CHAPTER 7

ANSWERS TO QUESTIONS

Q7.1 Both agree on the *change* in potential energy, and the kinetic energy. They may disagree on the value of gravitational potential energy, depending on their choice of a zero point.

Q7.2 (a) *mgh* is provided by the muscles.
(b) energy must be supplied to the muscles to keep the weight aloft.
(c) to lower slowly requires muscular work against gravity.

Q7.3 Lift a book from a low shelf to a high shelf. The net change in its kinetic energy is zero, but the book-Earth system increases in gravitational potential energy. Stretch a rubber band to encompass the ends of a ruler. It increases in elastic energy. Rub your hands together or let a pearl drift down at constant speed in a bottle of shampoo. Each system (two hands; pearl and shampoo) increases in internal energy.

Q7.4 Yes. Although to get started, a push on the ski poles is necessary to overcome friction.

Q7.5 Same amount of work. However, this is done over a much longer time (lower power output) in the first case.

Q7.6 No. The rock has an acceleration of larger magnitude on the way up, so its upward motion occupies less time than its downward motion. A net force of greater magnitude acts on it when it is moving up. Then gravity and air resistance are in the same direction.

Q7.7 The original kinetic energy of the skidding can into degraded into kinetic energy of random molecular motion in the tires and the road: it is internal energy. If the brakes are used properly, the same energy appears as internal energy in the brake shoes and drums.

Q7.8 Potential energy of plates under stress plus gravitational energy is released when the plates "slip".

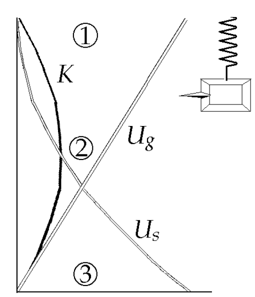
Q7.9 All the energy is supplied by foodstuffs that gained their energy from the sun.

Q7.10 The total energy of the ball-Earth system is conserved. Since the system initially has gravitational energy *mgh* and no kinetic energy, the ball will again have zero kinetic energy when it returns to its original position. Air resistance will cause the ball to come back to a point slightly below its initial position. On the other hand, if anyone gives a forward push to the ball anywhere along its path, the demonstrator will have to duck.

Q7.11 Gravitational energy is proportional to mass, so it doubles.

Q7.12 In stirring cake batter and in weightlifting, your body returns to the same conformation after each stroke. During each stroke chemical energy is irreversibly converted into output work (and internal energy). This observation proves that muscular forces are nonconservative.

Q7.13 Let the gravitational energy be zero at the lowest point in the motion. If you start the vibration by pushing down on the block (2), its kinetic energy becomes extra elastic potential energy in the spring (U_s). After the block starts moving up at its lower turning point (3), this energy becomes both kinetic energy (K) and gravitational potential energy (U_g), and then just gravitational energy when the block is at its greatest height (1). The energy then turns back into kinetic and elastic potential energy, and the cycle repeats.



Q7.14 A graph of potential energy versus position is a straight horizontal line for a particle in neutral equilibrium. The graph represents a constant function.

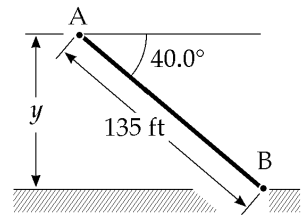
Q7.15 The ball is in neutral equilibrium.

Q7.16 Chemical energy in the fuel turns into internal energy as the fuel burns. Most of this leaves the car by heat through the walls of the engine and by matter transfer in the exhaust gases. Some leaves the system of fuel by work done to push down the piston. Of this work, a little results in internal energy in the bearings and gears, but most becomes work done on the air to push it aside. The work on the air immediately turns into internal energy in the air. If you use the windshield wipers, you take energy from the crankshaft and turn it into extra internal energy in the glass and wiper blades and wiper-motor coils. If you turn on the air conditioner, your end effect is to put extra energy out into the surroundings. You must apply the brakes at the end of your trip. As soon as the sound of the engine has died away, all you have to show for it is thermal pollution.

Q7.17 Kinetic energy is greatest at the starting point. Gravitational energy is a maximum at the top of the flight of the ball.

PROBLEM SOLUTIONS

7.1 (a) With our choice for the zero level for potential energy when the car is at point B,



$$\boxed{U_B = 0}$$

When the car is at point A, the potential energy of the car-Earth system is given by

$$U_A = mgy$$

where y is the vertical height above zero level. With 135 ft = 41.1 m,

this height is found as: $y = (41.1 \text{ m}) \sin 40.0° = 26.4 \text{ m}$

Thus, $U_A = (1000 \text{ kg})(9.80 \text{ m/s}^2)(26.4 \text{ m}) = \boxed{2.59 \times 10^5 \text{ J}}$

The change in potential energy as the car moves from A to B is

$$U_B - U_A = 0 - 2.59 \times 10^5 \text{ J} = \boxed{-2.59 \times 10^5 \text{ J}}$$

(b) With our choice of the zero level when the car is at point A, we have $\boxed{U_A = 0}$

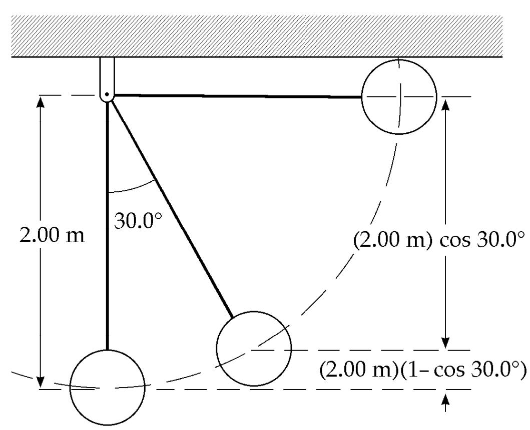
The potential energy when the car is at point B is given by $U_B = mgy$ where y is the vertical distance of point B below point A. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus, $U_B = (1000 \text{ kg})(9.80 \text{ m/s}^2)(-26.5 \text{ m}) = \boxed{-2.59 \times 10^5 \text{ J}}$

The change in potential energy when the car moves from A to B is

$$U_B - U_A = -2.59 \times 10^5 \text{ J} - 0 = \boxed{-2.59 \times 10^5 \text{ J}}$$

7.2 (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the string is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,



$$U_g = mgy = (400 \text{ N})(2.00 \text{ m}) = \boxed{800 \text{ J}}$$

(b) From the sketch, we see that at an angle of 30.0° the child is at a vertical height of $(2.00 \text{ m})(1 - \cos 30.0°)$ above the lowest point of the arc. Thus,

$$U_g = mgy = (400 \text{ N})(2.00 \text{ m})(1 - \cos 30.0°) = \boxed{107 \text{ J}}$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

***7.3** The volume flow rate is the volume of water going over the falls each second:

$$3\text{ m}(0.5\text{ m})(1.2\text{ m/s}) = 1.8\text{ m}^3/\text{s}$$

The mass flow rate is $\frac{m}{t} = \rho\frac{V}{t} = (1000\text{ kg/m}^3)(1.8\text{ m}^3/\text{s}) = 1800\text{ kg/s}$

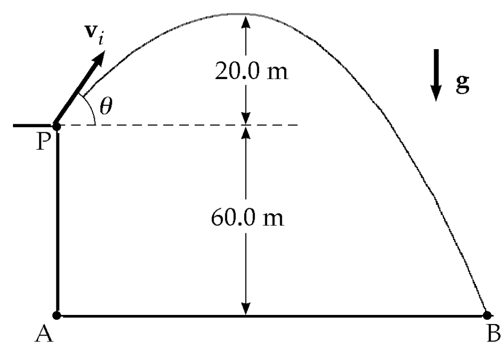
If the stream has uniform width and depth, the speed of the water below the falls is the same as the speed above the falls. Then no kinetic energy, but only gravitational energy is available for conversion into internal and electric energy.

The input power is $\mathcal{P}_{\text{in}} = \frac{\text{energy}}{t} = \frac{mgy}{t} = \frac{m}{t}gy = (1800\text{ kg/s})(9.8\text{ m/s}^2)(5\text{ m}) = 8.82\times10^4\text{ J/s}$

The output power is $\mathcal{P}_{\text{useful}} = (\text{efficiency})\mathcal{P}_{in} = 0.25(8.82\times10^4\text{ W}) = \boxed{2.20\times10^4\text{ W}}$

The efficiency of electric generation at Hoover Dam is about 85%, with a head of water (vertical drop) of 174 m. Intensive research is underway to improve the efficiency of low head generators.

7.4 (a) Energy of the particle-Earth system is conserved as the particle moves between point P and the apex of the trajectory.



Since the horizontal component of velocity is constant,

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_{ix}^2 + \frac{1}{2}mv_{iy}^2 = \frac{1}{2}mv_{ix}^2 + mgh$$

$$v_{iy} = \sqrt{2(9.80)(20.0)} = \boxed{19.8\text{ m/s}}$$

(b) $\Delta K|_{P\to B} = W_g = mg(60.0\text{ m}) = (0.500\text{ kg})(9.80\text{ m/s}^2)(60.0\text{ m}) = \boxed{294\text{ J}}$

(c) Now let the final point be point B.

$$v_{xi} = v_{xf} = 30.0\text{ m/s}$$

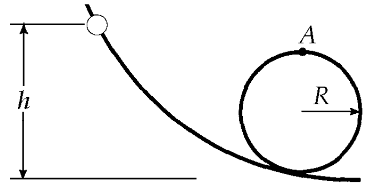
$$\Delta K|_{P\to B} = \frac{1}{2}mv_{yf}^2 - \frac{1}{2}mv_{yi}^2 = 294\text{ J}$$

$$v_{yf}^2 = \frac{2}{m}(294) + v_{yi}^2 = 1176 + 392$$

$$v_{yf} = -39.6\text{ m/s}$$

$$\mathbf{v}_B = \boxed{(30.0\text{ m/s})\mathbf{i} - (39.6\text{ m/s})\mathbf{j}}$$

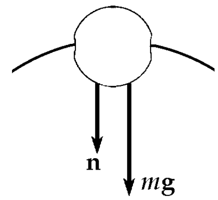
7.5 $U_i + K_i = U_f + K_f$: $mgh + 0 = mg(2R) + \frac{1}{2}mv^2$



$$g(3.50\ R) = 2\ g(R) + \tfrac{1}{2}v^2$$

$$\boxed{v = \sqrt{3.00\ g\ R}}$$

$\sum F = m\dfrac{v^2}{R}$: $n + mg = m\ \dfrac{v^2}{R}$



$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00\,g\,R}{R} - g\right] = 2.00\ mg$$

$$n = 2.00\left(5.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = \boxed{0.0980\ \text{N downward}}$$

7.6 From leaving ground to the highest point, $K_i + U_i = K_f + U_f$

$$\frac{1}{2}m(6.00\ \text{m/s})^2 + 0 = 0 + m\left(9.80\ \text{m/s}^2\right)y$$

The mass makes no difference: $\therefore\ y = \dfrac{(6.00\ \text{m/s})^2}{(2)\left(9.80\ \text{m/s}^2\right)} = \boxed{1.84\ \text{m}}$

***7.7** (a) Energy of the object-Earth system is conserved as the object moves between the release point and the lowest point. We choose to measure heights from $y = 0$ at the top end of the string.

$\left(K + U_g\right)_i = \left(K + U_g\right)_f$: $0 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$

$$\left(9.8\ \text{m/s}^2\right)\left(-2\ \text{m}\ \cos 30°\right) = \tfrac{1}{2}v_f^2 + \left(9.8\ \text{m/s}^2\right)(-2\ \text{m})$$

$$v_f = \sqrt{2\left(9.8\ \text{m/s}^2\right)(2\ \text{m})(1 - \cos 30°)} = \boxed{2.29\ \text{m/s}}$$

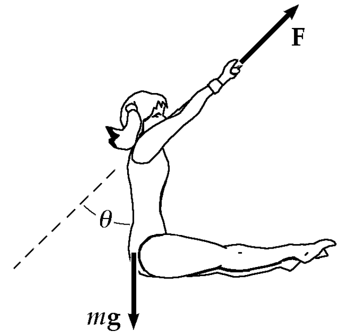
(b) Choose the initial point at $\theta = 30°$ and the final point at $\theta = 15°$:

$$0 + mg(-L\cos 30°) = \tfrac{1}{2}mv_f^2 + mg(-L\cos 15°)$$

$$v_f = \sqrt{2gL(\cos 15° - \cos 30°)} = \sqrt{2\left(9.8\ \text{m/s}^2\right)(2\ \text{m})(\cos 15° - \cos 30°)} = \boxed{1.98\ \text{m/s}}$$

***7.8** (a) The force needed to hang on is equal to the force F the trapeze bar exerts on the performer.

From the free-body diagram for the performer's body, as shown,



$$F - mg\cos\theta = m\frac{v^2}{\ell}$$

or $$F = mg\cos\theta + m\frac{v^2}{\ell}$$

Apply conservation of mechanical energy of the performer-Earth system as the performer moves between the starting point and any later point:

$$mg(\ell - \ell\cos\theta_i) = mg(\ell - \ell\cos\theta) + \tfrac{1}{2}mv^2$$

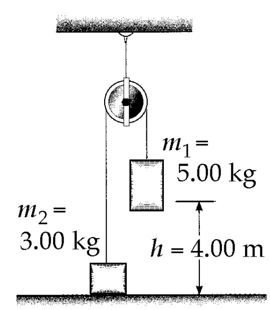
Solve for mv^2/ℓ and substitute into the force equation to obtain $F = \boxed{mg(3\cos\theta - 2\cos\theta_i)}$

(b) At the bottom of the swing, $\theta = 0°$ so $F = mg(3 - 2\cos\theta_i)$

$F = 2mg = mg(3 - 2\cos\theta_i)$

which gives $\theta_i = \boxed{60.0°}$

7.9 Using conservation of energy for the system of the Earth and the two objects



(a) $(5.00\text{ kg})g(4.00\text{ m}) = (3.00\text{ kg})g(4.00\text{ m}) + \tfrac{1}{2}(5.00 + 3.00)v^2$

$v = \sqrt{19.6} = \boxed{4.43\text{ m/s}}$

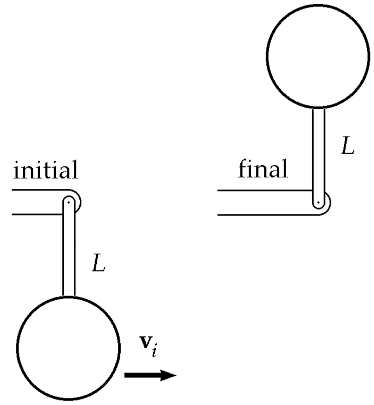
(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\tfrac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00g\,\Delta y$$

$$\Delta y = 1.00\text{ m}$$

$$y_{max} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

***7.10** The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation of the ball-Earth system between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there.



$K_i + U_{gi} = K_f + U_{gf}$: $\quad \frac{1}{2}mv_i^2 + 0 = 0 + mg(2L)$

$$v_i \sqrt{4gL} = \sqrt{4(9.80)(0.770)}$$

$$v_i \sqrt{4gL} = \boxed{5.49\text{ m/s}}$$

7.11 (a) $U_f = K_i - K_f + U_i$ $\quad U_f = 30.0 - 18.0 + 10.0 = \boxed{22.0\text{ J}}$

$$\boxed{E = 40.0\text{ J}}$$

(b) Yes, $\Delta E_{\text{mech}} = \Delta K + \Delta U$ is not equal to zero. For conservative forces $\Delta K + \Delta U = 0$.

***7.12** (a) $W = \int \mathbf{F} \cdot d\mathbf{r}$ and if the force is constant, this can be written as

$W = \mathbf{F} \cdot \int d\mathbf{r} = \boxed{\mathbf{F} \cdot (\mathbf{r}_\text{f} - \mathbf{r}_\text{i})\text{, which depends only on end points, not path.}}$

(b) $W = \int \mathbf{F} \cdot d\mathbf{r} = \int (3\mathbf{i} + 4\mathbf{j}) \cdot (dx\mathbf{i} + dy\mathbf{j}) = (3.00\text{ N})\int_0^{5.00\text{ m}} dx + (4.00\text{ N})\int_0^{5.00\text{ m}} dy$

$W = (3.00\text{ N})x\big|_0^{5.00\text{ m}} + (4.00\text{ N})y\big|_0^{5.00\text{ m}} = 15.0\text{ J} + 20.0\text{ J} = \boxed{35.0\text{ J}}$

The same calculation applies for all paths.

***7.13** (a) $W_{OA} = \int_0^{5.00\text{ m}} dx\,\mathbf{i}\cdot\left(2y\,\mathbf{i} + x^2\,\mathbf{j}\right) = \int_0^{5.00\text{ m}} 2y\,dx$

and since along this path, $y = 0$ $\quad W_{OA} = 0$

$$W_{AC} = \int_0^{5.00\text{ m}} dy\,\mathbf{j}\cdot\left(2y\,\mathbf{i} + x^2\,\mathbf{j}\right) = \int_0^{5.00\text{ m}} x^2 dy$$

For $x = 5.00\text{ m}$, $\quad W_{AC} = 125\text{ J}$

and $\quad W_{OAC} = 0 + 125 = \boxed{125\text{ J}}$

(b) $W_{OB} = \int_0^{5.00\text{m}} dy\mathbf{j}\cdot(2y\mathbf{i} + x^2\mathbf{j}) = \int_0^{5.00m} x^2 dy$

since along this path, $x = 0$, $\quad W_{OB} = 0$

$$W_{BC} = \int_0^{5.00\text{ m}} dx\,\mathbf{i}\cdot\left(2y\,\mathbf{i} + x^2\,\mathbf{j}\right) = \int_0^{5.00\text{ m}} 2ydx$$

since $y = 5.00\text{ m}$, $\quad W_{BC} = 50.0\text{ J}$

$$W_{OBC} = 0 + 50.0 = \boxed{50.0\text{ J}}$$

(c) $W_{OC} = \int\left(dx\,\mathbf{i} + dy\,\mathbf{j}\right)\cdot\left(2y\,\mathbf{i} + x^2\,\mathbf{j}\right) = \int\left(2ydx + x^2dy\right)$

Since $x = y$ along OC, $\quad W_{OC} = \int_0^{5.00\text{ m}}\left(2x + x^2\right)dx = \boxed{66.7\text{ J}}$

(d) F is $\boxed{\text{nonconservative}}$ since the work done is path dependent.

7.14 The distance traveled by the ball from the top of the arc to the bottom is πR. The work done by the non-conservative force, the force exerted by the pitcher,

is $\quad \Delta E = F\Delta r\cos 0° = F(\pi R)$.

We shall assign the gravitational energy of the ball-Earth system to be zero with the ball at the bottom of the arc.

Then $\quad \Delta E_{\text{mech}} = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2} + mgy_f - mgy_i$

becomes $\quad \frac{1}{2}mv_f^{\,2} = \frac{1}{2}mv_i^{\,2} + mgy_i + F(\pi R)$

or $\quad v_f = \sqrt{v_i^{\,2} + 2gy_i + \frac{2F(\pi R)}{m}} = \sqrt{(15.0)^2 + 2(9.80)(1.20) + \frac{2(30.0)\pi(0.600)}{0.250}}$

$$v_f = \boxed{26.5\text{ m/s}}$$

***7.15** As the locomotive moves up the hill at constant speed, its output power goes into internal energy plus gravitational energy of the locomotive-Earth system:

$$\mathcal{P}t = mgy + f\Delta r = mg\Delta r\sin\theta + f\Delta r \qquad \mathcal{P} = mgv_f\sin\theta + fv_f$$

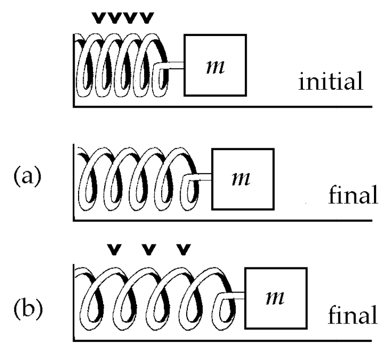
As the locomotive moves on level track,

$$\mathcal{P} = fv_i \qquad 1000\text{ hp}\left(\frac{746\text{ W}}{1\text{ hp}}\right) = f(27\text{ m/s}) \qquad f = 2.76\times10^4\text{ N}$$

Then also $746\,000\text{ W} = (160\,000\text{ kg})(9.8\text{ m/s}^2)v_f\left(\frac{5\text{ m}}{100\text{ m}}\right) + (2.76\times10^4\text{ N})v_f$

$$v_f = \frac{746\,000\text{ W}}{1.06\times10^5\text{ N}} = \boxed{7.04\text{ m/s}}$$

***7.16** (a) $\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$



$$0 + \frac{1}{2}(10\text{ N/m})(-0.18\text{ m})^2 = \frac{1}{2}(0.15\text{ kg})v_f^2 + 0$$

$$v_f = (0.18\text{ m})\sqrt{\left(\frac{10\text{ N}}{0.15\text{ kg}\cdot\text{m}}\right)\left(\frac{1\text{ kg}\cdot\text{m}}{1\text{ N}\cdot\text{s}^2}\right)} = \boxed{1.47\text{ m/s}}$$

(b) $K_i + U_{si} = K_f + U_{sf}$

$$0 + \frac{1}{2}(10\text{ N/m})(-0.18\text{ m})^2 = \frac{1}{2}(0.15\text{ kg})v_f^2 + \frac{1}{2}(10\text{ N/m})(0.25\text{ m} - 0.18\text{ m})^2$$

$$0.162\text{ J} = \frac{1}{2}(0.15\text{ kg})v_f^2 + 0.0245\text{ J} \qquad v_f = \sqrt{\frac{2(0.138\text{ J})}{0.15\text{ kg}}} = \boxed{1.35\text{ m/s}}$$

***7.17** Choose the zero point of gravitational potential energy of the object-spring-Earth system as the configuration in which the object comes to rest. Then because the incline is frictionless, we have $E_B = E_A$:

$$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

or

$$0 + mg(d + x)\sin\theta + 0 = 0 + 0 + \frac{1}{2}kx^2$$

Solving for d gives

$$d = \boxed{\frac{kx^2}{2mg\sin\theta} - x}$$

***7.18** (a) At the equilibrium position for the object, the tension in the spring equals the weight of the object. Thus, elongation of the spring when the object is at equilibrium is:

$kx_0 = mg$: $\quad x_0 = \frac{mg}{k} = \frac{(0.120)(9.80)}{40.0} = 0.0294 \text{ m}$

The object moves with maximum speed as it passes through the equilibrium position. Use energy conservation for the object-spring-Earth system, taking $U_g = 0$ at the initial position of the object, to find this speed:

$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$: $\quad \frac{1}{2}mv_{max}^2 + mg(-x_0) + \frac{1}{2}kx_0^2 = 0 + 0 + 0$

$$v_{max} = \sqrt{2gx_0 - \frac{kx_0^2}{m}} = \sqrt{2(9.80)(0.0294) - \frac{(40.0)(0.0294)^2}{0.120}} = \boxed{0.537 \text{ m/s}}$$

(b) When the object comes to rest, $K_f = 0$.

Therefore, $K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$

becomes $0 + mg(-x) + \frac{1}{2}kx^2 = 0 + 0 + 0$

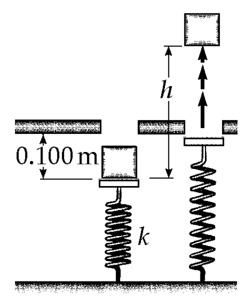
which becomes $x = \frac{2mg}{k} = 2x_0 = \boxed{0.0588 \text{ m}}$

***7.19** From conservation of energy for the block-spring-Earth system,

$U_{gt} = U_{si}$,

or $(0.250 \text{ kg})(9.80 \text{ m/s}^2)h = (1/2)(5000 \text{ N/m})(0.100 \text{ m})^2$

This gives a maximum height $h = \boxed{10.2 \text{ m}}$

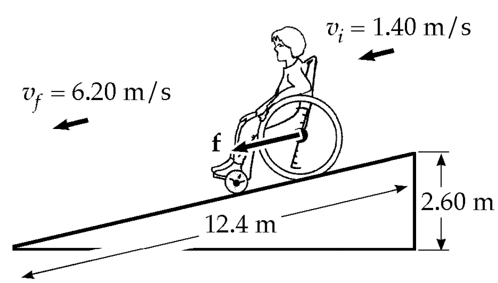


***7.20** $\Delta E_{mech} = (K_f - K_i) + (U_{gf} - U_{gi})$

But $\Delta E_{mech} = W_{app} - f\Delta x$, where W_{app} is the work the boy did pushing forward on the wheels.

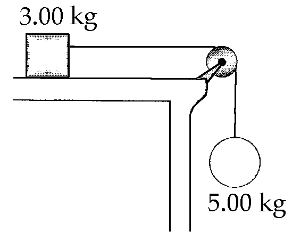
Thus, $W_{app} = (K_f - K_i) + (U_{gf} - U_{gi}) + f\Delta x$,

or $W_{app} = \frac{1}{2}m(v_f^2 - v_i^2) + mg(-h) + f\Delta x$



$$W_{app} = \frac{1}{2}(47.0)\left[(6.20)^2 - (1.40)^2\right] - (47.0)(9.80)(2.60) + (41.0)(12.4)$$

$W_{app} = \boxed{168 \text{ J}}$



7.21 $U_i + K_i + \Delta E_{\text{mech}} = U_f + K_f$: $\quad m_2gh - fh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$

$$f = \mu n = \mu m_1 g$$

$$m_2gh - \mu m_1 gh = \frac{1}{2}(m_1 + m_2)v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(1.50\text{ m})[5.00\text{ kg} - 0.400(3.00\text{ kg})]}{8.00\text{ kg}}} = \boxed{3.74\text{ m/s}}$$

7.22 Consider the whole motion: $K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

(a) $0 + mgy_i - f_1\Delta x_1 - f_2\Delta x_2 = \frac{1}{2}mv_f^2 + 0$

$$(80.0\text{ kg})(9.80\text{ m/s}^2)1000\text{ m} - (50.0\text{ N})(800\text{ m}) - (3600\text{ N})(200\text{ m}) = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$784\,000\text{ J - }40\,000\text{ J - }720\,000\text{ J} = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(24\,000\text{ J})}{80.0\text{ kg}}} = \boxed{24.5\text{ m/s}}$$

(b) $\boxed{\text{Yes}}$ this is too fast for safety.

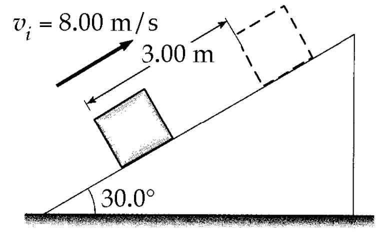
(c) Now in the same energy equation as in part (a), Δx_2 is unknown, and $\Delta x_1 = 1000\text{ m} - \Delta x_2$:

$$784\,000\text{ J} - (50.0\text{ N})(1000\text{ m} - \Delta x_2) - (3600\text{ N})\Delta x_2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2$$

$$784\,000\text{ J} - 50\,000\text{ J} - (3550\text{ N})\Delta x_2 = 1000\text{ J}$$

$$\Delta x_2 = \frac{733\,000\text{ J}}{3550\text{ N}} = \boxed{206\text{ m}}$$

(d) Really the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.



7.23 (a) $\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}mv_i^2 = \boxed{-160\text{ J}}$

(b) $\Delta U = mg(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$

(c) The mechanical energy converted due to friction is 86.5 J

$$f = \frac{86.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$$

(d) $f = \mu_k n = \mu_k mg\cos 30.0° = 28.8\text{ N}$

$$\mu_k = \frac{28.8\text{ N}}{(5.00\text{ kg})(9.80\text{ m/s}^2)\cos 30.0°} = \boxed{0.679}$$

7.24 (a) $(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f$:

$$0 + \tfrac{1}{2}kx^2 - f\Delta x = \tfrac{1}{2}mv^2 + 0$$

$$\tfrac{1}{2}(8.00\text{ N/m})\left(5.00\times10^{-2}\text{ m}\right)^2 - \left(3.20\times10^{-2}\text{ N}\right)(0.150\text{ m}) = \tfrac{1}{2}\left(5.30\times10^{-3}\text{ kg}\right)v^2$$

$$v = \sqrt{\frac{2\left(5.20\times10^{-2}\text{ J}\right)}{5.30\times10^{-3}\text{ kg}}} = \boxed{1.40\text{ m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $|F_s| = kx$; the spring is compressed by

$$\frac{3.20\times10^{-2}\text{ N}}{8.00\text{ N/m}} = 0.400\text{ cm}$$

and the ball has moved

$$5.00\text{ cm} - 0.400\text{ cm} = \boxed{4.60\text{ cm from the start.}}$$

(c) Between start and maximum speed points,

$$\tfrac{1}{2}kx_i^2 - f\Delta x = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx_f^2$$

$$\tfrac{1}{2}8.00\left(5.00\times10^{-2}\right)^2 - \left(3.20\times10^{-2}\right)\left(4.60\times10^{-2}\right) = \tfrac{1}{2}\left(5.30\times10^{-3}\right)v^2 + \tfrac{1}{2}8.00\left(4.00\times10^{-3}\right)^2$$

$$v = \boxed{1.79\text{ m/s}}$$

7.25 (a) The object moved down distance $1.20 \text{ m} + x$. Choose $y = 0$ at its lower point.

$$K_i + U_{gi} + U_{si} + \Delta E_{\text{mech}} = K_f + U_{gf} + U_{sf}$$

$$0 + mgy_i + 0 + 0 = 0 + 0 + \tfrac{1}{2}kx^2$$

$$(1.50 \text{ kg})(9.80 \text{ m/s}^2)(1.20 \text{ m} + x) = \tfrac{1}{2}(320 \text{ N/m})x^2$$

$$0 = (160 \text{ N/m})x^2 - (14.7 \text{ N})x - 17.6 \text{ J}$$

$$x = \frac{14.7 \text{ N} \pm \sqrt{(-14.7 \text{ N})^2 - 4(160 \text{ N/m})(-17.6 \text{ N}\cdot\text{m})}}{2(160 \text{ N/m})}$$

$$x = \frac{14.7 \text{ N} \pm 107 \text{ N}}{320 \text{ N/m}}$$

The negative root tells how high the object will rebound if it is instantly glued to the spring. We want

$$x = \boxed{0.381 \text{ m}}$$

(b) From the same equation,

$$(1.50 \text{ kg})(1.63 \text{ m/s}^2)(1.20 \text{ m} + x) = \tfrac{1}{2}(320 \text{ N/m})x^2$$

$$0 = 160x^2 - 2.44x - 2.93$$

The positive root is $x = \boxed{0.143 \text{ m}}$

(c) The equation expressing the energy version of the nonisolated system model has one more term:

$$mgy_i - f\Delta x = \tfrac{1}{2}kx^2$$

$$(1.50 \text{ kg})(9.80 \text{ m/s}^2)(1.20 \text{ m} + x) - 0.700 \text{ N}(1.20 \text{ m} + x) = \tfrac{1}{2}(320 \text{ N/m})x^2$$

$$17.6 \text{ J} + 14.7 \text{ N}\, x - 0.840 \text{ J} - 0.700 \text{ N}\, x = 160 \text{ N/m}\, x^2$$

$$160x^2 - 14.0x - 16.8 = 0$$

$$x = \frac{14.0 \pm \sqrt{(14.0)^2 - 4(160)(-16.8)}}{320}$$

$$x = \boxed{0.371 \text{ m}}$$

***7.26** The total mechanical energy of the skysurfer-Earth system is

$$E_{\text{mech}} = K + U_g = \frac{1}{2}mv^2 + mgh.$$

Since the skysurfer has constant speed,

$$\frac{dE_{\text{mech}}}{dt} = mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0 + mg(-v) = -mgv$$

The rate the system is losing mechanical energy is then

$$\left|\frac{dE_{\text{mech}}}{dt}\right| = mgv = (75.0 \text{ kg})(9.80 \text{ m/s}^2)(60.0 \text{ m/s}) = \boxed{44.1 \text{ kW}}$$

7.27 (a) $W = \int F_x\,dx = \int_1^{5.00\text{m}} (2x+4)dx = \left(\frac{2x^2}{2} + 4x\right)_1^{5.00\text{m}} = 25.0 + 20.0 - 1.00 - 4.00 = \boxed{40.0 \text{ J}}$

(b) $\Delta K + \Delta U = 0$ $\qquad \Delta U = -\Delta K = -W = \boxed{-40.0 \text{ J}}$

(c) $\Delta K = K_f - \frac{mv_1^2}{2}$ $\qquad K_f = \Delta K + \frac{mv_1^2}{2} = \boxed{62.5 \text{ J}}$

***7.28** (a) $U = -\int_0^x \left(-Ax + Bx^2\right)dx = \boxed{\frac{Ax^2}{2} - \frac{Bx^2}{3}}$

(b) $\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F dx = \frac{A\left[\left(3.00^2\right) - (2.00)^2\right]}{2} - \frac{B\left[(3.00)^3 - (2.00)^2\right]}{3} = \boxed{\frac{5.00}{2}A - \frac{19.0}{3}B}$

$\Delta K = \boxed{\left(-\frac{5.00}{2}A + \frac{19.0}{3}B\right)}$

7.29 $U(r) = \frac{A}{r}$ $\qquad F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$

***7.30** $F_x = -\frac{\partial U}{\partial x} = -\frac{\partial\left(3x^3y - 7x\right)}{\partial x} = -(9x^2y - 7) = 7 - 9x^2y$

$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial\left(3x^3y - 7x\right)}{\partial y} = -(3x^3 - 0) = -3x^3$

Thus, the force acting at the point (x, y) is $\qquad \mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} = \boxed{\left(7 - 9x^2y\right)\mathbf{i} - 3x^3\mathbf{j}}$

7.31 (a) $U = -\dfrac{GM_E m}{r} = -\dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(100\ \text{kg})}{(6.37+2.00)\times10^6\ \text{m}} = \boxed{-4.77\times10^{9}\ \text{J}}$

(b) and (c)

Planet and satellite exert forces of equal magnitude on each other, directed downward on the satellite and upward on the planet.

$$F = \frac{GM_E\, m}{r^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})(100\ \text{kg})}{(8.37\times10^6\ \text{m})^2} = \boxed{569\ \text{N}}$$

7.32 $U = -G\dfrac{Mm}{r}$ and $g = \dfrac{GM_E}{R_E{}^2}$

so that $\Delta U = -GMm\left(\dfrac{1}{3R_E} - \dfrac{1}{R_E}\right) = \tfrac{2}{3}mgR_E$

$$\Delta U = \tfrac{2}{3}(1000\ \text{kg})(9.80\ \text{m/s}^2)(6.37\times10^6\ \text{m}) = \boxed{4.17\times10^{10}\ \text{J}}$$

7.33 The height attained is not small compared to the radius of the Earth, so $U = mgy$ does not apply; $U = -GM_1M_2/r$ does. From launch to apogee at height h,

$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$: $\quad \tfrac{1}{2}M_p v_i{}^2 - \dfrac{GM_E M_p}{R_E} + 0 = 0 - \dfrac{GM_E M_p}{R_E + h}$

$$\tfrac{1}{2}(10.0\times10^3\,\text{m/s})^2 - (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left(\frac{5.98\times10^{24}\ \text{kg}}{6.37\times10^6\ \text{m}}\right)$$

$$= -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left(\frac{5.98\times10^{24}\ \text{kg}}{6.37\times10^6\ \text{m}+h}\right)$$

$$(5.00\times10^7\ \text{m}^2/\text{s}^2) - (6.26\times10^7\ \text{m}^2/\text{s}^2) = \frac{-3.99\times10^{14}\ \text{m}^3/\text{s}^2}{6.37\times10^6\ \text{m}+h}$$

$$6.37\times10^6\ \text{m} + h = \frac{3.99\times10^{14}\ \text{m}^3/\text{s}^2}{1.26\times10^7\ \text{m}^2/\text{s}^2} = 3.16\times10^7\ \text{m}$$

$$\boxed{h = 2.52\times10^7\ \text{m}}$$

7.34 (a) $U_{Tot} = U_{12} + U_{13} + U_{23} = 3U_{12} = 3\left(-\frac{Gm_1m_2}{r_{12}}\right)$

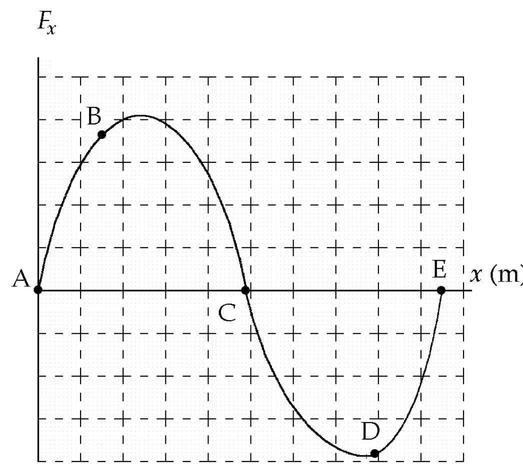
$$U_{Tot} = -\frac{3\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.00\times10^{-3}\ \text{kg}\right)^2}{0.300\ \text{m}} = \boxed{-1.67\times10^{-14}\ \text{J}}$$

(b) $\boxed{\text{At the center}}$ of the equilateral triangle

7.35 (a) F_x is zero at points A, C and E; F_x is positive at point B and negative at point D.

(b) A and E are unstable, and C is stable.

(c) See the picture to the right



***7.36** (a) There is an equilibrium point wherever the graph of potential energy is horizontal:

At $r = 1.5$ mm and 3.2 mm, the equilibrium is stable.
At $r = 2.3$ mm, the equilibrium is unstable.
A particle moving out toward $r \to \infty$ approaches neutral equilibrium.

(b) The system energy E cannot be less than –5.6 J. The particle is bound if $\boxed{-5.6\ \text{J} \le E < 1\ \text{J}}$

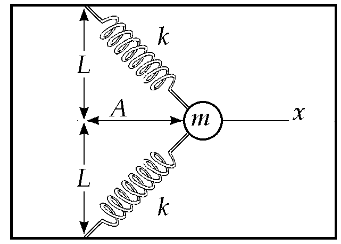
(c) If the system energy is –3 J, its potential energy must be less than or equal to –3 J. Thus, the particle's position is limited to $\boxed{0.6\ \text{mm} \le r \le 3.6\ \text{mm}}$

(d) $K + U = E$. Thus, $K_{max} = E - U_{min} = -3.0\ \text{J} - (-5.6\ \text{J}) = \boxed{2.6\ \text{J}}$

(e) Kinetic energy is a maximum when the potential energy is a minimum, at $\boxed{r = 1.5\ \text{mm}}$

(f) $-3\ \text{J} + W = 1\ \text{J}$. Hence, the binding energy is $W = \boxed{4\ \text{J}}$

***7.37** (a) When the mass moves distance x, the length of each spring changes from L to $\sqrt{x^2+L^2}$, so each exerts force $k\left(\sqrt{x^2-L^2}-L\right)$ towards its fixed end. The y-components cancel out and the x components add to:



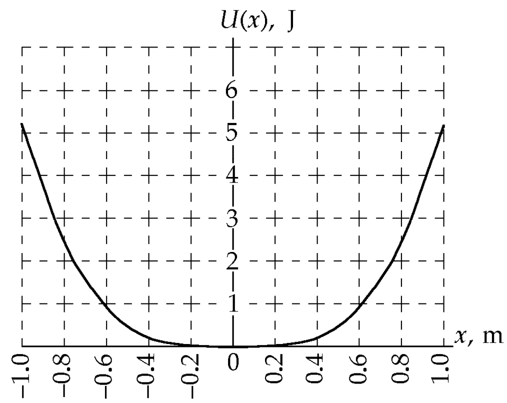
$$F_x = -2k\left(\sqrt{x^2-L^2}-L\right)\left(\frac{x}{\sqrt{x^2-L^2}}\right) = -2kx + \frac{2kLx}{\sqrt{x^2+L^2}}$$

Choose $U = 0$ at $x = 0$. Then at any point the potential energy of the system is

$$U(x) = -\int_0^x F_x dx = -\int_0^x \left(-2kx + \frac{2kLx}{\sqrt{x^2+L^2}}\right)dx = 2k\int_0^x x dx - 2kL\int_0^x \frac{x}{\sqrt{x^2+L^2}}dx$$

$$U(x) = \boxed{kx^2 + 2kL\left(L-\sqrt{x^2+L^2}\right)}$$

(b) $U(x) = 40.0x^2 + 96.0\left(1.20-\sqrt{x^2+1.44}\right)$



For negative *x*, *U(x)* has the same value as for positive *x*. The only equilibrium point (i.e., where $F_x = 0$) is $\boxed{x = 0}$

(c) $K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

$$0 + 0.400 \text{ J} + 0 = \tfrac{1}{2}(1.18 \text{ kg})v_f^2 + 0$$

$$v_f = \boxed{0.823 \text{ m/s}}$$

***7.38** The escape speed v is described by

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\tfrac{1}{2}mv^2 - \frac{GM_{sun}m}{r} = 0 + 0$$

$$r = \frac{2GM_{sun}}{v^2} = \frac{2\left(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30} \text{ kg}\right)}{\left(\dfrac{125\,000\,000 \text{ m}}{3600 \text{ s}}\right)^2} = \boxed{2.20\times10^{11} \text{ m}} = 1.47 \text{ AU}$$

Voyager I did in fact have this speed beyond the orbit of Mars, and has escaped from the solar system.

***7.39** For her jump on earth, $\frac{1}{2}mv_i^2 = mgy_f$ (1)

$$v_i = \sqrt{2gy_f} = \sqrt{2\left(9.8 \text{ m/s}^2\right)(0.5 \text{ m})} = 3.13 \text{ m/s}$$

We assume that she has the same takeoff speed on the asteroid. Here

$$\frac{1}{2}mv_i^2 - \frac{GM_A m}{R_A} = 0 + 0 \quad (2)$$

The equality of densities between planet and asteroid,

$$\rho = \frac{M_E}{\frac{4}{3}\pi R_E^2} = \frac{M_A}{\frac{4}{3}\pi R_A^2} \quad \text{implies} \quad M_A = \left(\frac{R_A}{R_E}\right)^3 M_E \quad (3)$$

Note also at Earth's surface $g = \dfrac{GM_E}{R_E^2}$ (4)

Combining the equations (2), (1), (3) and (4) by substitution gives

$$\frac{1}{2}v_i^2 = \frac{GM_A}{R_A} \qquad \frac{1}{2}\left(2gy_f\right) = \frac{G}{R_A}\left(\frac{R_A}{R_E}\right)^3 M_E$$

$$\frac{GM_E}{R_E^2}y_f = \frac{GM_E R_A^2}{R_E^3} \qquad R_A^2 = y_f R_E = (0.5 \text{ m})\left(6.37\times 10^6 \text{ m}\right)$$

$$R_A = \boxed{1.78\times 10^3 \text{ m}}$$

***7.40** (a) The daredevil falls through a height of 216 m. Let x represent the final extension of the rope beyond its unstressed length:

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf} \qquad 0 + mgy_i + 0 = 0 + 0 + \frac{1}{2}kx_f^2$$

$$(70 \text{ kg})\left(9.8 \text{ m/s}^2\right)(216 \text{ m}) = \frac{1}{2}(4900 \text{ N/m})x_f^2 \qquad x_f = \sqrt{\frac{2(148\,000 \text{ J})}{4900 \text{ N/m}}(\text{N}\cdot\text{m/J})} = 7.78 \text{ m}$$

The unstressed length of the rope is $216 \text{ m} - 7.78 \text{ m} = \boxed{208 \text{ m}}$

(b) $F_s = -kx = -(4900 \text{ N/m})(7.78 \text{ m downward}) = \boxed{3.81\times 10^4 \text{ N up}}$

(c) $F_g = mg = (70 \text{ kg})\left(9.8 \text{ m/s}^2\right) = 686 \text{ N}$

The rope force is larger by $\dfrac{3.81\times 10^4 \text{ N}}{686 \text{ N}} = \boxed{55.5 \text{ times}}$

So large a force will tear apart his body. The plan is $\boxed{\text{not good}}$.

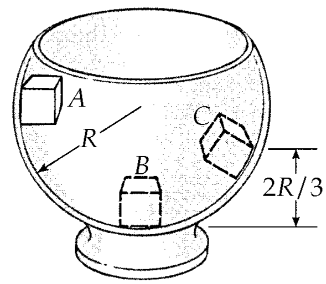
7.41 At a pace I could keep up for a half-hour exercise period, I climb two stories up, traversing forty steps each 18 cm high, in 20 s. My output work becomes the final gravitational energy of the system of the Earth and me,

$$mgy = (85\text{ kg})(9.80\text{ m/s}^2)(40 \times 0.18\text{ m}) = 6000\text{ J}$$

making my sustainable power

$$\frac{6000\text{ J}}{20\text{ s}} = \boxed{\sim 10^2\text{ W}}$$

7.42 (a) $U_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m}) = \boxed{0.588\text{ J}}$



(b) $K_A + U_A = K_B + U_B$

$K_B = K_A + U_A - U_B = mgR = \boxed{0.588\text{ J}}$

(c) $v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{2(0.588\text{ J})}{0.200\text{ kg}}} = \boxed{2.42\text{ m/s}}$

(d) $U_C = mgh_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) = \boxed{0.392\text{ J}}$

$K_C = K_A + U_A - U_C = mg(h_A - h_C)$

$K_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300 - 0.200)\text{ m} = \boxed{0.196\text{ J}}$

7.43 (a) $K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200\text{ kg})(1.50\text{ m/s})^2 = \boxed{0.225\text{ J}}$

(b) $\Delta E_{\text{mech}} = \Delta K + \Delta U = K_B - K_A + U_B - U_A$

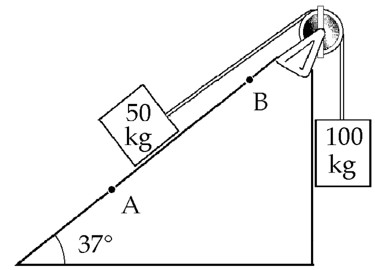
$$= K_B + mg(h_B - h_A)$$

$$= 0.225\text{ J} + (0.200\text{ kg})(9.80\text{ m/s}^2)(0 - 0.300\text{ m})$$

$$= 0.225\text{ J} - 0.558\text{ J} = \boxed{-0.363\text{ J}}$$

(c) It's possible to find an effective coefficient of friction, but not the actual value of μ since n and f vary with position.

***7.44** $\Sigma F_y = n - mg\cos 37.0° = 0$

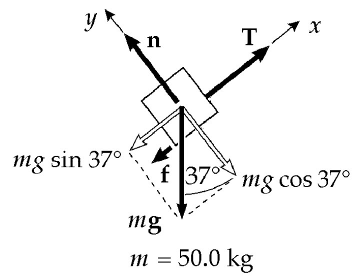


$\therefore n = mg\cos 37.0° = 400\text{ N}$

$f = \mu n = 0.250(400\text{ N}) = 100\text{ N}$

$-f\Delta x = \Delta E_{\text{mech}}$

$(-100)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$



$\Delta U_A = m_A g\left(h_f - h_i\right) = (50.0)(9.80)(20.0\sin 37.0°) = 5.90\times 10^3$

$\Delta U_B = m_B g\left(h_f - h_i\right) = (100)(9.80)(-20.0) = -1.96\times 10^4$

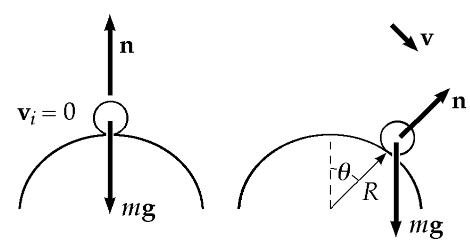
$\Delta K_A = \frac{1}{2}m_A\left(v_f^{\,2} - v_i^{\,2}\right)$

$\Delta K_B = \frac{1}{2}m_B\left(v_f^{\,2} - v_i^{\,2}\right) = \frac{m_B}{m_A}\Delta K_A = 2\Delta K_A$

Adding and solving, $\Delta K_A =$ $\boxed{3.92\text{ kJ}}$

***7.45** m = mass of pumpkin

R = radius of silo top



$\Sigma F_r = ma_r \Rightarrow n - mg\cos\theta = -m\frac{v^2}{R}$

When the pumpkin first loses contact with the surface, $n = 0$. Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

Choose $U_g = 0$ in the $\theta = 90.0°$ plane. Then applying conservation of energy for the pumpkin-Earth system between the starting point and the point where the pumpkin leaves the surface gives

$K_f + U_{gf} = K_i + U_{gi}$

$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$

Using the result from the force analysis, this becomes

$\frac{1}{2}mRg\cos\theta + mgR\cos\theta = mgR$, which reduces to

$\cos\theta = \frac{2}{3}$, and gives $\theta = \cos^{-1}(2/3) =$ $\boxed{48.2°}$

as the angle at which the pumpkin will lose contact with the surface.

7.46 $k = 2.50 \times 10^4$ N/m, $m = 25.0$ kg,

$x_A = -0.100$ m, $U_g|_{x=0} = U_s|_{x=0} = 0$

(a) $E_{\text{mech}} = K_A + U_{gA} + U_{sA}$ $E_{\text{mech}} = 0 + mgx_A + \frac{1}{2}kx_A^2$

$$E_{\text{mech}} = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(-0.100 \text{ m}) + \tfrac{1}{2}(2.50 \times 10^4 \text{ N/m})(-0.100 \text{ m})^2$$

$$E_{\text{mech}} = -24.5 \text{ J} + 125 \text{ J} = \boxed{100 \text{ J}}$$

(b) Since only conservative forces are involved, the total energy of the child-pogo-stick-Earth system at point *C* is the same as that at point *A*.

$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$: $0 + (25.0 \text{ kg})(9.80 \text{ m/s}^2)x_C + 0 = 0 - 24.5 \text{ J} + 125 \text{ J}$

$$x_C = \boxed{0.410 \text{ m}}$$

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$: $\frac{1}{2}(25.0 \text{ kg})v_B^2 + 0 + 0 = 0 + (-24.5 \text{ J}) + 125 \text{ J}$

$$v_B = \boxed{2.84 \text{ m/s}}$$

(d) K and v are at a maximum when $a = \Sigma F/m = 0$ (i.e., when the magnitude of the upward spring force equals the magnitude of the downward gravitational force).

This occurs at $x < 0$ where $k|x| = mg$

or $$|x| = \frac{(25.0 \text{ kg})(9.8 \text{ m/s}^2)}{2.50 \times 10^4 \text{ N/m}} = 9.80 \times 10^{-3} \text{ m}$$

Thus, $K = K_{\max}$ at $x = \boxed{-9.80 \text{ mm}}$

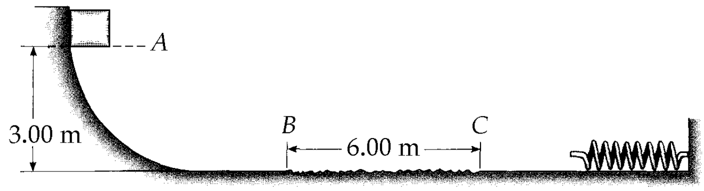
(e) $K_{\max} = K_A + \left(U_{gA} - U_g|_{x=-9.80 \text{ mm}}\right) + \left(U_{sA} - U_s|_{x=-9.80 \text{ mm}}\right)$

or $$\tfrac{1}{2}(25.0 \text{ kg})v_{\max}^2 = (25.0 \text{ kg})(9.80 \text{ m/s}^2)[(-0.100 \text{ m}) - (-0.0098 \text{ m})]$$

$$+\tfrac{1}{2}(2.50 \times 10^4 \text{ N/m})\left[(-0.100 \text{ m})^2 - (-0.0098 \text{ m})^2\right]$$

yielding $v_{\max} = \boxed{2.85 \text{ m/s}}$

7.47 $\Delta E_{\text{mech}} = -f\Delta x$

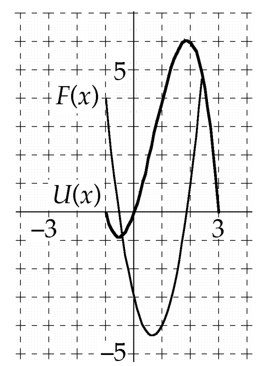


$E_f - E_i = -f \cdot d_{BC}$

$\frac{1}{2}kx^2 - mgh = -\mu mg d_{BC}$

$\mu = \dfrac{mgh - \frac{1}{2}kx^2}{mgd_{BC}} = \boxed{0.328}$

7.48 (a) $\mathbf{F} = -\dfrac{d}{dx}\left(-x^3 + 2x^2 + 3x\right)\mathbf{i} = \boxed{\left(3x^2 - 4x - 3\right)\mathbf{i}}$



(b) $F = 0$

when $x = \boxed{1.87 \text{ and } -0.535}$

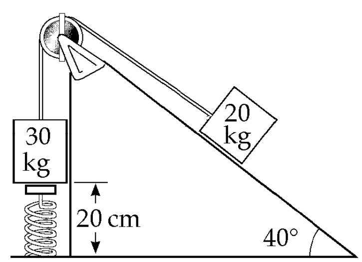
(c) The stable point is at

$x = -0.535$ point of minimum $U(x)$

The unstable point is at

$x = 1.87$ maximum in $U(x)$

7.49 $(K+U)_i = (K+U)_f$



$0 + (30.0 \text{ kg})(9.80 \text{ m/s}^2)(0.200 \text{ m}) + \frac{1}{2}(250 \text{ N/m})(0.200 \text{ m})^2$

$= \frac{1}{2}(50.0 \text{ kg})v^2 + (20.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0.200 \text{ m})\sin 40.0°$

$58.8 \text{ J} + 5.00 \text{ J} = (25.0 \text{ kg})\, v^2 + 25.2 \text{ J}$

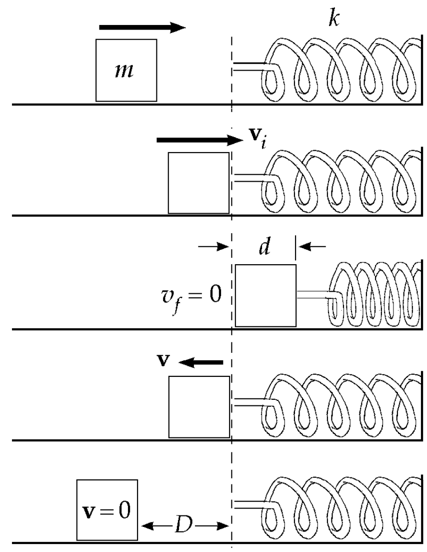
$\boxed{v = 1.24 \text{ m/s}}$

7.50 (a) Between the second and the third picture, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-\mu mgd = -\tfrac{1}{2}mv_i^2 + \tfrac{1}{2}kd^2$$

$$\tfrac{1}{2}(50.0\ \text{N/m})d^2 + 0.250(1.00\ \text{kg})\left(9.80\ \text{m/s}^2\right)d - \tfrac{1}{2}(1.00\ \text{kg})\left(3.00\ \text{m/s}^2\right) = 0$$

$$d = \frac{[-2.45 \pm 21.25]\ \text{N}}{50.0\ \text{N/m}} = \boxed{0.378\text{m}}$$



(b) Between picture two and picture four, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(2d) = -\tfrac{1}{2}mv^2 + \tfrac{1}{2}mv_i^2$$

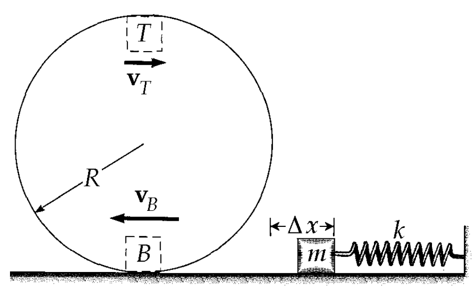
$$v = \sqrt{(3.00\ \text{m/s})^2 - \frac{2}{(1.00\ \text{kg})}(2.45\ \text{N})(2)(0.378\ \text{m})} = \boxed{2.30\ \text{m/s}}$$

(c) For the motion from picture two to picture five, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(D + 2d) = -\tfrac{1}{2}(1.00\ \text{kg})(3.00\ \text{m/s})^2$$

$$D = \frac{9.00\ \text{J}}{2(0.250)(1.00\ \text{kg})\left(9.80\ \text{m/s}^2\right)} - 2(0.378\ \text{m}) = \boxed{1.08\ \text{m}}$$

7.51 (a) Initial compression of spring: $\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2$



$$\tfrac{1}{2}(450\ \text{N/m})(\Delta x)^2 = \tfrac{1}{2}(0.500\ \text{kg})(12.0\ \text{m/s})^2$$

$$\therefore \Delta x = \boxed{0.400\ \text{m}}$$

(b) Speed of block at top of track: $\Delta E_{\text{mech}} = -f\Delta x$

$$\left(mgh_T + \tfrac{1}{2}mv_T^2\right) - \left(mgh_B + \tfrac{1}{2}mv_B^2\right) = -f(\pi R)$$

$$(0.500\ \text{kg})\left(9.80\ \text{m/s}^2\right)(2.00\ \text{m}) + \tfrac{1}{2}(0.500\ \text{kg})v_T^2 - \tfrac{1}{2}(0.500\ \text{kg})(12.0\ \text{m/s})^2 = -(7.00\ \text{N})(\pi)(1.00\ \text{m})$$

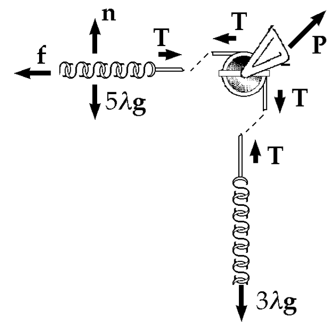
$$0.250v_T^2 = 4.21 \qquad \therefore v_T = \boxed{4.10\ \text{m/s}}$$

(c) Does block fall off at or before top of track? Block falls if $a_c < g$

$$a_c = \frac{v_T^2}{R} = \frac{(4.10)^2}{1.00} = 16.8\ \text{m/s}^2$$

Therefore $a_c > g$ and the $\boxed{\text{block stays on the track}}$.

7.52 Let λ represent the mass of each one meter of the chain and T represent the tension in the chain at the table edge. We imagine the edge to act like a frictionless and massless pulley.



(a) For the five meters on the table with motion impending,

$\Sigma F_y = 0$: $\quad +n - 5\lambda g = 0 \quad n = 5\lambda g$

$$f_s \le \mu_s n = 0.6(5\lambda g) = 3\lambda g$$

$\Sigma F_x = 0$: $\quad +T - f_s = 0 \quad T = f_s \quad T \le 3\lambda g$

The maximum value is barely enough to support the hanging segment according to

$\Sigma F_y = 0$: $\quad +T - 3\lambda g = 0 \quad T = 3\lambda g$

so it is at this point that the chain starts to slide.

(b) Let x represent the variable distance the chain has slipped since the start.

Then length $(5 - x)$ remains on the table, with now

$\Sigma F_y = 0$: $\quad +n - (5 - x)\lambda g = 0 \quad n = (5 - x)\lambda g$

$$f_k = \mu_k n = 0.4(5 - x)\lambda g = 2\lambda g - 0.4x\lambda g$$

Consider energies of the chain-Earth system at the initial moment when the chain starts to slip, and a final moment when $x = 5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8 - \frac{3}{2} = 6.5$ m.

$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$: $\quad 0 + (m_1 g y_1 + m_2 g y_2)_i - \int_i^f f_k dx = \left(\frac{1}{2}mv^2 + mgy\right)_f$

$$(5\lambda g)8 + (3\lambda g)6.5 - \int_0^5 (2\lambda g - 0.4x\lambda g)dx = \tfrac{1}{2}(8\lambda)v^2 + (8\lambda g)4$$

$$40.0g + 19.5g - 2.00g\int_0^5 dx + 0.400g\int_0^5 x\,dx = 4.00v^2 + 32.0g$$

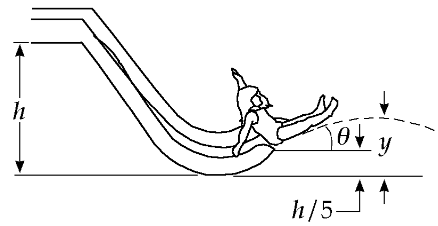
$$27.5g - 2.00gx\Big|_0^5 + 0.400g\frac{x^2}{2}\Big|_0^5 = 4.00v^2$$

$$27.5g - 2.00g(5.00) + 0.400g(12.5) = 4.00v^2$$

$$22.5g = 4.00v^2$$

$$v = \sqrt{\frac{(22.5\text{ m})(9.80\text{ m/s}^2)}{4.00}} = \boxed{7.42\text{ m/s}}$$

***7.53** Launch speed is found from



$mg\left(\frac{4}{5}h\right)=\frac{1}{2}mv^2$: $\qquad v=\sqrt{2g\left(\frac{4}{5}\right)h}$

$v_y = v\sin\theta$

The height y above the water (by conservation of energy for the child-Earth system) is found from

$mgy=\frac{1}{2}mv_y^{\,2}+mg\frac{h}{5}$ (since $\frac{1}{2}mv_x^{\,2}$ is constant in projectile motion)

$$y=\frac{1}{2g}\;v_y^{\,2}+\frac{h}{5}=\frac{1}{2g}\;v^2\sin^2\theta+\frac{h}{5}$$

$$y=\frac{1}{2g}\left[2g\left(\frac{4}{5}h\right)\right]\;\sin^2\theta+\frac{h}{5}=\boxed{\frac{4}{5}h\sin^2\theta+\frac{h}{5}}$$

***7.54** (a) The potential energy associated with the wind force is $+Fx$, where x is the horizontal distance traveled, with x positive when swinging into the wind and negative when swinging in the direction the wind is blowing. The initial energy of the Jane-wind-Earth system is, (using the pivot point of the swing as the point of zero gravitational energy),

where m is her mass, $\qquad E_i=(K+U_g+U_{\text{wind}})_i=\frac{1}{2}mv_i^{\,2}-mgL\cos\theta-FL\sin\theta$

At the end of her swing, the energy is $\qquad E_f=(K+U_g+U_{\text{wind}})_f=0-mgL\cos\phi+FL\sin\phi$

so conservation of energy ($E_i=E_f$) gives $\qquad \frac{1}{2}mv_i^{\,2}-mgL\cos\theta-FL\sin\theta=-mgL\cos\phi+FL\sin\phi$

This leads to $\qquad v_i=\sqrt{2gL(\cos\theta-\cos\phi)+2\frac{FL}{m}(\sin\theta+\sin\phi)}$

But $D=L\sin\phi+L\sin\theta$, so that $\qquad \sin\phi=\frac{D}{L}-\sin\theta=\frac{50.0}{40.0}-\sin 50.0°=0.484$

which gives $\qquad \phi=28.9°$

Using this, we have $\qquad \boxed{v_i=6.15\text{ m/s}}$

(b) Here (again using conservation of energy for the Jane-Tarzan-wind-Earth system) we have,

$$-MgL\cos\phi+FL\sin\phi+\frac{1}{2}Mv^2=-MgL\cos\theta-FL\sin\theta$$

where M is the combined mass of Jane and Tarzan.

Therefore, $\qquad v=\sqrt{2gL(\cos\phi-\cos\theta)-2\frac{FL}{M}(\sin\phi+\sin\theta)}$

which gives the minimum speed needed: $\qquad \boxed{v=9.87\text{ m/s}}$

***7.55** Case I: Surface is frictionless

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$k = \frac{mv^2}{x^2} = \frac{(5.00\text{ kg})(1.20\text{ m/s})^2}{10^{-2}\text{ m}^2} = 7.20\times10^2\text{ N/m}$$

Case II: Surface is rough, $\mu_k = 0.300$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 - \mu_k mgx$$

$$\frac{5.00\text{ kg}}{2}v^2 = \frac{1}{2}\left(7.20\times10^2\text{ N/m}\right)\left(10^{-1}\text{ m}\right)^2 - (0.300)(5.00\text{ kg})\left(9.80\text{ m/s}^2\right)\left(10^{-1}\text{ m}\right)$$

$$\boxed{v = 0.923\text{ m/s}}$$

***7.56** If the spring is just barely able to lift the lower block from the table, the spring lifts it through no noticeable distance, but exerts on the block a force equal to its weight Mg. The extension of the spring, from $|F_s| = kx$, must be Mg/k. Between an initial point at release and a final point when the moving block first comes to rest, we have

$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$:

$$0 + mg\left(-\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = 0 + mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k} + \frac{8m^2g^2}{k} = \frac{mMg^2}{k} + \frac{M^2g^2}{2k}$$

$$4m^2 = mM + \frac{M^2}{2}$$

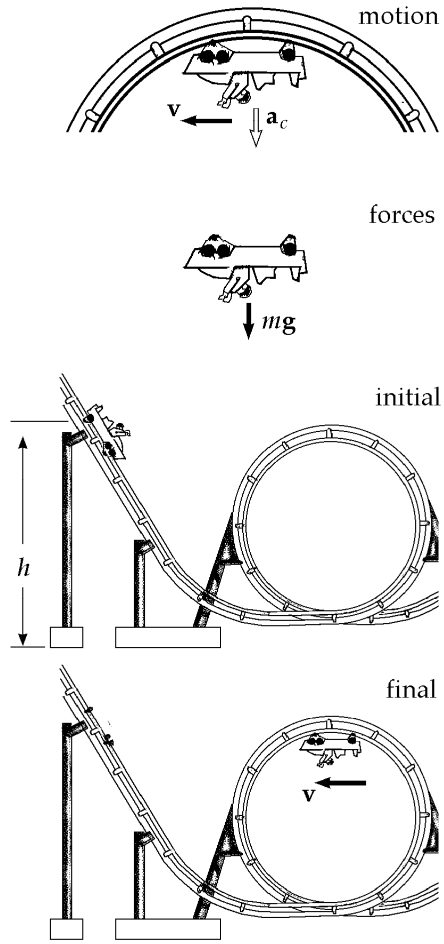
$$\frac{M^2}{2} + mM - 4m^2 = 0$$

$$M = \frac{-m \pm \sqrt{m^2 - 4(1/2)\left(-4m^2\right)}}{2(1/2)} = -m \pm \sqrt{9m^2}$$

Only a positive mass is physical, so we take

$$M = m(3-1) = \boxed{2m}$$

***7.57** (a) At the top of the loop the car and riders are in free fall:



$\Sigma F_y = ma_y$: $\qquad mg \text{ down} = \dfrac{mv^2}{R} \text{ down}$

$$v = \sqrt{Rg}$$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$K_i + U_{gi} = K_f + U_{gf}$: $\qquad 0 + mgh = \frac{1}{2}mv^2 + mg(2R)$

$$gh = \frac{1}{2}Rg + g(2R)$$

$$\boxed{h = 2.50\text{ R}}$$

(b) Let h now represent the height $\geq 2.5R$ of the release point. At the bottom of the loop we have

$mgh = \frac{1}{2}mv_b{}^2 \qquad$ or $\qquad v_b{}^2 = 2gh$

$\Sigma F_y = ma_y$: $\qquad n_b - mg = \dfrac{mv_b{}^2}{R}(\text{up})$

$$n_b = mg + \frac{m(2gh)}{R}$$

At the top of the loop, $\qquad mgh = \dfrac{1}{2}mv_t{}^2 + mg(2R)$

$$v_t{}^2 = 2gh - 4gR$$

$\Sigma F_y = ma_y$: $\qquad -n_t - mg = -\dfrac{mv_t{}^2}{R}$

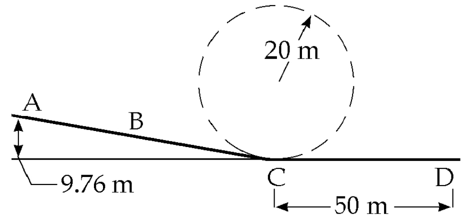
$$n_t = -mg + \frac{m}{R}(2gh - 4gR)$$

$$n_t = \frac{m(2gh)}{R} - 5mg$$

Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m(2gh)}{R} - \frac{m(2gh)}{R} + 5mg = \boxed{6mg}$$

***7.58** (a) Conservation of energy for the sled-rider-Earth system, between A and C:



$$K_i + U_{gi} = K_f + U_{gf}$$

$$\tfrac{1}{2}m(2.5\text{ m/s})^2 + m(9.80\text{ m/s}^2)(9.76\text{ m}) = \tfrac{1}{2}mv_C^{\,2} + 0$$

$$v_C = \sqrt{(2.5\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(9.76\text{ m})} = \boxed{14.1\text{ m/s}}$$

(b) Incorporating the loss of mechanical energy during the portion of the motion in the water, we have, for the entire motion between A and D (the rider's stopping point),

$K_i + U_{gi} - f_k\Delta x = K_f + U_{gf}$: $\quad \frac{1}{2}(80\text{ kg})(2.5\text{ m/s})^2 + (80\text{ kg})(9.80\text{ m/s}^2)(9.76\text{ m}) - f_k\,\Delta x = 0 + 0$

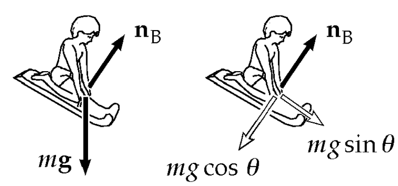
$$-f_k\Delta x = \boxed{-7.90\times10^3\text{ J}}$$

(c) The water exerts a frictional force $\quad f_k = \dfrac{7.90\times10^3\text{ J}}{\Delta x} = \dfrac{7.90\times10^3\text{ N}\cdot\text{m}}{50\text{ m}} = 158\text{ N}$

and also a normal force of $\quad n = mg = (80\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$

The magnitude of the water force is $\quad \sqrt{(158\text{ N})^2 + (784\text{ N})^2} = \boxed{800\text{ N}}$

(d) The angle of the slide is



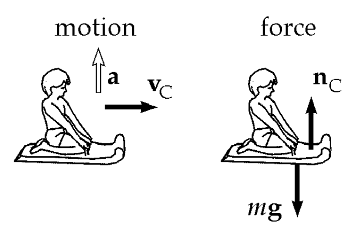
$$\theta = \sin^{-1}\frac{9.76\text{ m}}{54.3\text{ m}} = 10.4°$$

For forces perpendicular to the track at B,

$\Sigma F_y = ma_y$: $\quad n_B - mg\cos\theta = 0$

$$n_B = (80.0\text{ kg})(9.80\text{ m/s}^2)\cos 10.4° = \boxed{771\text{ N}}$$

(e) $\Sigma F_y = ma_y$: $\quad +n_C - mg = \dfrac{mv_C^{\,2}}{r}$



$$n_C = (80.0\text{ kg})(9.80\text{ m/s}^2) + \frac{(80.0\text{ kg})(14.1\text{ m/s})^2}{20\text{ m}}$$

$$n_C = \boxed{1.57\times10^3\text{ N up}}$$

The rider pays for the thrills of a giddy height at A, and a high speed and tremendous splash at C. As a bonus, he gets the quick change in direction and magnitude among the forces we found in parts (d), (e), and (c).

ANSWERS TO EVEN NUMBERED PROBLEMS

2. (a) 800 J (b) 107 J (c) 0

4. (a) 19.8 m/s (b) 294 J (c) (30.0**i** – 39.6**j**) m/s

6. 1.84 m

8. (a) See the solution. (b) 60.0°

10. 5.49 m/s

12. (a) See the solution. (b) 35.0 J

14. 26.5 m/s

16. (a) 1.47 m/s (b) 1.35 m/s

18. (a) 0.537 m/s (b) 0.058 8 m

20. 168 J

22. (a) 24.5 m/s (b) Yes
(c) 206 m (d) Unrealistic; see the solution

24. (a) 1.40 m/s (b) 4.60 cm after release (c) 1.79 m/s

26. 44.1 kW

28. (a) $Ax^2/2 - Bx^3/3$ (b) $\Delta U = 5A/2 - 19B/3$; $\Delta K = -5A/2 + 19B/3$

30. $\left(7 - 9x^2y\right)\mathbf{i} - 3x^3\mathbf{j}$

32. 4.17×10^{10} J

34. (a) -1.67×10^{-14} J (c) at the center

36. (a) $r = 1.5$ mm stable; 2.3 mm unstable; 3.2 mm stable; $r \to \infty$ neutral (b) $1\text{ J} > E > -5.6\text{ J}$
(c) $0.6\text{ mm} < r < 3.6\text{ mm}$ (d) 2.6 J
(e) 1.5 mm (f) 4 J

38. 2.20×10^{11} m (1.47 AU)

40. (a) 208 m (b) 3.81×10^{4} N up (c) 55.5 mg; no

42. (a) 0.588 J (b) 0.588 J
(c) 2.42 m/s (d) $U_C = 0.392$ J, $K_C = 0.196$ J

44. 3.92 kJ

46. (a) 100 J (b) 0.410 m (c) 2.84 m/s
(d) –9.80 mm (e) 2.85 m/s

48. (a) $\left(3x^2 - 4x - 3\right)\mathbf{i}$ (b) $x = 1.87$ and -0.535
(c) See the solution. $x = -0.535$ (stable), and $x = 1.87$ (unstable)

50. (a) 0.378 m (b) 2.30 m/s (c) 1.08 m

52. (a) See the solution. (b) 7.42 m/s

54. (a) 6.15 m/s (b) 9.87 m/s

56. $2m$

58. (a) 14.1 m/s (b) –7.90 kJ (c) 800 N
(d) 771 N (e) 1.57 kN up