Code: 9012

DEPARTMENT OF MATHEMATICS
University of Toronto
MAT 135Y
Term-Test #3
Wednesday, March 4, 2009
Time allowed: 1 hour, 45 minutes

Please PRINT in INK or BALL-POINT PEN:

NAME OF STUDENT:
(Please PRINT full name
and UNDERLINE surname): _____

STUDENT NO.: _____

SIGNATURE OF STUDENT
(in INK or BALL-POINT PEN): _____

TUTORIAL CODE (e.g. M4A, R5D, etc.): _____

TUTORIAL TIME (e.g. T4, R5, F3, etc.): _____

NAME OF YOUR T.A.: _____

NOTE:

1. Before you start, check that this test has 14 pages. There are NO blank pages.
2. This test has two parts:
   PART A [48 marks]: 12 multiple choice questions
   PART B [52 marks]: 7 written questions
   Answers to both PART A and PART B are to be given in this booklet. No computer cards will be used.
3. No aids allowed.
   No calculators!

DO NOT TEAR OUT ANY PAGES.

FOR MARKERS ONLY	
QUESTION	**MARK**
PART A	/ 48
B1	/ 7
B2	/ 7
B3	/ 7
B4	/ 7
B5	/ 8
B6	/ 8
B7	/ 8
TOTAL	/ 100

PART A *[48 marks]*

Please read carefully:
PART A consists of 12 multiple-choice questions, each of which has exactly one correct answer. Indicate your answer to each question by **completely filling in the appropriate circle with a dark pencil** .
MARKING SCHEME: 4 marks for a correct answer, 0 for no answer or a wrong answer. You are not required to justify your answers in PART A. Note that for PART A, **only your final answers (as indicated by the circles you darken) count; your computations and answers indicated elsewhere will NOT count.**
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1. If $f''(x) = 6x + 2$ and $f(1) = f'(1) = 7$, then $f(2) =$

   Ⓐ 14

   ● 19

   Ⓒ -12

   Ⓓ -15

   Ⓔ 15

2. Let

$$f(x) = \begin{cases} 2x & \text{if } x < 2 \\ x^2 & \text{if } x \geq 2. \end{cases}$$

   Then $\displaystyle\int_1^3 f(x)\,dx =$

   Ⓐ $\frac{22}{3}$

   Ⓑ $\frac{25}{3}$

   ● $\frac{28}{3}$

   Ⓓ $\frac{17}{2}$

   Ⓔ $\frac{19}{2}$

3. Find the Riemann Sum for $f(x) = 2x+1$ on $[1,7]$, by partitioning $[1,7]$ into 3 subintervals of equal length and choosing each sample point to be the left endpoint of the subinterval.

   Ⓐ 48

   Ⓑ 54

   Ⓒ 72

   ● 42

   Ⓔ 66

4. If $G(x) = \int_1^{2x} \sqrt{2+t+t^4}\, dt$, then $G'(-1) =$

   ● 8

   Ⓑ 5

   Ⓒ $2\sqrt{2}$

   Ⓓ undefined

   Ⓔ $\sqrt{2}$

5. Find the average value of $f(x) = 5 + 4x + 3x^2$ on the interval $[0,2]$.

   Ⓐ 12

   ● 13

   Ⓒ 11

   Ⓓ 15

   Ⓔ 14

6. Find the area of the region enclosed between the graphs of $y = x + 1$ and $y = x^2 - x + 1$.

Ⓐ $\frac{4}{5}$

Ⓑ $\frac{1}{2}$

Ⓒ $\frac{3}{5}$

Ⓓ $\frac{5}{4}$

● $\frac{4}{3}$

7. Let R be the region bounded by the curves $y = 2x$ and $y = x^2$. Find the volume of the solid generated by revolving R about the y-axis.

Ⓐ $\frac{6\pi}{5}$

Ⓑ $\frac{8\pi}{5}$

Ⓒ $\frac{5\pi}{3}$

Ⓓ $\frac{7\pi}{3}$

● $\frac{8\pi}{3}$

8. $\int_0^2 x \ln\left(\frac{x^2+2}{x^2+4}\right) dx =$

Ⓐ $3\ln 3 - 4\ln 2$

Ⓑ $4\ln 3 - 3\ln 2$

● $3\ln 3 - 6\ln 2$

Ⓓ $4\ln 3 - 6\ln 2$

Ⓔ $4\ln 3 - 8\ln 2$

Hint: First, derive the formula

$$\int \ln t \, dt = t\ln t - t + C.$$

Then $\int x \ln\left(\frac{x^2+2}{x^2+4}\right) dx$

$= \int x \{\ln(x^2+2) - \ln(x^2+4)\} dx$

$= \frac{1}{2}\int \{\ln(u+2) - \ln(u+4)\} du$ (by letting $u = x^2$)

Now, use the formula at the top.

9. Find the value of

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{4n^2+n}}+\frac{1}{\sqrt{4n^2+2n}}+\frac{1}{\sqrt{4n^2+3n}}+\cdots+\frac{1}{\sqrt{4n^2+(n-1)n}}+\frac{1}{\sqrt{4n^2+n\cdot n}}\right).$$

Ⓐ $\frac{\pi}{4}$

Ⓑ $2\sqrt{5}$

Ⓒ some number between 3 and 4

Ⓓ $2\ln 4$

● $2\sqrt{5}-4$

Hint: This is the limit of a certain Riemann sum.

10. $\displaystyle\int_{-1}^{0} \frac{x}{(1-x)^2 e^x}\,dx =$

Ⓐ $\frac{1}{6}(3-2e)$

● $\frac{1}{2}(2-e)$

Ⓒ $\frac{1}{3}(e-3)$

Ⓓ $\frac{1}{2}(e-3)$

Ⓔ $\frac{1}{3}(2-e)$

Hint: Use Integration by Parts.

Let $u = x e^{-x}$, $dv = \frac{dx}{(1-x)^2}$.

11. $\int_{-2}^{6} \left|3 - |x-2|\right| dx =$

● 10

Ⓑ 12

Ⓒ 9

Ⓓ 11

Ⓔ 7

Hint: Show that

$$\int_{-2}^{6} \left|3 - |x-2|\right| dx$$

$$= \int_{-2}^{-1} (-x-1)\,dx + \int_{-1}^{2} (x+1)\,dx + \int_{2}^{5} (5-x)\,dx + \int_{5}^{6} |-5+x|\,dx .$$

12. Find the area of the region enclosed between the curves $y = \dfrac{1}{8x}$ and $y = \dfrac{1}{4x^2+3}$.

Ⓐ $\dfrac{\pi}{8\sqrt{3}} - \dfrac{\ln 3}{6}$

Ⓑ $\dfrac{\pi}{8\sqrt{3}} - \dfrac{\ln 3}{4}$

Ⓒ $\dfrac{\pi}{6\sqrt{3}} - \dfrac{\ln 3}{4}$

● $\dfrac{\pi}{12\sqrt{3}} - \dfrac{\ln 3}{8}$

Ⓔ $\dfrac{\pi}{6\sqrt{3}} - \dfrac{\ln 3}{8}$

Hint: $\text{area} = \displaystyle\int_{1/2}^{3/2} \left(\frac{1}{4x^2+3} - \frac{1}{8x} \right) dx.$

PART B *[52 marks]*

Please read carefully:

Present your complete solutions to the following questions in the spaces provided, in a neat and logical fashion, showing all your computations and justifications. Any answer in PART B without proper justification may receive very little or no credit. Use the back of each page for rough work only. If you must continue your formal solution on the back of a page, you should indicate clearly, in LARGE letters, "SOLUTION CONTINUED ON THE BACK OF PAGE _____". In this case, you may get credit for what you write on the back of that page, but you may also be penalized for mistakes on the back of that page.

MARKS FOR EACH QUESTION ARE INDICATED BY [ ].

DO NOT TEAR OUT ANY PAGES.

1. Find $\int xe^{3x}\,dx$

[7]

Ans. $\frac{xe^{3x}}{3} - \frac{e^{3x}}{9} + C$.

2. Find $\int (4x + 8x^3)(1 + x^2 + x^4)^{45}\,dx$.

NOTE: This is an easy question.

[7]

Ans. $\frac{1}{23}(1 + x^2 + x^4)^{46} + C$.

3. Find $\int \sin^{76} x \cos^3 x \, dx$.

[7]

Ans. $\frac{\sin^{77} x}{77} - \frac{\sin^{79} x}{79} + C$.

4. Find $\int \frac{1}{x^2 + 6x + 25} \, dx$.

[7]

Ans. $\frac{1}{4} \arctan\left(\frac{x+3}{4}\right) + C$.

5. Find $\int \frac{1}{(4-x^2)^{3/2}}\,dx$.

[8]

Ans. $\frac{x}{4\sqrt{1-x^2}} + C$.

6. Find $\displaystyle\int \frac{x^2+8x-9}{x(x+1)(x-3)}\,dx$.

[8]

Ans. $3\ln|x| - 4\ln|x+1| + 2\ln|x-3| + C$.

7. **NOTE: This is a hard question and will be marked very strictly. Very little or no credit will be given unless your solution is <u>completely correct</u>.**

Suppose that $\displaystyle\int_{\pi/4}^{\pi/2} \frac{\cos^4 x}{\sin^5 x}\,dx = k$. Find the value $\displaystyle\int_{\pi/4}^{\pi/2} \frac{\cos^6 x}{\sin^7 x}\,dx$ and express your answer in terms of k. Simplify your final answer as much as possible.

[8]

Hint:

Use Integration by Parts.

Let $u = \cos^5 x$, $dv = \dfrac{\cos x\,dx}{\sin^7 x}$.

Ans. $\dfrac{1}{6}(\sqrt{2} - 5k)$.