CHAPTER 2

ANSWERS TO QUESTIONS

Q2.1 Yes. Yes, if the particle winds up in the $+x$ region at the end.

Q2.2 Zero.

Q2.3 Yes. Yes.

Q2.4 (a) Accelerating East (b) Braking East (c) Cruising East
(d) Braking West (e) Accelerating West (f) Cruising West
(g) Stopped but starting to move East
(h) Stopped but starting to move West

Q2.5 No. Constant acceleration only. Yes. Zero is a constant.

Q2.6 They are the same! (Solve the kinematic equations and see!)

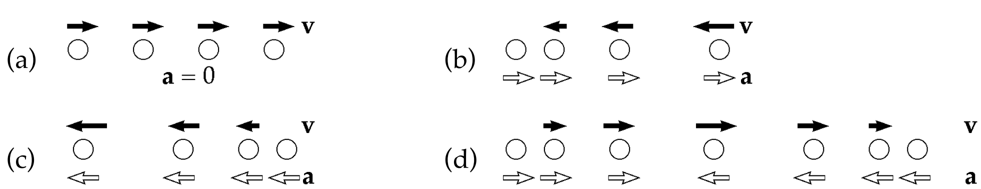
Q2.7 No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past.

Q2.8 No. Their velocities are different vectors, different in direction.

Q2.9 Above. Your ball has zero initial speed and smaller average speed during the time of flight to the passing point.

Q2.10 For $\theta = 90°$, the acceleration is g. For $\theta = 0$, the acceleration is zero. Consistent with these limits is $a = g\sin\theta$.

Q2.11 We use solid arrows for velocity and open arrows for acceleration. Pictures (a) and (d) read toward the right. Pictures (b) and (c) start at the right and read toward the left.



Q2.12 On the 11th day. Then two more weeks suffice for its height to get four times larger.

Q2.13 Ignoring air resistance, in 16 s the pebble would fall $x = \frac{1}{2}gt^2 = \frac{1}{2}\left(9.8 \text{ m/s}^2\right)\left(16 \text{ s}\right)^2 = 1.25 \text{ km}$.
Air resistance is an important force after the first few seconds, when the pebble has attained high speed. Also, part of the 16-s time must be occupied by the sound returning up the well. Thus the depth is less than 1.25 km.

Q2.14 To travel 300 km at 25 cm/yr requires time $t \sim 3 \times 10^5 \text{ m}\Big/\left(2.5 \times 10^{-2} \text{ m/yr}\right) \sim 10^7 \text{ yr}$

Q2.15 With $h = \frac{1}{2}gt^2$, (a) $0.5h = \frac{1}{2}g(0.707t)^2$. The time is later than $0.5t$.
(b) The distance fallen is $0.25h = \frac{1}{2}g(0.5t)^2$. The elevation is $0.75h$, greater than $0.5h$.

PROBLEM SOLUTIONS

2.1 (a) $\bar{v} = \boxed{2.30 \text{ m/s}}$

(b) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5 \text{ m} - 9.20 \text{ m}}{3.00 \text{ s}} = \boxed{16.1 \text{ m/s}}$

(c) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5 \text{ m} - 0 \text{ m}}{5.00 \text{ s}} = \boxed{11.5 \text{ m/s}}$

2.2 (a) Displacement $= \left(8.50\times 10^4 \text{ m/h}\right)\left(\dfrac{35.0}{60.0} \text{ h}\right) + 130\times 10^3 \text{ m}$

$x = (49.6 + 130)\times 10^3 \text{ m} = \boxed{180 \text{ km}}$

(b) Average velocity $= \dfrac{\text{displacement}}{\text{time}} = \dfrac{180 \text{ km}}{\left(\dfrac{35.0 + 15.0}{60.0} + 2.00\right) \text{h}} = \boxed{63.4 \text{ km/h}}$

2.3 (a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{10 \text{ m}}{2 \text{ s}} = \boxed{5 \text{ m/s}}$

(b) $\bar{v} = \dfrac{5 \text{ m}}{4 \text{ s}} = \boxed{1.2 \text{ m/s}}$

(c) $\bar{v} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{5 \text{ m} - 10 \text{ m}}{4 \text{ s} - 2 \text{ s}} = \boxed{-2.5 \text{ m/s}}$

(d) $\bar{v} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{-5 \text{ m} - 5 \text{ m}}{7 \text{ s} - 4 \text{ s}} = \boxed{-3.3 \text{ m/s}}$

(e) $\bar{v} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{0 - 0}{8 - 0} = \boxed{0 \text{ m/s}}$

***2.4** $x = 10t^2$: For

t(s)	=	2.0	2.1	3.0
x(m)	=	40	44.1	90

(a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{50 \text{ m}}{1.0 \text{ s}} = \boxed{50.0 \text{ m/s}}$

(b) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{4.1 \text{ m}}{0.1 \text{ s}} = \boxed{41.0 \text{ m/s}}$

2.5 (a) $\bar{v} = \dfrac{\text{Total distance}}{\text{Total time}}$

Let d be the distance from A to B.

Then the time required is $\dfrac{d}{v_1} + \dfrac{d}{v_2}$

And the average speed is $\bar{v} = \dfrac{2d}{\dfrac{d}{v_1} + \dfrac{d}{v_2}} = \boxed{\dfrac{2v_1 v_2}{v_1 + v_2}}$

(b) With total displacement zero, her average velocity is $\boxed{0}$

***2.6** (a) At any time, t, the position is given by $x = (3.00\ \text{m/s}^2)t^2$

Thus, at $t_i = 3.00$ s: $x_i = (3.00\ \text{m/s}^2)(3.00\ \text{s})^2 = \boxed{27.0\ \text{m}}$

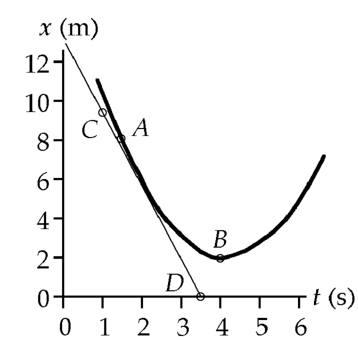
(b) At $t_f = 3.00\ \text{s} + \Delta t$: $x_f = (3.00\ \text{m/s}^2)(3.00\ \text{s} + \Delta t)^2$, or

$x_f = \boxed{27.0\ \text{m} + (18.0\ \text{m/s})\Delta t + \left(3.00\ \text{m/s}^2\right)(\Delta t)^2}$

(c) The instantaneous velocity at $t = 3.00$ s is:

$$v = \lim_{\Delta t \to 0}\left(\frac{x_f - x_i}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(18.0\ \text{m/s} + \left(3.00\ \text{m/s}^2\right)\Delta t\right) = \boxed{18.0\ \text{m/s}}$$

2.7 (a) at $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)



at $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$\bar{v} = \dfrac{x_f - x_i}{t_f - t_i} = \dfrac{(2.0 - 8.0)\ \text{m}}{(4 - 1.5)\ \text{s}} = -\dfrac{6.0\ \text{m}}{2.5\ \text{s}} = \boxed{-2.4\ \text{m/s}}$

(b) The slope of the tangent line is found from points C and D.

($t_C = 1.0$ s, $x_C = 9.5$ m) and ($t_D = 3.5$ s, $x_D = 0$),

$v \cong \boxed{-3.8\ \text{m/s}}$

(c) The velocity is zero when x is a minimum. This is at $t \cong \boxed{4\ \text{s}}$

2.8 (b) At $t = 5.0$ s, the slope is $v \cong \dfrac{58\text{ m}}{2.5\text{ s}} \cong \boxed{23\text{ m/s}}$

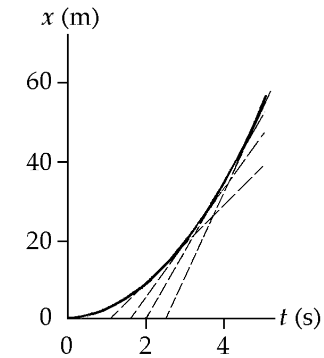
At $t = 4.0$ s, the slope is $v \cong \dfrac{54\text{ m}}{3\text{ s}} \cong \boxed{18\text{ m/s}}$

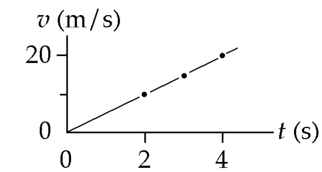
At $t = 3.0$ s, the slope is $v \cong \dfrac{49\text{ m}}{3.4\text{ s}} \cong \boxed{14\text{ m/s}}$

At $t = 2.0$ s, the slope is $v \cong \dfrac{36\text{ m}}{4.0\text{ s}} \cong \boxed{9.0\text{ m/s}}$

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} \cong \dfrac{23\text{ m/s}}{5.0\text{ s}} \cong \boxed{4.6\text{ m/s}^2}$

(d) Initial velocity of the car was $\boxed{\text{zero}}$



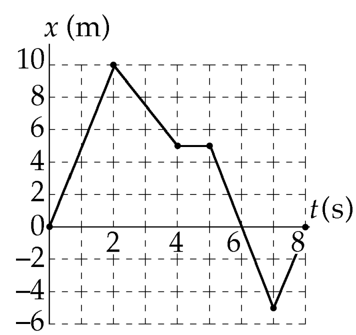


2.9 (a) $v = \dfrac{(5-0)\text{ m}}{(1-0)\text{ s}} = \boxed{5\text{ m/s}}$

(b) $v = \dfrac{(5-10)\text{ m}}{(4-2)\text{ s}} = \boxed{-2.5\text{ m/s}}$

(c) $v = \dfrac{(5\text{ m}-5\text{ m})}{(5\text{ s}-4\text{ s})} = \boxed{0}$

(d) $v = \dfrac{0-(-5\text{ m})}{(8\text{ s}-7\text{ s})} = \boxed{+5\text{ m/s}}$



***2.10** The speed is $$v_x = 100\ \frac{\text{km}}{\text{h}}\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 27.8\text{ m/s}$$

The length of the combination of truck and trailers is $x = v_x t = (27.8\text{ m/s})0.6\text{ s} = 16.7\text{ m}$

While some part of the rig is on the bridge, the front bumper moves forward by 416.7 m. This requires time

$$t = \frac{x}{v_x} = \frac{417\text{ m}}{27.8\text{ m/s}} = \boxed{15.0\text{ s}}$$

***2.11** Once it resumes the race, the hare will run for a time of $$t = \frac{x_f - x_i}{v_x} = \frac{1000\text{ m} - 800\text{ m}}{8\text{ m/s}} = 25\text{ s}$$

In this time, the tortoise can crawl a distance $$x_f - x_i = (0.2\text{ m/s})(25\text{ s}) = \boxed{5.00\text{ m}}$$

2.12 Choose the positive direction to be the outward direction, perpendicular to the wall.

$$v_f = v_i + at: \quad a = \frac{\Delta v}{\Delta t} = \frac{22.0 \text{ m/s} - (-25.0 \text{ m/s})}{3.50 \times 10^{-3} \text{ s}} = \boxed{1.34 \times 10^4 \text{ m/s}^2}$$

2.13 (a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{8.00 \text{ m/s}}{6.00 \text{ s}} = \boxed{1.3 \text{ m/s}^2}$

(b) Maximum positive acceleration is at $t = 3$ s, and is approximately $\boxed{2 \text{ m/s}^2}$

(c) $a = 0$, at $\boxed{t = 6 \text{ s}}$, and also for $\boxed{t > 10 \text{ s}}$

(d) Maximum negative acceleration is at $t = 8$ s, and is approximately $\boxed{-1.5 \text{ m/s}^2}$

***2.14** (a) At $t = 2.00$ s, $x = [3.00(2.00)^2 - 2.00(2.00) + 3.00]$ m $= 11.0$ m

At $t = 3.00$ s, $x = [3.00(9.00)^2 - 2.00(3.00) + 3.00]$ m $= 24.0$ m

so $\quad \bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{24.0 \text{ m} - 11.0 \text{ m}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{13.0 \text{ m/s}}$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}\left(3.00t^2 - 2.00t + 3.00\right) = \left(6.00t - 2.00\right) \text{ m/s}$$

At $t = 2.00$ s, $v = [6.00(2.00) - 2.00]$ m/s $= \boxed{10.0 \text{ m/s}}$

At $t = 3.00$ s, $v = [6.00(3.00) - 2.00]$ m/s $= \boxed{16.0 \text{ m/s}}$

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{16.0 \text{ m/s} - 10.0 \text{ m/s}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{6.00 \text{ m/s}^2}$

(d) At all times $a = \dfrac{d}{dt}(6.00 - 2.00) = \boxed{6.00 \text{ m/s}^2}$ (This includes both $t = 2.00$ s and $t = 3.00$ s).

2.15 $x = 2.00 + 3.00t - t^2$, $\quad v = \dfrac{dx}{dt} = 3.00 - 2.00t$, $\quad a = \dfrac{dv}{dt} = -2.00$

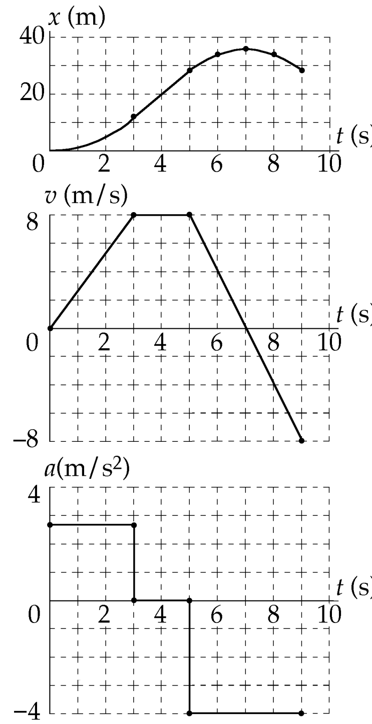
At $t = 3.00$ s:

(a) $x = (2.00 + 9.00 - 9.00)$ m $= \boxed{2.00 \text{ m}}$

(b) $v = (3.00 - 6.00)$ m/s $= \boxed{-3.00 \text{ m/s}}$

(c) $a = \boxed{-2.00 \text{ m/s}^2}$

***2.16** (a) See the graphs at the right.



Choose $x = 0$ at $t = 0$

At $t = 3$ s, $x = \frac{1}{2}(8\text{ m/s})(3\text{ s}) = 12$ m

At $t = 5$ s, $x = 12\text{ m} + (8\text{ m/s})(2\text{ s}) = 28$ m

At $t = 7$ s, $x = 28\text{ m} + \frac{1}{2}(8\text{ m/s})(2\text{ s}) = 36$ m

(b) For $0 < t < 3$ s, $a = (8\text{ m/s})/3\text{ s} = 2.67\text{ m/s}^2$

For $3 < t < 5$ s, $a = 0$

(c) For $5\text{ s} < t < 9$ s, $a = -(16\text{ m/s})/4\text{ s} = \boxed{-4\text{ m/s}^2}$

(d) At $t = 6$ s, $x = 28\text{ m} + (6\text{ m/s})(1\text{ s}) = \boxed{34\text{ m}}$

(e) At $t = 9$ s, $x = 36\text{ m} + \frac{1}{2}(-8\text{ m/s})\ 2\text{ s} = \boxed{28\text{ m}}$

***2.17** (a) Acceleration is constant over the first ten seconds, so at the end,

$$v_f = v_i + at = 0 + (2.00\text{ m/s}^2)(10.0\text{ s}) = \boxed{20.0\text{ m/s}}$$

Then $a = 0$ so v is constant from $t = 10.0$ s to $t = 15.0$ s. And over the last five seconds the velocity changes to

$$v_f = v_i + at = 20.0\text{ m/s} + (3.00\text{ m/s}^2)(5.00\text{ s}) = \boxed{5.00\text{ m/s}}$$

(b) In the first ten seconds,

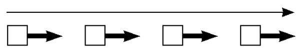
$$x_f = x_i + v_i t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}(2.00\text{ m/s}^2)(10.0\text{ s})^2 = 100\text{ m}$$

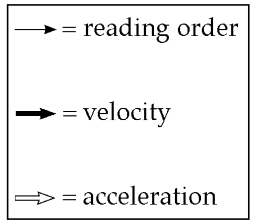
Over the next five seconds the position changes to

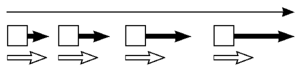
$$x_f = x_i + v_i t + \tfrac{1}{2}at^2 = 100\text{ m} + (20.0\text{ m/s})(5.00\text{ s}) + 0 = 200\text{ m}$$

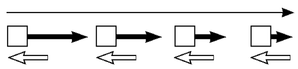
And at $t = 20.0$ s,

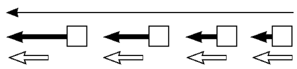
$$x_f = x_i + v_i t + \tfrac{1}{2}at^2 = 200\text{ m} + (20.0\text{ m/s})(5.00\text{ s}) + \tfrac{1}{2}(-3.00\text{ m/s}^2)(5.00\text{ s})^2 = \boxed{262\text{ m}}$$

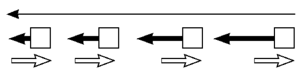
***2.18** (a) 



(b) 

(c) 

(d) 

(e) 

(f) One way of phrasing the answer: The spacing of the successive positions would change with less regularity.

Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the accelerations vectors would vary in magnitude and direction.

***2.19** (a) Choose the initial point where the pilot reduces the throttle and the final point where the boat passes the buoy:

$x_i = 0,\ x_f = 100\text{ m},\ v_{xi} = 30\text{ m/s},\ v_{xf} = ?,\ a_x = -3.5\text{ m/s}^2,\ t = ?$

$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$: $\qquad 100\text{ m} = 0 + (30\text{ m/s})t + \frac{1}{2}\left(-3.5\text{ m/s}^2\right)t^2$

$$\left(1.75\text{ m/s}^2\right)t^2 - 30\text{ m/s }t + 100\text{ m} = 0$$

We use the quadratic formula: $\qquad t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$t = \frac{30\text{ m/s} \pm \sqrt{900\text{ m}^2/\text{s}^2 - 4\left(1.75\text{ m/s}^2\right)(100\text{ m})}}{2\left(1.75\text{ m/s}^2\right)} = \frac{30\text{ m/s} \pm 14.1\text{ m/s}}{3.5\text{ m/s}^2} = 12.6\text{ s} \quad \text{or} \quad \boxed{4.53\text{ s}}$$

The smaller value is the physical answer. If the boat kept moving with the same acceleration, it would stop and move backward, then gain speed, and pass the buoy again at 12.6 s.

(b) $v_{xf} = v_{xi} + a_x t = 30\text{ m/s} - \left(3.5\text{ m/s}^2\right)4.53\text{ s} = \boxed{14.1\text{ m/s}}$

***2.20** (a) Assuming a constant acceleration: $\qquad a = \dfrac{v_f - v_i}{t} = \dfrac{42.0\text{ m/s}}{8.00\text{ s}} = \boxed{5.25\text{ m/s}^2}$

(b) Taking the origin at the original position of the car,

$$x_f = \frac{1}{2}\left(v_i + v_f\right)t = \frac{1}{2}(42.0\text{ m/s})(8.00\text{ s}) = \boxed{168\text{ m}}$$

(c) From $v_f = v_i + at$, the velocity 10.0 s after the car starts from rest is:

$$v_f = 0 + \left(5.25\text{ m/s}^2\right)(10.0\text{ s}) = \boxed{52.5\text{ m/s}}$$

2.21 Given $v_i = 12.0\ \text{cm/s}$ when $x_i = 3.00\ \text{cm}$ $(t = 0)$, and at $t = 2.00\ \text{s}$, $x_f = -5.00\ \text{cm}$

$$x_f - x_i = v_i t + \tfrac{1}{2}at^2: \qquad -5.00 - 3.00 = 12.0(2.00) + \tfrac{1}{2}a(2.00)^2$$

$$-8.00 = 24.0 + 2a \qquad a = -\frac{32.0}{2} = \boxed{-16.0\ \text{cm/s}^2}$$

2.22 Suppose the unknown acceleration is constant as a car moving at $v_i = 35.0\ \text{mi/h}$ comes to a $v_f = 0$ stop in $x_f - x_i = 40.0\ \text{ft}$. We find its acceleration from $v_f^2 = v_i^2 + 2a(x_f - x_i)$:

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{0 - (35.0\ \text{mi/h})^2}{2(40.0\ \text{ft})}\left(\frac{5280\ \text{ft}}{\text{mi}}\right)^2\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)^2 = -32.9\ \text{ft/s}^2$$

Now consider a car moving at $v_i = 70.0\ \text{mi/h}$ and stopping to $v_f = 0$ with $a = -32.9\ \text{ft/s}^2$. From the same equation its stopping distance is

$$x_f - x_i = \frac{v_f^2 - v_i^2}{2a} = \frac{0 - (70.0\ \text{mi/h})^2}{2(-32.9\ \text{ft/s}^2)}\left(\frac{5280\ \text{ft}}{1\ \text{mi}}\right)^2\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)^2 = \boxed{160\ \text{ft}}$$

***2.23** $v_i = 5.20\ \text{m/s}$

(a) $v(t = 2.50\ \text{s}) = v_i + at = 5.20\ \text{m/s} + (3.00\ \text{m/s}^2)(2.50\ \text{s}) = \boxed{12.7\ \text{m/s}}$

(b) $v(t = 2.50\ \text{s}) = v_i + at = 5.20\ \text{m/s} + (-3.00\ \text{m/s}^2)(2.50\ \text{s}) = \boxed{-2.30\ \text{m/s}}$

***2.24** (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form $x_f = x_i + v_i t + \frac{1}{2}at^2$ to recognize that $x_i = 2.00\ \text{m}$, $v_i = 3.00\ \text{m/s}$, and $a = -8.00\ \text{m/s}^2$

The velocity equation, $v_f = v_i + at$, is then $v_f = 3.00\ \text{m/s} - (8.00\ \text{m/s}^2)t$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}\ \text{s}$. The position at this time is:

$$x = 2.00\ \text{m} + (3.00\ \text{m/s})\left(\tfrac{3}{8}\ \text{s}\right) - (4.00\ \text{m/s}^2)\left(\tfrac{3}{8}\ \text{s}\right)^2 = \boxed{2.56\ \text{m}}$$

(b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -2v_i/a$. Thus, when the particle returns to its initial position,

the time is $t = \dfrac{-2(3.00\ \text{m/s})}{-8.00\ \text{m/s}^2} = \dfrac{3}{4}\ \text{s}$

and the velocity is $v_f = 3.00\ \text{m/s} - (8.00\ \text{m/s}^2)\left(\tfrac{3}{4}\ \text{s}\right) = \boxed{-3.00\ \text{m/s}}$

2.25 (a) $v_i = 100 \text{ m/s}$, $a = -5.00 \text{ m/s}^2$ $v_f = v_i + at$ so $0 = 100 - 5t$

$v_f^2 = v_i^2 + 2a(x_f - x_i)$ so $0 = (100)^2 - 2(5.00)(x_f - 0)$

Thus $x_f =$ 1000 m and $t = \boxed{20.0 \text{ s}}$

(b) At this acceleration the plane would overshoot the runway: $\boxed{\text{No}}$

***2.26** (a) Take $t_i = 0$ at the bottom of the hill where $x_i = 0$, $v_i = 30.0 \text{ m/s}$, $a = -2.00 \text{ m/s}^2$.

Use these values in the general equation $x_f = x_i + v_i t + \frac{1}{2}at^2$

to find $x_f = 0 + (30.0t \text{ m/s}) + \frac{1}{2}\left(-2.00 \text{ m/s}^2\right)t^2$

when t is in seconds $\boxed{x_f = \left(30.0t - t^2\right) \text{ m}}$

To find an equation for the velocity, use $v_f = v_i + at = 30.0 \text{ m/s} + \left(-2.00 \text{ m/s}^2\right)t$

$$\boxed{v_f = (30.0 - 2.00t) \text{ m/s}}$$

(b) The distance of travel x_f becomes a maximum, $x_{\max}$, when $v_f = 0$ (turning point in the motion). Use the expressions found in part (a) for v_f to find the value of t when x_f has its maximum value:

From $v_f = (3.00 - 2.00t) \text{ m/s}$, $v_f = 0$ when $t = 15.0 \text{ s}$

Then $x_{\max} = \left(30.0t - t^2\right) \text{ m} = (30.0)(15.0) - (15.0)^2 = \boxed{225 \text{ m}}$

2.27 In the simultaneous equations:

$$\left\{\begin{matrix} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \end{matrix}\right\} \quad \text{we have} \quad \left\{\begin{matrix} v_{xf} = v_{xi} - \left(5.60 \text{ m/s}^2\right)(4.20 \text{ s}) \\ 62.4 \text{ m} = \frac{1}{2}\left(v_{xi} + v_{xf}\right)(4.20 \text{ s}) \end{matrix}\right\}$$

So substituting for v_{xi} gives $62.4 \text{ m} = \dfrac{1}{2}\left[v_{xf} + \left(5.60 \text{ m/s}^2\right)(4.20 \text{ s}) + v_{xf}\right](4.20 \text{ s})$

$$14.9 \text{ m/s} = v_{xf} + \tfrac{1}{2}\left(5.60 \text{ m/s}^2\right)(4.20 \text{ s})$$

Thus $v_{xf} = \boxed{3.10 \text{ m/s}}$

2.28 Take any two of the standard four equations, such as $\begin{Bmatrix} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \end{Bmatrix}$

Solve one for v_{xi}, and substitute into the other: $v_{xi} = v_{xf} - a_x t$

$$x_f - x_i = \frac{1}{2}\left(v_{xf} - a_x t + v_{xf}\right)t$$

Thus $\boxed{x_f - x_i = v_{xf}\,t - \frac{1}{2}a_x t^2}$

Back in problem 27, $62.4\text{ m} = v_{xf}(4.20\text{ s}) - \frac{1}{2}\left(-5.60\text{ m/s}^2\right)(4.20\text{ s})^2$

$$v_{xf} = \frac{62.4\text{ m} - 49.4\text{ m}}{4.20\text{ s}} = \boxed{3.10\text{ m/s}}$$

2.29 (a) $a = \frac{v_f - v_i}{t} = \frac{632(5280/3600)}{1.40} = \boxed{-662\text{ ft/s}^2} = -202\text{ m/s}^2$

(b) $x_f = v_i t + \frac{1}{2}at^2 = (632)(5280/3600)(1.40) - \frac{1}{2}662(1.40)^2 = \boxed{649\text{ ft}} = \boxed{198\text{ m}}$

2.30 (a) $v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$: $\left[0.01\left(3\times10^8\text{ m/s}\right)\right]^2 = 0 + 2a_x(40\text{ m})$

$$a_x = \frac{\left(3\times10^6\text{ m/s}\right)^2}{80\text{ m}} = \boxed{1.12\times10^{11}\text{ m/s}^2}$$

(b) We must find separately the time t_1 for speeding up and the time t_2 for coasting:

$x_f - x_i = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t_1$: $40\text{ m} = \frac{1}{2}\left(3\times10^6\text{ m/s} + 0\right)t_1$

$$t_1 = 2.67\times10^{-5}\text{ s}$$

$x_f - x_i = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t_2$: $60\text{ m} = \frac{1}{2}\left(3\times10^6\text{ m/s} + 3\times10^6\text{ m/s}\right)t_2$

$$t_2 = 2.00\times10^{-5}\text{ s}$$

$$\text{total time} = \boxed{4.67\times10^{-5}\text{ s}}$$

***2.31** (a) The time it takes the truck to reach 20.0 m/s is found from $v_f = v_i + at$.

Solving for t yields $$t = \frac{v_f - v_i}{a} = \frac{20.0\text{ m/s} - 0\text{ m/s}}{2.00\text{ m/s}^2} = 10.0\text{ s}$$

The total time is thus $$10.0\text{ s} + 20.0\text{ s} + 5.00\text{ s} = \boxed{35.0\text{ s}}$$

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

$$x_1 = \bar{v}t = \left(\frac{0 + 20.0}{2}\right)(10.0) = 100\text{ m}$$

With a being 0 for this interval, the distance traveled during the next 20.0 s is

$$x_2 = v_i t + \tfrac{1}{2}at^2 = (20.0)(20.0) + 0 = 400\text{ m}$$

The distance traveled in the last 5.00 s is $$x_3 = \bar{v}t = \left(\frac{20.0 + 0}{2}\right)(5.00) = 50.0\text{ m}$$

The total distance $x = x_1 + x_2 + x_3 = 100 + 400 + 50 = 550\text{ m}$, and the average velocity is given by

$$\bar{v} = \frac{x}{t} = \frac{550}{35.0} = \boxed{15.7\text{ m/s}}$$

2.32 (a) Take initial and final points at top and bottom of the incline.

If the ball starts from rest, $$v_i = 0,\ a = 0.500\text{ m/s}^2,\ \ x_f - x_i = 9.00\text{ m}$$

Then $$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 0^2 + 2\left(0.500\text{ m/s}^2\right)(9.00\text{ m})$$

$$v_f = \boxed{3.00\text{ m/s}}$$

(b) $x_f - x_i = v_i t + \tfrac{1}{2}at^2$ $$9.00 = 0 + \tfrac{1}{2}\left(0.500\text{ m/s}^2\right)t^2$$

$$t = \boxed{6.00\text{ s}}$$

(c) Take initial and final points at the bottom of the planes and the top of the second plane, respectively:

$$v_i = 3.00\text{ m/s},\ v_f = 0,\ x_f - x_i = 15.00\text{ m}$$

$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$ gives $$a = \frac{v_f^2 - v_i^2}{2\left(x_f - x_i\right)} = \frac{\left[0 - (3.00\text{ m/s})^2\right]}{2(15.0\text{ m})} = \boxed{-0.300\text{ m/s}^2}$$

(d) Take the initial point at the bottom of the planes and the final point 8.00 m along the second:

$v_i = 3.00\text{ m/s},\qquad x_f - x_i = 8.00\text{ m},\qquad a = -0.300\text{ m/s}^2$

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = (3.00\text{ m/s})^2 + 2\left(-0.300\text{ m/s}^2\right)(8.00\text{ m}) = 4.20\text{ m}^2/\text{s}^2$$

$$v_f = \boxed{2.05\text{ m/s}}$$

2.33 Take the original point to be when Sue notices the van. Choose the origin of the x-axis at Sue's car. For her we have

$$x_{is} = 0 \qquad v_{is} = 30.0\ \mathrm{m/s} \qquad a_s = -2.00\ \mathrm{m/s^2}$$

so her position is given by

$$x_s(t) = x_{is} + v_{is}t + \tfrac{1}{2}a_s t^2 = (30.0\ \mathrm{m/s})t + \tfrac{1}{2}\left(-2.00\ \mathrm{m/s^2}\right)t^2$$

For the van, $x_{iv} = 155\ \mathrm{m}$ $\quad v_{iv} = 5.00\ \mathrm{m/s}$ $\quad a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \tfrac{1}{2}a_v t^2 = 155 + (5.00\ \mathrm{m/s})t + 0$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^{\,2} = 155 + 5.00t_c$$

$$0 = t_c^{\,2} - 25.0t_c + 155$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6\ \mathrm{s} \quad \text{or} \quad \boxed{11.4\ \mathrm{s}}$$

The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position $155\ \mathrm{m} + (5.00\ \mathrm{m/s})(11.4\ \mathrm{s}) = \boxed{212\ \mathrm{m}}$

***2.34** Choose the origin ($y = 0,\ t = 0$) at the starting point of the ball and take upward as positive. Then $y_i = 0,\ v_i = 0$, and $a = -g = -9.80\ \mathrm{m/s^2}$. The position and the velocity at time t become:

$$y_f - y_i = v_i t + \tfrac{1}{2}at^2: \quad y_f = -\tfrac{1}{2}gt^2 = \tfrac{1}{2}\left(9.80\ \mathrm{m/s^2}\right)t^2$$

and $v_f = v_i + at$: $\quad v_f = -gt = -\left(9.80\ \mathrm{m/s^2}\right)t$

(a) at $t = 1.00\ \mathrm{s}$: $y_f = -\tfrac{1}{2}\left(9.80\ \mathrm{m/s^2}\right)(1.00\ \mathrm{s})^2 = \boxed{-4.90\ \mathrm{m}}$

at $t = 2.00\ \mathrm{s}$: $y_f = -\tfrac{1}{2}\left(9.80\ \mathrm{m/s^2}\right)(2.00\ \mathrm{s})^2 = \boxed{-19.6\ \mathrm{m}}$

at $t = 3.00\ \mathrm{s}$: $y_f = -\tfrac{1}{2}\left(9.80\ \mathrm{m/s^2}\right)(3.00\ \mathrm{s})^2 = \boxed{-44.1\ \mathrm{m}}$

at $t = 1.00\ \mathrm{s}$: $v_f = -\left(9.80\ \mathrm{m/s^2}\right)(1.00\ \mathrm{s}) = \boxed{-9.80\ \mathrm{m/s}}$

at $t = 2.00\ \mathrm{s}$: $v_f = -\left(9.80\ \mathrm{m/s^2}\right)(2.00\ \mathrm{s}) = \boxed{-19.6\ \mathrm{m/s}}$

at $t = 3.00\ \mathrm{s}$: $v_f = -\left(9.80\ \mathrm{m/s^2}\right)(3.00\ \mathrm{s}) = \boxed{-29.4\ \mathrm{m/s}}$

2.35 (a) $y_f - y_i = v_i t + \frac{1}{2}at^2$: $4.00 = (1.50)v_i - (4.90)(1.50)^2$

and $v_i = \boxed{10.0 \text{ m/s upward}}$

(b) $v_f = v_i + at = 10.0 - (9.80)(1.50) = -4.68 \text{ m/s}$ $\quad v_f = \boxed{4.68 \text{ m/s downward}}$

2.36 We have $y_f = -\frac{1}{2}gt^2 + v_i t + y_i$

$$0 = -\left(4.90 \text{ m/s}^2\right)t^2 - (8.00 \text{ m/s})t + 30.0 \text{ m}$$

Solving for t, $t = \dfrac{8.00 \pm \sqrt{64.0 + 588}}{-9.80}$

Using only the positive value for t, we find that $t = \boxed{1.79 \text{ s}}$

2.37 (a) $v_f = v_i - gt$: $\quad v_f = 0$ when $t = 3.00$ s, $g = 9.80 \text{ m/s}^2$

Therefore, $v_i = gt = \left(9.80 \text{ m/s}^2\right)(3.00 \text{ s}) = \boxed{29.4 \text{ m/s}}$

(b) $y_f - y_i = \frac{1}{2}\left(v_f + v_i\right)t$ $\quad y_f - y_i = \frac{1}{2}(29.4 \text{ m/s})(3.00 \text{ m/s}) = \boxed{44.1 \text{ m}}$

***2.38** Assume that air resistance may be neglected. Then, the acceleration at all times during the flight is that due to gravity, $a = -g = -9.80 \text{ m/s}^2$. During the flight, Goff went 1 mile (1609 m) up and then 1 mile back down. Determine his speed just after launch by considering his upward flight:

$v_f^2 = v_i^2 + 2a\left(y_f - y_i\right)$: $\quad 0 = v_i^2 - 2\left(9.80 \text{ m/s}^2\right)(1609 \text{ m})$

$$v_i = 178 \text{ m/s}$$

His time in the air may be found by considering his motion from just after launch to just before impact:

$y_f - y_i = v_i t + \frac{1}{2}at^2$: $\quad 0 = (178 \text{ m/s})t - \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)t^2$

The root $t = 0$ describes launch; the other root, $t = 36.2$ s, describes his flight time. His rate of pay may then be found from

$$\text{pay rate} = \frac{\$1.00}{36.2 \text{ s}} = (0.0276 \text{ \$/s})(3600 \text{ s/h}) = \boxed{\$99.4/\text{h}}$$

We have assumed that the workman's flight time, "a mile", and "a dollar", were measured to three-digit precision. We have interpreted "up in the sky" as referring to the free fall time, not to the launch and landing times. Both the takeoff and landing times must be several seconds away from the job, in order for Goff to survive to resume work.

2.39 Time to fall 3.00 m is found from Eq. 2.11 with $v_i = 0$, $3.00\text{ m} = \frac{1}{2}\left(9.80\text{ m/s}^2\right)t^2$

$t = 0.782\text{ s}$

(a) With the horse galloping at 10.0 m/s, the horizontal distance is $vt = \boxed{7.82\text{ m}}$

(b) $t = \boxed{0.782\text{ m}}$

***2.40** (a) For the stone thrown down, $v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$

$$v_{xf}^2 = (-12\text{ m/s})^2 + 2\left(-9.8\text{ m/s}^2\right)(-20\text{ m} - 0)$$

$$v_{xf} = \pm\sqrt{536\text{ m}^2/\text{s}^2} = \boxed{-23.2\text{ m/s}}$$

For the stone thrown up, the same equation gives $v_{xf}^2 = (+12\text{ m/s})^2 + 2\left(-9.8\text{ m/s}^2\right)(-20\text{ m} - 0)$

$$v_{xf} = \pm\sqrt{536\text{ m}^2/\text{s}^2} = \boxed{-23.2\text{ m/s}}$$

(b) For the stone thrown down, $\bar{v}_x = \frac{1}{2}\left(v_{xi} + v_{xf}\right) = \frac{1}{2}(-12\text{ m/s} - 23.2\text{ m/s})$

$$\bar{v}_x = \boxed{-17.6\text{ m/s}}$$

For the stone thrown up, $\bar{v}_x = \frac{1}{2}(12\text{ m/s} - 23.2\text{ m/s}) = \boxed{-5.58\text{ m/s}}$

2.41 (a) $v_f = v_i - gt = 0$ where $v_i = 15.0\text{ m/s}$: $t = \dfrac{v_i}{g} = \dfrac{15.0\text{ m/s}}{9.80\text{ m/s}^2} = \boxed{1.53\text{ s}}$

(b) $h = v_i t - \frac{1}{2}gt^2 = \dfrac{v_i^2}{2g}$: $h = \dfrac{225}{19.6}\text{ m} = \boxed{11.5\text{ m}}$

(c) At $t = 2.00\text{ s}$, $v_f = v_i - gt$: $v_f = 15.0 - 19.6 = \boxed{-4.60\text{ m/s}}$

$$a = -g = \boxed{-9.80\text{ m/s}^2}$$

2.42 $y = 3.00t^3$: At $t = 2.00$ s, $y = 3.00(2.00)^3 = 24.0$ m

and $v_y = \dfrac{dy}{dt} = 9.00t^2 = 36.0 \text{ m/s} \uparrow$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \tfrac{1}{2}gt^2 = 24.0 + 36.0t - \tfrac{1}{2}(9.80)t^2$$

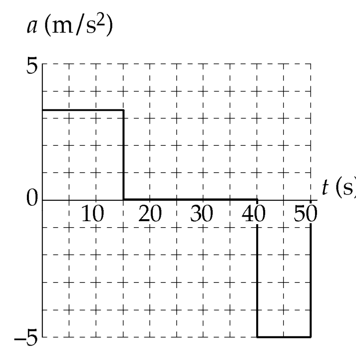
Setting $y_b = 0$, $0 = 24.0 + 36.0t - 4.90t^2$

Solving for t, (only positive values of t count), $\boxed{t = 7.96 \text{ s}}$

***2.43** From $v_f^2 = v_i^2 + 2ax$, we have $\left(10.97 \times 10^3 \text{ m/s}\right)^2 = 0 + 2a(220 \text{ m})$, so that

$\boxed{a = 2.74 \times 10^5 \text{ m/s}^2}$ which is $\boxed{a = 2.79 \times 10^4 \text{ times } g}$

***2.44** (a) Total displacement = area under the (v, t) curve from $t = 0$ to 50 s.



$$\Delta x = \frac{1}{2}(50 \text{ m/s})(15 \text{ s}) + (50 \text{ m/s})(40 - 15)\text{s} + \frac{1}{2}(50 \text{ m/s})(10 \text{ s})$$

$\Delta x = \boxed{1875 \text{ m}}$

(b) From $t = 10$ s to $t = 40$ s, displacement is

$$\Delta x = \frac{1}{2}(50 \text{ m/s} + 33 \text{ m/s})(5 \text{ s}) + (50 \text{ m/s})(25 \text{ s}) = \boxed{1457 \text{ m}}$$

(c) $0 \le t \le 15$ s: $a_1 = \dfrac{\Delta v}{\Delta t} = \dfrac{(50 - 0) \text{ m/s}}{15 \text{ s} - 0} = \boxed{3.3 \text{ m/s}^2}$

$15 \text{ s} < t < 40$ s: $\boxed{a_2 = 0}$

$40 \text{ s} \le t \le 50$ s: $a_3 = \dfrac{\Delta v}{\Delta t} = \dfrac{(0 - 50) \text{ m/s}}{50 \text{ s} - 40 \text{ s}} = \boxed{-5.0 \text{ m/s}^2}$

(d) (i) $x_1 = 0 + \tfrac{1}{2}a_1t^2 = \tfrac{1}{2}\left(3.3 \text{ m/s}^2\right)t^2$, or $\boxed{x_1 = \left(1.67 \text{ m/s}^2\right)t^2}$

(ii) $x_2 = \tfrac{1}{2}(15 \text{ s})[50 \text{ m/s} - 0] + (50 \text{ m/s})(t - 15 \text{ s})$ or $\boxed{x_2 = (50 \text{ m/s})t - 375 \text{ m}}$

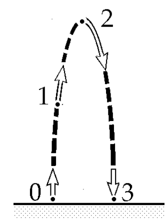
(iii) For $40 \text{ s} \le t \le 50$ s, $x_3 = \begin{pmatrix}\text{area under } v \text{ vs } t \\ \text{from } t = 0 \text{ to } 40 \text{ s}\end{pmatrix} + \dfrac{1}{2}a_3(t - 40 \text{ s})^2 + (50 \text{ m/s})(t - 40 \text{ s})$

or $x_3 = 375 \text{ m} + 1250 \text{ m} + \tfrac{1}{2}\left(-5.0 \text{ m/s}^2\right)(t - 40 \text{ s})^2 + (50 \text{ m/s})(t - 40 \text{ s})$

which reduces to $\boxed{x_3 = (250 \text{ m/s})t - \left(2.5 \text{ m/s}^2\right)t^2 - 4375 \text{ m}}$

(e) $\bar{v} = \dfrac{\text{total displacement}}{\text{total elapsed time}} = \dfrac{1875 \text{ m}}{50 \text{ s}} = \boxed{37.5 \text{ m/s}}$

2.45 Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. The data in the table are found for each phase of the rocket's motion.



(0 to 1) $v_f^2 - (80.0)^2 = 2(4.00)(1000)$ so $v_f = 120 \text{ m/s}$

$120 = 80.0 + (4.00)t$ giving $t = 10.0 \text{ s}$

(1 to 2) $0 - (120)^2 = 2(-9.80)(x_f - x_i)$ giving $x_f - x_i = 735 \text{ m}$

$0 - 120 = -9.80t$ giving $t = 12.2 \text{ s}$

This is the time of maximum height of the rocket.

(2 to 3) $v_f^2 - 0 = 2(-9.80)(-1735)$

$v_f = -184 = (-9.80)t$ giving $t = 18.8 \text{ s}$

(a) $t_{\text{total}} = 10 + 12.2 + 18.8 = \boxed{41.0 \text{ s}}$

(b) $(x_f - x_i)_{\text{total}} = \boxed{1.73 \text{ km}}$

(c) $v_{\text{final}} = \boxed{-184 \text{ m/s}}$

		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1000	120	+4.00
#2	Rise Upwards	22.2	1735	0	–9.80
#3	Fall to Earth	41.0	0	–184	–9.80

***2.46** Distance traveled by motorist = $(15.0 \text{ m/s})t$

Distance traveled by policeman = $\frac{1}{2}(2.00 \text{ m/s}^2)t^2$

(a) intercept occurs when $15.0t = t^2$ $t = \boxed{15.0 \text{ s}}$

(b) $v \text{ (officer)} = (2.00 \text{ m/s}^2)t = \boxed{30.0 \text{ m/s}}$

(c) $x \text{ (officer)} = \frac{1}{2}(2.00 \text{ m/s}^2)t^2 = \boxed{225 \text{ m}}$

2.47 (a) Let x be the distance traveled at acceleration a until maximum speed v is reached. If this is achieved in time t_1 we can use the following three equations:

$$x = \tfrac{1}{2}(v + v_i)\, t_1, \qquad 100 - x = v(10.2 - t_1) \qquad \text{and} \qquad v = v_i + a t_1$$

The first two give $100 = \left(10.2 - \tfrac{1}{2}t_1\right)v = \left(10.2 - \tfrac{1}{2}t_1\right)a t_1$ $\qquad a = \dfrac{200}{(20.4 - t_1)t_1}$

For Maggie $a = \dfrac{200}{(18.4)(2.00)} = \boxed{5.43\ \text{m/s}^2}$

For Judy $a = \dfrac{200}{(17.4)(3.00)} = \boxed{3.83\ \text{m/s}^2}$

(b) $v = a_1 t$

Maggie: $v = (5.43)(2.00) = \boxed{10.9\ \text{m/s}}$

Judy: $v = (3.83)(3.00) = \boxed{11.5\ \text{m/s}}$

(c) At the six-second mark $x = \tfrac{1}{2}a t_1^2 + v(6.00 - t_1)$

Maggie: $x = \tfrac{1}{2}(5.43)(2.00)^2 + (10.9)(4.00) = 54.3\ \text{m}$

Judy: $x = \tfrac{1}{2}(3.83)(3.00)^2 + (11.5)(3.00) = 51.7\ \text{m}$

Maggie is ahead by $\boxed{2.62\ \text{m}}$

***2.48** $a_1 = 0.100\ \text{m/s}^2$ $\qquad a_2 = -0.500\ \text{m/s}^2$

$x = 1000\ \text{m} = \tfrac{1}{2}a_1 t_1^2 + v_1 t_2 + \tfrac{1}{2}a_2 t_2^2$ $\qquad t = t_1 + t_2$ and $v_1 = a_1 t_1 = -a_2 t_2$

$$1000 = \tfrac{1}{2}a_1 t_1^2 + a_1 t_1\left(-\frac{a_1 t_1}{a_2}\right) + \tfrac{1}{2}a_2\left(\frac{a_1 t_1}{a_2}\right)^2 \qquad 1000 = \tfrac{1}{2}a_1\left(1 - \frac{a_1}{a_2}\right)t_1^2$$

$$t_1 = \sqrt{\frac{20\,000}{1.20}} = \boxed{129\ \text{s}}$$

$t_2 = \dfrac{a_1 t_1}{-a_2} = \dfrac{12.9}{0.500} \approx 26\ \text{s}$ $\qquad$ Total time $= t = \boxed{155\ \text{s}}$

2.49 Let the ball fall 1.50 m. It strikes at speed given by $v_{xf}^2 = v_{xi}^2 + 2a(x_f - x_i)$:

$$v_{xf}^2 = 0 + 2(-9.80 \text{ m/s}^2)(-1.50 \text{ m}) \qquad v_{xf} = -5.42 \text{ m/s}$$

and its stopping is described by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$0 = (-5.42 \text{ m/s})^2 + 2a_x(-10^{-2} \text{ m})$$

$$a_x = \frac{-29.4 \text{ m}^2/\text{s}^2}{-2.00 \times 10^{-2} \text{ m}} = +1.47 \times 10^3 \text{ m/s}^2$$

Its maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3 \text{ m/s}^2}$

***2.50** (a) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$. We assume the package starts from rest.

$$-145 \text{ m} = 0 + 0 + \tfrac{1}{2}(-9.80 \text{ m/s}^2)t^2 \qquad t = \sqrt{\frac{2(-145 \text{ m})}{-9.80 \text{ m/s}^2}} = \boxed{5.44 \text{ s}}$$

(b) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 0 + 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)(5.18 \text{ s})^2 = -131 \text{ m}$

distance fallen $= |x_f| = \boxed{131 \text{ m}}$

(c) speed $= |v_{xf}| = |v_{xi} + a_x t| = |0 + (-9.8 \text{ m/s}^2)5.18 \text{ s}| = \boxed{50.8 \text{ m/s}}$

(d) The remaining distance is $145 \text{ m} - 131.5 \text{ m} = 13.5 \text{ m}$

During deceleration, $v_{xi} = -50.8 \text{ m/s}$, $v_{xf} = 0$, $x_f - x_i = -13.5 \text{ m}$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i): \qquad 0 = (-50.8 \text{ m/s})^2 + 2a_x(-13.5 \text{ m})$$

$$a_x = \frac{-2580 \text{ m}^2/\text{s}^2}{2(-13.5 \text{ m})} = +95.3 \text{ m/s}^2 = \boxed{95.3 \text{ m/s}^2 \text{ up}}$$

2.51 (a) $y_f = v_{i1}t + \frac{1}{2}at^2 = 50.0 = 2.00t + \frac{1}{2}(9.80)t^2$

$$4.90t^2 + 2.00t - 50.0 = 0 \qquad t = \frac{-2.00 + \sqrt{2.00^2 - 4(4.90)(-50.0)}}{2(4.90)}$$

Only the positive root is physically meaningful: $t = \boxed{3.00 \text{ s}}$ after the first stone is thrown.

(b) $y_f = v_{i2}t + \frac{1}{2}at^2$ and $t = 3.00 - 1.00 = 2.00 \text{ s}$

substitute $50.0 = v_{i2}(2.00) + \frac{1}{2}(9.80)(2.00)^2$: $v_{i2} = \boxed{15.3 \text{ m/s}}$ downward

(c) $v_{1f} = v_{i1} + at = 2.00 + (9.80)(3.00) = \boxed{31.4 \text{ m/s}}$ downward

$v_{2f} = v_{i2} + at = 15.3 + (9.80)(2.00) = \boxed{34.8 \text{ m/s}}$ downward

2.52 (a) $d = \frac{1}{2}9.80t_1^2$ $\quad d = 336t_2$

$t_1 + t_2 = 2.40$ $\quad 336t_2 = 4.90(2.40 - t_2)^2$

$4.90t_2^2 - 359.5t_2 + 28.22 = 0$ $\quad t_2 = \frac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80}$

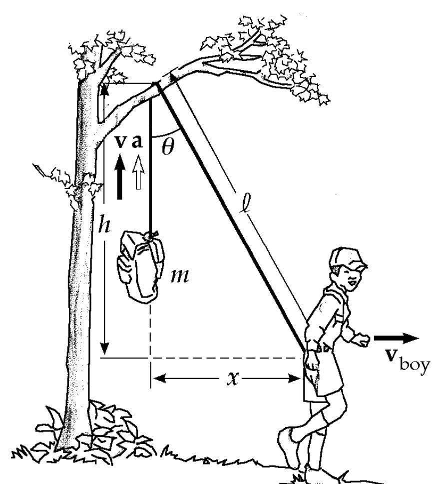
$t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.0765 \text{ s}$ so $d = 336t_2 = \boxed{26.4 \text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2$ m, an error of $\boxed{6.82\%}$

2.53 (a) In walking a distance Δx, in a time Δt, the length of rope ℓ is only increased by $\Delta x \sin\theta$.

$\therefore$ The pack lifts at a rate $\frac{\Delta x}{\Delta t}\sin\theta$.

$$v = \frac{\Delta x}{\Delta t}\sin\theta = v_{boy}\frac{x}{\ell} = \boxed{v_{boy}\frac{x}{\sqrt{x^2 + h^2}}}$$



(b) $a = \frac{dv}{dt} = \frac{v_{boy}}{\ell}\frac{dx}{dt} + v_{boy}x\frac{d}{dt}\left(\frac{1}{\ell}\right)$

$a = v_{boy}\frac{v_{boy}}{\ell} - \frac{v_{boy}x}{\ell^2}\frac{d\ell}{dt}$, but $\frac{d\ell}{dt} = v = v_{boy}\frac{x}{\ell}$

$$\therefore a = \frac{v_{boy}^2}{\ell}\left(1 - \frac{x^2}{\ell^2}\right) = \frac{v_{boy}^2}{\ell}\frac{h^2}{\ell^2} = \boxed{\frac{h^2 v_{boy}^2}{\left(x^2 + h^2\right)^{3/2}}}$$

(c) $\frac{v_{boy}^2}{h}$, 0

(d) v_{boy}, 0

2.54 $h = 6.00\text{ m},\ v_{boy} = 2.00\text{ m/s}$

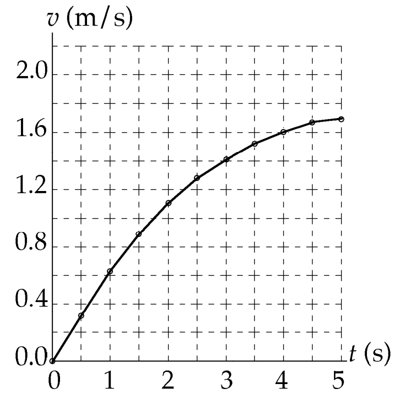
$$v = \frac{\Delta x}{\Delta t}\sin\theta = v_{boy}\frac{x}{\ell} = \frac{v_{boy}x}{\left(x^2+h^2\right)^{1/2}}$$

However, $x = v_{boy}t$:

$$\therefore v = \frac{v_{boy}{}^2 t}{\left(v_{boy}{}^2t^2+h^2\right)^{1/2}} = \frac{4t}{\left(4t^2+36\right)^{1/2}}$$

(a)

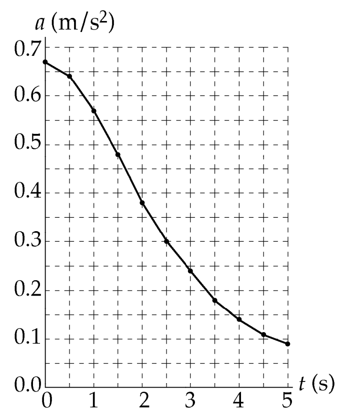
t(s)	v(m/s)
0	0
0.5	0.32
1	0.63
1.5	0.89
2	1.11
2.5	1.28
3	1.41
3.5	1.52
4	1.60
4.5	1.66
5	1.71



(b) From problem 2.53 above,

$$a = \frac{h^2 v_{boy}{}^2}{\left(x^2+h^2\right)^{3/2}} = \frac{h^2 v_{boy}{}^2}{\left(v_{boy}{}^2t^2+h^2\right)^{3/2}} = \frac{144}{\left(4t^2+36\right)^{3/2}}$$

t(s)	a(m/s^2)
0	0.67
0.5	0.64
1	0.57
1.5	0.48
2	0.38
2.5	0.30
3.	0.24
3.5	0.18
4.	0.14
4.5	0.11
5	0.09



***2.55** Average speed of every point on the train as the first car passes Liz:

$$\frac{\Delta x}{\Delta t} = \frac{8.60\text{ m}}{1.50\text{ s}} = 5.73\text{ m/s}$$

The train has this as its instantaneous speed halfway through the 1.50 s time. Similarly, halfway through the next 1.10 s, the speed of the train is $8.60\text{ m}/1.10\text{ s} = 7.82\text{ m/s}$. The time required for the speed to change from 5.73 m/s to 7.82 m/s is

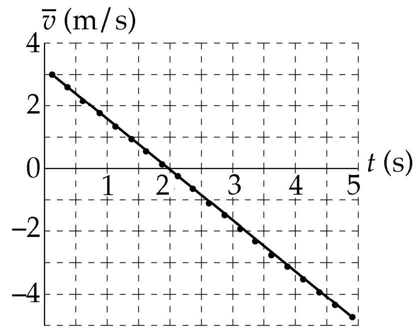
$$\tfrac{1}{2}(1.50\text{ s}) + \tfrac{1}{2}(1.10\text{ s}) = 1.30\text{ s}$$

so the acceleration is:

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{7.82\text{ m/s} - 5.73\text{ m/s}}{1.30\text{ s}} = \boxed{1.60\text{ m/s}^2}.$$

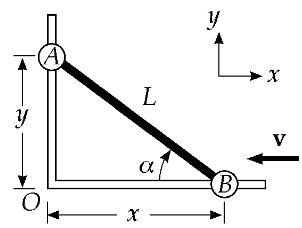
***2.56**

Time t(s)	Height h (m)	Δh (m)	Δt (s)	$\overline{v}$ (m/s)	midpt time t (s)
0.00	5.00				
		0.75	0.25	3.00	0.13
0.25	5.75				
		0.65	0.25	2.60	0.38
0.50	6.40				
		0.54	0.25	2.16	0.63
0.75	6.94				
		0.44	0.25	1.76	0.88
1.00	7.38				
		0.34	0.25	1.36	1.13
1.25	7.72				
		0.24	0.25	0.96	1.38
1.50	7.96				
		0.14	0.25	0.56	1.63
1.75	8.10				
		0.03	0.25	0.12	1.88
2.00	8.13				
		–0.06	0.25	–0.24	2.13
2.25	8.07				
		–0.17	0.25	–0.68	2.38
2.50	7.90				
		–0.28	0.25	–1.12	2.63
2.75	7.62				
		–0.37	0.25	–1.48	2.88
3.00	7.25				
		–0.48	0.25	–1.92	3.13
3.25	6.77				
		–0.57	0.25	–2.28	3.38
3.50	6.20				
		–0.68	0.25	–2.72	3.63
3.75	5.52				
		–0.79	0.25	–3.16	3.88
4.00	4.73				
		–0.88	0.25	–3.52	4.13
4.25	3.85				
		–0.99	0.25	–3.96	4.38
4.50	2.86				
		–1.09	0.25	–4.36	4.63
4.75	1.77				
		–1.19	0.25	–4.76	4.88
5.00	0.58				



acceleration = slope of line is constant. $\overline{a} = -1.63\ \mathrm{m/s^2} = \boxed{1.63\ \mathrm{m/s^2}\ \text{downward}}$

2.57 The distance x and y are always related by $x^2 + y^2 = L^2$. Differentiating this equation with respect to time, we have



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Now $\frac{dy}{dt}$ is v_B, the unknown velocity of B; and $\frac{dx}{dt} = -v$.

From the equation resulting from differentiation, we have $\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\frac{x}{y}(-v)$

But $\frac{y}{x} = \tan\alpha$ so $v_B = \left(\frac{1}{\tan\alpha}\right)v$

When $\alpha = 60.0°$, $v_B = \frac{v}{\tan 60.0°} = \frac{v\sqrt{3}}{3} = \boxed{0.577v}$

ANSWERS TO EVEN NUMBERED PROBLEMS

2. (a) 180 km (b) 63.4 km/h north

4. (a) 50.0 m/s (b) 41.0 m/s

6. (a) 27.0 m (b) $x_f = 27.0\text{ m} + (18.0\text{ m/s})\Delta t + \left(3.00\text{ m/s}^2\right)(\Delta t)^2$
(c) 18.0 m/s

8. (a) See the solution. (b) $v_{5\text{ s}} = 23\text{ m/s},\ v_{4\text{ s}} = 18\text{ m/s},\ v_{3\text{ s}} = 14\text{ m/s},\ v_{2\text{ s}} = 9.0\text{ m/s}$
(c) 4.6 m/s^2 (d) 0

10. 15.0 s

12. $1.34 \times 10^4\text{ m/s}^2$

14. (a) 13.0 m/s (b) 10.0 m/s, 16.0 m/s
(c) 6.00 m/s^2 (d) 6.00 m/s^2

16. (a) and (b) See the solution. (c) -4 m/s^2
(d) 34 m (e) 28 m

18. (a) – (e) See the solution.
(f) The spacing of the successive positions would change with less regularity.

20. (a) 5.25 m/s^2 (b) 168 m (c) 52.5 m/s

22. 160 ft

24. (a) 2.56 m (b) –3.00 m/s

26. (a) $x = \left(30.0t - t^2\right)\text{ m},\ v = \left(30.0 - 2.00t\right)\text{ m/s}$ (b) 225 m

28. $x_f - x_i = v_{xf}\,t - \frac{1}{2}a_x\,t^2;\ 3.10\text{ m/s}$

30. (a) $1.12 \times 10^{11}\text{ m/s}^2$ (b) 46.7 μs

32. (a) 3.00 m/s (b) 6.00 s
(c) -0.300 m/s^2 (d) 2.05 m/s

34. (a) –4.90 m, –19.6 m, –44.1 m (b) –9.80 m/s, –19.6 m/s, –29.4 m/s

36. 1.79 s

38. \$99.4/h

40. (a) –23.2 m/s for both (b) downward throw: –17.6 m/s; upward throw: –5.58 m/s

42. 7.96 s

44. (a) 1.87 km (b) 1.46 km
(c) $a_1 = 3.3 \text{ m/s}^2$ $(0 < t < 15 \text{ s})$, $a_2 = 0$ $(15 \text{ s} < t < 40 \text{ s})$, $a_3 = -5.0 \text{ m/s}^2$ $(40 \text{ s} < t < 50 \text{ s})$
(d) (i) $x_1 = \left(1.67 \text{ m/s}^2\right)t^2$, (ii) $x_2 = (50 \text{ m/s})t - 375 \text{ m}$, (iii) $x_3 = (250 \text{ m/s})t - \left(2.5 \text{ m/s}^2\right)t^2 - 4375 \text{ m}$
(e) 37.5 m/s

46. (a) 15.0 s (b) 30.0 m/s (c) 225 m

48. 155 s, 129 s

50. (a) 5.44 s (b) 131 m
(c) 50.8 m/s (d) 95.3 m/s^2 up

52. (a) 26.4 m (b) 6.82%

54. See the solution.

56. See the solution. $a = 1.63 \text{ m/s}^2$ downward