CHAPTER 35

Interference and Diffraction

1* · When destructive interference occurs, what happens to the energy in the light waves?
The energy is distributed nonuniformly in space; in some regions the energy is below average (destructive interference), in others it is higher than average (constructive interference).

2 · Which of the following pairs of light sources are coherent: (*a*) two candles; (*b*) one point source and its image in a plane mirror; (*c*) two pinholes uniformly illuminated by the same point source; (*d*) two headlights of a car; (*e*) two images of a point source due to reflection from the front and back surfaces of a soap film.
(*b*), (*c*), and (*e*)

3 · (*a*) What minimum path difference is needed to introduce a phase shift of 180° in light of wavelength 600 nm? (*b*) What phase shift will that path difference introduce in light of wavelength 800 nm?

(*a*), (*b*) Use Equ. 35-1	(*a*) 300 nm (*b*) $\delta = 135^{\circ}$

4 · Light of wavelength 500 nm is incident normally on a film of water 10^{-4} cm thick. The index of refraction of water is 1.33. (*a*) What is the wavelength of the light in the water? (*b*) How many wavelengths are contained in the distance 2*t*, where *t* is the thickness of the film? (*c*) What is the phase difference between the wave reflected from the top of the air–water interface and the one reflected from the bottom of the water–air interface after it has traveled this distance?

(*a*) $\lambda_n = \lambda/n$	$\lambda_n = 376$ nm
(*b*) $N = 2t/\lambda_n$	$N = 5.32$
(*c*) $\delta = \pi + 2N\pi$	$\delta = 6.32\pi$ rad $= 0.32\pi$ rad

5* ·· Two coherent microwave sources that produce waves of wavelength 1.5 cm are in the *xy* plane, one on the *y* axis at $y = 15$ cm and the other at $x = 3$ cm, $y = 14$ cm. If the sources are in phase, find the difference in phase between the two waves from these sources at the origin.

1. Find $\Delta r = r_2 - r_1$	$r_1 = 15$ cm, $r_2 = 14.318$ cm; $\Delta r = 0.682$ cm
2. Use Equ. 35-1	$\delta = (0.682/1.5)360^{\circ} = 164^{\circ}$

6 · The spacing between Newton's rings decreases rapidly as the diameter of the rings increases. Explain qualitatively why this occurs.
The thickness of the air space between the flat glass and the lens is approximately proportional to the square of *d*, the diameter of the ring. Consequently, the separation between adjacent rings is proportional to 1/*d*.

7 ·· If the angle of a wedge-shaped air film such as that in Example 35-2 is too large, fringes are not observed. Why?

The distance between adjacent fringes is so small that the fringes are not resolved by the eye.

8 ·· Why must a film used to observe interference colors be thin?

If the film is thick, the various colors (i.e., different wavelengths) will give constructive and destructive interference at that thickness. Consequently, what one observes is the reflected intensity of white light (see Problem 33-75).

9* · A loop of wire is dipped in soapy water and held so that the soap film is vertical. (*a*) Viewed by reflection with white light, the top of the film appears black. Explain why. (*b*) Below the black region are colored bands. Is the first band red or violet? (*c*) Describe the appearance of the film when it is viewed by *transmitted* light.

(*a*) The phase change on reflection from the front surface of the film is 180°; the phase change on reflection from the back surface of the film is 0°. As the film thins toward the top, the phase change associated with the film's thickness becomes negligible and the two reflected waves interfere destructively.

(*b*) The first constructive interference will arise when $t = \lambda/4$. Therefore, the first band will be violet (shortest visible wavelength).

(*c*) When viewed in transmitted light, the top of the film is white, since no light is reflected. The colors of the bands are those complimentary to the colors seen in reflected light; i.e., the top band will be red.

10 · A wedge-shaped film of air is made by placing a small slip of paper between the edges of two flat plates of glass. Light of wavelength 700 nm is incident normally on the glass plates, and interference bands are observed by reflection. (*a*) Is the first band near the point of contact of the plates dark or bright? Why? (*b*) If there are five dark bands per centimeter, what is the angle of the wedge?

(*a*) The first band is dark because the phase difference due to reflection by the back surface of the top plate and the top surface of the bottom plate is 180°.

(*b*) $\theta = m\lambda/2x$ (see Example 35-2) $\theta = 1.75\times10^{-4}$ rad

11 ·· The diameters of fine wires can be accurately measured using interference patterns. Two optically flat pieces of glass of length *L* are arranged with the wire between them as shown in Figure 35-37. The setup is illuminated by monochromatic light, and the resulting interference fringes are detected. Suppose L = 20 cm and yellow sodium light ($\lambda \approx 590$ nm) is used for illumination. If 19 bright fringes are seen along this 20-cm distance, what are the limits on the diameter of the wire? *Hint:* The nineteenth fringe might not be right at the end, but you do not see a twentieth fringe at all.

1. Find t for the 19th and 20th bright fringe $t_{19} = (19 - 1/2)\lambda/2 = 5457$ nm; $t_{20} = 5753$ nm
2. Give the limits on d $5.46\ \mu\text{m} < d < 5.75\ \mu\text{m}$

12 ·· Light of wavelength 600 nm is used to illuminate normally two glass plates 22 cm in length that touch at one end and are separated at the other end by a wire of radius 0.025 mm. How many bright fringes appear along the total length of the plates?

For a bright fringe, $2t = (m + 1/2)\lambda$; solve for m $t = 0.05$ mm; $m = 166$

13* ·· A thin film having an index of refraction of 1.5 is surrounded by air. It is illuminated normally by white light and is viewed by reflection. Analysis of the resulting reflected light shows that the wavelengths 360, 450, and 602 nm are the only missing wavelengths in or near the visible portion of the spectrum. That is, for these wavelengths, there is destructive interference. (*a*) What is the thickness of the film? (*b*) What visible wavelengths are brightest in the reflected interference pattern? (*c*) If this film were resting on glass with an index of refraction of 1.6, what wavelengths in the visible spectrum would be missing from the reflected light?

(*a*)	1. Destructive interference condition: $\lambda_m = 2nt/m$	$450/360 = (m + 1)/m$; $m = 4$ for $\lambda = 450$ nm
	2. $t = m\lambda_m/2n$	$t = (900/1.5)$ nm = 600 nm
(*b*)	Constructive interference: $2nt/\lambda_m = m + 1/2$	For $m = 2$, 3, and 4, $\lambda_m = 720$ nm, 514 nm, and 400 nm, respectively. These are the only λ's in the visible range.
(*c*)	Now destructive interference for $2nt/\lambda_m = m + 1/2$	Missing wavelengths are 720 nm, 514 nm, and 400 nm.

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14 ·· A drop of oil ($n = 1.22$) floats on water ($n = 1.33$). When reflected light is observed from above as shown in Figure 35-38, what is the thickness of the drop at the point where the second red fringe, counting from the edge of the drop, is observed? Assume red light has a wavelength of 650 nm.

Note that in this case there is no phase change on reflection at either interface.

Interference maxima at $t = m\lambda_n/2$	$\lambda_n = 533$ nm; $t = 533$ nm

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15 ·· A film of oil of index of refraction $n = 1.45$ rests on an optically flat piece of glass of index of refraction $n = 1.6$. When illuminated with white light at normal incidence, light of wavelengths 690 nm and 460 nm is predominant in the reflected light. Determine the thickness of the oil film.

1. $t = m_1\lambda_{n1}/2 = m_2\lambda_{n2}/2$ (see Problem 14)	$m_1/m_2 = 690/460 = 1.5$; $m_1 = 3$, $m_2 = 2$
2. Solve for t	$t = (3\times460/2\times1.45)$ nm = 476 nm

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16 ·· A film of oil of index of refraction $n = 1.45$ floats on water ($n = 1.33$). When illuminated with white light at normal incidence, light of wavelengths 700 and 500 nm is predominant in the reflected light. Determine the thickness of the oil film.

Note that here there is a phase of π rad between the two reflected rays at the interfaces.

1. $2t = (m_1 + 1/2)\lambda_{n1} = (m_2 + 1/2)\lambda_{n2}$	$(m_1 + 1/2)/(m_2 + 1/2) = 7/5$; $m_1 = 3$, $m_2 = 2$
2. Solve for t	$t = 603$ nm

17* ·· A Newton's-ring apparatus consists of a glass lens with radius of curvature R that rests on a flat glass plate as shown in Figure 35-39. The thin film is air of variable thickness. The pattern is viewed by reflected light. (*a*) Show that for a thickness t the condition for a bright (constructive) interference ring is

$$t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2},\ m = 0,\ 1,\ 2,\ \ldots$$

(*b*) Apply the Pythagorean theorem to the triangle of sides r, $R - t$, and hypotenuse R to show that for $t << R$, the radius of a fringe is related to t by

$$r = \sqrt{2tR}$$

(*c*) How would the transmitted pattern look in comparison with the reflected one? (*d*) Use $R = 10$ m and a diameter of 4 cm for the lens. How many bright fringes would you see if the apparatus were illuminated by yellow sodium light ($\lambda \approx 590$ nm) and were viewed by reflection? (*e*) What would be the diameter of the sixth bright fringe? (*f*) If the glass used in the apparatus has an index of refraction $n = 1.5$ and water ($n_W = 1.33$) is placed between the two pieces of glass, what change will take place in the bright fringes?

(*a*) This arrangement is essentially identical to the "thin film" configuration, except that the "film" is air. Now the phase change of 180° occurs at the lower reflection. So the condition for constructive interference is $2t/\lambda = m + 1/2$ or $t = (m + 1/2)\lambda/2$. Note that the first bright fringe corresponds to $m = 0$.

(*b*) From Figure 35-39 we have $r^2 + (R - t)^2 = R^2 = r^2 + R^2 - 2Rt + t^2$. For $t << R$ we neglect the last term; solving for r one finds $r = \sqrt{2Rt}$.

(*c*) As discussed in Problem 9, the transmitted pattern is complimentary to the reflected pattern.

(*d*) From (*a*) and (*b*), $r^2 = (m + 1/2)\lambda R$; solve for m $m = 67$; there will be 68 bright fringes

(*e*) $D = 2\sqrt{(m + 1/2)\lambda R}$; solve for $m = 5$ $D = 1.14$ cm

(*f*) Now λ in the film is $\lambda_{air}/n = 444$ nm. So the separation between fringes is reduced and the number of fringes that will be seen is increased by the factor $n = 1.33$.

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18 ·· A plano-convex glass lens of radius of curvature 2.0 m rests on an optically flat glass plate. The arrangement is illuminated from above with monochromatic light of 520-nm wavelength. The indexes of refraction of the lens and plate are 1.6. Determine the radii of the first and second bright fringe in the reflected light. (Use Equation 35-29 from Problem 17 to relate r to t.)

Use Equ. 35-29 and $t = (m + 1/2)\lambda/2$ $m = 0$, $r = 0.721$ mm; $m = 1$, $r = 1.25$ mm

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19 ·· Suppose that before the lens of Problem 18 is placed on the plate a film of oil of refractive index 1.82 is deposited on the plate. What will then be the radii of the first and second bright fringes? (Use Equation 35-29 from Problem 17 to relate r to t.)

Repeat Problem 35-18 with $\lambda_n = \lambda/n$ $m = 0$, $r = 0.534$ mm; $m = 1$, $r = 0.926$ mm

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20 · A double-slit interference experiment is set up in a chamber that can be evacuated. Using monochromatic light, an interference pattern is observed when the chamber is open to air. As the chamber is evacuated one will note that

(*a*) the interference fringes remain fixed.

(*b*) the interference fringes move closer together.

(*c*) the interference fringes move farther apart.

(*d*) the interference fringes disappear completely.

(*b*)

21* · Two narrow slits separated by 1 mm are illuminated by light of wavelength 600 nm, and the interference pattern is viewed on a screen 2 m away. Calculate the number of bright fringes per centimeter on the screen.

1. From Equ. 35-5, $\Delta y = \lambda L/d$; $N = 1/\Delta y$ $\Delta y = 1.2$ mm $= 0.12$ cm; $N = 8.33$/cm

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22 · Using a conventional two-slit apparatus with light of wavelength 589 nm, 28 bright fringes per centimeter are observed on a screen 3 m away. What is the slit separation?

Use Equ. 35-5 $d = m\lambda L/y_m$; $d = 4.95$ mm

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23 · Light of wavelength 633 nm from a helium–neon laser is shone normally on a plane containing two slits. The first interference maximum is 82 cm from the central maximum on a screen 12 m away. (*a*) Find the separation of the slits. (*b*) How many interference maxima can be observed?

(*a*) Use Equ. 35-2 — $\sin\theta_1 = 0.06817$; $d = 9.29\ \mu$m

(*b*) $\sin\theta_m \le 1$; $m_{max} = d/\lambda$ and must be integer — $d/\lambda = 14.7$; $m_{max} = 14$; $N = 2m_{max} + 1 = 29$

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24 ·· Two narrow slits are separated by a distance *d*. Their interference pattern is to be observed on a screen a large distance *L* away. (*a*) Calculate the spacing *y* of the maxima on the screen for light of wavelength 500 nm when $L = 1$ m and $d = 1$ cm. (*b*) Would you expect to observe the interference of light on the screen for this situation? (*c*) How close together should the slits be placed for the maxima to be separated by 1 mm for this wavelength and screen distance?

(*a*), (*b*) Use Equ. 35-5 — $\Delta y = \lambda L/d = 0.05$ mm; the separation is too small to be observed with the naked eye

(*c*) Use Equ. 35-5 — $d = 0.5$ mm

25* ·· Light is incident at an angle ϕ with the normal to a vertical plane containing two slits of separation *d* (Figure 35-40). Show that the interference maxima are located at angles θ given by $\sin\theta + \sin\phi = m\lambda/d$.

Note that the total path difference is $d\sin\phi + d\sin\theta$. For constructive interference, $d\sin\phi + d\sin\theta = m\lambda$. Thus, $\sin\phi + \sin\theta = m\lambda/d$ is the condition for interference maxima.

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26 ·· White light falls at an angle of 30° to the normal of a plane containing a pair of slits separated by 2.5 μm. What visible wavelengths give a bright interference maximum in the transmitted light in the direction normal to the plane? (See Problem 25.)

Use the result of Problem 25; here $\theta = 0$ — $m\lambda = 1250$ nm; $\lambda = 625$ nm, $\lambda = 417$ nm are in the visible range.

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27 ·· Laser light falls normally on three evenly spaced, very narrow slits. When one of the side slits is covered, the first-order maximum is at 0.60° from the normal. If the center slit is covered and the other two are open, find (*a*) the angle of the first-order maximum and (*b*) the order number of the maximum that now occurs at the same angle as the fourth-order maximum did before.

(*a*) Covering the center, $d' = 2d$, so $\theta_1' = \theta_1/2$ — $\theta_1' = 0.30°$

(*b*) $m'\theta_1' = m\theta_1$ — $m' = 8$

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28 · As the width of a slit producing a single-slit diffraction pattern is slowly and steadily reduced, how will the diffraction pattern change?

The diffraction pattern becomes wider.

29* · Equation 35-2, $d\sin\theta = m\lambda$, and Equation 35-11, $a\sin\theta = m\lambda$, are sometimes confused. For each equation, define the symbols and explain the equation's application.

Equ. 35-2 expresses the condition for an intensity maximum in two-slit interference. Here *d* is the slit separation, λ the wavelength of the light, *m* an integer, and θ the angle at which the interference maximum appears.

Equ. 35-11 expresses the condition for the first minimum in single-slit diffraction. Here *a* is the width of the slit, λ the wavelength of the light, and θ the angle at which the first minimum appears.

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30 · Light of wavelength 600 nm is incident on a long, narrow slit. Find the angle of the first diffraction minimum if the width of the slit is (*a*) 1 mm, (*b*) 0.1 mm, and (*c*) 0.01 mm.

(a), (b), (c) Use Equ. 35-9; $\theta = \sin^{-1}(\lambda/a)$ — (a) $\theta = 0.6$ mrad (b) $\theta = 6$ mrad (c) $\theta = 60$ mrad

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31 • The single-slit diffraction pattern of light is observed on a screen a large distance L from the slit. Note from Equation 35-12 that the width $2y$ of the central maximum varies inversely with the width a of the slit. Calculate the width $2y$ for $L = 2$ m, $\lambda = 500$ nm, and (a) $a = 0.1$ mm, (b) $a = 0.01$ mm, and (c) $a = 0.001$ mm.

(a), (b), (c) $2y = 2L \tan\theta = 2L \tan[\sin^{-1}(\lambda/a)]$ — (a) $\lambda/a = 5\times10^{-3}$; $2y = 2$ cm (b) $\lambda/a = 0.05$; $2y = 20$ cm

Note: For $\lambda/a << 1$, $2y = 2L\lambda/a$ — (c) $\lambda/a = 0.5$; $\theta = 30°$, $2y = 2.31$ m

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32 • Plane microwaves are incident on a long, narrow metal slit of width 5 cm. The first diffraction minimum is observed at $\theta = 37°$. What is the wavelength of the microwaves?

Use Equ. 35-9 — $\lambda = 3.01$ cm

33* •• For a ruby laser of wavelength 694 nm, the end of the ruby crystal is the aperture that determines the diameter of the light beam emitted. If the diameter is 2 cm and the laser is aimed at the moon, 380,000 km away, find the approximate diameter of the light beam when it reaches the moon, assuming the spread is due solely to diffraction.

Use Equ. 35-25; the diameter at the moon = θR_{EM} — $\theta = 4.23\times10^{-5}$ rad; $d = 2\times3.8\times10^{8}\times4.23\times10^{-5}$ m $= 32.2$ km

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34 • How many interference maxima will be contained in the central diffraction maximum in the diffraction–interference pattern of two slits if the separation d of the slits is 5 times their width a? How many will there be if $d = Na$ for any value of N?

See Example 35-5 — $N = 9$ if $d = 5a$; $N = 2n - 1$ if $d = na$

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35 •• A two-slit Fraunhofer interference–diffraction pattern is observed with light of wavelength 500 nm. The slits have a separation of 0.1 mm and a width of a. (a) Find the width a if the fifth interference maximum is at the same angle as the first diffraction minimum. (b) For this case, how many bright interference fringes will be seen in the central diffraction maximum?

(a) $\lambda/a << 1$; given $\lambda/a = 5\lambda/d$, find a — $a = d/5 = 20\ \mu$m

(b) Use Equ. 35-13 — $N = 9$

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36 •• A two-slit Fraunhofer interference–diffraction pattern is observed with light of wavelength 700 nm. The slits have widths of 0.01 mm and are separated by 0.2 mm. How many bright fringes will be seen in the central diffraction maximum?

Use Equ. 35-13 with $m = d/a$ — $N = 39$

37* •• Suppose that the *central* diffraction maximum for two slits contains 17 interference fringes for some wavelength of light. How many interference fringes would you expect in the first *secondary* diffraction maximum?

There are 8 interference fringes on each side of the central maximum. The secondary diffraction diffraction maximum is half as wide as the central one. It follows that it will contain 8 interference maxima.

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38 •• Light of wavelength 550 nm illuminates two slits of width 0.03 mm and separation 0.15 mm. (a) How many interference maxima fall within the full width of the central diffraction maximum? (b) What is the ratio of the intensity of the third interference maximum to the side of the centerline (not counting the center interference maximum) to the intensity of the center interference maximum?

(*a*) Use Equ. 35-13 — $m = 5$; $N = 9$

(*b*) Find ϕ for $m = 3$; note that for $m = 5$ $\phi = 2\pi$ rad — $\phi = 6\pi/5$ rad = 216°

Use Equ. 35-20 to find I/I_0 — $I/I_0 = 0.255$

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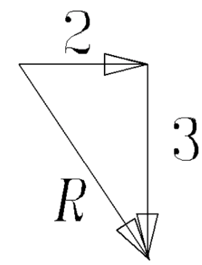
39 · Find the resultant of the two waves $E_1 = 2 \sin \omega t$ and $E_2 = 3 \sin (\omega t + 270°)$.

The phasor diagram is shown at the right. The magnitude of the resultant, R, is $13^{1/2} = 3.61$ and the phase angle between R and E_1 is $\delta = \tan^{-1}(-3/2) = -56.3°$. $E = 3.61 \sin (\omega t - 56.3°)$



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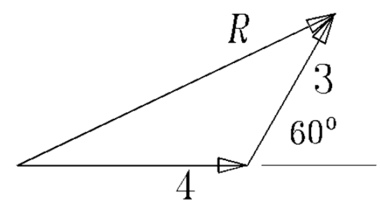
40 · Find the resultant of the two waves $E_1 = 4 \sin \omega t$ and $E_2 = 3 \sin (\omega t + 60°)$.

The phasor diagram is shown at the right. Using standard methods of vector addition one obtains $\mathbf{R} = 5.50\,\mathbf{i} + 2.60\,\mathbf{j}$. Thus $R = 6.08$ and the phase angle between $\mathbf{R}$ and $\mathbf{E}_1$ is 25.3°. $E = 6.08 \sin (\omega t + 25.3°)$



41* ·· At the second secondary maximum of the diffraction pattern of a single slit, the phase difference between the waves from the top and bottom of the slit is approximately 5π. The phasors used to calculate the amplitude at this point complete 2.5 circles. If I_0 is the intensity at the central maximum, find the intensity I at this second secondary maximum.

1. Let A_0 be the amplitude at the central maximum — $I_0 = CA_0^2$; C is a constant

2. Find A such that $(5\pi/2)A = A_0$; $I = CA^2$ — $A = 2A_0/5\pi$; $I = (4/25\pi^2)I_0 = 0.0162I_0$

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42 ·· (*a*) Show that the positions of the interference minima on a screen a large distance L away from three equally spaced sources (spacing d, with $d >> \lambda$) are given approximately by

$$y = \frac{n\lambda L}{3d}, \quad \text{where } n = 1, 2, 4, 5, 7, 8, 10, \ldots$$

that is, n is not a multiple of 3. (*b*) For $L = 1$ m, $\lambda = 5\times10^{-7}$ m, and $d = 0.1$ mm, calculate the width of the principal interference maxima (the distance between successive minima) for three sources.

(*a*) We can use the phasor concept here. The first minimum will appear when $\delta = 360°/3 = 120°$, corresponding to a path difference between adjacent slits of $\lambda/3$; for the second minimum $\delta = 240°$, corresponding to a path difference between adjacent slits of $2\lambda/3$. When the path difference is $3\lambda/3 = \lambda$ we have an interference maximum, not a minimum. If the small angle approximation is valid then

$y_{\min} = n\lambda L/3d$, where $n = 1, 2, 4, 5, 7, 8, \ldots$

(*b*) Use the above result to find $2y_{\min}$ for $n = 1$ — $2y_{\min} = 3.33$ mm

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43 ·· (*a*) Show that the positions of the interference minima on a screen a large distance L away from four equally spaced sources (spacing d, with $d >> \lambda$) are given approximately by

$$y = \frac{n\lambda L}{4d}, \quad \text{where } n = 1, 2, 3, 5, 6, 7, 9, 10, \ldots$$

that is, n is not a multiple of 4. (*b*) For $L = 2$ m, $\lambda = 6\times10^{-7}$ m, and $d = 0.1$ mm, calculate the width of the principal interference maxima (the distance between successive minima) for four sources. Compare this width with that for two sources with the same spacing.

(*a*) Proceed as in the preceding problem. In this case the first minimum corresponds to a phase difference of 90° between the phasors from adjacent slits. Minima will then appear (in the small angle approximation) at $y_{min} = n\lambda L/4d$, where $n = 1, 2, 3, 5, 6, 7, 9, \ldots$

(*b*) Use the above to find $2y_{min}$ for $n = 1$ — $2y_{min} = 6.0$ mm; this is half the width for 2 slits

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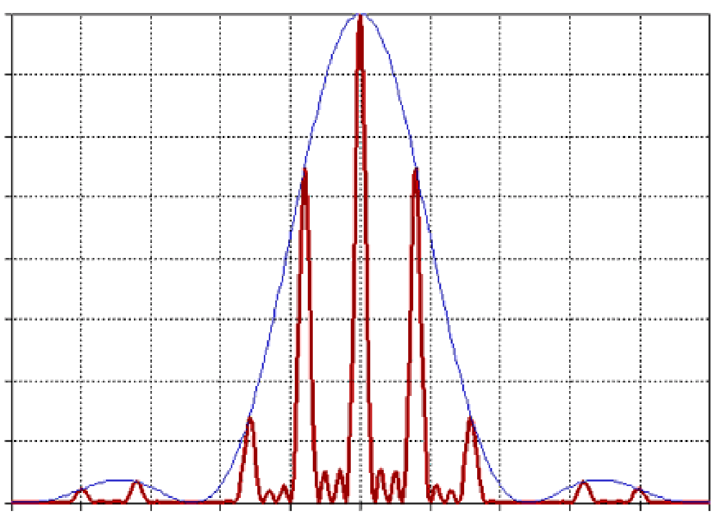
44 ·· Light of wavelength 480 nm falls normally on four slits. Each slit is 2 μm wide and is separated from the next by 6 μm. (*a*) Find the angle from the center to the first point of zero intensity of the single–slit diffraction pattern on a distant screen. (*b*) Find the angles of any bright interference maxima that lie inside the central diffraction maximum. (*c*) Find the angular spread between the central interference maximum and the first interference minimum on either side of it. (*d*) Sketch the intensity as a function of angle.

(a) Use Equ. 39-5 — $\theta = \sin^{-1}(0.24) = 0.242$ rad

(b) Find θ_m using Equ. 35-2 Note that the maximum at $m = 3$ will not be seen — $\theta_m = \sin^{-1}(0.08m)$; $\theta_0 = 0$, $\theta_1 = 0.08$ rad, $\theta_2 = 0.161$ rad, $\theta_3 = 0.242$ rad $= \theta$ (diffraction minimum)

(*c*) See Problem 35-43; $\theta_{min} = n\lambda/4d$ — For $n = 1$, $\theta_{min} = 0.02$ rad

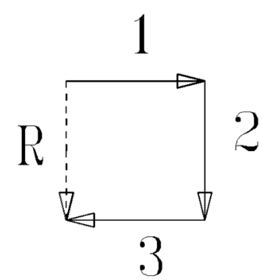
(*d*) The intensity as a function of θ is shown below.



45* ··· Three slits, each separated from its neighbor by 0.06 mm, are illuminated by a coherent light source of wavelength 550 nm. The slits are extremely narrow. A screen is located 2.5 m from the slits. The intensity on the centerline is 0.05 W/m^2. Consider a location 1.72 cm from the centerline. (*a*) Draw the phasors, according to the phasor model for the addition of harmonic waves, appropriate for this location. (*b*) From the phasor diagram, calculate the intensity of light at this location.

(*a*) 1. Determine δ for adjacent slits — From Equs. 35-4 and 35-5, $\delta = 2\pi dy/\lambda L = 3\pi/2$ rad

2. The three phasors, 270° apart, are shown in the diagram. Note that they form three sides of a square. Consequently, their sum, here shown as the resultant R, equals the



(*b*) Note that $I \propto R^2$ and $I_0 \propto 9R^2$ — $I = I_0/9 = 0.00556\ \text{W/m}^2$

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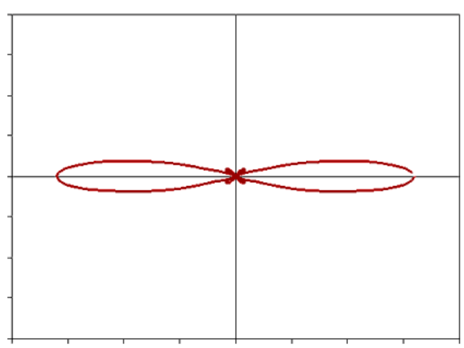
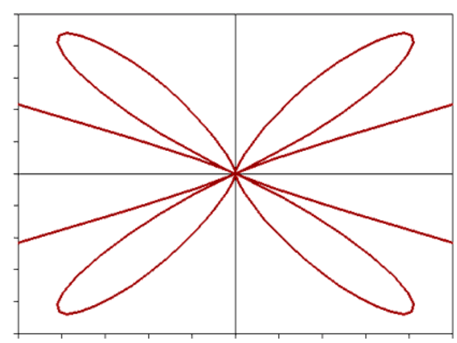
46 ··· Four coherent sources are located on the y axis at $+3\lambda/4$, $+\lambda/4$, $-\lambda/4$, and $-3\lambda/4$. They emit waves of wavelength λ and intensity I_0. (*a*) Calculate the net intensity I as a function of the angle θ measured from the $+x$ axis. (*b*) Make a polar plot of $I(\theta)$.

This is a four-slit arrangement with the distance between adjacent slits equal to $\lambda/2$; i.e., $d = \lambda/2$. For any angle θ from the x axis, the path difference between waves from adjacent slits is $d \sin \theta$, and the phase difference is then $\delta = \pi \sin \theta$. Using the phasor procedure, the four phasors add as vectors. Their x and y components are $A_x = A(1 + \cos \delta + \cos 2\delta + \cos 3\delta)$ and $A_y = A(\sin \delta + \sin 2\delta + \sin 3\delta)$, where A is the amplitude of each wave emerging from the slits. The intensity is proportional to $A^2 = A_x^2 + A_y^2$, and this is shown in the figures below. There is a principal maximum for $\theta = 0$ and π as well as four subsidiary maxima; these are shown in greater detail in the second figure.





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47 ··· For single-slit diffraction, calculate the first three values of ϕ (the total phase difference between rays from each edge of the slit) that produce subsidiary maxima by (*a*) using the phasor model and (*b*) setting $dI/d\phi = 0$, where I is given by Equation 35-20.

(*a*) From Figure 35-26 we can see that the first subsidiary maximum occurs when $\phi \approx 3\pi$; when $\phi = 4\pi$ there is a minimum, and when $\phi \approx 5\pi$ there is another maximum. Thus subsidiary maxima appear when $\phi \approx (2n + 1)\pi$. The first three subsidiary maxima are at $\phi \approx 3\pi$, 5π, and 7π.

(*b*) To find the exact values of ϕ, differentiate Equ. 35-20 with respect to ϕ and set the derivative equal to zero.

$\frac{dI}{d\phi} = 2I_0\left(\frac{\sin(\phi/2)}{\phi/2}\right)\left[\frac{(\phi/4)\cos(\phi/2) - \sin(\phi/2)/2}{(\phi/2)^2}\right]$ = 0. The equation is satisfied for tan (ϕ/2) = ϕ/2. This transcendental equation must be solved numerically; the results for the first three solutions are $\phi = 2.86\pi$, $\phi = 4.92\pi$, and $\phi = 6.94\pi$. Note that these are in fair agreement with the approximate result obtained in part (*a*), and that the agreement improves as *n* increases.

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48 • Light of wavelength 700 nm is incident on a pinhole of diameter 0.1 mm. (*a*) What is the angle between the central maximum and the first diffraction minimum for a Fraunhofer diffraction pattern? (*b*) What is the distance between the central maximum and the first diffraction minimum on a screen 8 m away?

(*a*) Use Equ. 35-25	θ = 8.54 mrad
(*b*) $y_{min} = L \tan\theta$	y_{min} = 6.83 cm

49* • Two sources of light of wavelength 700 nm are 10 m away from the pinhole of Problem 48. How far apart must the sources be for their diffraction patterns to be resolved by Rayleigh's criterion?

Use Equ. 35-26 to find α_c; then $\Delta y = L\alpha_c$	$\alpha_c = 8.54\times10^{-3}$ rad; Δy = 8.54 cm

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50 • Two sources of light of wavelength 700 nm are separated by a horizontal distance *x*. They are 5 m from a vertical slit of width 0.5 mm. What is the least value of *x* for which the diffraction pattern of the sources can be resolved by Rayleigh's criterion?

1. For slits, Rayleigh's criterion is $\alpha_c = \lambda/D$	α_c = 1.4 mrad
2. $x/L = \alpha_c$	x = 7 mm

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51 • The headlights on a small car are separated by 112 cm. At what maximum distance could you resolve them if the diameter of your pupil is 5 mm and the effective wavelength of the light is 550 nm?

1. Use Equ. 35-26	α_c = 0.134 mrad
2. $y/L = \alpha_c$	L = 8.35 km

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52 • You are told not to shoot until you see the whites of their eyes. If their eyes are separated by 6.5 cm and the diameter of your pupil is 5 mm, at what distance can you resolve the two eyes using light of wavelength 550 nm?

1. Use Equ. 35-26	α_c = 0.1342 mrad
2. $y/L = \alpha_c$	L = 484 m

53* •• (*a*) How far apart must two objects be on the moon to be resolved by the eye? Take the diameter of the pupil of the eye to be 5 mm, the wavelength of the light to be 600 nm, and the distance to the moon to be 380,000 km. (*b*) How far apart must the objects on the moon be to be resolved by a telescope that has a mirror of diameter 5 m?

(*a*) Proceed as in Problem 49	$\alpha_c = 1.46\times10^{-4}$ rad; Δy = 55.6 km
(*b*) Repeat with D = 5 m	$\alpha_c = 1.46\times10^{-7}$ rad; Δy = 55.6 m

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54 •• The ceiling of your lecture hall is probably covered with acoustic tile, which has small holes separated by about 6 mm. (*a*) Using light with a wavelength of 500 nm, how far could you be from this tile and still resolve these holes? The diameter of the pupil of your eye is about 5 mm. (*b*) Could you resolve these holes better with red light or with violet light?

(*a*) 1. Use Equ. 35-26 $\alpha_c = 0.122$ mrad

2. $y/L = \alpha_c$ $L = 49.2$ m

(*b*) The holes can be resolved better with violet light which has a shorter wavelength; note that α_c is proportional

to λ.

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55 ·· The telescope on Mount Palomar has a diameter of 200 inches. Suppose a double star were 4 lightyears away. Under ideal conditions, what must be the minimum separation of the two stars for their images to be resolved using light of wavelength 550 nm?

1. Use Equ. 35-26; $D = 5.08$ m $\alpha_c = 1.321\times10^{-7}$ rad

2. $y = L\alpha_c$; $L = 3.784\times10^{16}$ m $y = 5\times10^{9}$ m

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56 ·· The star Mizar in Ursa Major is a binary system of stars of nearly equal magnitudes. The angular separation between the two stars is 14 seconds of arc. What is the minimum diameter of the pupil that allows resolution of the two stars using light of wavelength 550 nm?

Use Equ. 35-26; $D_{min} = 1.22\lambda/\theta$ $\theta = 6.79\times10^{-5}$ rad; $D_{min} = 9.9$ mm ≈ 1 cm

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57* · When a diffraction grating is illuminated with white light, the first-order maximum of green light

(*a*) is closer to the central maximum than that of red light.

(*b*) is closer to the central maximum than that of blue light.

(*c*) overlaps the second order maximum of red light.

(*d*) overlaps the second order maximum of blue light.

(*a*)

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58 · A diffraction grating with 2000 slits per centimeter is used to measure the wavelengths emitted by hydrogen gas. At what angles θ in the first-order spectrum would you expect to find the two violet lines of wavelengths 434 and 410 nm?

Use Equ. 35-27; $d = 5$ μm, $m = 1$ $\theta_{434} = 86.8$ mrad; $\theta_{410} = 82.0$ mrad

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59 · With the grating used in Problem 58, two other lines in the first-order hydrogen spectrum are found at angles $\theta_1 = 9.72\times10^{-2}$ rad and $\theta_2 = 1.32\times10^{-1}$ rad. Find the wavelengths of these lines.

Use Equ. 35-27 $\lambda_1 = 485$ nm, $\lambda_2 = 658$ nm

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60 · Repeat Problem 58 for a diffraction grating with 15,000 slits per centimeter.

Use Equ. 35-27; $d = 0.667$ μm $\theta_{434} = \sin^{-1}(0.651) = 0.709$ rad $= 40.6°$; $\theta_{410} = 38.0°$

61* · What is the longest wavelength that can be observed in the fifth-order spectrum using a diffraction grating with 4000 slits per centimeter?

Find largest λ such that $5\lambda/d = \sin\theta = 1$ $d = (1/4000)$ cm; $\lambda = d/5 = 500$ nm

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62 ·· A diffraction grating of 2000 slits per centimeter is used to analyze the spectrum of mercury. (*a*) Find the angular separation in the first-order spectrum of the two lines of wavelength 579.0 and 577.0 nm. (*b*) How wide must the beam on the grating be for these lines to be resolved?

(*a*) Use Equ. 35-27; $d = 5\ \mu$m — $\theta_{579} = \sin^{-1}(0.1158) = 6.65°$; $\theta_{577} = 6.627°$; $\Delta\theta = 0023°$

(*b*) Use Equ. 35-28; $w = Nd$ — $N = 289$; $w = 1.45$ mm

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63 ·· A diffraction grating with 4800 lines per centimeter is illuminated at normal incidence with white light (wavelength range 400 nm to 700 nm). For how many orders can one observe the complete spectrum in the transmitted light? Do any of these orders overlap? If so, describe the overlapping regions.

1. To see complete spectrum, $\sin\theta \leq 1$ — $m_{max} = d/\lambda_{max} = 2.97$; one can see the complete spectrum only for $m = 1$ and 2

2. For overlap, $m_1\lambda_1 \geq m_2\lambda_2$ — Since $700 < 2\times400$, there is no overlap of the second order spectrum into the first order spectrum; however, there is overlap of long wavelengths in the second order with short wavelengths in the third order spectrum.

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64 ·· A square diffraction grating with an area of 25 cm^2 has a resolution of 22,000 in the fourth order. At what angle should you look to see a wavelength of 510 nm in the fourth order?

1. Use Equ. 35-28 to find N — $N = 22{,}000/4 = 5500$ lines

2. Find d; width of grating is 5 cm — $d = 9.09\ \mu$m

3. Use Equ. 35-27 to find θ — $\theta = \sin^{-1}(0.2244) = 13.0°$

65* ·· Sodium light of wavelength 589 nm falls normally on a 2-cm-square diffraction grating ruled with 4000 lines per centimeter. The Fraunhofer diffraction pattern is projected onto a screen at 1.5 m by a lens of focal length 1.5 m placed immediately in front of the grating. Find (*a*) the positions of the first two intensity maxima on one side of the central maximum, (*b*) the width of the central maximum, and (*c*) the resolution in the first order.

(*a*) From Equ. 35.3, $y_m = m\lambda L/d$; $d = 1/n$ — $d = 2.5\times10^{-6}$ m; $y_1 = 0.353$ m; $y_2 = 0.707$ m

(*b*) $\theta_{min} = \lambda/Nd$ (see p. 1130); $\Delta y = 2\theta_{min}L$ — $\theta_{min} = 2.95\times10^{-5}$ rad; $\Delta y = 88.4\ \mu$m

(*c*) Use Equ. 35-27 with $m = 1$ — $R = N = 8000$

---

66 ·· The spectrum of neon is exceptionally rich in the visible region. Among the many lines are two at wavelengths of 519.313 nm and 519.322 nm. If light from a neon discharge tube is normally incident on a transmission grating with 8400 lines per centimeter and the spectrum is observed in second order, what must be the width of the grating that is illuminated so that these two lines can be resolved?

1. Use Equ 35-28 to find N — $N = 28{,}850$

2. Find $w = Nd$ — $w = 3.43$ cm

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67 ·· Mercury has several stable isotopes, among them ^{198}Hg and ^{202}Hg. The strong spectral line of mercury at about 546.07 nm is a composite of spectral lines from the various mercury isotopes. The wavelengths of this line for ^{198}Hg and ^{202}Hg are 546.07532 and 546.07355 nm, respectively. What must be the resolving power of a grating capable of resolving these two isotopic lines in the third-order spectrum? If the grating is illuminated over a 2-cm-wide region, what must be the number of lines per centimeter of the grating?

(*a*) Use Equ. 35-28 — $R = 3.085\times10^5$

(*b*) $N = R/m$; $n = N/w = R/mw$ — $n = 51{,}400$ lines/cm

68 ·· A transmission grating is used to study the spectral region extending from 480 to 500 nm. The angular spread of this region is 12° in third order. (*a*) Find the number of lines per centimeter. (*b*) How many orders are visible?

(*a*) 1. Approximate $\Delta\theta$ by $(d\theta/d\lambda)\Delta\lambda$; find $d\theta/d\lambda$ — $d\theta/d\lambda = m/(d\cos\theta) = mn/\cos\theta$

2. Solve for n; evaluate n for $m = 3$, $\lambda = 490$ nm, and $\Delta\lambda = 20$ nm — $n = \frac{1}{m}\frac{\Delta\theta}{\Delta\lambda}\cos\theta = \frac{1}{m}\frac{\Delta\theta}{\Delta\lambda}(1 - n^2m^2\lambda^2)^{1/2}$

$n = 1/m\sqrt{\lambda^2 + (\Delta\lambda/\Delta\theta)^2} = 6.677\times10^5\ \text{m}^{-1} = 6677\ \text{cm}^{-1}$

(*b*) $m_{max} = d/\lambda_{max} = 1/n\lambda_{max}$ — $m_{max} = 3$

69* ·· White light is incident normally on a transmission grating and the spectrum is observed on a screen 8.0 m from the grating. In the second-order spectrum, the separation between light of 520- and 590-nm wavelength is 8.4 cm. (*a*) Determine the number of lines per centimeter of the grating. (*b*) What is the separation between these two wavelengths in the first-order and third-order spectra?

We will assume that the angle θ_2 is small and then verify that this is a justified assumption.

(*a*) 1. From Equ. 35-3, $y_2 - y_1 = mL(\lambda_2 - \lambda_1)/d$ — $d = mL(\lambda_2 - \lambda_1)/(y_2 - y_1) = 1.333\times10^{-5}$ m

2. $n = 1/d$ — $n = 750$ lines/cm

3. Find θ_2 for $\lambda = 590$ nm — $\theta_2 = \sin^{-1}(2\lambda/d) = 8.85\times10^{-2} << 1$; $\sin\theta \cong \theta$

(*b*) For $m = 1$, $\Delta y = \Delta y(m = 2)/2$ — $m = 1$, $\Delta y = 4.2$ cm; $m = 3$, $\Delta y = 3\times4.2$ cm $= 12.6$ cm

70 ··· A diffraction grating has n lines per meter. Show that the angular separation of two lines of wavelengths λ and $\lambda + \Delta\lambda$ meters is approximately

$$\Delta\theta = \frac{\Delta\lambda}{\sqrt{(1/\text{nm})^2 - \lambda^2}}$$

$\Delta\theta = nm\Delta\lambda/(1 - n^2m^2\lambda^2)^{1/2}$ (see Problem 35-68). This can be rewritten as $\Delta\theta = \frac{\Delta\lambda}{\sqrt{(1/nm)^2 - \lambda^2}}$.

71 ··· When assessing a diffraction grating, we are interested not only in its resolving power R, which is the ability of the grating to separate two close wavelengths, but also in the dispersion D of the grating. This is defined by

$D = \Delta\theta_m/\Delta\lambda$ in the mth order. (*a*) Show that D can be written

$$D = \frac{m}{\sqrt{d^2 - m^2\lambda^2}}$$

where d is the slit spacing. (*b*) If a diffraction grating with 2000 slits per centimeter is to resolve the two yellow sodium lines in the second order (wavelengths 589.0 and 589.6 nm), how many slits must be illuminated by the beam? (*c*) What would the separation be between these resolved yellow lines if the pattern were viewed on a screen 4 m from the grating?

(*a*) Since $d = 1/n$, the result of Problem 35-70 reduces to $(\Delta\theta_m/\Delta\lambda) = D = \frac{m}{\sqrt{d^2 - m^2\lambda^2}}$.

(*b*) Use Equ. 35-28 — $N = (589.3/0.6)/2 = 491$

(*c*) $\Delta y = L\Delta\theta_m$; use the result from part (*a*) — $\Delta y = 0.988$ mm

72 ··· For a diffraction grating in which all the surfaces are normal to the incident radiation, most of the energy goes into the zeroth order, which is useless from a spectroscopic point of view since in zeroth order all the wavelengths are at 0°. Therefore, modern gratings have shaped, or *blazed,* grooves as shown in Figure 35-41. This shifts the specular reflection, which contains most of the energy, from the zeroth order to some higher order. (*a*) Calculate the blaze angle ϕ in terms of a (the groove separation), λ (the wavelength), and m (the order in which specular reflection is to occur). (*b*) Calculate the proper blaze angle for the specular reflection to occur in the second order for light of wavelength 450 nm incident on a grating with 10,000 lines per centimeter.

(*a*) From Figure 35-41 it is apparent that $\theta_i = 2\phi$. Therefore, from Equ. 35-27,we require $a \sin 2\phi = m\lambda$. Thus $\phi = [\sin^{-1}(m\lambda/a)]/2$.

(*b*) Use the result of part (*a*); $a = 1\ \mu$m, $\lambda = 450$ nm For $m = 2$, $\phi = 32.1°$

73* ··· In this problem you will derive Equation 35-28 for the resolving power of a diffraction grating containing N slits separated by a distance d. To do this you will calculate the angular separation between the maximum and minimum for some wavelength λ and set it equal to the angular separation of the mth-order maximum for two nearby wavelengths. (*a*) Show that the phase difference ϕ between the light from two adjacent slits is given by

$$\phi = \frac{2\pi d}{\lambda} \sin \theta$$

(*b*) Differentiate this expression to show that a small change in angle $d\theta$ results in a change in phase of $d\phi$ given by

$$d\phi = \frac{2\pi d}{\lambda} \cos \theta \ d\theta$$

(*c*) For N slits, the angular separation between an interference maximum and interference minimum corresponds to a phase change of $d\phi = 2\pi/N$. Use this to show that the angular separation $d\theta$ between the maximum and minimum for some wavelength λ is given by

$$d\theta = \frac{\lambda}{Nd \cos \theta}$$

(*d*) The angle of the mth-order interference maximum for wavelength λ is given by Equation 35-27. Compute the differential of each side of this equation to show that angular separation of the mth-order maximum for two nearly equal wavelengths differing by $d\lambda$ is given by

$$d\theta \approx \frac{m\ d\lambda}{d \cos \theta}$$

(*e*) According to Rayleigh’s criterion, two wavelengths will be resolved in the mth order if the angular separation of the wavelengths given by Equation 35-31 equals the angular separation of the interference maximum and interference minimum given by Equation 35-30. Use this to derive Equation 35-28 for the resolving power of a grating.

(*a*) The path difference for two adjacent slits for an angle θ is $\Delta = d \sin \theta$. The phase difference is $\phi = 2\pi\Delta/\lambda = (2\pi d/\lambda) \sin \theta$.

(*b*) $d\phi/d\theta = (2\pi d/\lambda) \cos \theta$; so $d\phi = (2\pi d/\lambda) \cos \theta\, d\theta$.

(*c*) From (*b*), with $d\phi = 2\pi/N$ we have $d\theta = \lambda/(Nd \cos \theta)$.

(*d*) $m\lambda = d \sin \theta$; differentiate with respect to λ. $m = d \cos \theta\, (d\theta/d\lambda)$ and $d\theta = (m\ d\lambda)/(d \cos \theta)$.

(*e*) We now have two expressions for $d\theta$. Equating these gives $\lambda/d\lambda = R = mN$.

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74 · True or false:

(a) When waves interfere destructively, the energy is converted into heat energy.
(b) Interference is observed only for waves from coherent sources.
(c) In the Fraunhofer diffraction pattern for a single slit, the narrower the slit, the wider the central maximum of the diffraction pattern.
(d) A circular aperture can produce both a Fraunhofer and a Fresnel diffraction pattern.
(e) The ability to resolve two point sources depends on the wavelength of the light.
(a) False (b) True (c) True (d) True (e) True

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75 • In a lecture demonstration, laser light is used to illuminate two slits separated by 0.5 mm, and the interference pattern is observed on a screen 5 m away. The distance on the screen from the centerline to the thirty-seventh bright fringe is 25.7 cm. What is the wavelength of the light?

Use Equ. 35-5 $\lambda = y_m d/mL = 695$ nm

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76 • A long, narrow, horizontal slit lies 1 mm above a plane mirror, which is in the horizontal plane. The interference pattern produced by the slit and its image is viewed on a screen 1 m from the slit. The wavelength of the light is 600 nm. (a) Find the distance from the mirror to the first maximum. (b) How many dark bands per centimeter are seen on the screen?

Note that, due to reflection, the wave from the image is 180° out of phase with that from the source. The condition for an interference maximum is therefore $(m + 1/2)\lambda = d \sin \theta \approx d\theta$. In this case, d = 2 mm. The first maximum is for m = 0, and the distance from the mirror is $y_0 = L\theta_0 = Ld/2\lambda = 0.15$ mm for λ = 600 nm and L = 1 m.

77* • In a lecture demonstration, a laser beam of wavelength 700 nm passes through a vertical slit 0.5 mm wide and hits a screen 6 m away. Find the horizontal length of the principal diffraction maximum on the screen; that is, find the distance between the first minimum on the left and the first minimum on the right of the central maximum.

Use Equ. 35-12; width = $2y = 2\lambda L/a$ $2y = 1.68$ cm

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78 • What minimum aperture, in millimeters, is required for opera glasses (binoculars) if an observer is to be able to distinguish the soprano's individual eyelashes (separated by 0.5 mm) at an observation distance of 25 m? Assume the effective wavelength of the light to be 550 nm.

Use Equ. 35-26; $\alpha_c = 2\times10^{-5}$ rad $D = 3.36$ cm

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79 • The diameter of the aperture of the radio telescope at Arecibo, Puerto Rico, is 300 m. What is the resolving power of the telescope when tuned to detect microwaves of 3.2 cm wavelength?

Use Equ. 35-26 $\alpha_c = 0.13$ mrad

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80 •• A thin layer of a transparent material with an index of refraction of 1.30 is used as a nonreflective coating on the surface of glass with an index of refraction of 1.50. What should the thickness of the material be for it to be nonreflecting for light of wavelength 600 nm?

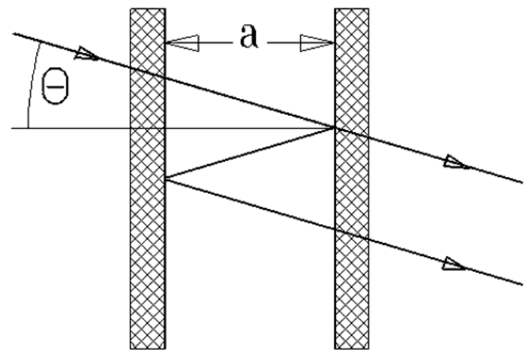
Note that reflection at both surfaces involves a phase reversal.

1. Write the condition for destructive interference $2t = \lambda_n(m + 1/2)$
2. Evaluate t for m = 0 $t = 115$ nm

81* ·· A *Fabry–Perot interferometer* consists of two parallel, half-silvered mirrors separated by a small distance a. Show that when light is incident on the interferometer with an angle of incidence θ, the transmitted light will have maximum intensity when $a = (m\lambda/2)\cos\theta$.

The *Fabry-Perot interferometer* is shown in the figure.

The path difference between the two rays that emerge from the interferometer is $\Delta r = 2a/\cos\theta$. For constructive interference we require $\Delta r = m\lambda$. It follows that the intensity will be a maximum when $a = (m\lambda/2)\cos\theta$.



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82 ·· A mica sheet 1.2 μm thick is suspended in air. In reflected light, there are gaps in the visible spectrum at 421, 474, 542, and 633 nm. Find the index of refraction of the mica sheet.

1. Write the condition for destructive interference	$2t = \lambda_n m$; $474m = 421(m + 1)$
2. Solve for m	$m = 8$ for $\lambda = 474$ nm
3. For $m = 8$, find λ_n and $n = \lambda/\lambda_n$	$\lambda_n = 300$ nm; $n = 1.58$

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83 ·· A camera lens is made of glass with an index of refraction of 1.6. This lens is coated with a magnesium fluoride film ($n = 1.38$) to enhance its light transmission. This film is to produce zero reflection for light of wavelength 540 nm. Treat the lens surface as a flat plane and the film as a uniformly thick flat film. (*a*) How thick must the film be to accomplish its objective in the first order? (*b*) Would there be destructive interference for any other visible wavelengths? (*c*) By what factor would the reflection for light of wavelengths 400 and 700 nm be reduced by this film? Neglect the variation in the reflected light amplitudes from the two surfaces.

(*a*) $t = \lambda/4n$ (see Problem 80)	$t = 97.8$ nm
(*b*) Find λ for $m = 1$	$\lambda = 180$ nm; this is not in the visible range; no
(*c*) 1. Find δ for 400 and 700 nm; here δ is the phase difference between the two reflected waves	$\delta = 2\pi(2t/\lambda_n) = 4\pi tn/\lambda$; for $\lambda = 400$ nm, $\delta = 4.24$ rad; for $\lambda = 700$ nm, $\delta = 2.42$ rad
2. Reduction factor $f = \cos^2 \delta/2$ (see Equ. 35-8)	$f_{400} = 0.273$; $f_{700} = 0.124$

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84 ·· In a pinhole camera, the image is fuzzy because of geometry (rays arrive at the film through different parts of the pinhole) and because of diffraction. As the pinhole is made smaller, the fuzziness due to geometry is reduced, but the fuzziness due to diffraction is increased. The optimum size of the pinhole for the sharpest possible image occurs when the spread due to diffraction equals that due to the geometric effects of the pinhole. Estimate the optimum size of the pinhole if the distance from it to the film is 10 cm and the wavelength of the light is 550 nm.

The angular width of a distant object at the film is 2θ, where $\theta = D/2L$. As D is reduced, so is θ. However the angular width of the diffraction pattern is $2\theta_d$, where $\theta_d \approx 1.22\lambda/D$. Set $\theta_d = \theta$ and solve for D.

$D \approx \sqrt{2.44\,L\lambda} = 0.366$ mm.

85* ·· The Impressionist painter Georges Seurat used a technique called "pointillism," in which his paintings are composed of small, closely spaced dots of pure color, each about 2 mm in diameter. The illusion of the colors blending together smoothly is produced in the eye of the viewer by diffraction effects. Calculate the minimum viewing distance for this effect to work properly. Use the wavelength of visible light that requires the *greatest* distance, so that you're sure the effect will work for *all* visible wavelengths. Assume the pupil of the eye has a diameter of 3 mm.

1. Write the angle subtended by adjacent dots in terms of the viewing distance L and dot separation d	$\theta = d/L$
2. Set $\theta = \alpha_c$ for shortest λ (λ = 400 nm); solve for L	$L = Dd/1.22\lambda = \dfrac{3\times10^{-3}\times2\times10^{-3}}{1.22\times4\times10^{-9}}$ m = 12.3 m

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86 ··· A *Jamin refractometer* is a device for measuring or comparing the indexes of refraction of fluids. A beam of monochromatic light is split into two parts, each of which is directed along the axis of a separate cylindrical tube before being recombined into a single beam that is viewed through a telescope. Suppose that each tube is 0.4 m long and that sodium light of wavelength 589 nm is used. Both tubes are initially evacuated, and constructive interference is observed in the center of the field of view. As air is slowly allowed to enter one of the tubes, the central field of view changes to dark and back to bright a total of 198 times. (*a*) What is the index of refraction of air? (*b*) If the fringes can be counted to ±0.25 fringe, where one fringe is equivalent to one complete cycle of intensity variation at the center of the field of view, to what accuracy can the index of refraction of air be determined by this experiment?

(*a*) 1. Write the optical length of each tube	$L_{vac} = (0.4\text{ m})/\lambda = N$; $L_{air} = (0.4\text{ m})n/\lambda = (N + 198) = n$
2. Solve for n	$n = 1 + 198\lambda/(0.4\text{ m}) = 1.00029156$
(*b*) Find the range of n	$1 + 198.25\lambda/(0.4\text{ m}) > n > 1 + 197.75\lambda/(0.4\text{ m})$
	$1.00029192 > n > 1.00029119$; $\Delta n/n = 7.3\times10^{-7}$

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87 ··· Light of wavelength λ is diffracted through a single slit of width a, and the resulting pattern is viewed on a screen a long distance L away from the slit. (*a*) Show that the width of the central maximum on the screen is approximately $2L\lambda/a$. (*b*) If a slit of width $2L\lambda/a$ is cut in the screen and is illuminated, show that the width of its central diffraction maximum at the same distance L is a to the same approximation.

(*a*) Width = $2y$; use Equ. 35-12	width = $2L\lambda/a$
(*b*) In Equ. 35-12 set $a = 2L\lambda/a$	width = $2L\lambda/(2L\lambda/a) = a$

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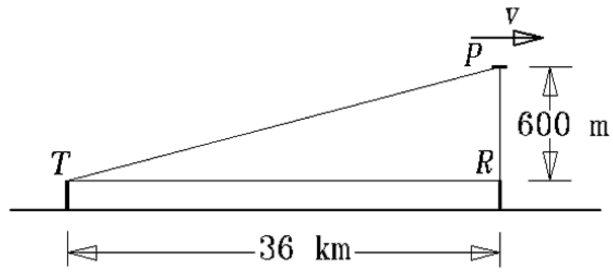
88 ··· Television viewers in rural areas often find that the picture flickers (fades in and out) as an airplane flies across the sky in the vicinity. The flickering arises from the interference between the signal directly from the transmitter and that reflected to the antenna from the airplane. Suppose the receiver is 36 km from the transmitter broadcasting at a frequency of 86.0 MHz and an airplane is flying at a height of about 600 m above the receiver toward the transmitter. The rate of oscillation of the picture's intensity is 4 Hz. (*a*) Determine the speed of the plane. (b) If the picture's intensity is a maximum when the plane is directly overhead, what is the exact height of the plane above the receiving antenna?

(a) The situation is displayed in the figure to the right. There are two paths. One path is from the transmitter, *T*, to the receiver, *R*. The other path is from the transmitter to the plane, *P*, and hence to the receiver, *R*. The horizontal distance from *T* to *P* is given by $x = 3.6\times10^4$ m + *vt*.



We proceed by finding the path difference for the two paths as a function of time.

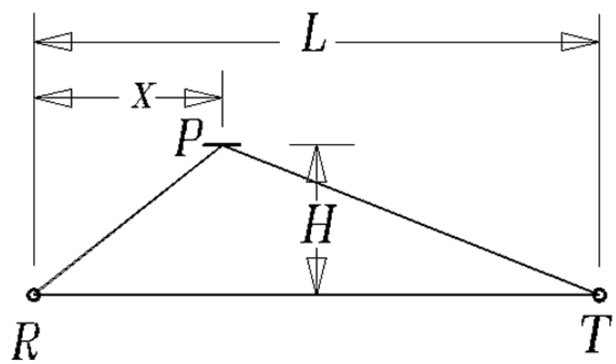
The path length of the second path is $L_2 = \sqrt{(3.6\_10^4 + vt)^2 + 600^2} + \sqrt{(vt)^2 + 600^2}$ m. The first path length is $L_1 = 3.6\times10^4$ m. The frequency of the oscillation is $f_{osc} = (1/\lambda)(d\Delta r/dt)$, where Δr is the path difference $L_2 - L_1$. Thus, $f_{osc} = (1/\lambda)(dL_2/dt)$. We evaluate (dL_2/dt) at $t = 0$ and obtain, to within 0.02% accuracy, $dL_2/dt = v$. Therefore, $v = \lambda f_{osc} = cf_{osc}/f = (3\times10^8\times4/86\times10^6)$ m/s = 14 m/s = 50.2 km/h.

(*b*) Let *h* be the height. Then for constructive interference $[(3.6\times10^4)^2 + h^2]^{1/2} + h - 3.6\times10^4 = n\lambda$. Since $h << L_1$, the condition for constructive interference is very closely given by $h = n\lambda$, where *n* is an integer. Here $\lambda = c/f$ so $h/\lambda = 600\times86\times10^6/3\times10^8 = 172$. The nearest integer is $n = 172$; the exact height is $h = 600$ m to better than 1 mm.

89* ··· For the situation described in Problem 88, show that the rate of oscillation of the picture's intensity is a minimum when the airplane is directly above the midpoint between the transmitter and receiving antenna.

There are two paths from the transmitter, *T*, to the receiver, *R*, the direct path of length *L* and the path from *T* to *P*, the plane, and from *P* to *R*. The path difference is $\Delta r = \sqrt{x^2 + H^2} + \sqrt{(L-x)^2 + H^2} - L$.

The rate of oscillation, assuming the plane travels at constant speed, will be a minimum when $d(\Delta r)/dx$ is a minimum. We set the derivative equal to zero and solve for *x*.



$$\frac{d(\Delta r)}{dx} = \frac{x}{\sqrt{x^2 + H^2}} - \frac{L-x}{\sqrt{(L-x)^2 + H^2}} = 0;$$ $x = L/2$. The rate of oscillation is a minimum when the plane is midway between the transmitter and receiver.

90 ··· A double-slit experiment uses a helium–neon laser with a wavelength of 633 nm and a slit separation of 0.12 mm. When a thin sheet of plastic is placed in front of one of the slits, the interference pattern shifts by 5.5 fringes. When the experiment is repeated under water, the shift is 3.5 fringes. Calculate (*a*) the thickness of the plastic sheet and (*b*) the index of refraction of the plastic sheet.

(*a*), (*b*) A shift of *P* fringes means that the optical length in the plastic differs from that in the other path by *P* wavelengths. Let the thickness of the plastic be $t = N\lambda$.

(*a*) 1. Write the condition for the system in air $t = (N + 5.5)\lambda/n = N\lambda$

2. Write the condition for the system in water $t = (N' + 3.5)\lambda/n = N'\lambda/1.33$; $N' = 1.33N$

3. Eliminate *t* from the two equations; solve for *N* $N + 5.5 = nN$; $1.33N + 3.5 = nN$; $N = 6.06$

4. Evaluate t	$t = 6.06\lambda = 3.84\ \mu\text{m}$
(*b*) Evaluate n	$n = 11.56/6.06 = 1.91$

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91 ••• Two coherent sources are located on the y axis at $+\lambda/4$ and $-\lambda/4$. They emit waves of wavelength λ and intensity I_0. (*a*) Calculate the net intensity I as a function of the angle θ measured from the $+x$ axis. (*b*) Make a polar plot of $I(\theta)$.

(*a*) 1. Express the phase difference δ in terms of θ	$\delta = (2\pi d/\lambda)\sin\theta = \pi\sin\theta$
2. Use Equ. 35-8	$I(\theta) = 4I_0\cos^2[(\pi/2)\sin\theta]$

(b) The plot of $I(\theta)$ versus θ is shown below.