CHAPTER 6

ANSWERS TO QUESTIONS

Q6.1 The kinetic energy of the object decreases.

Q6.2 The force is perpendicular to every increment of displacement. Therefore, $\mathbf{F} \cdot \Delta\mathbf{r} = 0$.

Q6.3 Yes. Force times distance over which the toe is in contact with the ball. No. Yes. Air friction and gravity.

Q6.4 No. Less force $mg\sin\theta$ is required if the angle θ of the ramp is reduced, but the distance over which the movers must push is increased by the same factor.

Q6.5 Force of tension on a ball rotating on the end of a string. Normal force and gravitational force on an object at rest or moving across a level floor.

Q6.6 (a) Tension (b) Air resistance
(c) Positive in increasing velocity on the downswing. Negative in decreasing velocity on the upswing.

Q6.7 No. If the vectors are in the same direction, their dot product is positive.

Q6.8 Suppose that Bobby Orr gives the puck a large velocity horizontally toward the north. In the reference frame of the ice, he does work on the puck by exerting a northward force through the distance the stick moves while in contact with the puck. The work - kinetic energy theorem is satisfied. The theorem identifies the joules of work he does, one by one, as the joules of kinetic energy gained by the puck. The ant clinging to the puck is in an accelerated, non-inertial reference frame. The ant sees the puck as always stationary in spite of the large northward force on it. The ant sees the athlete, ice, spectators, and scoreboard as suddenly taking off toward the south with a huge acceleration. Newton's first and second laws are not valid in the reference frame of the ant, and we should not expect the work - kinetic energy theorem to be true. The theorem does not describe the scoreboard, which gains a large amount of kinetic energy without work input. Nevertheless, the kinetic energy of the puck is always zero in the ant's frame of reference, and the work on the puck is zero because the stick acts through zero distance. Thus it happens that the work - kinetic energy theorem correctly describes the puck in the puck's frame with the tautology 0 = 0.

Q6.9 Kinetic energy is always positive. Mass and squared speed are both positive. A moving object can always do positive work in striking another object and causing it to move along the same direction of motion.

Q6.10 Kinetic energy is proportional to mass. The first bullet has twice as much kinetic energy.

Q6.11 Kinetic energy is proportional to squared speed. Doubling the speed makes an object's kinetic energy four times larger.

Q6.12 $2W$ for each half of the spring, if it is described by Hooke's law. The spring constant for each half is twice as large as that of the uncut spring, because the same tension stretches the cut section only half as much. Therefore, $\frac{1}{2}(2k)x_{\max}^2 = 2\left(\frac{1}{2}kx_{\max}^2\right) = 2W$.

Q6.13 No violation. Choose the book as the system. You did work and the earth did work on the book. The average force you exerted just counterbalanced the weight of the book. The total work on the book is zero, and is equal to its overall change in kinetic energy.

Q6.14 If the total work on an object is zero in some process, its speed must be the same at the final point as it was at the initial point.

Q6.15 If the instantaneous power output by some agent changes continuously, its average power in a process must be equal to its instantaneous power at at least one instant. If its power output is constant, its instantaneous power is always equal to its average power.

Q6.16 Decreases, as the force required to lift the car decreases.

Q6.17 The rock increases in speed. The farther it has fallen, the more force it might exert on the sand at the bottom; but it might instead make a deeper crater with an equal-size average force. The farther it falls, the more work it will do in stopping. Its kinetic energy is increasing due to the work that the gravitational force does on it.

PROBLEM SOLUTIONS

6.1 (a) $W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b) and (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

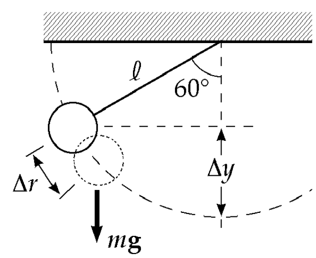
6.2 The component of force along the direction of motion is

$$F\cos\theta = (35.0\text{ N})\cos 25.0° = 31.7\text{ N}$$

The work done by this force is

$$W = (F\cos\theta)\Delta r = (31.7\text{ N})(50.0\text{ m}) = \boxed{1.59\times 10^3\text{ J}}$$

6.3 **Method One.**



Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then $\cos\theta = \cos(90° + \phi) = -\sin\phi$.

The work done by the gravitational force on Batman is

$$W = \int_{\text{i}}^{\text{f}} F\cos\theta\, d\text{r} = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12\text{ m})d\phi$$

$$= -mg(12\text{ m})\int_0^{60°}\sin\phi\, d\phi = (-80\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})\,(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times 10^3\text{ J}}$$

Method Two.

The force of gravity on Batman is $mg = (80\text{ kg})(9.8\text{ m/s}^2) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y-coordinate below the tree limb is -12 m. His final y-coordinate is $(-12\text{ m})\cos 60° = -6\text{ m}$. His change in elevation is $-6\text{ m} - (-12\text{ m}) = 6\text{ m}$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}$$

6.4 (a) $W = mgh = \left(3.35\times10^{-5}\right)(9.80)(100)\text{ J} = \boxed{3.28\times10^{-2}\text{ J}}$

(b) Since $R = mg$, $W_{\text{air resistance}} = \boxed{-3.28\times10^{-2}\text{ J}}$

6.5 $A = 5.00$; $B = 9.00$; $\theta = 50.0°$

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$

6.6 $\mathbf{A}\cdot\mathbf{B} = \left(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\right)\cdot\left(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}\right)$

$\mathbf{A}\cdot\mathbf{B} = A_xB_x\,(\mathbf{i}\cdot\mathbf{i}) + A_xB_y\,(\mathbf{i}\cdot\mathbf{j}) + A_xB_z\,(\mathbf{i}\cdot\mathbf{k}) +$

$A_yB_x\,(\mathbf{j}\cdot\mathbf{i}) + A_yB_y\,(\mathbf{j}\cdot\mathbf{j}) + A_yB_z\,(\mathbf{j}\cdot\mathbf{k}) +$

$A_zB_x\,(\mathbf{k}\cdot\mathbf{i}) + A_zB_y\,(\mathbf{k}\cdot\mathbf{j}) + A_zB_z\,(\mathbf{k}\cdot\mathbf{k})$

$\mathbf{A}\cdot\mathbf{B} = \boxed{A_xB_x + A_yB_y + A_zB_z}$

6.7 (a) $W = \mathbf{F}\cdot\Delta\mathbf{r} = F_xx + F_yy = (6.00)(3.00)\text{ N}\cdot\text{m} + (-2.00)(1.00)\text{ N}\cdot\text{m} = \boxed{16.0\text{ J}}$

(b) $\theta = \cos^{-1}\left(\dfrac{\mathbf{F}\cdot\Delta\mathbf{r}}{F\Delta r}\right) = \cos^{-1}\dfrac{16}{\sqrt{\left((6.00)^2 + (-2.00)^2\right)\left((3.00)^2 + (1.00)^2\right)}} = \boxed{36.9°}$

***6.8** We must first find the angle between the two vectors. It is:

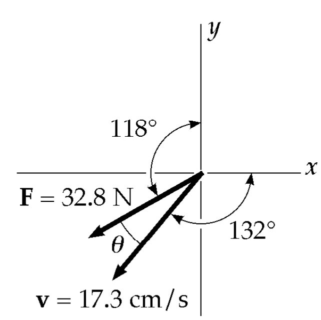
$\theta = 360° - 118° - 90.0° - 132° = 20.0°$

Then

$\mathbf{F}\cdot\mathbf{v} = Fv\cos\theta = (32.8\text{ N})(0.173\text{ m/s})\cos 20.0°$

or

$\mathbf{F}\cdot\mathbf{v} = 5.33\,\dfrac{\text{N}\cdot\text{m}}{\text{s}} = 5.33\,\dfrac{\text{J}}{\text{s}} = \boxed{5.33\text{ W}}$



6.9 (a) $\mathbf{A} = 3.00\mathbf{i} - 2.00\mathbf{j}$

$\mathbf{B} = 4.00\mathbf{i} - 4.00\mathbf{j}$ $\qquad \theta = \cos^{-1}\dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \cos^{-1}\dfrac{12.0 + 8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3°}$

(b) $\mathbf{B} = 3.00\mathbf{i} - 4.00\mathbf{j} + 2.00\mathbf{k}$

$\mathbf{A} = -2.00\mathbf{i} + 4.00\mathbf{j}$ $\qquad \cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \dfrac{-6.00 - 16.0}{\sqrt{(20.0)(29.0)}}$ $\qquad \theta = \boxed{156°}$

(c) $\mathbf{A} = \mathbf{i} - 2.00\mathbf{j} + 2.00\mathbf{k}$

$\mathbf{B} = 3.00\mathbf{j} + 4.00\mathbf{k}$ $\qquad \theta = \cos^{-1}\left(\dfrac{A\cdot B}{AB}\right) = \cos^{-1}\left(\dfrac{-6.00 + 8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$

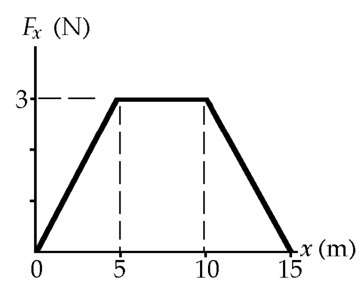
6.10 $\mathbf{A} - \mathbf{B} = (3.00\mathbf{i} + \mathbf{j} - \mathbf{k}) - (-\mathbf{i} + 2.00\mathbf{j} + 5.00\mathbf{k})$

$\mathbf{A} - \mathbf{B} = 4.00\mathbf{i} - \mathbf{j} - 6.00\mathbf{k}$

$\mathbf{C}\cdot(\mathbf{A} - \mathbf{B}) = (2.00\mathbf{j} - 3.00\mathbf{k})\cdot(4.00\mathbf{i} - \mathbf{j} - 6.00\mathbf{k}) = 0 + (-2.00) + (+18.0) = \boxed{16.0}$

6.11 $W = \int F_x\,dx$

and W equals the area under the Force-Displacement curve



(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00\text{ N})(5.00\text{ m}) = \boxed{15.0\text{ J}}$$

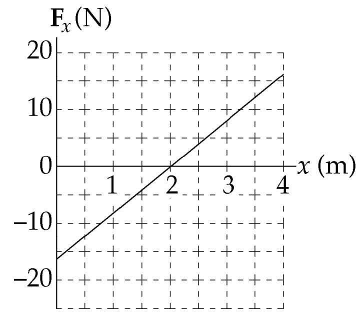
(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\text{ J} = \boxed{30.0\text{ J}}$$

6.12 $F_x = (8x - 16)$ N



(a) See figure to the right

(b) $W_{net} = \dfrac{-(2.00 \text{ m})(16.0 \text{ N})}{2} + \dfrac{(1.00 \text{ m})(8.00 \text{ N})}{2} = \boxed{-12.0 \text{ J}}$

6.13 $k = \dfrac{F}{y} = \dfrac{Mg}{y} = \dfrac{(4.00)(9.80) \text{ N}}{2.50 \times 10^{-2} \text{ m}} = 1.57 \times 10^3 \text{ N/m}$

(a) For 1.50 kg mass $y = \dfrac{mg}{k} = \dfrac{(1.50)(9.80)}{1.57 \times 10^3} = \boxed{0.938 \text{ cm}}$

(b) Work $= \frac{1}{2}ky^2$

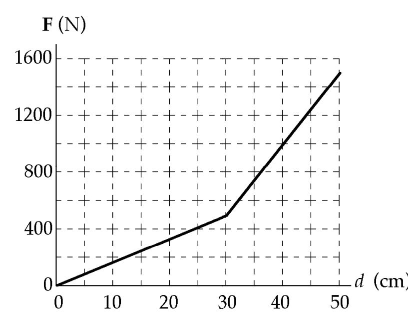
Work $= \frac{1}{2}\left(1.57 \times 10^3 \text{ N} \cdot \text{m}\right)\left(4.00 \times 10^{-2} \text{ m}\right)^2 = \boxed{1.25 \text{ J}}$

6.14 (a) Spring constant is given by $F = kx$

$k = \dfrac{F}{x} = \dfrac{(230 \text{ N})}{(0.400 \text{ m})} = \boxed{575 \text{ N/m}}$

(b) Work $= F_{avg}\, x = \frac{1}{2}(230 \text{ N})(0.400 \text{ m}) = \boxed{46.0 \text{ J}}$

***6.15** Compare an initial picture of the rolling car with a final picture with both springs compressed



$K_i + \sum W = K_f$

Use equation 6.15

$K_i + \frac{1}{2}k_1\left(x_{1i}{}^2 - x_{1f}{}^2\right) + \frac{1}{2}k_2\left(x_{2i}{}^2 - x_{2f}{}^2\right) = K_f$

$\frac{1}{2}mv_i{}^2 + 0 - \frac{1}{2}(1600 \text{ N/m})(0.500 \text{ m})^2 + 0 - \frac{1}{2}(3400 \text{ N/m})(0.200 \text{ m})^2 = 0$

$\frac{1}{2}(6000 \text{ kg})v_i{}^2 - 200 \text{ J} - 68.0 \text{ J} = 0$

$v_i = \sqrt{2(268 \text{ J})/(6000 \text{ kg})} = \boxed{0.299 \text{ m/s}}$

6.16 (a) $W = \int_i^f \mathbf{F} \cdot d\mathbf{r}$

$$W = \int_0^{0.600\text{ m}} \left(15000\text{ N} + 10000x\text{ N/m} - 25000x^2\text{ N/m}^2\right) dx \cos 0°$$

$$W = 15000x + \frac{10000x^2}{2} - \frac{25000x^3}{3}\Bigg|_0^{0.600}$$

$$W = 9.00\text{ kJ} + 1.80\text{ kJ} - 1.80\text{ kJ} = \boxed{9.00\text{ kJ}}$$

(b) Similarly,

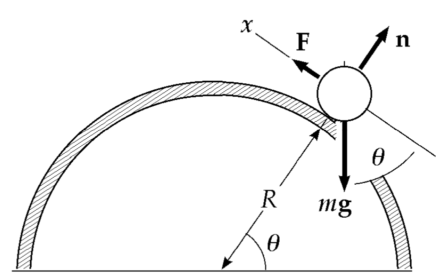
$$W = (15.0\text{ kN})(1.00\text{ m}) + \frac{(10.0\text{ kN/m})(1.00\text{ m})^2}{2} - \frac{\left(25.0\text{ kN/m}^2\right)(1.00\text{ m})^3}{3}$$

$W = \boxed{11.7\text{ kJ}}$, larger by 29.6%

6.17 $$W = \int_i^f \mathbf{F} \cdot d\mathbf{r} = \int_0^{5\text{ m}} \left(4x\,\mathbf{i} + 3y\,\mathbf{j}\right)\text{ N} \cdot dx\,\mathbf{i}$$

$$\int_0^{5\text{ m}} (4\text{ N/m})x\,dx + 0 = (4\text{ N/m})x^2/2\,\Big|_0^{5\text{ m}} = \boxed{50.0\text{ J}}$$

***6.18** (a) The radius to the object makes angle θ with the horizontal, so its weight makes angle θ with the negative side of the x–axis, when we take the x–axis in the direction of motion tangent to the cylinder.



$$\sum F_x = ma_x$$

$$F - mg\cos\theta = 0$$

$$F = \boxed{mg\cos\theta}$$

(b) $W = \int_i^f \mathbf{F} \cdot d\mathbf{r}$

We use radian measure to express the next bit of displacement as $dr = Rd\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg\cos\theta R\,d\theta = mgR\sin\theta\,\Big|_0^{\pi/2}$$

$$W = mgR(1-0) = \boxed{mgR}$$

6.19 (a) $K_A = \frac{1}{2}(0.600 \text{ kg})(2.00 \text{ m/s})^2 = \boxed{1.20 \text{ J}}$

(b) $\frac{1}{2}\ mv_B{}^2 = K_B$: $\qquad v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50)}{0.600}} = \boxed{5.00 \text{ m/s}}$

(c) $\sum W = \Delta K = K_B - K_A = \frac{1}{2}m\left(v_B{}^2 - v_A{}^2\right) = 7.50 \text{ J} - 1.20 \text{ J} = \boxed{6.30 \text{ J}}$

6.20 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.300 \text{ kg})(15.0 \text{ m/s})^2 = \boxed{33.8 \text{ J}}$

(b) $K = \frac{1}{2}(0.300)(30.0)^2 = \frac{1}{2}(0.300)(15.0)^2(4) = 4(33.8) = \boxed{135 \text{ J}}$

6.21 $\mathbf{v}_i = (6.00\mathbf{i} - 2.00\mathbf{j}) \text{ m/s}$

(a) $v_i = \sqrt{v_{ix}{}^2 + v_{iy}{}^2} = \sqrt{40.0} \text{ m/s}$

$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00 \text{ kg})\left(40.0 \text{ m}^2/\text{s}^2\right) = \boxed{60.0 \text{ J}}$

(b) $\mathbf{v}_f = 8.00\,\mathbf{i} + 4.00\,\mathbf{j}$

$v_f^2 = \mathbf{v}_f \cdot \mathbf{v}_f = 64.0 + 16.0 = 80.0 \text{ m}^2/\text{s}^2$

$\Delta K = K_f - K_i = \frac{1}{2}m\left(v_f{}^2 - v_i^2\right) = \frac{3.00}{2}(80.0) - 60.0 = \boxed{60.0 \text{ J}}$

***6.22** (a) $\Delta K = K_f - K_i = \frac{1}{2}\ mv_f{}^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 5.00$ m)

$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50 \text{ J})}{4.00 \text{ kg}}} = \boxed{1.94 \text{ m/s}}$

(b) $\Delta K = K_f - K_i = \frac{1}{2}\ mv_f{}^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 10.0$ m)

$v_f = \sqrt{\frac{2(\text{area})}{\text{m}}} = \sqrt{\frac{2(22.5 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.35 \text{ m/s}}$

(c) $\Delta K = K_f - K_i = \frac{1}{2}\ mv_f{}^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 15.0$ m)

$v_f = \sqrt{\frac{2(\text{area})}{\text{m}}} = \sqrt{\frac{2(30.0 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.87 \text{ m/s}}$

6.23 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d = 5.00$ m represent the distance over which the driver falls freely, and $h = 0.12$ m the distance it moves the piling.

$W = \Delta K$: $\quad \sum W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

so $\quad (mg)(h+d)\cos 0° + (\bar{F})(d)\cos 180° = 0 - 0$

Thus, $\quad \bar{F} = \dfrac{(mg)(h+d)}{d} = \dfrac{(2100\text{ kg})(9.80\text{ m/s}^2)(5.12\text{ m})}{0.120\text{ m}} = \boxed{8.78 \times 10^5\text{ N}}$

6.24 (a) $K_i + \sum W = K_f = \dfrac{1}{2}mv_f^2$

$0 + \sum W = \frac{1}{2}(15.0 \times 10^{-3}\text{ kg})(780\text{ m/s})^2 = \boxed{4.56\text{ kJ}}$

(b) $F = \dfrac{W}{\Delta r \cos\theta} = \dfrac{4.56 \times 10^3\text{ J}}{(0.720\text{ m})\cos 0°} = \boxed{6.34\text{ kN}}$

(c) $a = \dfrac{v_f^2 - v_i^2}{2x_f} = \dfrac{(780\text{ m/s})^2 - 0}{2(0.720\text{ m})} = \boxed{422\text{ km/s}^2}$

(d) $\sum F = ma = (15 \times 10^{-3}\text{ kg})(422 \times 10^3\text{ m/s}^2) = \boxed{6.34\text{ kN}}$

6.25 $\Sigma W = \Delta K = 0$: $\quad \int_0^L mg\sin 35.0°\, dl - \int_0^d kx\, dx = 0$

$$mg \sin 35.0° (L) = \frac{1}{2} kd^2$$

$$d = \sqrt{\frac{2mg\sin 35.0°(L)}{k}}$$

$$d = \sqrt{\frac{2(12.0\text{ kg})(9.80\text{ m/s}^2)(\sin 35.0°)(3.00\text{ m})}{3.00 \times 10^4\text{ N/m}}} = \boxed{0.116\text{ m}}$$

***6.26** (a) $v_f = 0.096\left(3\times10^8 \text{ m/s}\right) = 2.88\times10^7 \text{ m/s}$

$$K_f = \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(2.88\times10^7 \text{ m/s}\right)^2 = \boxed{3.78\times10^{-16}\text{ J}}$$

(b) $K_i + W = K_f$: $\quad 0 + F\Delta r\cos\theta = K_f$

$$F(0.028\text{ m})\cos 0° = 3.78\times10^{-16}\text{ J}$$

$$F = \boxed{1.35\times10^{-14}\text{ N}}$$

(c) $\sum F = ma;$ $\quad a = \dfrac{\sum F}{m} = \dfrac{1.35\times10^{-14}\text{ N}}{9.11\times10^{-31}\text{ kg}} = \boxed{1.48\times10^{+16}\text{ m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t$ $\quad 2.88\times10^7\text{ m/s} = 0 + \left(1.48\times10^{16}\text{ m/s}^2\right)t$

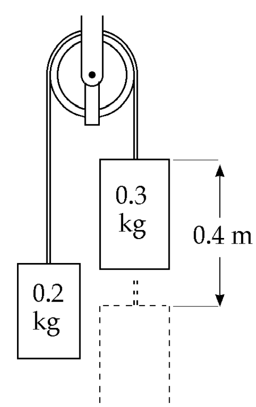
$$t = \boxed{1.94\times10^{-9}\text{ s}}$$

Check: $\quad x_f = x_i + \tfrac{1}{2}(v_{xi}+v_{xf})t$

$$0.028\text{ m} = 0 + \tfrac{1}{2}\left(0 + 2.88\times10^7\text{ m/s}\right)t$$

$$t = 1.94\times10^{-9}\text{ s}$$

***6.27** $\sum W = \Delta K$:

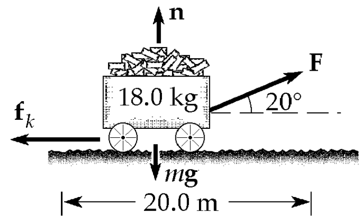


$$m_1gh - m_2gh = \tfrac{1}{2}(m_1+m_2)v_f^2 - 0$$

$$v_f^2 = \frac{2(m_1-m_2)gh}{m_1+m_2} = \frac{2(0.300-0.200)(9.80)(0.400)}{0.300+0.200}\,\frac{\text{m}^2}{\text{s}^2}$$

$$v_f = \sqrt{1.57}\text{ m/s} = \boxed{1.25\text{ m/s}}$$

6.28 (a) $\sum F_y = F\sin\theta + n - mg = 0 \qquad n = mg - F\sin\theta$



$\sum F_x = F\cos\theta - \mu_k n = 0 \qquad n = \dfrac{F\cos\theta}{\mu_k}$

$\therefore \qquad mg - F\sin\theta = \dfrac{F\cos\theta}{\mu_k}$

$$F = \frac{\mu_k mg}{\mu_k \sin\theta + \cos\theta}$$

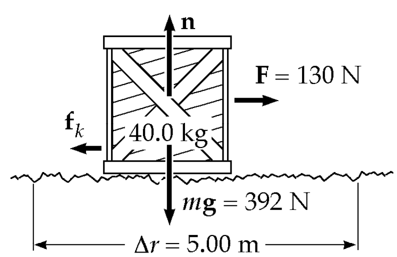
$$F = \frac{(0.500)(18.0)(9.80)}{0.500\sin 20.0° + \cos 20.0°} = \boxed{79.4\text{ N}}$$

(b) $W_F = F\Delta r\cos\theta = (79.4\text{ N})(20.0\text{ m})\cos 20.0° = \boxed{1.49\text{ kJ}}$

(c) $f_k = F\cos\theta = 74.6\text{ N}$

$\Delta E_{\text{int}} = f_k\Delta x = (74.6\text{ N})(20.0\text{ m}) = \boxed{1.49\text{ kJ}}$

6.29 $\sum F_y = ma_y: \qquad n - 392\text{ N} = 0$



$n = 392\text{ N}$

$f_k = \mu_k n = (0.300)(392\text{ N}) = 118\text{ N}$

(a) $W_F = F\Delta r\cos\theta = (130)(5.00)\cos 0° = \boxed{650\text{ J}}$

(b) $\Delta E_{\text{int}} = f_k\,\Delta x = (118)(5.00) = \boxed{588\text{ J}}$

(c) $W_n = n\Delta r\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\Delta r\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W_{\text{other}} - \Delta E_{\text{int}}$

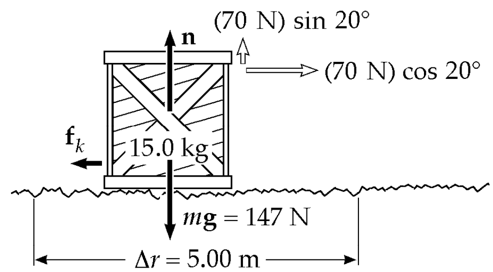
$\frac{1}{2}mv_f^{\,2} - 0 = 650\text{ J} - 588\text{ J} + 0 + 0 = \boxed{62.0\text{ J}}$

(f) $v_f = \sqrt{\dfrac{2K_f}{m}} = \sqrt{\dfrac{2(62.0\text{ J})}{40.0\text{ kg}}} = \boxed{1.76\text{ m/s}}$

***6.30** $\Sigma F_y = ma_y$: $n + (70.0\text{ N})\sin 20.0° - 147\text{ N} = 0$

$n = 123\text{ N}$

$f_k = \mu_k n = 0.300\,(123\text{ N}) = 36.9\text{ N}$



(a) $W = F\Delta r\cos\theta = (70.0\text{ N})(5.00\text{ m})\cos 20.0° = \boxed{329\text{ J}}$

(b) $W = F\Delta r\cos\theta = (123\text{ N})(5.00\text{ m})\cos 90.0° = \boxed{0\text{ J}}$

(c) $W = F\Delta r\cos\theta = (147\text{ N})(5.00\text{ m})\cos 90.0° = \boxed{0}$

(d) $\Delta E_{\text{int}} = F\Delta x = (36.9\text{ N})(5.00\text{ m}) = \boxed{185\text{ J}}$

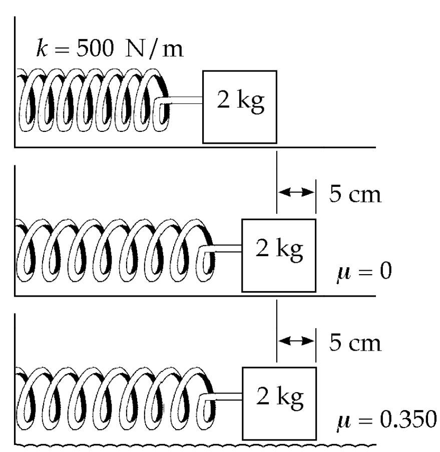
(e) $\Delta K = K_f - K_i = \Sigma W - \Delta E_{\text{int}} = 329\text{ J} - 185\text{ J} = \boxed{+144\text{ J}}$

6.31 $v_i = 2.00\text{ m/s}$ $\quad \mu_k = 0.100$

$K_i - f_k\Delta x + W_{\text{other}} = K_f$: $\quad \frac{1}{2}mv_i^2 - f_k\Delta x = 0$

$\frac{1}{2}mv_i^2 = \mu_k mg\Delta x$ $\quad \Delta x = \dfrac{v_i^2}{2\mu_k g} = \dfrac{(2.00\text{ m/s})^2}{2(0.100)(9.80)} = \boxed{2.04\text{ m}}$

***6.32** (a) $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}(500)\left(5.00\times10^{-2}\right)^2 - 0 = 0.625\text{ J}$



$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0$

so $\quad v_f = \sqrt{\dfrac{2\left(\sum W\right)}{m}} = \sqrt{\dfrac{2(0.625)}{2.00}}\text{ m/s} = \boxed{0.791\text{ m/s}}$

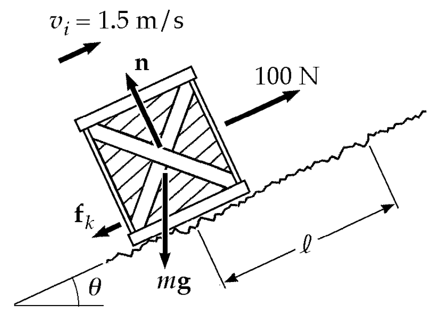
(b) $\frac{1}{2}mv_i^2 - f_k\Delta x + W_s = \frac{1}{2}mv_f^2$

$0 - (0.350)(2.00)(9.80)(0.0500)\text{ J} + 0.625\text{ J} = \frac{1}{2}mv_f^2$

$0.282\text{ J} = \frac{1}{2}(2.00\text{ kg})\,v_f^2$

$v_f = \sqrt{\dfrac{2(0.282)}{2.00}}\text{ m/s} = \boxed{0.531\text{ m/s}}$

6.33 (a) $W_g = mg\ell\cos(90.0° + \theta)$



$$W_g = (10.0\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m})\cos 110° = \boxed{-168\text{ J}}$$

(b) $f_k = \mu_k n = \mu_k\, mg\cos\theta$

$$\Delta E_{\text{int}} = \ell f_k = \ell\mu_k mg\cos\theta$$

$$\Delta E_{\text{int}} = (5.00\text{ m})(0.400)(10.0)(9.80)\cos 20.0° = \boxed{184\text{ J}}$$

(c) $W_f = F\ell = (100)(5.00) = \boxed{500\text{ J}}$

(d) $\Delta K = \Sigma W_{\text{other}} - \Delta E_{\text{int}} = W_F + W_g - \Delta E_{\text{int}} = \boxed{148\text{ J}}$

(e) $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$$v_f = \sqrt{\frac{2(\Delta K)}{m} + v_i^2} = \sqrt{\frac{2(148)}{10.0} + (1.50)^2} = \boxed{5.65\text{ m/s}}$$

6.34 A 1300-kg car speeds up from rest to 55.0 mi/h = 24.6 m/s in 15.0 s. The output work of the engine is equal to its final kinetic energy,

$$\frac{1}{2}(1300\text{ kg})(24.6\text{ m/s})^2 = 390\text{ kJ}$$

with power $\mathcal{P} = \dfrac{390\,000\text{ J}}{15.0\text{ s}}\ \boxed{\sim 10^4\text{ W}}$ around 30 horsepower.

6.35 $\text{Power} = \dfrac{W}{t}$ $\quad \mathcal{P} = \dfrac{mgh}{t} = \dfrac{(700\text{ N})(10.0\text{ m})}{8.00\text{ s}} = \boxed{875\text{ W}}$

6.36 $\mathcal{P}_a = f_a v:$ $\quad f_a = \dfrac{\mathcal{P}_a}{v} = \dfrac{2.24\times 10^4}{27.0} = \boxed{830\text{ N}}$

6.37 (a) $\Sigma W = \Delta K$, but $\Delta K = 0$ because he moves at constant speed. The skier rises a vertical distance of (60.0 m) sin 30.0° = 30.0 m. Thus,

$$W_{\text{in}} = -W_g = (70.0\text{ kg})g(30.0\text{ m}) = \boxed{2.06\times 10^4\text{ J}} = \boxed{20.6\text{ kJ}}$$

(b) The time to travel 60.0 m at a constant speed of 2.00 m/s is 30.0 s. Thus,

$$\mathcal{P}_{\text{input}} = \frac{W}{\Delta t} = \frac{2.06\times 10^4\text{ J}}{30.0\text{ s}} = \boxed{686\text{ W}} = 0.919\text{ hp}$$

6.38 (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}t = \left[\frac{0 + 1.75\text{ m/s}}{2}\right](3.00\text{ s}) = 2.63\text{ m}$$

The motor and the earth's gravity do work on the elevator car:

$$\tfrac{1}{2}mv_i^2 + W_{\text{motor}} + mg\Delta y\cos 180° = \tfrac{1}{2}mv_f^2$$

$$W_{\text{motor}} = \tfrac{1}{2}(650\text{ kg})(1.75\text{ m/s})^2 - 0 + (650\text{ kg})g(2.63\text{ m}) = 1.77\times10^4\text{J}$$

Also, $W = \bar{\mathcal{P}}t$ so $\bar{\mathcal{P}} = \dfrac{W}{t} = \dfrac{1.77\times10^4\text{ J}}{3.00\text{ s}} = \boxed{5.91\times10^3\text{ W}} = 7.92\text{ hp}$

(b) When moving upward at constant speed $(v = 1.75\text{ m/s})$

the applied force equals the weight $= (650\text{ kg})(9.80\text{ m/s}^2) = 6.37\times10^3\text{ N}$

Therefore, $\mathcal{P} = Fv = (6.37\times10^3\text{ N})(1.75\text{ m/s}) = \boxed{1.11\times10^4\text{ W}} = 14.9\text{ hp}$

***6.39** (a) Burning 1 lb of fat releases energy $1\text{ lb}\left(\dfrac{454\text{ g}}{1\text{ lb}}\right)\left(\dfrac{9\text{ kcal}}{1\text{ g}}\right)\left(\dfrac{4186\text{ J}}{1\text{ kcal}}\right) = 1.71\times10^7\text{ J}$

The mechanical energy output is $(1.71\times10^7\text{ J})(0.20) = nF\Delta r\cos\theta$

Then $3.42\times10^6\text{ J} = nmg\Delta y\cos 0°$

$$3.42\times10^6\text{ J} = n(50\text{ kg})(9.8\text{ m/s}^2)(80\text{ steps})(0.150\text{ m})$$

$$3.42\times10^6\text{ J} = n(5.88\times10^3\text{ J})$$

where the number of times she must climb the steps is $n = \dfrac{3.42\times10^6\text{ J}}{5.88\times10^3\text{ J}} = \boxed{582}$

This method is impractical compared to limiting food intake.

(b) Her mechanical power output is

$$\mathcal{P} = \frac{W}{t} = \frac{5.88\times10^3\text{ J}}{65\text{ s}} = \boxed{90.5\text{ W}} = 90.5\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{0.121\text{ hp}}$$

***6.40** (a) The fuel economy for walking is $\dfrac{1\text{ h}}{220\text{ kcal}}\left(\dfrac{3\text{ mi}}{\text{h}}\right)\left(\dfrac{1\text{ kcal}}{4186\text{ J}}\right)\left(\dfrac{1.30\times10^8\text{ J}}{1\text{ gal}}\right) = \boxed{423\text{ mi/gal}}$

(b) For bicycling $\dfrac{1\text{ h}}{400\text{ kcal}}\left(\dfrac{10\text{ mi}}{\text{h}}\right)\left(\dfrac{1\text{ kcal}}{4186\text{ J}}\right)\left(\dfrac{1.30\times10^8\text{ J}}{1\text{ gal}}\right) = \boxed{776\text{ mi/gal}}$

6.41 $\frac{1}{2}mv_i^2 + GM_E m\left(\frac{1}{r_f} - \frac{1}{r_i}\right) = \frac{1}{2}mv_f^2$ $\qquad$ $\frac{1}{2}v_i^2 + GM_E\left(0 - \frac{1}{R_E}\right) = \frac{1}{2}v_f^2$

or $\qquad$ $v_f^2 = v_1^2 - \frac{2GM_E}{R_E}$

and $\qquad$ $v_f = \left(v_1^2 - \frac{2GM_E}{R_E}\right)^{1/2}$

$$v_f = \left[\left(2.00\times10^4\right)^2 - 1.25\times10^8\right]^{1/2} = \boxed{1.66\times10^4\ \text{m/s}}$$

6.42 (a) From the calculation in section 6.9, the work done by the gravitational force is

$W_g = GM_E m_p\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$, a negative quantity.

The work input from a lifting agent is

$$-W_g = GM_E m_p\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = GM_E m_p\left(\frac{1}{R_E} - \frac{1}{R_E + y}\right)$$

$$-W_g = \left(\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)\left(\frac{1}{6.37\times10^6\ \text{m}} - \frac{1}{7.37\times10^6\ \text{m}}\right)$$

$$\boxed{W = 850\ \text{MJ}}$$

(b) In a circular orbit, gravity causes the centripetal acceleration. $\qquad$ $\frac{GM_E m_p}{\left(R_E + y\right)^2} = \frac{m_p v^2}{\left(R_E + y\right)}$

Then $\qquad$ $\frac{1}{2}m_p v^2 = \frac{1}{2}\frac{GM_E m_p}{\left(R_E + y\right)}$

So, additional work = kinetic energy required

$$W_{\text{additional}} = \frac{1}{2}\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)}{\left(\text{kg}^2\right)\left(7.37\times10^6\ \text{m}\right)}$$

$$W_{\text{additional}} = \boxed{2.71\times10^9\ \text{J}}$$

6.43 At start, $\mathbf{v} = (40.0\ \text{m/s})\cos 30.0°\ \mathbf{i} + (40.0\ \text{m/s})\sin 30.0°\ \mathbf{j}$

At apex, $\mathbf{v} = (40.0\ \text{m/s})\cos 30.0°\ \mathbf{i} + 0\ \mathbf{j} = (34.6\ \text{m/s})\ \mathbf{i}$

And $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150\ \text{kg})(34.6\ \text{m/s})^2 = \boxed{90.0\ \text{J}}$

6.44 (a) $\mathbf{A} \cdot \mathbf{i} = (A)(1)\cos\alpha$ But also, $\mathbf{A} \cdot \mathbf{i} = A_x$.

Thus, $(A)(1)\cos\alpha = A_x$ or $\boxed{\cos\alpha = \dfrac{A_x}{A}}$

Similarly, $\boxed{\cos\beta = \dfrac{A_y}{A}}$

and $\boxed{\cos\gamma = \dfrac{A_z}{A}}$

where $A = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$

(b) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\dfrac{A_x}{A}\right)^2 + \left(\dfrac{A_y}{A}\right)^2 + \left(\dfrac{A_z}{A}\right)^2 = \dfrac{A^2}{A^2} = 1$

6.45 (a) $x = t + 2.00\ t^3$

Therefore, $v = \dfrac{dx}{dt} = 1 + 6.00\ t^2$

$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.00)\left(1 + 6.00t^2\right)^2 = \boxed{\left(2.00 + 24.0t^2 + 72.0t^4\right)\ \text{J}}$

(b) $a = \dfrac{dv}{dt} = \boxed{(12.0t)\ \text{m/s}^2}$

$F = ma = 4.00(12.0t) = \boxed{(48.0t)\ \text{N}}$

(c) $\mathcal{P} = Fv = (48.0t)\left(1 + 6.00t^2\right) = \boxed{\left(48.0t + 288t^3\right)\ \text{W}}$

(d) $W = \int_0^{2.00} \mathcal{P}\,dt = \int_0^{2.00} \left(48.0t + 288t^3\right)dt = \boxed{1250\ \text{J}}$

***6.46** (a) The work done by the traveler is mgh_sN where N is the number of steps he climbs during the ride.

$$N = (\text{time on escalator})(n)$$

where $$(\text{time on escalator}) = \frac{h}{\text{vertical velocity of person}}$$

and $$\text{vertical velocity of person} = v + nh_s$$

Then, $$N = \frac{nh}{v + nh_s}$$

and the work done by the person becomes $$W_{\text{person}} = \boxed{\frac{mgnhh_s}{v + nh_s}}$$

(b) The work done by the escalator is $W_e = (\text{power})(\text{time}) = [(\text{force exerted})(\text{speed})](\text{time}) = mgvt$

where $$t = \frac{h}{v + nh_s} \text{ as above.}$$

Thus, $$W_e = \boxed{\frac{mgvh}{v + nh_s}}$$

As a check, the total work done on the person's body must add up to mgh, the work an elevator would do in lifting him.

It does add up as follows: $$\Sigma W = W_{\text{person}} + W_e = \frac{mgnhh_s}{v + nh_s} + \frac{mgvh}{v + nh_s} = \frac{mgh(nh_s + v)}{v + nh_s} = mgh$$

6.47 $$W = \int_{x_i}^{x_f} F\,dx = \int_0^{x_f} (-kx + \beta x^3)\,dx$$

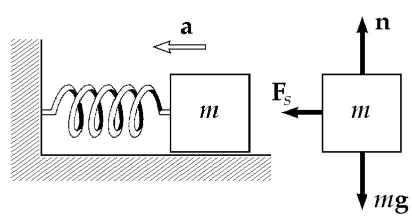
$$W = -\frac{kx^2}{2} + \frac{\beta x^4}{4}\bigg|_0^{x_f} = -\frac{kx_f^{\,2}}{2} + \frac{\beta x_f^{\,2}}{4}$$

$$W = \frac{(-10.0\text{ N/m})(0.100\text{ m})^2}{2} + \frac{(100\text{ N/m}^3)(0.100\text{ m})^4}{4}$$

$$W = -5.00 \times 10^{-2}\text{ J} + 2.50 \times 10^{-3}\text{ J} = \boxed{-4.75 \times 10^{-2}\text{ J}}$$

***6.48** $\Sigma F_x = ma_x: \quad kx = ma$

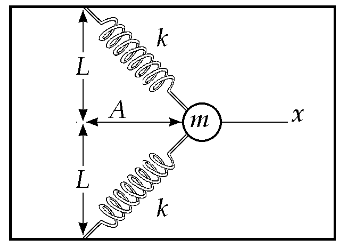
$$k = \frac{ma}{x} = \frac{(4.70 \times 10^{-3}\text{ kg})0.800(9.80\text{ m/s}^2)}{0.500 \times 10^{-2}\text{ m}} = \boxed{7.37\text{ N/m}}$$



***6.49** (a) $\mathscr{P} = Fv = F(v_i + at) = F\left(0 + \frac{F}{m}t\right) = \boxed{\left(\frac{F^2}{m}\right)t}$

(b) $\mathscr{P} = \left[\frac{(20.0\text{ N})^2}{5.00\text{ kg}}\right](3.00\text{ s}) = \boxed{240\text{ W}}$

***6.50** (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2} - L$ and the force it exerts is $k\,(\sqrt{x^2+L^2} - L)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components add to



$$\mathbf{F} = -2\mathbf{i}k\,(\sqrt{x^2+L^2} - L)x/\sqrt{x^2+L^2} = \boxed{-2kx\mathbf{i}\left(1 - L\big/\sqrt{x^2+L^2}\right)}$$

(b) $W = \int_i^f F_x\,dx$ $\qquad W = \int_A^0 -2kx\left(1 - L\big/\sqrt{x^2+L^2}\right)dx$

$$W = -2k\int_A^0 x\,dx + kL\int_A^0 \left(x^2+L^2\right)^{-1/2} 2x\,dx \qquad W = -2k\frac{x^2}{2}\Bigg|_A^0 + kL\frac{\left(x^2+L^2\right)^{1/2}}{(1/2)}\Bigg|_A^0$$

$$W = -0 + kA^2 + 2kL^2 - 2kL\sqrt{A^2+L^2} \qquad W = \boxed{2kL^2 + kA^2 - 2kL\sqrt{A^2+L^2}}$$

6.51 (a) $\sum W = \Delta K$: $\qquad W_s + W_g = 0$

$$\tfrac{1}{2}kx_i^2 - 0 + mg\Delta x\cos(90° + 60°) = 0$$

$$\tfrac{1}{2}\left(1.40\times10^3\text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x = 0 \qquad \Delta x = \boxed{4.12\text{ m}}$$

(b) $\sum W = \Delta K + \Delta E_{\text{int}}$: $\quad W_s + W_g - \Delta E_{\text{int}} = 0$

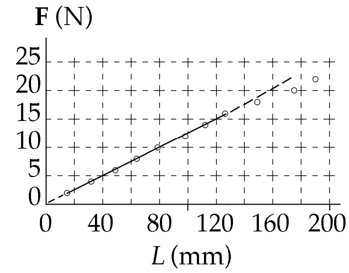
$$\tfrac{1}{2}kx_i^2 + mg\Delta x\cos 150° - \mu_k mg\cos 60°\Delta x = 0$$

$$\tfrac{1}{2}\left(1.40\times10^3\text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x - (0.200)(9.80)(0.400)(\cos 60.0°)\Delta x = 0$$

$$\Delta x = \boxed{3.35\text{ m}}$$

6.52 (a)

F (N)	L (mm)	F (N)	L (mm)
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190
12.0	98.0		



(b) A straight line fits the first eight points, together with the origin. By least-square fitting, its slope is

$0.125\ \text{N/mm} \pm 2\% = \boxed{125\ \text{N/m}} \pm 2\%$.

In $F = kx$, the spring constant is $k = F/x$, the same as the slope of the F-versus-x graph.

(c) $F = kx = (125\ \text{N/m})(0.105\ \text{m}) = \boxed{13.1\ \text{N}}$

***6.53** (a) $\mathbf{F}_1 = (25.0\ \text{N})(\cos 35.0°\mathbf{i} + \sin 35.0°\mathbf{j}) = \boxed{(20.5\mathbf{i} + 14.3\mathbf{j})\ \text{N}}$

$\mathbf{F}_2 = (42.0\ \text{N})(\cos 150°\mathbf{i} + \sin 150°\mathbf{j}) = \boxed{(-36.4\mathbf{i} + 21.0\mathbf{j})\ \text{N}}$

(b) $\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \boxed{(-15.9\mathbf{i} + 35.3\mathbf{j})\ \text{N}}$

(c) $\mathbf{a} = \dfrac{\sum \mathbf{F}}{m} = \boxed{(-3.18\mathbf{i} + 7.07\mathbf{j})\ \text{m/s}^2}$

(d) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = (4.00\mathbf{i} + 2.50\mathbf{j})\ \text{m/s} + (-3.18\mathbf{i} + 7.07\mathbf{j})(\text{m/s}^2)(3.00\ \text{s})$

$\mathbf{v}_f = \boxed{(-5.54\mathbf{i} + 23.7\mathbf{j})\ \text{m/s}}$

(e) $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{r}_f = 0 + (4.00\mathbf{i} + 2.50\mathbf{j})(\text{m/s})(3.00\ \text{s}) + \frac{1}{2}(-3.18\mathbf{i} + 7.07\mathbf{j})(\text{m/s}^2)(3.00\ \text{s})^2$

$\Delta\mathbf{r} = \mathbf{r}_f = \boxed{(-2.30\mathbf{i} + 39.3\mathbf{j})\ \text{m}}$

(f) $K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(5.00\ \text{kg})\left[(5.54)^2 + (23.7)^2\right](\text{m/s})^2 = \boxed{1.48\ \text{kJ}}$

(g) $K_f = \frac{1}{2}mv_i^2 + \sum \mathbf{F}\cdot\Delta\mathbf{r}$

$K_f = \frac{1}{2}(5.00\ \text{kg})\left[(4.00)^2 + (2.50)^2\right](\text{m/s})^2 + [(-15.9\ \text{N})(-2.30\ \text{m}) + (35.3\ \text{N})(39.3\ \text{m})]$

$K_f = 55.6\ \text{J} + 1426\ \text{J} = \boxed{1.48\ \text{kJ}}$

6.54 (a) $\Delta E_{\text{int}} = -\Delta K = -\frac{1}{2}m\left(v_f^2 - v_i^2\right)$: $\quad \Delta E_{\text{int}} = -\frac{1}{2}(0.400\text{ kg})\left((6.00)^2 - (8.00)^2\right)(\text{m/s})^2 = \boxed{5.60\text{ J}}$

(b) $\Delta E_{\text{int}} = f\Delta r = \mu_k mg(2\pi r)$: $\quad 5.60\text{ J} = \mu_k(0.400\text{ kg})\left(9.80\text{ m/s}^2\right)2\pi(1.50\text{ m})$

Thus, $\quad \mu_k = \boxed{0.152}$

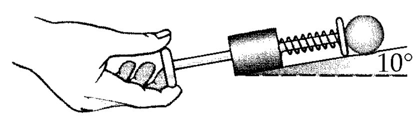
(c) After N revolutions, the object comes to rest and $K_f = 0$.

Thus, $\quad \Delta E_{\text{int}} = -\Delta K = -0 + K_i = \frac{1}{2}mv_i^2$

or $\quad \mu_k mg\left[N(2\pi r)\right] = \frac{1}{2}mv_i^2$

This gives $\quad N = \dfrac{\frac{1}{2}mv_i^2}{\mu_k mg(2\pi r)} = \dfrac{\frac{1}{2}(8.00\text{ m/s})^2}{(0.152)(9.80\text{ m/s}^2)2\pi(1.50\text{ m})} = \boxed{2.28\text{ rev}}$

6.55 $K_i + W_s + W_g = K_f$



$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 + mg\Delta x\cos\theta = \frac{1}{2}mv_f^2$

$0 + \frac{1}{2}kx_i^2 - 0 + mgx_i\cos 100° = \frac{1}{2}mv_f^2$

$\frac{1}{2}\left(1.20\ \dfrac{\text{N}}{\text{cm}}\right)(5.00\text{ cm})(0.0500\text{ m}) - (0.100\text{ kg})\left(9.80\text{ m/s}^2\right)(0.0500\text{ m})\sin 10.0° = \frac{1}{2}(0.100\text{ kg})\ v^2$

$0.150\text{ J} - 8.51\times 10^{-3}\text{ J} = (0.0500\text{ kg})v^2$

$v = \sqrt{\dfrac{0.141}{0.0500}} = \boxed{1.68\text{ m/s}}$

6.56 If positive F represents an outward force, (same as direction as r), then

$$W = \int_i^f \mathbf{F}\cdot d\mathbf{r} = \int_{r_i}^{r_f}\left(2F_0\sigma^{13}r^{-13} - F_0\sigma^7 r^{-7}\right)dr$$

$$W = \frac{2F_0\sigma^{13}r^{-12}}{-12} - \frac{F_0\sigma^7 r^{-6}}{-6}\Bigg|_{r_i}^{r_f}$$

$$W = \frac{-F_0\sigma^{13}\left(r_f^{-12} - r_i^{-12}\right)}{6} + \frac{F_0\sigma^7\left(r_f^{-6} - r_i^{-6}\right)}{6} = \frac{F_0\sigma^7}{6}\left[r_f^{-6} - r_i^{-6}\right] - \frac{F_0\sigma^{13}}{6}\left[r_f^{-12} - r_i^{-12}\right]$$

$$W = 1.03\times 10^{-77}\left[r_f^{-6} - r_i^{-6}\right] - 1.89\times 10^{-134}\left[r_f^{-12} - r_i^{-12}\right]$$

$$W = 1.03\times 10^{-77}\left[1.88\times 10^{-6} - 2.44\times 10^{-6}\right]10^{60} - 1.89\times 10^{-134}\left[3.54\times 10^{-12} - 5.96\times 10^{-8}\right]10^{120}$$

$$W = -2.49\times 10^{-21}\text{ J} + 1.12\times 10^{-21}\text{ J} = \boxed{-1.37\times 10^{-21}\text{ J}}$$

***6.57** (a) $W_{\text{other}} - f_k \Delta x = \Delta K: \quad m_2 gh - \mu m_1 gh = \frac{1}{2}(m_1 + m_2)\left(v_f^2 - v_i^2\right)$

$$v_f = \sqrt{\frac{2gh(m_2 - \mu m_1)}{(m_1 + m_2)}} = \sqrt{\frac{2(9.80)(20.0)[0.400 - (0.200)(0.250)]}{(0.400 + 0.250)}} = \boxed{14.5 \text{ m/s}}$$

(b) $-f_k \Delta x + W_g = \Delta K = 0: \qquad -\mu(\Delta m_1 + m_1)gh + m_2 gh = 0$

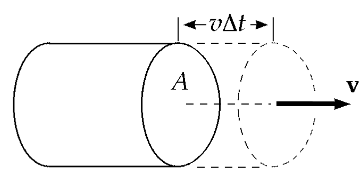
$$\mu(\Delta m_1 + m_1) = m_2$$

$$\Delta m_1 = \frac{m_2}{\mu} - m_1 = \frac{0.400 \text{ kg}}{0.200} - 0.250 \text{ kg} = \boxed{1.75 \text{ kg}}$$

(c) $-f_k \Delta x + W_g = \Delta K = 0: \qquad -\mu m_1 gh + (m_2 - \Delta m_2)gh = 0$

$$\Delta m_2 = m_2 - \mu m_1 = 0.400 \text{ kg} - (0.200)(0.250 \text{ kg}) = \boxed{0.350 \text{ kg}}$$

6.58 $\mathcal{P}\Delta t = W = \Delta K = \dfrac{(\Delta m)v^2}{2}$



The density is $\qquad \rho = \dfrac{\Delta m}{\text{vol}} = \dfrac{\Delta m}{A\,\Delta x}$

Substituting this into the first equation and solving for $\mathcal{P}$, since $\Delta x / \Delta t = v$,

for a constant speed, we get $\qquad \boxed{\mathcal{P} = \dfrac{\rho A v^3}{2}}$

Also, since $\mathcal{P} = Fv$, $\qquad \boxed{F = \dfrac{\rho A v^2}{2}}$

Our model predicts the same proportionalities as the empirical equation, and gives $D = 1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

6.59 We evaluate $\int_{12.8}^{23.7} \dfrac{375 dx}{x^3 + 3.75x}$ by calculating

$$\frac{375(0.100)}{(12.8)^3 + 3.75(12.8)} + \frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \ldots \frac{375(0.100)}{(23.6)^3 + 3.75(23.6)} = 0.806$$

and $$\frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \frac{375(0.100)}{(13.0)^3 + 3.75(13.0)} + \ldots \frac{375(0.100)}{(23.7)^3 + 3.75(23.7)} = 0.791$$

The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799 \text{ N} \cdot \text{m}}$.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 1.59×10^3 J

4. (a) 32.8 mJ (b) –32.8 mJ

6. See the solution.

8. 5.33 W

10. 16.0

12. (a) See the solution. The graph is a straight line passing through points (2 m, 0 N) and (3 m, 8 N)
(b) –12.0 J

14. (a) 575 N/m (b) 46.0 J

16. (a) 9.00 kJ (b) 11.7 kJ, larger by 29.6%

18. (a) See the solution. (b) mgR

20. (a) 33.8 J (b) 135 J

22. (a) 1.94 m/s (b) 3.35 m/s (c) 3.87 m/s

24. (a) 4.56 kJ (b) 6.34 kN (c) 422 km/s^2
(d) 6.34 kN

26. (a) 3.78×10^{-16} J (b) 1.35×10^{-14} N (c) 1.48×10^{16} m/s^2
(d) 1.94×10^{-9} s

28. (a) 79.4 N (b) 1.49 kJ (c) 1.49 kJ

30. (a) 329 J (b) 0 (c) 0
(d) 185 J (e) 144 J

32. (a) 0.791 m/s (b) 0.531 m/s

34. $\sim 10^4$ W

36. 830 N

38. (a) 5910 W (b) It is 53.0% of 11 100 W

40. (a) 423 mi/gal (b) 776 mi/gal

42. (a) 8.50×10^8 J (b) 2.71×10^9 J

44. (a) $\cos\alpha = \frac{A_x}{A}, \cos\beta = \frac{A_y}{A}, \cos\gamma = \frac{A_z}{A}$ (b) See the solution.

46. (a) $\frac{mgnhh_s}{v + nh_s}$ (b) $\frac{mgvh}{v + nh_s}$

48. 7.37 N/m

50. (a) See the solution. (b) $2kL^2 + kA^2 - 2kL\sqrt{A^2 + L^2}$

52. (a) See the solution. (b) 125 N/m (c) 13.1 N

54. (a) 5.60 J (b) 0.152 (c) 2.28 rev

56. -1.37×10^{-21} J

58. See the solution.