsilon_0$ and points radially inward. Adding these fields we see that just inside the hole the field is continuous and has the value $\sigma/2\varepsilon_0 = kQ/2r^2$.

61* · True or false:

(a) If there is no charge in a region of space, the electric field on a surface surrounding the region must be zero everywhere.

(b) The electric field inside a uniformly charged spherical shell is zero.

(c) In electrostatic equilibrium, the electric field inside a conductor is zero.

(d) If the net charge on a conductor is zero, the charge density must be zero at every point on the surface of the conductor.

(a) False (b) True (assuming there are no charges inside the shell) (c) True (d) False

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62 · If the electric field E is zero everywhere on a closed surface, is the net flux through the surface necessarily zero? What, then, is the net charge inside the surface?

Yes, because $\varphi = \int_S E\, dA$. The net charge inside the surface must be zero by Gauss's law.

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63 · A point charge $-Q$ is at the center of a spherical conducting shell of inner radius R_1 and outer radius R_2 as shown in Figure 23-37. The charge on the inner surface of the shell is (a) $+Q$. (b) zero. (c) $-Q$. (d) dependent on the total charge carried by the shell.

(*a*)

64 • For the configuration of Figure 23-37, the charge on the outer surface of the shell is (*a*) $+Q$. (*b*) zero. (*c*) $-Q$. (*d*) dependent on the total charge carried by the shell.

(*d*)

65* •• Suppose that the total charge on the conducting shell of Figure 23-37 is zero. It follows that the electric field for $r < R_1$ and $r > R_2$ points

(*a*) away from the center of the shell in both regions.

(*b*) toward the center of the shell in both regions.

(*c*) toward the center of the shell for $r < R_1$ and is zero for $r > R_2$.

(*d*) away from the center of the shell for $r < R_1$ and is zero for $r > R_2$.

(*b*)

66 •• If the conducting shell in Figure 23-37 is grounded, which of the following statements is then correct?

(*a*) The charge on the inner surface of the shell is $+Q$ and that on the outer surface is $-Q$.

(*b*) The charge on the inner surface of the shell is $+Q$ and that on the outer surface is zero.

(*c*) The charge on both surfaces of the shell is $+Q$.

(*d*) The charge on both surfaces of the shell is zero.

(*b*)

67 •• For the configuration described in Problem 66, in which the conducting shell is grounded, the electric field for

$r < R_1$ and $r > R_2$ points

(*a*) away from the center of the shell in both regions.

(*b*) toward the center of the shell in both regions.

(*c*) toward the center of the shell for $r < R_1$ and is zero for $r > R_2$.

(*d*) toward the center of the shell for $r < R_1$ and is zero for $r > R_1$.

(*c*)

68 •• If the net flux through a closed surface is zero, does it follow that the electric field E is zero everywhere on the surface? Does it follow that the net charge inside the surface is zero?

The electric field need not be zero everywhere on the surface, but the net charge inside the surface is zero.

69* •• Equation 23-8 for the electric field on the perpendicular bisector of a finite line charge is different from Equation 23-9 for the electric field near an infinite line charge, yet Gauss's law would seem to give the same result for these two cases. Explain.

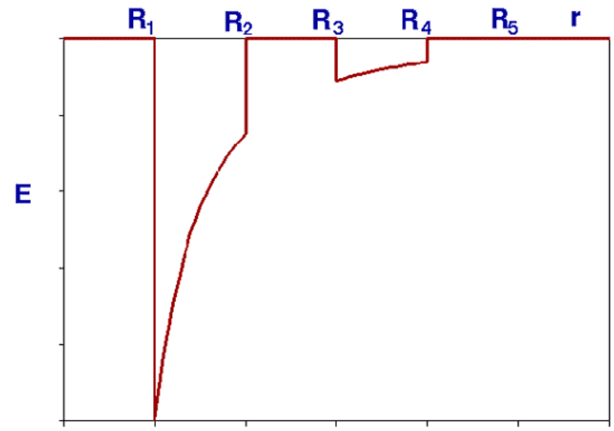
The two expressions agree if $r << L$, where L is the length of the line charge of finite length. For r of the same order of magnitude as L or greater, the electric field does not have cylindrical symmetry, and one cannot use Gauss's law to determine E.

70 •• True or false: The electric field is discontinuous at all points at which the charge density is discontinuous.

False; see, for example, the field of a uniformly charged sphere. ρ is discontinuous at the surface, E is not.

71 •• Consider the three concentric metal spheres shown in Figure 23-38. Sphere I is solid, with radius R_1. Sphere II is hollow, with inner radius R_2 and outer radius R_3. Sphere III is hollow, with inner radius R_4 and outer radius R_5. Initially, all three spheres have zero excess charge. Then a negative charge $-Q_0$ is placed on sphere I and a positive charge $+Q_0$ is placed on sphere III. (*a*) After the charges have reached equilibrium, will the electric field in the space between spheres I and II point *toward* the center, *away* from the center, or neither? (b) How much charge will be on the inner surface of sphere II? Give the correct sign. (c) How much charge will be on the outer surface of sphere II? (d) How much charge will be on the inner surface of sphere III? (e) How much charge will be on the outer surface of sphere III? (f) Plot E versus r.

(a) toward the center
(b) $+Q_0$
(c) $-Q_0$
(d) $+Q_0$
(e) 0
(f) See the adjacent figure. Here E and r are in arbitrary units.



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72 ·· An early model of the hydrogen atom considered the atom to consist of a proton, which is a uniform charged sphere of radius R, with an electron in an orbit of radius r_0 *inside* the proton as shown in Figure 23-39. (a) Use Gauss's law to obtain the magnitude of E (the field due to the proton) at the position of the electron. Give your answer in terms of e (the charge on a proton), r_0, and R. (b) Find the frequency of revolution f in terms of r_0 and the velocity of the electron v. (c) What is the force on the electron in terms of m,v, and r_0? (d) What is the frequency f in terms of m, e, R, ε_0 , and r_0? (Each of your answers need not include all of the specified quantities.)

(a) 1. Find the charge within r_0; $q \propto r$	$q = er_0^3/R^3$
2. Use Gauss's law	$E = er_0^3/4\pi r_0^2\varepsilon_0 R^3 = er_0/4\pi\varepsilon_0 R^3$
(b) $f = 1/T$; $T = 2\pi r_0/v$	$f = v/2\pi r_0$
(c) $F = ma_c$	$F = mv^2/r_0$
(d) $eE = F$; express f in terms of variables	$f = (e/2\pi)\sqrt{1/4\pi\,\varepsilon_0\, m\, R^3}$

73* ·· A nonuniform surface charge lies in the yz plane. At the origin, the surface charge density is $\sigma = 3.10$ μC/m^2. Other charged objects are present as well. Just to the right of the origin, the x component of the electric field is $E_x = 4.65\times10^5$ N/C. What is E_x just to the left of the origin?

Use Equ. 23-24	$E_{x,\text{left}} = E_{x,\text{right}} - \sigma/\varepsilon_0 = 1.15\times10^5$ N/C

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74 ·· An infinite line charge of uniform linear charge density $\lambda = -1.5$ μC/m lies parallel to the y axis at $x = -2$ m. A point charge of 1.3 μC is located at $x = 1$ m, $y = 2$ m. Find the electric field at $x = 2$ m, $y = 1.5$ m.

1. Find the field at (2, 1.5) due to line charge	$\boldsymbol{E}_\lambda = (2k\lambda/r)\,\boldsymbol{i} = -6.74\times10^3$ N/C $\boldsymbol{i}$
2. Find E due to point charge	$E_p = kq/r^2 = 9.35$ kN/C
3. Determine the x and y components of E_p	$E_{px} = 9.35\times1/1.25^{1/2} = 8.36$ kN/C; $E_{py} = -4.18$ kN/C
4. $\boldsymbol{E} = \boldsymbol{E}_\lambda + \boldsymbol{E}_p$	$\boldsymbol{E} = 1.62$ kN/C $\boldsymbol{i} - 4.18$ kN/C $\boldsymbol{j}$

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75 ·· Two infinite planes of charge lie parallel to each other and to the *yz* plane. One is at $x = -2$ m and has a surface charge density of $\sigma = -3.5\ \mu C/m^2$. The other is at $x = 2$ m and has a surface charge density of $\sigma = 6.0\ \mu C/m^2$. Find the electric field for (*a*) $x < -2$ m, (*b*) $-2\text{ m} < x < 2$ m, and (*c*) $x > 2$ m.

(*a*) Find E of each plane; use Equ. 23-12	$E_{-2} = 1.98$ kN/C, pointing toward this plane
	$E_2 = 3.39$ kN/C, pointing away from this plane
For $x < -2$ m, $E = E_{-2} - E_2$	$\boldsymbol{E} = -1.41$ kN/C $\boldsymbol{i}$
(*b*) For $-2\text{ m} < x < 2$ m, $E = E_{-2} + E_2$	$\boldsymbol{E} = -5.37$ kN/C $\boldsymbol{i}$
(*c*) For $x > 2$ m, $E = E_{-2} - E_2$	$\boldsymbol{E} = 1.41$ kN/C $\boldsymbol{i}$

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76 ·· An infinitely long cylindrical shell is coaxial with the *y* axis and has a radius of 15 cm. It carries a uniform surface charge density $\sigma = 6\ \mu C/m^2$. A spherical shell of radius 25 cm is centered on the *x* axis at $x = 50$ cm and carries a uniform surface charge density $\sigma = -12\ \mu C/m^2$. Calculate the magnitude and direction of the electric field at (*a*) the origin; (*b*) $x = 20$ cm, $y = 10$ cm; and (*c*) $x = 50$ cm, $y = 20$ cm. (See Problem 42.)

(*a*) For $x < 15$ cm, $E_{\text{cyl}} = 0$	$\boldsymbol{E}(0, 0) = (4\pi \times 0.25^2 \times 12\times10^{-6} \times 8.99\times10^9/0.5^2)$ N/C $\boldsymbol{i}$
	$= 3.39\times10^5$ N/C $\boldsymbol{i}$;
(*b*) Both cylinder and sphere contribute to $\boldsymbol{E}$	$\boldsymbol{E}_{\text{cyl}} = 5.08\times10^5$ N/C $\boldsymbol{i}$; $E_{\text{sph}} = 8.47\times10^5$ N/C
Find $E_{\text{sph},x}$, $E_{\text{sph},y}$ and $\boldsymbol{E}$	$E_{\text{sph},x} = 8.04\times10^5$ N/C, $E_{\text{sph},y} = -2.68\times10^5$ N/C
	$\boldsymbol{E} = \boldsymbol{E}_{\text{cyl}} + \boldsymbol{E}_{\text{sph}} = 1.31\times10^6$ N/C $\boldsymbol{i}$ - 2.68×10^5 N/C $\boldsymbol{j}$
(*c*) At (0.5, 0.2) $E_{\text{sph}} = 0$	$\boldsymbol{E} = 2.03\times10^5$ N/C $\boldsymbol{I}$

77* ·· An infinite plane in the *xz* plane carries a uniform surface charge density $\sigma_1 = 65\ nC/m^2$. A second infinite plane carrying a uniform charge density $\sigma_2 = 45\ nC/m^2$ intersects the *xz* plane at the *z* axis and makes an angle of 30° with the *xz* plane as shown in Figure 23-40. Find the electric field in the *xy* plane at (*a*) $x = 6$ m, $y = 2$ m and (*b*) $x = 6$ m, $y = 5$ m.

Find $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ for (6, 2) and (6, 5)	$\boldsymbol{E}_1(6, 2) = \boldsymbol{E}_1(6, 5) = (\sigma_1/2\varepsilon_0)\ \boldsymbol{j}$;
	$\boldsymbol{E}_2(6, 2) = (\sigma_2/2\varepsilon_0)(\sin 30^\circ\ \boldsymbol{i} - \cos 30^\circ\ \boldsymbol{j})$;
	$\boldsymbol{E}_2(6, 5) = -\boldsymbol{E}_2(6, 2)$
(*a*) $\boldsymbol{E}(6, 2) = \boldsymbol{E}_1(6, 2) + \boldsymbol{E}_2(6, 2)$	$\boldsymbol{E}(6, 2) = 1.27$ kN/C $\boldsymbol{i}$ + 1.47 kN/C $\boldsymbol{j}$
(*b*) $\boldsymbol{E}(6, 5) = \boldsymbol{E}(6, 5) + \boldsymbol{E}_2(6, 5)$	$\boldsymbol{E}(6, 5) = -1.27$ kN/C $\boldsymbol{i}$ + 5.87 kN/C $\boldsymbol{j}$

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78 ·· A ring of radius *R* carries a uniform, positive, linear charge density λ. Figure 23-41 shows a point *P* in the plane of the ring but not at the center. Consider the two elements of the ring of lengths s_1 and s_2 shown in the figure at distances r_1 and r_2, respectively, from point *P*. (*a*) What is the ratio of the charges of these elements? Which produces the greater field at point *P*? (*b*) What is the direction of the field at point *P* due to each element? What is the direction of the total electric field at point *P*? (*c*) Suppose that the electric field due to a point charge varied as $1/r$ rather than $1/r^2$. What would the electric field be at point *P* due to the elements shown? (*d*) How would your answers to parts (*a*), (*b*), and (*c*) differ if point *P* were inside a spherical shell of uniform charge and the elements were of areas s_1 and s_2?

(*a*) The charges on the two segments are proportional to r_1 and r_2, respectively, so $q_1/q_2 = r_1/r_2$. The fields are proportional to $1/r_1^2$ and $1/r_2^2$, respectively. Consequently, $E_1/E_2 = r_2/r_1$ and $E_1 > E_2$.

(*b*) The two fields point away from their segments of arc. $\boldsymbol{E}$ points toward s_2

(*c*) In this case, $E_1 = E_2$, and the total field at *P* would be zero.

(*d*) For a spherical shell, $q_1 \propto r_1^2$ and $q_2 \propto r_2^2$, so $q_1/q_2 = r_1^2/r_2^2$. Since the fields are proportional to $1/r^2$, $E_1/E_2 = 1$. The two fields are of equal magnitude and oppositely directed. Hence $E = 0$.
If $E \propto 1/r$, then s_2 would produce the stronger field at *P*, and **E** would point toward s_1.

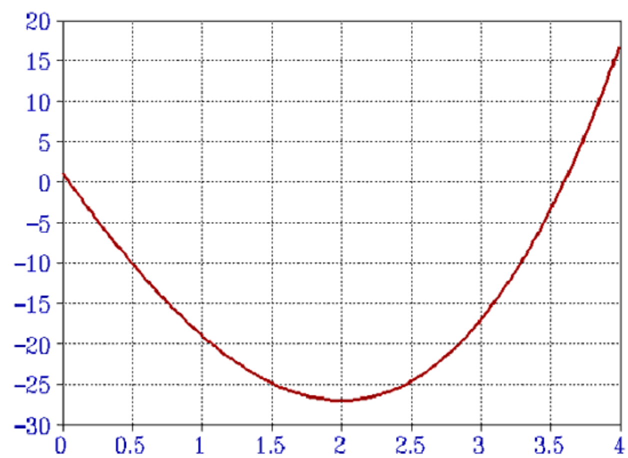
79 ·· A ring of radius *R* that lies in the horizontal (*xy*) plane carries a charge *Q* uniformly distributed over its length. A mass *m* carries a charge *q* whose sign is opposite that of *Q*. (*a*) What is the minimum value of $|q|/m$ such that the mass will be in equilibrium under the action of gravity and the electrostatic force on the charge *q*? (*b*) If $|q|/m$ is twice that calculated in (*a*), where will the mass be when it is in equilibrium?

(*a*) The minimum value of $|q|/m$ will be where the field due to the ring is greatest, i.e. at $z = -R/\sqrt{2}$ (see Problem 11). Substituting this value into Equ. 23-10 one obtains $E = 2kQ/R^2\sqrt{27}$. At equilibrium, $mg = qE$, so $|q|/m = g/E = R^2 g\sqrt{27}/2kQ$.

(*b*) If $|q|/m$ is twice as great as in (*a*), the *E* should be half the value in (*a*), i.e., $E = 2kQ/R^2\sqrt{27}$. The condition to be satisfied is

$$\frac{1}{27R^4} = \frac{z^2}{(z^2+R^2)^3}. \text{ Let } a = z^2/R^2.$$

The equation for *a* is $a^3 + 3a^2 - 24a + 1 = 0$. This cubic equation is plotted in the adjacent figure. The solutions (by trial and error) are $a = 0.042$ and 3.6. The distances for equilibrium are $z = -0.205R$ and $z = -1.9R$. However only $z = -1.9R$ is a location of stable equilibrium.



80 ·· A long, thin, nonconducting plastic rod is bent into a loop with radius *R*. Between the ends of the rod, a small gap of length ℓ ($\ell \ll R$) remains. A charge *Q* is equally distributed on the rod. (*a*) Indicate the direction of the electric field at the center of the loop. (*b*) Find the magnitude of the electric field at the center of the loop.

(*a*) The loop with the small gap is equivalent to a closed loop and a charge of $-Q\ell/2\pi R$ at the gap. The field at the center of a closed loop of uniform line charge is zero. Thus the field is entirely due to the charge $-Q\ell/2\pi R$. If *Q* is positive, the field at the origin points radially outward.

(*b*) $E = Q\ell/2\pi R^3$

81* ·· A rod of length *L* lies perpendicular to an infinitely long uniform line charge of charge density λ C/m (Figure 23-42). The near end of the rod is a distance *d* above the line charge. The rod carries a total charge *Q* uniformly distributed along its length. Find the force that the infinitely long line charge exerts on the rod.
Let *y* be the distance from the infinite line charge. The element of charge on the finite rod is $dq = (Q/L)dy$, and the field at the charge dq is $2k\lambda/y$. The force on the rod is

$$F = \frac{2k\lambda Q}{L}\int_{d}^{d+L}\frac{dy}{y} = \frac{2kQ\lambda}{L}\text{1n}\left(\frac{L+d}{d}\right)0$$

82 ·· A nonconducting sphere 1.2 m in diameter with its center on the x axis at x = 4 m carries a uniform volume charge of density ρ = 5 μC/m^3. Surrounding the sphere is a spherical shell with a diameter of 2.4 m and a uniform surface charge density σ = –1.5 μC/m^2. Calculate the magnitude and direction of the electric field at (*a*) x = 4.5 m, y = 0; (*b*) x = 4.0 m, y = 1.1 m; and (*c*) x = 2.0 m, y = 3.0 m.

(a) Point (4.5, 0) is inside the shell and inside the sphere. $E_{\text{shell}} = 0$; $E_{\text{sph}} = (4\pi/3)kr\rho$.

(b) Point (4, 1.1) is inside the shell but outside the sphere. $E = (4\pi/3)kr_1^3\rho/r^2$, r_1 = 0.6 m, r = 1.1 m

(c) Point (2, 3) is outside shell and sphere. Find the equivalent point charge at (4, 0). Find E(2, 3) and its components

$\boldsymbol{E} = 9.41\times10^4$ N/C $\boldsymbol{i}$

$\boldsymbol{E} = 3.36\times10^4$ N/C $\boldsymbol{j}$

$$Q = \frac{4\pi}{3}r_1^3\rho + 4\pi r_2^2\sigma = -26.2\,\mu\text{C}\,0$$

E = 18.1 kN/C; $\boldsymbol{E}$ = 10.0 kN/C $\boldsymbol{i}$ – 15.1 kN/C $\boldsymbol{j}$

83 ·· An infinite plane of charge with surface charge density σ_1 = 3 μC/m^2 is parallel to the xz plane at y = –0.6 m. A second infinite plane of charge with surface charge density σ_2 = –2 μC/m^2 is parallel to the yz plane at x = 1 m. A sphere of radius 1 m with its center in the xy plane at the intersection of the two charged planes (x = 1 m, y = –0.6 m) has a surface charge density σ_3 = –3 μC/m^2. Find the magnitude and direction of the electric field on the x axis at (*a*) x = 0.4 m and (*b*) x = 2.5 m.

(a) Point (0.4, 0) is within the sphere's surface. Find $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ due to the two planes at (0.4, 0) and add.	$\boldsymbol{E}_1 = 2\pi k\sigma_1\,\boldsymbol{j}$ = 169 kN/C $\boldsymbol{j}$; $\boldsymbol{E}_2$ = 113 kN/C $\boldsymbol{i}$ $\boldsymbol{E}$ = (113 $\boldsymbol{i}$ + 169 $\boldsymbol{j}$) kN/C; E = 203 kN/C, θ = 56.3°
(b) Point (2.5, 0) is outside the sphere; find Q on the sphere; find $\boldsymbol{E}_{\text{sph}}$ at (2.5, 0)	$Q = 4\pi\sigma$ = –37.7 μC $\boldsymbol{E}_{\text{sph}} = (kQ/r^2)[(1.5\text{ m})/r\,\boldsymbol{i} + (0.6\text{ m})/r\,\boldsymbol{j}]$; r = 1.62 m $\boldsymbol{E}_{\text{sph}}$ = –120 kN/C $\boldsymbol{i}$ – 47.8 kN/C $\boldsymbol{j}$ $\boldsymbol{E}$ = –233 kN/C $\boldsymbol{i}$ + 121 kN/C $\boldsymbol{j}$
$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 + \boldsymbol{E}_{\text{sph}}$ (note: here $\boldsymbol{E}_2$ = –113 kN/C $\boldsymbol{i}$)	E = 262 kN/C, θ = 153°

84 ·· An infinite plane lies parallel to the yz plane at x = 2 m and carries a uniform surface charge density σ = 2 μC/m^2. An infinite line charge of uniform linear charge density λ = 4 μC/m passes through the origin at an angle of 45° with the x axis in the xy plane. A sphere of volume charge density ρ = –6 μC/m^3 and radius 0.8 m is centered on the x axis at x = 1 m. Calculate the magnitude and direction of the electric field in the xy plane at

$x = 1.5$ m, $y = 0.5$ m.

1. Find $\boldsymbol{E}_{\rm plane}$ at P, where P is at (1.5, 0.5) — $\boldsymbol{E}_{\rm plane} = -(2\times10^{-6}/2\times8.85\times10^{-12})$ N/C $\boldsymbol{i}$ = 113 kN/C $\boldsymbol{i}$
2. Find $\boldsymbol{E}_{\rm line}$ at P; $E_{\rm line} = 2k\lambda/r$; $r = 0.707$ m — $\boldsymbol{E}_{\rm line}$ = 102 kN/C (cos 45° $\boldsymbol{i}$ – sin 45° $\boldsymbol{j}$)
3. r' from center of sphere to $P = 0.707$ m — $E_{\rm sphere} = (4\pi/3)kr'\rho\rho$ = 160 kN/C, directed to center of sphere; $\boldsymbol{E}_{\rm sphere}$ = –113 kN/C $\boldsymbol{i}$ – 113 kN/C $\boldsymbol{j}$
4. $\boldsymbol{E} = \boldsymbol{E}_{\rm plane} + \boldsymbol{E}_{\rm line} + \boldsymbol{E}_{\rm sphere}$ — $\boldsymbol{E}$ = –154 kN/C $\boldsymbol{i}$ – 185 kN/C $\boldsymbol{j}$; E = 241 kN/C, $\theta = 230^{\circ}$

85* ·· An infinite line charge λ is located along the z axis. A mass m that carries a charge q whose sign is opposite to that of λ is in a circular orbit in the xy plane about the line charge. Obtain an expression for the period of the orbit in terms of m, q, R, and λ, where R is the radius of the orbit.

For a circular orbit $mR\omega^2 = 2k\lambda q/R$; so $T = 1/f = 2\pi/\omega = 2\pi R\sqrt{\dfrac{m}{2kq\lambda}}$

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86 ·· A ring of radius R that lies in the yz plane carries a positive charge Q uniformly distributed over its length. A particle of mass m that carries a negative charge of magnitude q is at the center of the ring. (a) Show that if $x \ll R$, the electric field along the axis of the ring is proportional to x. (b) Find the force on the mass m as a function of x. (c) Show that if m is given a small displacement in the x direction, it will perform simple harmonic motion. Calculate the period of that motion.

(a) For $x \ll R$, $E_x \cong kQx/R^3$ (see Equ. 23-10). (b) $F = qE_x = kqQx/R^3$. (c) Assume that q and Q are of opposite sign. Then $F_x = -(kQq/R^3)x = m(d^2x/dt^2)$. This is the SHO equation of motion; the period is given by

$$T = 2\pi\sqrt{\frac{m R^3}{k\, q\, Q}}$$

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87 ·· When the charges Q and q of Problem 86 are 5 μC and –5 μC, respectively, and the radius of the ring is 8.0 cm, the mass m oscillates about its equilibrium position with an angular frequency of 21 rad/s. Find the angular frequency of oscillation of the mass if the radius of the ring is doubled to 16 cm and all other parameters remain unchanged.

Note that $T \propto R^{3/2}$ (see Problem 86) and $\omega \propto 1/T$. Therefore doubling R must change ω by the factor $2^{-3/2}$. The new angular frequency is $\omega = 21/2^{3/2}$ rad/s = 7.42 rad/s.

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88 ·· Given the initial conditions of Problem 87, find the angular frequency of oscillation of the mass if the radius of the ring is doubled to 16 cm while keeping the linear charge density on the ring constant.

If the linear charge density remains constant, doubling R doubles Q. Consequently, since ω is now proportional to $1/R$ (see Problem 86), the new angular frequency is $\omega = 21/2$ rad/s = 10.5 rad/s.

89* ·· A nonconducting cylinder of radius 1.2 m and length 2.0 m carries a charge of 50 μC uniformly distributed throughout the cylinder. Find the electric field *on the cylinder axis* at a distance of (a) 0.5 m, (b) 2.0 m, and (c) 20 m from the center of the cylinder.

We shall first solve in general terms and then insert appropriate numerical values. Let the origin of coordinates be at the center of the cylinder. Now consider a disk of radius R, the radius of the cylinder, and thickness dx. The charge carried by that disk is $dq = (Q/L)dx$, where Q is the total charge of the cylinder and L its length.

The disk has an effective surface charge density $\sigma = Q/\pi R^2 L$. We can now use Equ. 23-11 to find the field due to this disk along its axis.

If the point of interest, P, is within the cylinder, the charge to the left of P will result in a field to the right; the charge to the right of P will give a field to the left. Thus,

$$E = 2\pi k\sigma\left[\int_0^{L/2+x}\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)dx - \int_0^{L/2-x}\left(1-\frac{x}{\sqrt{x^2+R^2}}dx\right)\right]$$. Performing the indicated integrations, one obtains

$$E = 2\pi k\sigma[2x - \sqrt{(L/2+x)^2+R^2} + \sqrt{(L/2-x)^2+R^2}\,].$$

If P is beyond the end of the cylinder, the field at that point is given by

$$E = 2\pi k\sigma\int_{x-L/2}^{x+L/2}\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)dx = 2\pi k\sigma[L - \sqrt{(L/2+x)^2+R^2} + \sqrt{(L/2-x)^2+R^2}\,]$$ as before.

We can now substitute numerical values.

(*a*) For $x = 0.5$ m, $E = 119$ kN/C. (*b*) For $x = 2$ m, $E = 103$ kN/C. (*c*) For $x = 20$ m, $E = 1.12$ kN/C; note that since the distance of 20 m is much greater than the length of the rod, we could have used $E_x \cong kQ/x^2 = 1.12$ kN/C.

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90 ·· A uniform line charge of density λ lies on the x axis between $x = 0$ and $x = L$. Its total charge is $Q = 8$ nC. The electric field at $x = 2L$ is 600 N/C $\boldsymbol{i}$. Find the electric field at $x = 3L$.

Note that for a uniform line charge $E_x = kQ/[x_0(x_0 - L)]$. If $x_0 = 2L$, $E_x(2L) = kQ/2L^2$; if $x_0 = 3L$, $E_x(3L) = kQ/6L^2$. Thus, $E_x(3L) = E_x(2L)/3 = 200$ N/C.

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91 ·· Find the linear charge density λ (in C/m) of the line charge of Problem 90.

1. $E_x(2L) = kQ/2L^2$ (see Problem 90); solve for L	$L = \sqrt{kQ/2E_x}$
2. $\lambda = Q/L = \sqrt{2E_xQ/k}$; evaluate λ	$\lambda = 32.7$ nC/m

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92 ··· A uniformly charged sphere of radius R is centered at the origin with a charge of Q. Find the force on a uniformly charged line oriented radially having a total charge q with its ends at $r = R$ and $r = R + d$.

1. Find the field of sphere at distance $r \geq R$	$E = kQ/r^2$
2. Find force on line element dr	$dF = (kQ/r^2)\lambda\, dr = (kQq/r^2 d)\, dr$
3. Integrate dF from $r = R$ to $r = R + d$	$F = kQq/[R(R + d)]$

93* ··· Two equal uniform line charges of length L lie on the x axis a distance d apart as shown in Figure 23-43. (*a*) What is the force that one line charge exerts on the other line charge? (*b*) Show that when $d >> L$, the force tends toward the expected result of $k(\lambda L)^2/d^2$.

(*a*) Take $x = 0$ to be at the left hand end of the left rod. Then the field at $x > L$ is $kQ/[x(x - L)]$. Now consider the right hand line charge. An element of charge in dx is $\lambda\, dx$ and experiences a force $E_x \lambda\, dx$. The total force due to the left hand line charge on the right hand line charge is therefore given by

$$F = k\lambda L\int_{L+d}^{2L+d}\frac{\lambda}{x(x-L)}dx = k\lambda^2 \ln\left[\frac{(d+L)^2}{d(2L+d)}\right]$$

(*b*) For $d >> L$, the expression in the square brackets reduces to $1 + L^2/d^2$ to lowest order in L/d. We can now use the expansion $\ln(1 + \varepsilon) = \varepsilon - \varepsilon^2/2 + \cdots$, and again keeping only the first term, obtain $F = k\lambda^2L^2/d^2 = kQ^2/d^2$.

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94 ··· A dipole $\boldsymbol{p}$ is located at a distance r from an infinitely long line charge with a uniform linear charge density

λ. Assume that the dipole is aligned with the field due to the line charge. Determine the force that acts on the dipole.

The force acting on a dipole is given by $F = p(dE/dr)$. Here $E = 2k\lambda/r$, so $F = -2k\lambda p/r^2$, the minus sign indicating that the dipole is attracted to the line charge.

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95 ••• Suppose that the charge on the rod in Problem 81 is given by $\lambda(y) = ay^2$, where y is the distance from the midpoint of the rod, and that the total charge on the rod is Q. (a) Determine the constant a. (b) Find the force $d\mathbf{F}$ that acts on an element of charge $\lambda(y)\,dy$. (c) Integrate the force obtained in part (b) between $-L/2$ and $L/2$ to obtain the total force that acts on the rod.

(a) The element of charge is $dq = ay^2\,dy$. So $Q = \int_{-L/2}^{L/2} ay^2\,dy = aL^3/12$ and $a = 12Q/L^3$.

(b) The field at y due to the infinite line charge is given by $E(y) = 2k\lambda/(b + y)$, where $b = L/2 + d$. The force dF is then $E(y)ay^2\,dy = [2ka\lambda y^2/(y + b)]\,dy$.

(c) $F = 2ka\lambda \int_{-c}^{c} \frac{y^2}{y+b}\,dy$, where $c = L/2$. The integral between the stated limits is

$$I = b^2 \ln\left(\frac{b+c}{b-c}\right) - 2bc = \left(\frac{L}{2}+d\right)^2 \ln\left(\frac{L+d}{2}\right) - L\left(\frac{L}{2}+d\right)0 = \left(\frac{L}{2} + d\right)\left[\left(\frac{L}{2} + d\right)\ln\left(\frac{L+d}{d}\right) - L\right]$$

and $F = 2ka\lambda\left(\frac{L}{2} + d\right)\left[\left(\frac{L}{2} + d\right)\ln\left(\frac{L+d}{d}\right) - L\right]0$, where $a = 12Q/L^3$.

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96 ••• Repeat Problem 95 with the charge on the rod being $\lambda(y) = by$, where y is measured from the midpoint of the rod with the positive y direction up.

Following the same procedure as in Problem 95, we find:

(a) $Q = 0$ and the constant b cannot be related to the total charge as in Problem 95.

(b) The field is given by the same expression as in Problem 95; i.e., $E(y) = 2k\lambda/(p + y)$, where $p = L/2 + d$.

(c) $dF = [2kb\lambda y/(y + p)]\,dy$.

(d) Let $c = L/2$. Then $F = 2kb\lambda \int_{-c}^{c} \frac{y}{y+p}\,dy = 2kb\lambda \ln\left(\frac{p+c}{p-c}\right) = 2kb\lambda \ln\left(\frac{L+d}{d}\right)$.
