CHAPTER 1

ANSWERS TO QUESTIONS

Q1.1 Density varies with temperature and pressure. It would be necessary to measure both mass and volume very accurately in order to use the density of water as a standard.

Q1.2 Atomic clocks are based on electromagnetic waves which atoms emit. Also, pulsars are highly regular astronomical clocks.

Q1.3 People have different size hands.

Q1.4 (a) 0.3 millimeters (b) 50 microseconds (c) 7.2 kilograms

Q1.5 Different elements have different crystal structures because of differences in electron configurations.

Q1.6 (b) and (d). You cannot add or subtract quantities of different dimension.

Q1.7 Zero digits. An order-of-magnitude calculation is accurate only within a factor of 10.

Q1.8 If I were a runner, I might walk or run 10^1 miles per day. Since I am a college professor, I walk about 10^0 miles per day. I drive about 40 miles per day on workdays and up to 200 miles per day on vacation.

Q1.9 On February 7, 2001, I am 55 years and 39 days old.

$$55\text{ yr}\left(\frac{365.25\text{ d}}{1\text{ yr}}\right)+39\text{ d}=20128\text{ d}\left(\frac{86\,400\text{ s}}{1\text{ d}}\right)=1.74\times10^9\text{ s}$$

Many college students are just approaching 1 Gs.

Q1.10 Force and velocity are vectors. The others are scalars. Height would be a vector if we defined it to have an upward direction.

Q1.11 If the direction-angle of **A** is between 180 degrees and 270 degrees, its components are both negative. If a vector is in the second quadrant or the fourth quadrant, its components have opposite signs.

Q1.12 Vector displacement equals zero. Distance is six meters.

Q1.13 Your first displacement is 110 miles in the direction of decreasing numbers. Your second displacement is 25 miles in the opposite direction. Your total displacement is 85 miles in the direction of decreasing numbers, with a magnitude of 85 miles.

Q1.14 They are perpendicular.

Q1.15 Each component of **A** must be equal to the corresponding component of **B**.

Q1.16 Addition of a vector to a scalar is not defined. Think of apples and oranges.

Q1.17 Its horizontal displacement component is (135 ft) cos 40° = 103 ft. It attains a height of (135 ft) sin 40°, or 86.8 ft.

Q1.18 Determining the magnitude and direction of a vector from its components, or vice versa, would be more difficult. So would working out dot and cross products later in the course. On the other hand, using unit-vector notation, addition, subtraction, and multiplication by a scalar would not get more difficult. If you make your own graph paper, drawing vectors would still be easy.

Q1.19 (a) Rotation does not noticeably affect revolution: simplification.
(b) Geometric (c) Structural (d) Simplification
(e) Geometric (f) Simplification (g) Structural

PROBLEM SOLUTIONS

1.1 With $V =$ (base area)(height) $V = \left(\pi r^2\right)h$

and $\rho = m/V$, we have $\rho = \dfrac{m}{\pi r^2 h} = \dfrac{1\text{ kg}}{\pi(19.5\text{ mm})^2(39.0\text{ mm})}\left(\dfrac{10^9\text{ mm}^3}{1\text{ m}^3}\right)$

$$\rho = \boxed{2.15\times10^4\text{ kg/m}^3}$$

1.2 $\rho = \dfrac{M}{V} = \dfrac{M}{\frac{4}{3}\pi R^3} = \dfrac{3\left(5.64\times10^{26}\text{ kg}\right)}{4\pi\left(6.00\times10^7\text{ m}\right)^3} = \boxed{623\text{ kg/m}^3}$

***1.3** Let V represent the volume of the model, the same in $\rho = m/V$ for both.

Then $\rho_{\text{iron}} = 9.35\text{ kg}/V$ and $\rho_{\text{gold}} = m_{\text{gold}}/V$

Next, $\dfrac{\rho_{\text{gold}}}{\rho_{\text{iron}}} = \dfrac{m_{\text{gold}}}{9.35\text{ kg}}$ $m_{\text{gold}} = 9.35\text{ kg}\left(\dfrac{19.3\times10^3\text{ kg/m}^3}{7.86\times10^3\text{ kg/m}^3}\right) = \boxed{23.0\text{ kg}}$

***1.4** $V = V_o - V_i = \dfrac{4}{3}\pi\left(r_2^3 - r_1^3\right)$

$\rho = \dfrac{m}{V}$, so $m = \rho V = \rho\left(\dfrac{4}{3}\pi\right)\left(r_2^3 - r_1^3\right) = \boxed{\dfrac{4\pi\rho\left(r_2^3 - r_1^3\right)}{3}}$

***1.5** Mass of gold abraded: $|\Delta m| = 3.80\text{ g} - 3.35\text{ g} = 0.45\text{ g} = (0.45\text{ g})\left(\dfrac{1\text{ kg}}{10^3\text{ g}}\right) = 4.5\times10^{-4}\text{ kg}$

Each atom has mass $m_0 = 197\text{ u} = 197\text{ u}\left(\dfrac{1.66\times10^{-27}\text{ kg}}{1\text{ u}}\right) = 3.27\times10^{-25}\text{ kg}$

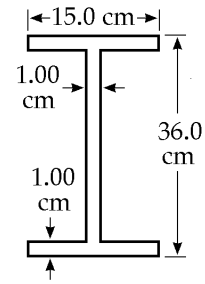
Now, $|\Delta m| = |\Delta N| m_0$, and the number of atoms missing is

$$|\Delta N| = \frac{|\Delta m|}{m_0} = \frac{4.5\times10^{-4}\text{ kg}}{3.27\times10^{-25}\text{ kg}} = 1.38\times10^{21}\text{ atoms}$$

The rate of loss is $\dfrac{|\Delta N|}{\Delta t} = \dfrac{1.38\times10^{21}\text{ atoms}}{50\text{ yr}}\left(\dfrac{1\text{ yr}}{365.25\text{ d}}\right)\left(\dfrac{1\text{ d}}{24\text{ h}}\right)\left(\dfrac{1\text{ h}}{60\text{ min}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)$

$$\frac{|\Delta N|}{\Delta t} = \boxed{8.72\times10^{11}\text{ atoms/s}}$$

***1.6** (a) The cross-sectional area is



$$A = 2(0.150 \text{ m})(0.010 \text{ m}) + (0.340 \text{ m})(0.010 \text{ m}) = 6.40\times10^{-3} \text{ m}^2$$

The volume of the beam is

$$V = AL = \left(6.40\times10^{-3} \text{ m}^2\right)(1.50 \text{ m}) = 9.60\times10^{-3} \text{ m}^3$$

Thus, its mass is $m = \rho V = \left(7.56\times10^3 \text{ kg/m}^3\right)\left(9.60\times10^{-3} \text{ m}^3\right) = \boxed{72.6 \text{ kg}}$

(b) The mass of one typical atom is $m_0 = (55.9 \text{ u})\left(\dfrac{1.66\times10^{-27} \text{ kg}}{1 \text{ u}}\right) = 9.28\times10^{-26} \text{ kg}$

Now $m = Nm_0$ and the number of atoms is $N = \dfrac{m}{m_0} = \dfrac{72.6 \text{ kg}}{9.28\times10^{-26} \text{ kg}} = \boxed{7.82\times10^{26} \text{ atoms}}$

***1.7** Consider one cubic meter of gold. Its mass from Table 1.5 is 19 300 kg.

One atom of gold has mass $m_0 = (197 \text{ u})\left(\dfrac{1.66\times10^{-27} \text{ kg}}{1 \text{ u}}\right) = 3.27\times10^{-25} \text{ kg}$

So, the number of atoms in the cube is $N = \dfrac{19\,300 \text{ kg}}{3.27\times10^{-25} \text{ kg}} = 5.90\times10^{28}$

The imagined cubical volume of each atom is $d^3 = \dfrac{1 \text{ m}^3}{5.90\times10^{28}} = 1.69\times10^{-29} \text{ m}^3$

So $d = \boxed{2.57\times10^{-10} \text{ m}}$

***1.8** (a) $\boxed{\text{This is incorrect}}$ since the units of $[ax]$ are m^2/s^2, while the units of $[v]$ are m/s.

(b) $\boxed{\text{This is correct}}$ since the units of $[y]$ are m, and $\cos(kx)$ is dimensionless if $[k]$ is in m^{-1}.

***1.9** The term x has dimensions of L, a has dimensions of LT^{-2}, and t has dimensions of T. Therefore, the equation $x = ka^m t^n$ has dimensions of

$$\text{L} = \left(\text{LT}^{-2}\right)^m (\text{T})^n \qquad \text{or} \qquad \text{L}^1\text{T}^0 = \text{L}^m\text{T}^{n-2m}$$

The powers of L and T must be the same on each side of the equation. Therefore,

$$\text{L}^1 = \text{L}^m \qquad \text{and} \qquad \boxed{m = 1}$$

Likewise, equating terms in T, we see that $n - 2m$ must equal 0. Thus, $\boxed{n = 2}$

The value of k, a dimensionless constant, $\boxed{\text{cannot be obtained by dimensional analysis}}$.

***1.10** (a) Circumference has dimensions of L.
(b) Volume has dimensions of L^3.
(c) Area has dimensions of L^2.

Expression (i) has dimension $L\left(L^2\right)^{1/2} = L^2$, so this must be area (c).
Expression (ii) has dimension L, so it is (a).
Expression (iii) has dimension $L\left(L^2\right) = L^3$, so it is (b). Thus, $\boxed{\text{(a)} = \text{ii}; \quad \text{(b)} = \text{iii}, \ \text{(c)} = \text{i}}$.

1.11 One month is $1 \text{ mo} = (30 \text{ day})(24 \text{ h/day})(3600 \text{ s/h}) = 2.592 \times 10^6 \text{ s}$

Applying units to the equation, $V = \left(1.50 \text{ Mft}^3/\text{mo}\right)t + \left(0.00800 \text{ Mft}^3/\text{mo}^2\right)t^2$

Since $1 \text{ Mft}^3 = 10^6 \text{ ft}^3$, $V = \left(1.50 \times 10^6 \text{ ft}^3/\text{mo}\right)t + \left(0.00800 \times 10^6 \text{ ft}^3/\text{mo}^2\right)t^2$

Converting months to seconds,
$$V = \frac{1.50 \times 10^6 \text{ ft}^3/\text{mo}}{2.592 \times 10^6 \text{ s/mo}}t + \frac{0.00800 \times 10^6 \text{ ft}^3/\text{mo}^2}{\left(2.592 \times 10^6 \text{ s/mo}\right)^2}t^2$$

Thus,
$$\boxed{V\,[\text{ft}^3] = \left(0.579 \text{ ft}^3/\text{s}\right)t + \left(1.19 \times 10^{-9} \text{ ft}^3/\text{s}^2\right)t^2}$$

***1.12** Apply the following conversion factors:

$$1 \text{ in} = 2.54 \text{ cm}, \ 1 \text{ d} = 86\,400 \text{ s}, \ 100 \text{ cm} = 1 \text{ m}, \ \text{and} \ 10^9 \text{ nm} = 1 \text{ m}$$

$$\left(\frac{1}{32} \text{ in/day}\right)\frac{(2.54 \text{ cm/in})\left(10^{-2} \text{ m/cm}\right)\left(10^9 \text{ nm/m}\right)}{86\,400 \text{ s/day}} = \boxed{9.19 \text{ nm/s}}$$

This means the proteins are assembled at a rate of many layers of atoms each second!

1.13 (a) Seven minutes is 420 seconds, so the rate is $r = \dfrac{30.0 \text{ gal}}{420 \text{ s}} = \boxed{7.14 \times 10^{-2} \text{ gal/s}}$

(b) Converting gallons first to liters, then to m^3, $r = \left(7.14 \times 10^{-2} \dfrac{\text{gal}}{\text{s}}\right)\left(\dfrac{3.786 \text{ L}}{1 \text{ gal}}\right)\left(\dfrac{10^{-3} \text{ m}^3}{1 \text{ L}}\right)$

$$r = \boxed{2.70 \times 10^{-4} \text{ m}^3/\text{s}}$$

(c) At that rate, to fill a 1-m^3 tank would take $t = \left(\dfrac{1 \text{ m}^3}{2.70 \times 10^{-4} \text{ m}^3/\text{s}}\right)\left(\dfrac{1 \text{ h}}{3600}\right) = \boxed{1.03 \text{ h}}$

***1.14** The weight flow rate is $1200\ \dfrac{\text{ton}}{\text{h}}\left(\dfrac{2000\text{ lb}}{\text{ton}}\right)\left(\dfrac{1\text{ h}}{60\text{ min}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)=\boxed{667\text{ lb/s}}$

***1.15** It is often useful to remember that the 1600-m race at track and field events is approximately 1 mile in length. To be precise, there are 1609 meters in a mile. Thus, 1 acre is equal in area to

$$(1\text{ acre})\left(\frac{1\text{ mi}^2}{640\text{ acres}}\right)\left(\frac{1609\text{ m}}{\text{mi}}\right)^2=\boxed{4.05\times10^3\text{ m}^2}$$

***1.16** (a) $\left(\dfrac{6\times10^{12}\ \$}{1000\ \$/\text{s}}\right)\left(\dfrac{1\text{ h}}{3600\text{ s}}\right)\left(\dfrac{1\text{ day}}{24\text{ h}}\right)\left(\dfrac{1\text{ yr}}{365\text{ days}}\right)=\boxed{190\text{ years}}$

(b) The circumference of the Earth at the equator is $2\pi(6378\times10^3\text{ m})=4.01\times10^7\text{ m}$. The length of one dollar bill is 0.155 m so that the length of 6 trillion bills is $9.30\times10^{11}\text{ m}$. Thus, the 6 trillion dollars would encircle the Earth

$$\frac{9.30\times10^{11}\text{ m}}{4.01\times10^7\text{ m}}=\boxed{2.32\times10^4\text{ times}}$$

1.17 $V=At$ so $t=\dfrac{V}{A}=\dfrac{3.78\times10^{-3}\text{ m}^3}{25.0\text{ m}^2}=\boxed{1.51\times10^{-4}\text{ m}\quad(\text{or }151\ \mu\text{m})}$

***1.18** (a) $d_{\text{nucleus, scale}}=d_{\text{nucleus, real}}\left(\dfrac{d_{\text{atom, scale}}}{d_{\text{atom, real}}}\right)=\left(2.40\times10^{-15}\text{ m}\right)\left(\dfrac{300\text{ ft}}{1.06\times10^{-10}\text{ m}}\right)=6.70\times10^{-3}\text{ ft}$

or $d_{\text{nucleus, scale}}=\left(6.79\times10^{-3}\text{ ft}\right)(304.8\text{ mm}/1\text{ ft})=\boxed{2.07\text{ mm}}$

(b) $\dfrac{V_{\text{atom}}}{V_{\text{nucleus}}}=\dfrac{4\pi r_{\text{atom}}^3/3}{4\pi r_{\text{nucleus}}^3/3}=\left(\dfrac{r_{\text{atom}}}{r_{\text{nucleus}}}\right)^3=\left(\dfrac{d_{\text{atom}}}{d_{\text{nucleus}}}\right)^3$

$$\frac{V_{\text{atom}}}{V_{\text{nucleus}}}=\left(\frac{1.06\times10^{-10}\text{ m}}{2.40\times10^{-15}\text{ m}}\right)^3=\boxed{8.62\times10^{13}\text{ times as large}}$$

1.19 To balance, $m_{\text{Fe}}=m_{\text{Al}}$ or $\rho_{\text{Fe}}V_{\text{Fe}}=\rho_{\text{Al}}V_{\text{Al}}$

$\rho_{\text{Fe}}\left(\frac{4}{3}\right)\pi r_{\text{Fe}}{}^3=\rho_{\text{Al}}\left(\frac{4}{3}\right)\pi r_{\text{Al}}{}^3$ $\qquad r_{\text{Al}}=r_{\text{Fe}}\left(\dfrac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}\right)^{1/3}=(2.00\text{ cm})\left(\dfrac{7.86}{2.70}\right)^{1/3}=\boxed{2.86\text{ cm}}$

1.20 The mass of each sphere is $m_{\text{Al}} = \rho_{\text{Al}} V_{\text{Al}} = \dfrac{4\pi \rho_{\text{Al}} r_{\text{Al}}^3}{3}$

and $m_{\text{Fe}} = \rho_{\text{Fe}} V_{\text{Fe}} = \dfrac{4\pi \rho_{\text{Fe}} r_{\text{Fe}}^3}{3}$

Setting these masses equal, $\dfrac{4\pi \rho_{\text{Al}} r_{\text{Al}}^3}{3} = \dfrac{4\pi \rho_{\text{Fe}} r_{\text{Fe}}^3}{3}$ and $\boxed{r_{\text{Al}} = r_{\text{Fe}} \sqrt[3]{\rho_{\text{Fe}} / \rho_{\text{Al}}}}$

***1.21** Model the room as a rectangular solid with dimensions 4 m by 4 m by 3 m, and each ping-pong ball as a sphere of diameter 0.038 m. The volume of the room is $4 \times 4 \times 3 = 48\ \text{m}^3$, while the volume of one ball is

$$\frac{4\pi}{3}\left(\frac{0.038\ \text{m}}{2}\right)^3 = 2.87 \times 10^{-5}\ \text{m}^3.$$

Therefore, one can fit about $\dfrac{48}{2.87 \times 10^{-5}} \sim \boxed{10^6}$ ping-pong balls in the room.

As an aside, the actual number is smaller than this because there will be a lot of space in the room that cannot be covered by balls. In fact, even in the best arrangement, the so-called "best packing fraction" is $\frac{1}{6}\pi\sqrt{2} = 0.74$ so that at least 26% of the space will be empty. Therefore, the above estimate reduces to $1.67 \times 10^6 \times 0.740 \sim 10^6$.

***1.22** A reasonable guess for the diameter of a tire might be 2.5 ft, with a circumference of about 8 ft. Thus, the tire would make $(50\,000\ \text{mi})(5280\ \text{ft/mi})(1\ \text{rev}/8\ \text{ft}) = 3 \times 10^7\ \text{rev} \sim \boxed{10^7\ \text{rev}}$

1.23 Assume the tub measures 1.3 m by 0.5 m by 0.3 m. One-half of its volume is then

$$V = (0.5)(1.3\ \text{m})(0.5\ \text{m})(0.3\ \text{m}) = 0.10\ \text{m}^3$$

The mass of this volume of water is

$$m_{\text{water}} = \rho_{\text{water}} V = \left(1000\ \text{kg/m}^3\right)\left(0.10\ \text{m}^3\right) = 100\ \text{kg} \qquad \boxed{\sim 10^2\ \text{kg}}$$

Pennies are now mostly zinc, but consider copper pennies filling 50% of the volume of the tub. The mass of copper required is

$$m_{\text{copper}} = \rho_{\text{copper}} V = \left(8930\ \text{kg/m}^3\right)\left(0.10\ \text{m}^3\right) = 893\ \text{kg} \qquad \boxed{\sim 10^3\ \text{kg}}$$

1.24 The typical person probably drinks 2 to 3 soft drinks daily. Perhaps half of these were in aluminum cans. Thus, we will estimate 1 aluminum can disposal per person per day. In the U.S. there are ~250 million people, and 365 days in a year,

so $$\left(250\times10^{6}\ \text{cans/day}\right)\left(365\ \text{days/year}\right)\cong10^{10}\ \text{cans}$$

are thrown away or recycled each year. Guessing that each can weighs around 1/10 of an ounce, we estimate this represents

$$\left(10^{10}\ \text{cans}\right)(0.1\ \text{oz/can})(1\ \text{lb/16 oz})(1\ \text{ton/2000 lb})\approx3.1\times10^{5}\ \text{tons/year}. \quad \boxed{\sim10^{5}\ \text{tons}}$$

1.25 (a)

$$\begin{array}{r} 756.?? \\ 37.2? \\ 0.83 \\ \underline{+\ 2.5?} \\ 796.\cancel{5}\cancel{3} \end{array} = \boxed{797}$$

(b) $0.0032\ (2\ \text{s.f.})\times356.3\ (4\ \text{s.f.})=1.14016=(2\ \text{s.f.})\ \boxed{1.1}$

(c) $5.620\ (4\ \text{s.f.})\times\pi\ (>4\ \text{s.f.})=17.656=(4\ \text{s.f.})\ \boxed{17.66}$

***1.26** We work to nine significant digits:

$$1\ \text{yr}=1\ \text{yr}\left(\frac{365.242199\ \text{d}}{1\ \text{yr}}\right)\left(\frac{24\ \text{h}}{1\ \text{d}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right)=\boxed{31556926.0\ \text{s}}$$

1.27 (a) $\boxed{3}$ (b) $\boxed{4}$ (c) $\boxed{3}$ (d) $\boxed{2}$

1.28 $r=(6.50\pm0.20)\ \text{cm}=(6.50\pm0.20)\times10^{-2}\ \text{m}$ $\qquad m=(1.85\pm0.02)\ \text{kg}$

$$\rho=\frac{m}{\left(\frac{4}{3}\right)\pi r^{3}}$$

also, $$\frac{\delta\rho}{\rho}=\frac{\delta m}{m}+\frac{3\delta r}{r}$$

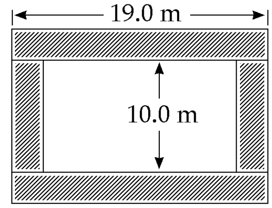
In other words, the percentages of uncertainty are cumulative.

Therefore, $$\frac{\delta\rho}{\rho}=\frac{0.02}{1.85}+\frac{3(0.20)}{6.50}=0.103$$

$$\rho=\frac{1.85}{\left(\frac{4}{3}\right)\pi\left(6.5\times10^{-2}\ \text{m}\right)^{3}}=\boxed{1.61\times10^{3}\ \text{kg/m}^{3}}$$

and $$\rho\pm\delta\rho=\boxed{(1.61\pm0.17)\times10^{3}\ \text{kg/m}^{3}}$$

***1.29** $V = 2V_1 + 2V_2 = 2(V_1 + V_2)$



$$V_1 = (17.0\text{ m} + 1.0\text{ m} + 1.0\text{ m})(1.0\text{ m})(0.09\text{ m}) = 1.70\text{ m}^3$$

$$V_2 = (10.0\text{ m})(1.0\text{ m})(0.090\text{ m}) = 0.900\text{ m}^3$$

$$V = 2\left(1.70\text{ m}^3 + 0.900\text{ m}^3\right) = \boxed{5.2\text{ m}^3}$$

$$\left.\begin{array}{l} \dfrac{\delta \ell_1}{\ell_1} = \dfrac{0.12\text{ m}}{19.0\text{ m}} = 0.0063 \\ \dfrac{\delta w_1}{w_1} = \dfrac{0.01\text{ m}}{1.0\text{ m}} = 0.010 \\ \dfrac{\delta t_1}{t_1} = \dfrac{0.1\text{ cm}}{9.0\text{ cm}} = 0.011 \end{array}\right\} \dfrac{\delta V}{V} = 0.006 + 0.010 + 0.011 = 0.027 = \boxed{2.7\%}$$

1.30 $x = r\cos\theta = (5.50\text{ m})\cos 240° = (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$

$$y = r\sin\theta = (5.50\text{ m})\sin 240° = (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$$

***1.31** The x distance out to the fly is 2.00 m and the y distance up to the fly is 1.00 m.

(a) We can use the Pythagorean theorem to find the distance from the origin to the fly.

$$\text{distance} = \sqrt{x^2 + y^2} = \sqrt{(2.00\text{ m})^2 + (1.00\text{ m})^2} = \sqrt{5.00\text{ m}^2} = \boxed{2.24\text{ m}}$$

(b) $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$; $\mathbf{r} = \boxed{2.24\text{ m, } 26.6°}$

***1.32** (a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2.00 - [-3.00])^2 + (-4.00 - 3.00)^2}$

$$d = \sqrt{25.0 + 49.0} = \boxed{8.60\text{ m}}$$

(b) $r_1 = \sqrt{(2.00)^2 + (-4.00)^2} = \sqrt{20.0} = \boxed{4.47\text{ m}}$ $\qquad \theta_1 = \tan^{-1}\left(-\dfrac{4.00}{2.00}\right) = \boxed{-63.4°}$

$r_2 = \sqrt{(-3.00)^2 + (3.00)^2} = \sqrt{18.0} = \boxed{4.24\text{ m}}$ $\qquad \theta_2 = \boxed{135°}$ measured from the $+x$ axis.

1.33 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(y/x)$

(a) The radius for this new point is $\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$

and its angle is $\tan^{-1}\left(\frac{y}{-x}\right) = \boxed{180° - \theta}$

(b) $\sqrt{(-2x)^2 + (-2y)^2} = \boxed{2r}$. This point is in the third quadrant if (x, y) is in the first quadrant or in the fourth quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{180° + \theta}$.

(c) $\sqrt{(3x)^2 + (-3y)^2} = \boxed{3r}$. This point is in the fourth quadrant if (x, y) is in the first quadrant or in the third quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{-\theta}$.

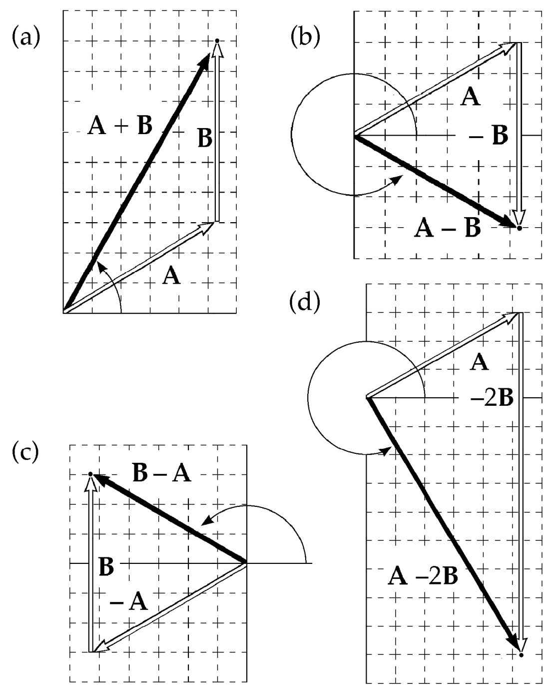
1.34 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) **A** + **B** = 5.2 m at 60°

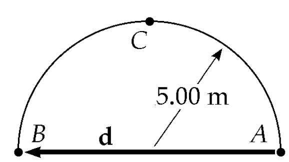
(b) **A** – **B** = 3.0 m at 330°

(c) **B** – **A** = 3.0 m at 150°

(d) **A** – 2**B** = 5.2 m at 300°.



1.35 (a) $|\mathbf{d}| = |-10.0\mathbf{i}| = \boxed{10.0 \text{ m}}$ since the displacement is in a straight line from point A to point B.



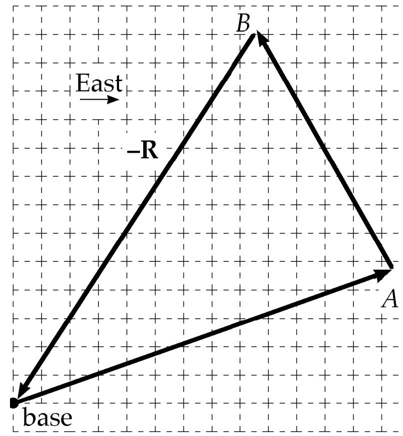
(b) The actual distance skated is not equal to the straight-line displacement. The distance follows the curved path of the semi-circle (ACB).

$$s = \tfrac{1}{2}(2\pi r) = 5\pi = \boxed{15.7 \text{ m}}$$

(c) If the circle is complete, **d** begins and ends at point **A**. Hence, $|\mathbf{d}| = \boxed{0}$

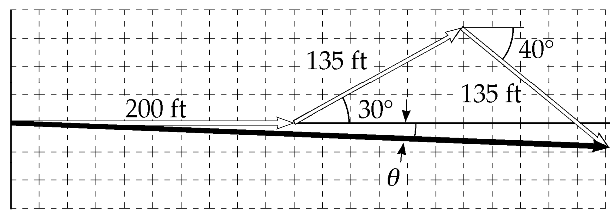
***1.36** $-\mathbf{R} = \boxed{310\text{ km at }57°\text{ S of W}}$

(Scale: 1 unit = 20 km)



***1.37** The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be

$$\boxed{d = 420\text{ ft}\quad\text{and}\quad \theta = -3°}$$

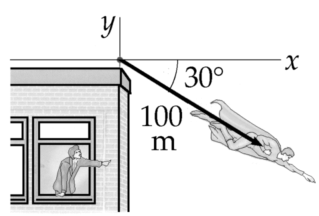


(Scale: 1 unit = 20 ft)

1.38 Coordinates of the super-hero are:

$$x = (100\text{ m})\cos(-30.0°) = \boxed{86.6\text{ m}}$$

$$y = (100\text{ m})\sin(-30.0°) = \boxed{-50.0\text{ m}}$$

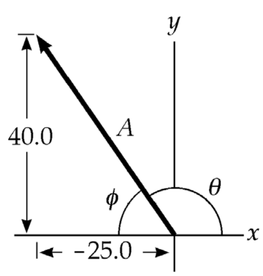


1.39 $A_x = -25.0 \qquad A_y = 40.0$

$$A = \sqrt{A_x{}^2 + A_y{}^2} = \sqrt{(-25.0)^2 + (40.0)^2} = \boxed{47.2\text{ units}}$$

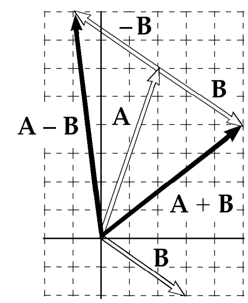
We observe that $\quad \tan\phi = |A_y|/|A_x|$

So $\quad \phi = \tan^{-1}\left(\dfrac{A_y}{|A_x|}\right) = \tan\dfrac{40.0}{25.0} = \tan^{-1}(1.60) = 58.0°$



The diagram shows that the angle from the +x axis can be found by subtracting from 180°: $\quad \theta = 180° - 58° = \boxed{122°}$

1.40 (a) See figure to the right.



(b) $\mathbf{C} = \mathbf{A} + \mathbf{B} = 2.00\mathbf{i} + 6.00\mathbf{j} + 3.00\mathbf{i} - 2.00\mathbf{j} = \boxed{5.00\mathbf{i} + 4.00\mathbf{j}}$

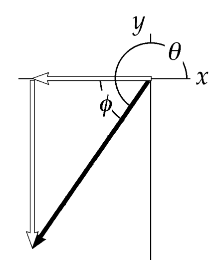
$C = \sqrt{25.0 + 16.0}$ at $\tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$

$\mathbf{D} = \mathbf{A} - \mathbf{B} = 2.00\mathbf{i} + 6.00\mathbf{j} - 3.00\mathbf{i} + 2.00\mathbf{j} = \boxed{-1.00\mathbf{i} + 8.00\mathbf{j}}$

$D = \sqrt{(-1.00)^2 + (8.00)^2}$ at $\tan^{-1}\left(\frac{8.00}{-1.00}\right)$

$D = 8.06$ at $(180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$

1.41 $\mathbf{d}_1 = 100\mathbf{i}$ $\quad\quad$ $\mathbf{d}_2 = -300\mathbf{j}$



$\mathbf{d}_3 = -150\cos(30.0°)\mathbf{i} - 150\sin(30.0°)\mathbf{j} = -130\mathbf{i} - 75.0\mathbf{j}$

$\mathbf{d}_4 = -200\cos(60.0°)\mathbf{i} + 200\sin(60.0°)\mathbf{j} = -100\mathbf{i} + 173\mathbf{j}$

$\mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \mathbf{d}_4 = \boxed{(-130\mathbf{i} - 202\mathbf{j}) \text{ m}}$

$|\mathbf{R}| = \sqrt{(-130)^2 + (-202)^2} = \boxed{240 \text{ m}}$

$\phi = \tan^{-1}\left(\frac{202}{130}\right) = 57.2°$ $\quad\quad$ $\theta = 180 + \phi = \boxed{237°}$

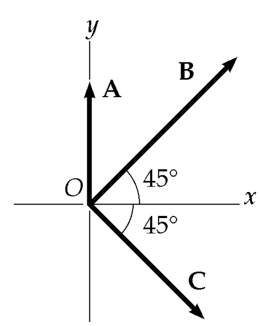
***1.42** $\mathbf{A} = -8.70\mathbf{i} + 15.0\mathbf{j}$ and $\mathbf{B} = 13.2\mathbf{i} - 6.60\mathbf{j}$

$\mathbf{A} - \mathbf{B} + 3\mathbf{C} = 0$: $\quad$ $3\mathbf{C} = \mathbf{B} - \mathbf{A} = 21.9\mathbf{i} - 21.6\mathbf{j}$

$\mathbf{C} = 7.30\mathbf{i} - 7.20\mathbf{j}$

or $\quad$ $C_x = \boxed{7.30 \text{ cm}}$; $C_y = \boxed{-7.20 \text{ cm}}$

1.43 (a) $R_x = 40.0\cos 45.0° + 30.0\cos 45.0° = 49.5$



$R_y = 40.0\sin 45.0° - 30.0\sin 45.0° + 20.0 = 27.1$

$\mathbf{R} = \boxed{49.5\mathbf{i} + 27.1\mathbf{j}}$

(b) $|\mathbf{R}| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$

$\theta = \tan^{-1}\left(\frac{27.1}{49.5}\right) = \boxed{28.7°}$

***1.44** (a) $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C} = 2\mathbf{i} + 4\mathbf{j}$

$$|\mathbf{D}| = \sqrt{2^2 + 4^2} = \boxed{4.47 \text{ m at } \theta = 63.4°}$$

(b) $\mathbf{E} = -\mathbf{A} - \mathbf{B} + \mathbf{C} = -6\mathbf{i} + 6\mathbf{j}$

$$|\mathbf{E}| = \sqrt{6^2 + 6^2} = \boxed{8.49 \text{ m at } \theta = 135°}$$

***1.45** (a) $(\mathbf{A} + \mathbf{B}) = (3\mathbf{i} - 2\mathbf{j}) + (-\mathbf{i} - 4\mathbf{j}) = \boxed{2\mathbf{i} - 6\mathbf{j}}$

(b) $(\mathbf{A} - \mathbf{B}) = (3\mathbf{i} - 2\mathbf{j}) - (-\mathbf{i} - 4\mathbf{j}) = \boxed{4\mathbf{i} + 2\mathbf{j}}$

(c) $|\mathbf{A} + \mathbf{B}| = \sqrt{2^2 + 6^2} = \boxed{6.32}$

(d) $|\mathbf{A} - \mathbf{B}| = \sqrt{4^2 + 2^2} = \boxed{4.47}$

(e) $\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\dfrac{6}{2}\right) = -71.6° = \boxed{288°}$

$$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6°}$$

1.46 (a) $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$$\mathbf{F} = 120\cos(60.0°)\mathbf{i} + 120\sin(60.0°)\mathbf{j} - 80.0\cos(75.0°)\mathbf{i} + 80.0\sin(75.0°)\mathbf{j}$$

$$\mathbf{F} = 60.0\mathbf{i} + 104\mathbf{j} - 20.7\mathbf{i} + 77.3\mathbf{j} = (39.3\mathbf{i} + 181\mathbf{j}) \text{ N}$$

$$|\mathbf{F}| = \sqrt{39.3^2 + 181^2} = \boxed{185 \text{ N}} \qquad \theta = \tan^{-1}\left(\frac{181}{39.3}\right) = \boxed{77.8°}$$

(b) $\mathbf{F}_3 = -\mathbf{F} = \boxed{(-39.3\mathbf{i} - 181\mathbf{j}) \text{ N}}$

***1.47** $\mathbf{A} = 3.00 \text{ m}, \ \theta_A = 30.0°$ $\qquad$ $\mathbf{B} = 3.00 \text{ m}, \ \theta_B = 90.0°$

$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60 \text{ m}$ $\qquad$ $A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50 \text{ m}$

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} = (2.60\mathbf{i} + 1.50\mathbf{j}) \text{ m}$$

$B_x = 0, \ B_y = 3.00 \text{ m}$ $\quad$ so $\quad$ $\mathbf{B} = 3.00\mathbf{j} \text{ m}$

$$\mathbf{A} + \mathbf{B} = (2.60\mathbf{i} + 1.50\mathbf{j}) + 3.00\mathbf{j} = \boxed{(2.60\mathbf{i} + 4.50\mathbf{j}) \text{ m}}$$

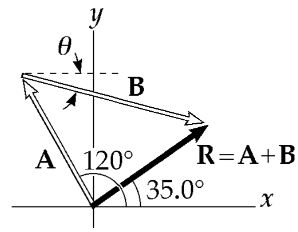
***1.48** We have $\mathbf{B} = \mathbf{R} - \mathbf{A}$:

$$A_x = 150\cos 120° = -75.0 \text{ cm}$$

$$A_y = 150\sin 120° = 130 \text{ cm}$$

$$R_x = 140\cos 35.0° = 115 \text{ cm}$$

$$R_y = 140\sin 35.0° = 80.3 \text{ cm}$$



Therefore,

$$\mathbf{B} = [115 - (-75)]\mathbf{i} + [80.3 - 130]\mathbf{j} = (190\,\mathbf{i} - 49.7\,\mathbf{j}) \text{ cm}$$

$$|\mathbf{B}| = \sqrt{190^2 + 49.7^2} = \boxed{196 \text{ cm}}$$

$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}$$

1.49 (a) $\mathbf{A} = \boxed{8.00\,\mathbf{i} + 12.0\,\mathbf{j} - 4.00\,\mathbf{k}}$

(b) $\mathbf{B} = \mathbf{A}/4 = \boxed{2.00\,\mathbf{i} + 3.00\,\mathbf{j} - 1.00\,\mathbf{k}}$

(c) $\mathbf{C} = -3\mathbf{A} = \boxed{-24.0\,\mathbf{i} - 36.0\,\mathbf{j} + 12.0\,\mathbf{k}}$

***1.50** Let the positive x-direction be eastward, the positive y-direction be vertically upward, and the positive z-direction be southward. The total displacement is then

$$\mathbf{d} = (4.80\,\mathbf{i} + 4.80\,\mathbf{j}) \text{ cm} + (3.70\,\mathbf{j} - 3.70\,\mathbf{k}) \text{ cm} = (4.80\,\mathbf{i} + 8.50\,\mathbf{j} - 3.70\,\mathbf{k}) \text{ cm}$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2} \text{ cm} = \boxed{10.4 \text{ cm}}$

(b) Its angle with the y-axis follows from $\cos\theta = \dfrac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$

***1.51** $\mathbf{B} = B_x\,\mathbf{i} + B_y\,\mathbf{j} + B_z\,\mathbf{k} = 4.00\,\mathbf{i} + 6.00\,\mathbf{j} + 3.00\,\mathbf{k}$

$$|\mathbf{B}| = \sqrt{4.00^2 + 6.00^2 + 3.00^2} = \boxed{7.81}$$

$$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2°}$$

$$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8°}$$

$$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4°}$$

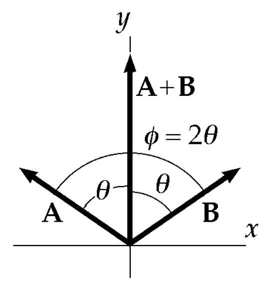
***1.52** Taking components along **i** and **j**, we get two equations:

$$6.00a - 8.00b + 26.0 = 0 \qquad -8.00a + 3.00b + 19.0 = 0$$

Solving simultaneously, $\boxed{a = 5.00,\ \ b = 7.00}$

Therefore, $5.00\mathbf{A} + 7.00\mathbf{B} + \mathbf{C} = 0$

***1.53** Since $\mathbf{A} + \mathbf{B} = 6.00\,\mathbf{j}$,



we have $\left(A_x + B_x\right)\mathbf{i} + \left(A_y + B_y\right)\mathbf{j} = 0\,\mathbf{i} + 6.00\,\mathbf{j}$

giving $A_x + B_x = 0$ or $A_x = -B_x$ [1]

and $A_y + B_y = 6.00$ [2]

Since both vectors have a magnitude of 5.00, we also have

$$A_x{}^2 + A_y{}^2 = B_x{}^2 + B_y{}^2 = 5.00^2$$

From $A_x = -B_x$, it is seen that $A_x{}^2 = B_x{}^2$

Therefore, $A_x{}^2 + A_y{}^2 = B_x{}^2 + B_y{}^2$ gives $A_y{}^2 = B_y{}^2$

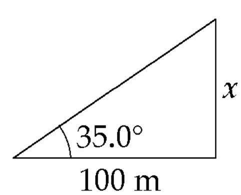
Then, $A_y = B_y$ and Eq. [2] gives $A_y = B_y = 3.00$

Defining θ as the angle between either **A** or **B** and the y axis, it is seen that

$$\cos\theta = \frac{A_y}{A} = \frac{B_y}{B} = \frac{3.00}{5.00} = 0.600 \qquad \text{and} \qquad \theta = 53.1°$$

The angle between **A** and **B** is then $\boxed{\phi = 2\theta = 106°}$

***1.54** $\tan 35.0° = \dfrac{x}{100\text{ m}}$



$x = (100\text{ m})\tan 35.0° = \boxed{70.0\text{ m}}$

***1.55** From the figure, we may see that the spacing between diagonal planes is half the distance between diagonally adjacent atoms on a flat plane. This diagonal distance may be obtained from the Pythagorean theorem, $L_{\text{diag}} = \sqrt{L^2 + L^2}$. Thus, since the atoms are separated by a distance $L = 0.200$ nm, the diagonal planes are separated by $\frac{1}{2}\sqrt{L^2 + L^2} = \boxed{0.141\text{ nm}}$

***1.56** It is desired to find the distance x such that $\dfrac{x}{100\text{ m}} = \dfrac{1000\text{ m}}{x}$

(i.e., such that x is the same multiple of 100m as the multiple that 1000 m is of x)

Thus, it is seen that $x^2 = (100\text{ m})(1000\text{ m}) = 1.00\times10^5\text{ m}^2$

and therefore $x = \sqrt{1.00\times10^5\text{ m}^2} = \boxed{316\text{ m}}$

1.57

$$A_{\text{total}} = (N)\left(A_{\text{drop}}\right) = \left(\frac{V_{\text{total}}}{V_{\text{drop}}}\right)\left(A_{\text{drop}}\right) = \left(\frac{V_{\text{total}}}{4\pi r^3/3}\right)\left(4\pi r^2\right)$$

$$A_{\text{total}} = \left(\frac{3V_{\text{total}}}{r}\right) = 3\left(\frac{30.0\times10^{-6}\text{ m}^3}{2.00\times10^{-5}\text{ m}}\right) = \boxed{4.50\text{ m}^2}$$

***1.58**

α' (deg)	α (rad)	$\tan(\alpha)$	$\sin(\alpha)$	difference
15.0	0.262	0.268	0.259	3.47%
20.0	0.349	0.364	0.342	6.43%
25.0	0.436	0.466	0.423	10.2%
24.0	0.419	0.445	0.407	9.34%
24.4	0.426	0.454	0.413	9.81%
24.5	0.428	0.456	0.415	9.87%
24.6	0.429	0.458	0.416	9.98%
24.7	0.431	0.460	0.418	10.1%

$\boxed{24.6°}$

1.59 The actual number of seconds in a year is $(86\,400\text{ s/day})(365.25\text{ day/yr}) = 31\,557\,600\text{ s/yr}$

The percent error in the approximation is

$$\frac{\left|\left(\pi\times10^7\text{ s/yr}\right) - \left(31\,557\,600\text{ s/yr}\right)\right|}{31\,557\,600\text{ s/yr}}\times100\% = \boxed{0.449\%}$$

***1.60** (a) The speed of flow may be found from $v = \dfrac{(\text{Vol rate of flow})}{(\text{Area}:\ \pi D^2/4)} = \dfrac{16.5\text{ cm}^3/\text{s}}{\pi(6.30\text{ cm})^2/4} = \boxed{0.529\text{ cm/s}}$

(b) Likewise, at a 1.35 cm diameter, $v = \dfrac{16.5\text{ cm}^3/\text{s}}{\pi(1.35\text{ cm})^2/4} = \boxed{11.5\text{ cm/s}}$

1.61 The volume of the galaxy is $\pi r^2 t = \pi\left(10^{21}\ \text{m}\right)^2\left(10^{19}\ \text{m}\right) \sim 10^{61}\ \text{m}^3$

If the distance between stars is 4×10^{16} m, then there is one star in a volume on the order of $\left(4\times10^{16}\ \text{m}\right)^3 \sim 10^{50}\ \text{m}^3$

The number of stars is about $\dfrac{10^{61}\ \text{m}^3}{10^{50}\ \text{m}^3/\text{star}} \sim \boxed{10^{11}\ \text{stars}}$

1.62 The position vector from the ground under the controller of the first airplane is

$$\mathbf{r}_1 = (19.2\ \text{km})(\cos 25°)\mathbf{i} + (19.2\ \text{km})(\sin 25°)\mathbf{j} + (0.8\ \text{km})\mathbf{k} = (17.4\mathbf{i} + 8.11\mathbf{j} + 0.8\mathbf{k})\ \text{km}$$

The second is at

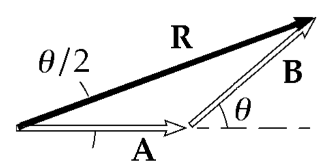
$$\mathbf{r}_2 = (17.6\ \text{km})(\cos 20°)\mathbf{i} + (17.6\ \text{km})(\sin 20°)\mathbf{j} + (1.1\ \text{km})\mathbf{k} = (16.5\mathbf{i} + 6.02\mathbf{j} + 1.1\mathbf{k})\ \text{km}$$

Now the displacement from the first plane to the second is

$$\mathbf{r}_2 - \mathbf{r}_1 = (-0.863\mathbf{i} - 2.09\mathbf{j} + 0.3\mathbf{k})\ \text{km}$$

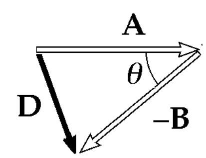
with magnitude $\sqrt{(0.863)^2 + (2.09)^2 + (0.3)^2} = \boxed{2.29\ \text{km}}$

***1.63** Let θ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\theta/2$, and $\theta/2$.



The magnitude of **R** is then $R = 2A\cos(\theta/2)$

[*Hint*: apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]



Again, **A**, **–B**, and **D** = **A – B** form an isosceles triangle with apex angle θ.

Applying the law of cosines and the identity $(1 - \cos\theta) = 2\sin^2(\theta/2)$

gives the magnitude of **D** as $D = 2A\sin(\theta/2)$

The problem requires that $R = nD$

or $\cos(\theta/2) = n\sin(\theta/2)$

giving $\theta = 2\tan^{-1}(1/n)$

***1.64** (a) You start at point A: $\mathbf{r}_1 = \mathbf{r}_A = (30.0\,\mathbf{i} - 20.0\,\mathbf{j})$ m

The displacement to B is $\mathbf{r}_B - \mathbf{r}_A = 60.0\,\mathbf{i} + 80.0\,\mathbf{j} - 30.0\,\mathbf{i} + 20.0\,\mathbf{j} = 30.0\,\mathbf{i} + 100\,\mathbf{j}$

You cover half of this, $(15.0\,\mathbf{i} + 50.0\,\mathbf{j})$

to move to $\mathbf{r}_2 = 30.0\,\mathbf{i} - 20.0\,\mathbf{j} + 15.0\,\mathbf{i} + 50.0\,\mathbf{j} = 45.0\,\mathbf{i} + 30.0\,\mathbf{j}$

Now the displacement from your current position to C is

$$\mathbf{r}_C - \mathbf{r}_2 = -10.0\,\mathbf{i} - 10.0\,\mathbf{j} - 45.0\,\mathbf{i} - 30.0\,\mathbf{j} = -55.0\,\mathbf{i} - 40.0\,\mathbf{j}$$

You cover one-third, moving to

$$\mathbf{r}_3 = \mathbf{r}_2 + \Delta\mathbf{r}_{23} = 45.0\,\mathbf{i} + 30.0\,\mathbf{j} + \frac{1}{3}(-55.0\,\mathbf{i} - 40.0\,\mathbf{j}) = 26.7\,\mathbf{i} + 16.7\,\mathbf{j}$$

The displacement from where you are to D is

$$\mathbf{r}_D - \mathbf{r}_3 = 40.0\,\mathbf{i} - 30.0\,\mathbf{j} - 26.7\,\mathbf{i} - 16.7\,\mathbf{j} = 13.3\,\mathbf{i} - 46.7\,\mathbf{j}$$

You traverse one-quarter of it, moving to

$$\mathbf{r}_4 = \mathbf{r}_3 + \tfrac{1}{4}(\mathbf{r}_D - \mathbf{r}_3) = 26.7\,\mathbf{i} + 16.7\,\mathbf{j} + \tfrac{1}{4}(13.3\,\mathbf{i} - 46.7\,\mathbf{j}) = 30.0\,\mathbf{i} + 5.00\,\mathbf{j}$$

The displacement from your new location to **E** is

$$\mathbf{r}_E - \mathbf{r}_4 = -70.0\,\mathbf{i} + 60.0\,\mathbf{j} - 30.0\,\mathbf{i} - 5.00\,\mathbf{j} = -100\,\mathbf{i} + 55.0\,\mathbf{j}$$

of which you cover one-fifth the distance, $-20.0\,\mathbf{i} + 11.0\,\mathbf{j}$, moving to

$$\mathbf{r}_4 + \Delta\mathbf{r}_{45} = 30.0\,\mathbf{i} + 5.00\,\mathbf{j} - 20.0\,\mathbf{i} + 11.0\,\mathbf{j} = 10.0\,\mathbf{i} + 16.0\,\mathbf{j}$$

The treasure is at $\boxed{(10.0\text{ m},\ 16.0\text{ m})}$

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\mathbf{r}_A + \frac{1}{2}(\mathbf{r}_B - \mathbf{r}_A) = \left(\frac{\mathbf{r}_A + \mathbf{r}_B}{2}\right)$$

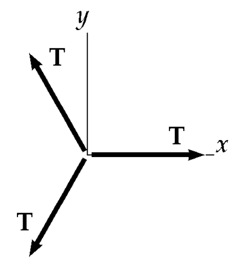
then to $$\frac{(\mathbf{r}_A + \mathbf{r}_B)}{2} + \frac{\mathbf{r}_C - (\mathbf{r}_A + \mathbf{r}_B)/2}{3} = \frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C}{3}$$

then to $$\frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)}{3} + \frac{\mathbf{r}_D - (\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)/3}{4} = \frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D}{4}$$

and last to $$\frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)}{4} + \frac{\mathbf{r}_E - (\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)/4}{5} = \frac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D + \mathbf{r}_E}{5}$$

This center of mass of the tree distribution is the same location whatever order we take the trees in.

***1.65** (a) Let T represent the force exerted by each child.



The x-component of the resultant force is

$$T\cos 0 + T\cos 120° + T\cos 240° = T(1) + T(-0.5) + T(-0.5) = 0$$

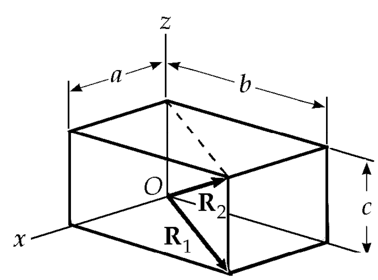
The y-component is

$$T\sin 0 + T\sin 120 + T\sin 240 = 0 + 0.866T - 0.866T = 0$$

Thus, $\Sigma\mathbf{F} = 0$

(b) If the total force is not zero, it must point in some direction. When each child moves one space clockwise, the total must turn clockwise by that angle, $360°/N$. Since each child exerts the same force, the new situation is identical to the old and the net force on the tire must still point in the original direction. The contradiction indicates that we were wrong in supposing that the total force is not zero. The total force *must* be zero.

1.66 (a) From the picture, $\mathbf{R}_1 = a\mathbf{i} + b\mathbf{j}$ and $|\mathbf{R}_1| = \sqrt{a^2 + b^2}$



(b) $\mathbf{R}_2 = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$;

its magnitude is $\sqrt{|\mathbf{R}_1|^2 + c^2} = \sqrt{a^2 + b^2 + c^2}$

ANSWERS TO EVEN NUMBERED PROBLEMS

2. $623\ \text{kg/m}^3$

4. $\dfrac{4\pi\rho\left(r_2^{\,3}-r_1^{\,3}\right)}{3}$

6. (a) 72.6 kg (b) 7.82×10^{26} atoms

8. (b) only

10. (a) ii (b) iii (c) i

12. 9.19 nm/s

14. 667 lb/s

16. (a) 190 yr (b) 2.32×10^{4} times

18. (a) 2.07 mm (b) 8.62×10^{13} times as large

20. $r_{\text{Al}}=r_{\text{Fe}}\sqrt[3]{\rho_{\text{Fe}}/\rho_{\text{Al}}}$

22. $\sim10^{7}$ or 10^{8} rev

24. $\sim10^{10}$ cans; $\sim10^{5}$ tons

26. 31 556 926.0 s

28. $(1.61\pm0.17)\times10^{3}\ \text{kg/m}^3$

30. (–2.75, –4.76) m

32. (a) 8.60 m (b) 4.47 m, –63.4°, 4.24 m, 135°

34. (a) 5.2 m at 60° (b) 3.0 m at 330°
(c) 3.0 m at 150° (d) 5.2 m at 300°

36. 310 km at 57° S of W

38. 86.6 m, –50.0 m

40. (a) See the solution. (b) $5.00\mathbf{i} + 4.00\mathbf{j}$, 6.40 at 38.7°, $-1.00\mathbf{i} + 8.00\mathbf{j}$, 8.06 at 97.2°

42. $C_x = 7.30$ cm, $C_y = -7.20$ cm

44. (a) 4.47 m at 63.4° (b) 8.49 m at 135°

46. (a) 185 N at 77.8° from the $+x$ axis (b) $(-39.3\,\mathbf{i} - 181\,\mathbf{j})$ N

48. 196 cm at –14.7°

50. (a) 10.4 cm (b) 35.5°

52. $a = 5.00$, $b = 7.00$

54. 70.0 m

56. 316 m

58. 24.6°

60. (a) 0.529 cm/s (b) 11.5 cm/s

62. 2.29 km

64. (10.0 m, 16.0 m)

66. (a) $\mathbf{R}_1 = a\,\mathbf{i} + b\,\mathbf{j}$; $R_1 = \sqrt{a^2 + b^2}$ (b) $\mathbf{R}_2 = a\,\mathbf{i} + b\,\mathbf{j} + c\,\mathbf{k}$; $R_2 = \sqrt{a^2 + b^2 + c^2}$