Midterm - MAE143B, Spring 2003, May 2, 3:00-5:00pm

1 Prob. 1

The followings are "true" or "false" questions. Check true or false and show (explain) why.

(a) [5 points] The system below is asymptotically stable.

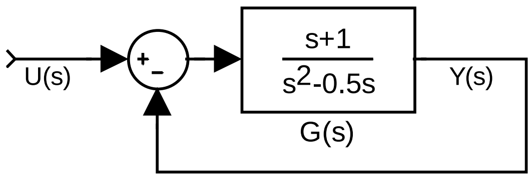

Figure 1: Block Diagram

TRUE FALSE

(b) [10 points] There exists a proper compensation $C(s)$ which causes the system below to have the closed loop transfer function

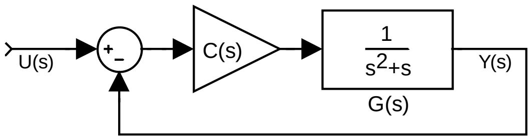

Figure 2: Block Diagram

$$\frac{Y(s)}{U(s)} = H(s) = \frac{s+2}{s^2+2s+2}$$

TRUE FALSE

(c) [10 points] The steady state response of

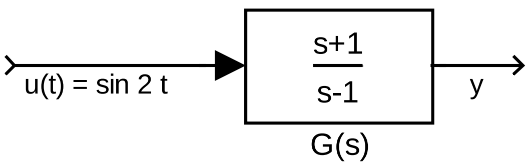

Figure 3: Block Diagram

is $y(t) = sin(t + 2\pi + 2\tan^{-1}(2))$

TRUE FALSE

2 Prob. 2

Write the ordinary differential equation that relates $y(t)$ to $u(t)$. [5 points]

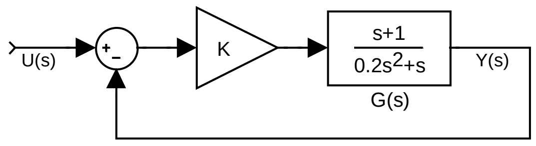

Figure 4: Block Diagram

3 Prob. 3

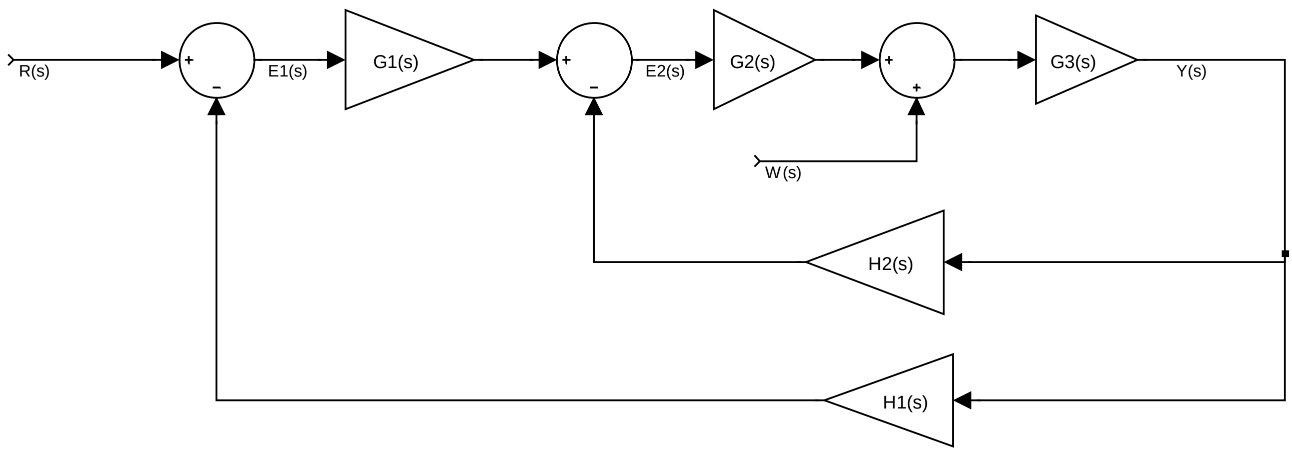

Figure 5: Block Diagram

Consider the feedback control system shown in the above. Derive the following transfer functions [5 points each] : (a) $\frac{Y(s)}{R(s)}$, (b) $\frac{Y(s)}{W(s)}$, (c) $\frac{E2(s)}{R(s)}$, (d) $\frac{E2(s)}{W(s)}$

4 Prob. 4

(a) [10 points] Find the values of K and T for which the system shown below is stable, if it is possible.

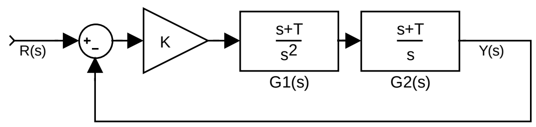

Figure 6: Block Diagram

(b) [10 points] When $K = \frac{1}{2}$, find the values of $T > 0$ such that all poles lie strictly to the left of the vertical line $s = -\frac{T}{2} \pm j\omega$, $0 \le \omega \le \infty$, if it is possible.

5 Prob. 5

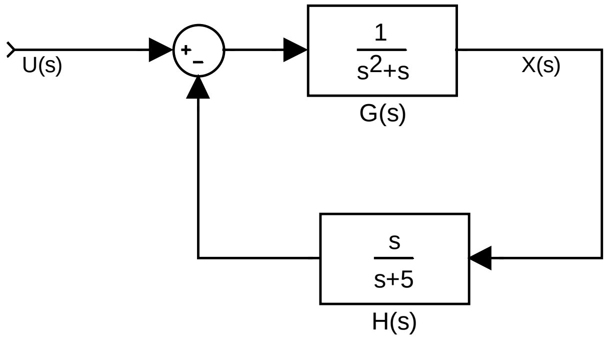

Figure 7: Block Diagram

Consider the feedback control system shown in the above.

(a) [10 points] Find $x(t)$ if $u(t)$ is a unit impulse function.

(b) [10 points] Explain how you would change $H(s)$ to guarantee that $lim_{t \to \infty} x(t) \to 0$ in the presence of a ramp input $u(t) \triangleq kt$.

6 Prob. 6

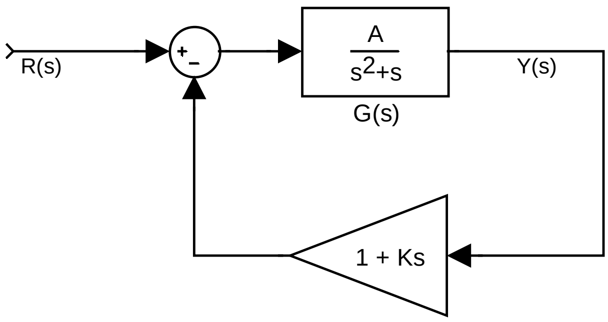

Figure 8: Block Diagram

Consider the feedback control system shown in the above.

(a) [10 points] Determine A and K to satisfy the following specifications.

(i) The transfer function $\frac{\mathbf{Y}(s)}{R(s)}$ is stable.

(ii) maximum overshoot (M_p) for a unit step input of less than 17%

(iii) 3% settling time (t_s) of less than 3.5 sec.

(b) [10 points] Determine the system type and error constant with respect to the error($e = r - y$) .

7 Prob. 7

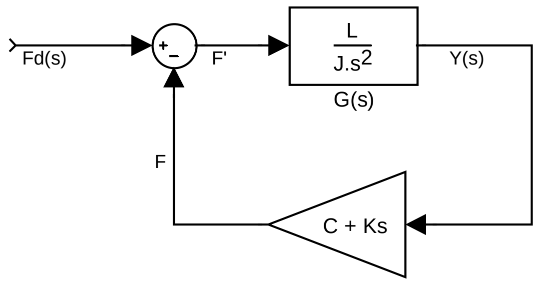

Figure 9: Block Diagram

A rigid spacecraft is controlled by reaction jets which operate in pairs to produce the torque FL . A position plus rate feedback (PD) controller is employed for the controller $H(s)$. The rate gyro gain, K, and the position gain ,C, ae to be determined.

(a) [20 points] When $J = 1$ and $L = 4$, determine K and C such that (i) and (ii) hold.

(i) The impulse response shows no oscillations.

(ii) The steady state position error, $\theta_e(\infty)$, is less than 0.01 in the presence of an effective bias disturbance of magnitude $F_d(t) = 1$.

(b) [5 points] Let $C = 1$ and $K = 0.707$. Find a proper location (L) for the jets so that the system damping ratio ζ is 0.707. That is, find L such that $\zeta = 0.707$.

(c) [10 points] Suggest a feedback controller $H(s)$ which will guarantee $\lim_{t\to\infty} \theta_e(t) = 0$ regardless of the magnitude of the bias disturbance $F_d(t) = F_o$. (Show this result of your design) .

(d) [5 points] Write a set of state equations for the system in the above block diagram. Define the state variables as $x_1 = \theta$ and $x_2 = \dot{\theta}$ and the input as $u = F_d$. Find the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$, and D such that

$$P \left\{ \begin{array}{rcl} \dot{\mathbf{x}} & = & \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y & = & \mathbf{C}\mathbf{x} + Du \end{array} \right. \tag{1}$$

where $\mathbf{x} \triangleq \left[\begin{array}{cc} x_1 & x_2 \end{array} \right]^T$ and $y \triangleq x_1$.

(e) [5 points] As a check on your answer in (e), compute the transfer function

$$\frac{Y(s)}{U(s)} = \mathbf{C}\left(s\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B} + D$$

and compare this answer with

$$\frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)}.$$