CHAPTER 8

ANSWERS TO QUESTIONS

Q8.1 No. Impulse, $F\Delta t$, depends on the force and the time for which it is applied.

Q8.2 No. Only in a precise head-on collision with momenta with equal magnitudes and opposite directions can both balls wind up at rest. Yes. Assume that ball 2, originally at rest, is struck squarely by an equal-mass ball 1. Then ball 2 will take off with the velocity of ball 1, leaving ball 1 at rest.

Q8.3 Interestingly, it is gravitation that brings the ball and the Earth together. As the ball moves downward, the Earth moves upward, although 10^{25} times more slowly. The two objects meet, rebound, and separate. Momentum of the ball-Earth system is conserved.

Q8.4 Suppose car and truck move along the same line. If one vehicle overtakes the other, the faster-moving one loses more energy than the slower one gains. In a head-on collision, if the speed of the truck is less than $\frac{m_T + 3m_c}{3m_T + m_c}$ times the speed of the car, the car will lose more energy.

Q8.5 Choose the same momentum. Then the velocity of the bowling ball is at a minimum.

Q8.6 The superhero has finite mass. When he throws the piano, he must recoil in the opposite direction. To hover in midair, he must continuously fly by giving downward momentum to air, rocket exhaust or something that will disturb things below.

Q8.7 Yes. A boomerang, a kitchen stool.

Q8.8 If the mountain is very tall, like Everest, the gravitational acceleration will be greater toward the base. Thus the CG of the mountain will be slightly below its CM.

Q8.9 Longer contact time with the ball, in $F\Delta t$, will give greater momentum and longer distance.

Q8.10 The rifle has a much lower speed than the bullet and much less kinetic energy. The butt distributes the recoil force over an area much larger than that of the bullet.

Q8.11 His impact speed is determined by the acceleration of gravity and the distance of fall, in $v_f^2 = v_i^2 - 2g(0 - y_i)$. The force exerted by the pad depends also on the unknown stiffness of the pad.

Q8.12 The product of the mass flow rate and velocity of the water determines the force the firefighters must exert.

Q8.13 In empty space, the center of mass of a rocket-plus-fuel system does not accelerate during a burn, because no outside force acts on this system. According to Equation 8.42, the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

Q8.14 The sheet stretches and pulls the two students toward each other. These effects are larger for a faster-moving egg. The time over which the egg stops is extended so that the force stopping it is never too large.

Q8.15 There was a time when the English favored position (a), the Germans position (b), and the French position (c). A Frenchman, Jean D'Alembert, is most responsible for showing that each theory is consistent with the others. All are equally correct. Each is useful for giving a mathematically simple solution for some problems.

Chapter 8

PROBLEM SOLUTIONS

8.1 $m = 3.00$ kg, $\mathbf{v} = (3.00\mathbf{i} - 4.00\mathbf{j})$ m/s

(a) $\mathbf{p} = m\mathbf{v} = (9.00\mathbf{i} - 12.0\mathbf{j})$ kg · m/s

Thus, $\boxed{p_x = 9.00 \text{ kg} \cdot \text{m/s}}$

and $\boxed{p_y = -12.0 \text{ kg} \cdot \text{m/s}}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0 \text{ kg} \cdot \text{m/s}}$

$$\theta = \tan^{-1}\left(\frac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$$

8.2 The initial momentum is zero. Therefore, the final momentum, p_f, must also be zero.

We have, (taking eastward as the positive direction),

$$p_f = (40.0 \text{ kg})\, v_c + (0.500 \text{ kg})(5.00 \text{ m/s}) = 0$$

$$v_c = \boxed{-6.25 \times 10^{-2} \text{ m/s}}$$

(The child recoils westward.)

8.3 I have mass 85.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$$v_f^2 - v_i^2 = 2a\left(x_f - x_i\right): \qquad 0 - v_i^2 = 2\left(-9.80 \text{ m/s}^2\right)(0.250 \text{ m})$$

$$v_i = 2.20 \text{ m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the earth down and myself up:

$$0 = \left(5.98 \times 10^{24} \text{ kg}\right)v_e + (85.0 \text{ kg})(2.20 \text{ m/s})$$

$$v_e \sim \boxed{10^{-23} \text{ m/s}}$$

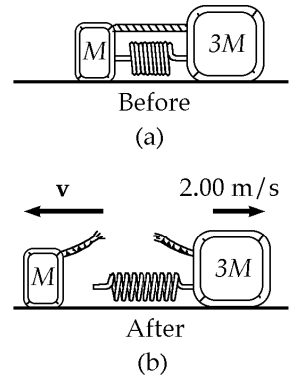
8.4 (a) For the system of two blocks $\Delta p = 0$,

or $p_i = p_f$

Therefore, $0 = Mv_m + (3M)(2.00\ \text{m/s})$

Solving gives $v_m = \boxed{-6.00\ \text{m/s}}$ (motion toward the left)

(b) $\frac{1}{2}kx^2 = \frac{1}{2}Mv_M{}^2 + \frac{1}{2}(3M)v_{3M}{}^2 = \boxed{8.40\ \text{J}}$



***8.5** (a) The momentum is $p = mv$, so $v = p/m$ and the kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{2K/m}$, so $p = mv = m\sqrt{2K/m} = \boxed{\sqrt{2mK}}$

***8.6** Assume the velocity of the blood is constant over the 0.160 s. Then the patient's body and pallet will have a constant velocity of $\left(6\times10^{-5}\ \text{m}\right)/0.160\ \text{s} = 3.75\times10^{-4}\ \text{m/s}$ in the opposite direction. Momentum conservation gives

$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$: $0 = m_{\text{blood}}\,0.5\ \text{m/s} + 54\ \text{kg}\left(-3.75\times10^{-4}\ \text{m/s}\right)$

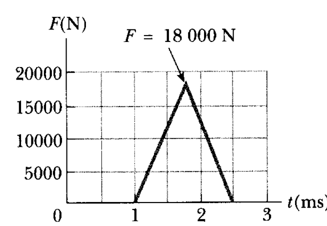
$m_{\text{blood}} = 0.0405\ \text{kg} = \boxed{40.5\ \text{g}}$

8.7 (a) $I = \int F dt$ = area under curve

$I = \frac{1}{2}\left(1.50\times10^{-3}\ \text{s}\right)(18000\ \text{N}) = \boxed{13.5\ \text{N}\cdot\text{s}}$

(b) $F = \dfrac{13.5\ \text{N}\cdot\text{s}}{1.50\times10^{-3}\ \text{s}} = \boxed{9.00\ \text{kN}}$

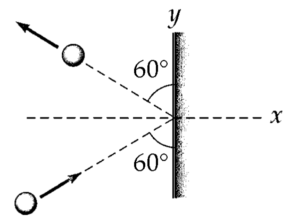
(c) From the graph, we see that $F_{\max} = \boxed{18.0\ \text{kN}}$



8.8 $I = \Delta p = m\,\Delta v = (70.0\ \text{kg})(5.20\ \text{m/s}) = \boxed{364\ \text{kg}\cdot\text{m/s}}$

$F = \dfrac{\Delta p}{\Delta t} = \dfrac{364\ \text{kg}\cdot\text{m/s}}{0.832\ \text{s}} = \boxed{438\ \text{N}}$

8.9 $\Delta\mathbf{p} = \mathbf{F}\,\Delta t$



$$\Delta p_y = m\left(v_{fy} - v_{iy}\right) = m(v\cos 60.0°) - mv\cos 60.0° = 0$$

$$\Delta p_x = m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0°$$

$$= -2(3.00\text{ kg})(10.0\text{ m/s})(0.866)$$

$$= -52.0\text{ kg}\cdot\text{m/s}$$

$$F_{\text{ave}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0\text{ kg}\cdot\text{m/s}}{0.200\text{ s}} = \boxed{-260\text{ N}}$$

8.10 Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse, $\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = (0.0600)(40.0)\mathbf{i} - (0.0600)(50.0)(-\mathbf{i}) = \boxed{5.40\mathbf{i}\text{ N}\cdot\text{s}}$

(b) Work $= K_f - K_i = \frac{1}{2}(0.0600)\left[(40.0)^2 - (50.0)^2\right] = \boxed{-27.0\text{ J}}$

8.11 Take x-axis toward the pitcher

(a) $p_{ix} + I_x = p_{fx}$: $(0.200\text{ kg})(15.0\text{ m/s})(-\cos 45.0°) + I_x = (0.200\text{ kg})(40.0\text{ m/s})\cos 30.0°$

$$I_x = 9.05\text{ N}\cdot\text{s}$$

$p_{iy} + I_y = p_{fy}$: $(0.200\text{ kg})(15.0\text{ m/s})(-\sin 45.0°) + I_y = (0.200\text{ kg})(40.0\text{ m/s})\sin 30.0°$

$$\mathbf{I} = \boxed{(9.05\mathbf{i} + 6.12\mathbf{j})\text{ N}\cdot\text{s}}$$

(b) $\mathbf{I} = \frac{1}{2}(0 + \mathbf{F}_m)(4.00\text{ ms}) + \mathbf{F}_m(20.0\text{ ms}) + \frac{1}{2}\mathbf{F}_m(4.00\text{ ms})$

$$\mathbf{F}_m \times 24.0\times 10^{-3}\text{ s} = (9.05\mathbf{i} + 6.12\mathbf{j})\text{ N}\cdot\text{s}$$

$$\mathbf{F}_m = \boxed{(377\mathbf{i} + 255\mathbf{j})\text{ N}}$$

***8.12** The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600\text{ kg})(25.0\text{ m/s}) - 0}{1.00\text{ s}} = \boxed{15.0\text{ N}}$$

According to Newton's 3rd law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0 N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

***8.13** (a) Energy is conserved for the spring-mass system:

$$K_i + U_{si} = K_f + U_{sf}: \qquad 0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0$$

$$v = x\sqrt{k/m}$$

(b) From the equation, a $\boxed{\text{smaller}}$ value of m makes $v = x\sqrt{k/m}$ larger.

(c) $I = \left|\mathbf{p}_f - \mathbf{p}_i\right| = mv_f - 0 = mx\sqrt{k/m} = x\sqrt{km}$

(d) From the equation, a $\boxed{\text{larger}}$ value of m makes $I = x\sqrt{km}$ larger.

(e) For the glider, $W = K_f - K_i = \frac{1}{2}mv^2 - 0 = \frac{1}{2}kx^2$

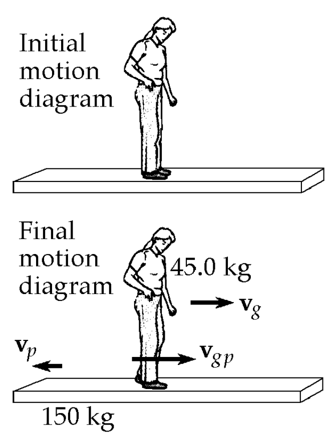
The mass makes $\boxed{\text{no difference}}$ to the work.

8.14 We call the initial speed of the bowling ball v_i, and from momentum conservation for the pin-ball system,

$$(7.00 \text{ kg})(v_i) + (2.00 \text{ kg})(0) = (7.00 \text{ kg})(1.80 \text{ m/s}) + (2.00 \text{ kg})(3.00 \text{ m/s})$$

gives $v_i = \boxed{2.66 \text{ m/s}}$

***8.15** (a) and (b) Let v_g and v_p be the velocity of the girl and the plank relative to the ice surface. Then we may say that $v_g - v_p$ is the velocity of the girl relative to the plank, so that



$$v_g - v_p = 1.50 \qquad (1)$$

But also we must have $m_g v_g + m_p v_p = 0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0\, v_g + 150\, v_p = 0, \qquad \text{or} \qquad v_g = -3.33\, v_p$$

Putting this into the equation (1) above gives

$$-3.33\ v_p - v_p = 1.50 \qquad \text{or} \qquad v_p = \boxed{-0.346 \text{ m/s}}$$

Then $v_g = -3.33(-0.346) = \boxed{1.15 \text{ m/s}}$

8.16 Gayle jumps on the sled: $p_{1i} + p_{2i} = p_{1f} + p_{2f}$

$$(50.0\text{ kg})(4.00\text{ m/s}) = (50.0\text{ kg} + 5.00\text{ kg})\,v_2$$

$$v_2 = 3.64\text{ m/s}$$

They glide down 5.00 m: $K_i + U_i = K_f + U_f$

$$\tfrac{1}{2}(55.0\text{ kg})(3.64\text{ m/s})^2 + (55.0\text{ kg})\left(9.8\text{ m/s}^2\right)(5.00\text{ m}) = \tfrac{1}{2}(55.0\text{ kg})v_3^{\,2}$$

$$v_3 = 10.5\text{ m/s}$$

Brother jumps on: $(55.0\text{ kg})(10.5\text{ m/s}) + 0 = (85.0\text{ kg})\,v_4$

$$v_4 = 6.82\text{ m/s}$$

All slide 10.0 m down: $\tfrac{1}{2}(85.0\text{ kg})(6.82\text{ m/s})^2 + (85.0\text{ kg})\left(9.80\text{ m/s}^2\right)(10.0\text{ m}) = \tfrac{1}{2}(85.0\text{ kg})v_5^{\,2}$

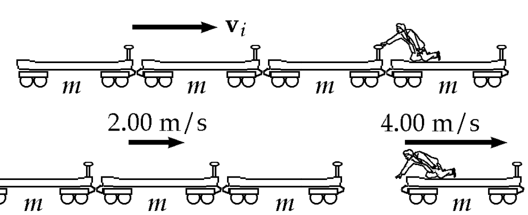
$$v_5 = \boxed{15.6\text{ m/s}}$$

8.17 (a) $mv_{1i} + 3mv_{2i} = 4mv_f$ where $m = 2.50 \times 10^4\text{ kg}$

$$v_f = \frac{4.00 + 3(2.00)}{4} = \boxed{2.50\text{ m/s}}$$

(b) $K_f - K_i = \tfrac{1}{2}(4m)v_f^{\,2} - \left[\tfrac{1}{2}mv_{1i}^{\,2} + \tfrac{1}{2}(3m)v_{2i}^{\,2}\right] = \left(2.50 \times 10^4\right)(12.5 - 8.00 - 6.00) = \boxed{-3.75 \times 10^4\text{ J}}$

***8.18** (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor



$$(4m)v_i = (3m)(2.00\text{ m/s}) + m(4.00\text{ m/s})$$

$$v_i = \frac{6.00\text{ m/s} + 4.00\text{ m/s}}{4} = \boxed{2.50\text{ m/s}}$$

(b) $W_{\text{actor}} = K_f - K_i = \tfrac{1}{2}\left[(3m)(2.00\text{ m/s})^2 + m(4.00\text{ m/s})^2\right] - \tfrac{1}{2}(4\text{ m})(2.50\text{ m/s})^2$

$$W_{\text{actor}} = \frac{\left(2.50 \times 10^4\text{ kg}\right)}{2}(12.0 + 16.0 - 25.0)(\text{m/s})^2 = \boxed{37.5\text{ kJ}}$$

(c) The event considered here is the time reversal of the perfectly inelastic collision in Problem 8.17. The same momentum conservation equation describes both processes.

8.19 (a) Following Example 8.7, the fraction of total kinetic energy transferred to the moderator is

$$f_2 = \frac{4m_1m_2}{(m_1+m_2)^2}$$

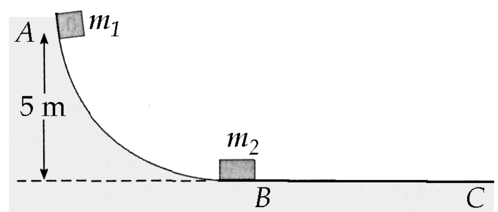
where m_2 is the moderator nucleus and in this case, $m_2 = 12m_1$

$$f_2 = \frac{4m_1(12m_1)}{(13m_1)^2} = \frac{48}{169} = \boxed{0.284 \text{ or } 28.4\%}$$

of the neutron energy is transferred to the carbon nucleus.

(b) $K_C = (0.284)\left(1.6\times10^{-13}\text{ J}\right) = \boxed{4.54\times10^{-14}\text{ J}}$ $\quad K_n = (0.716)\left(1.6\times10^{-13}\text{ J}\right) = \boxed{1.15\times10^{-13}\text{ J}}$

8.20 v_1, speed of m_1 at B before collision.



$$\tfrac{1}{2}m_1v_1^2 = m_1gh$$

$$v_1 = \sqrt{2(9.80)(5.00)} = 9.90\text{ m/s}$$

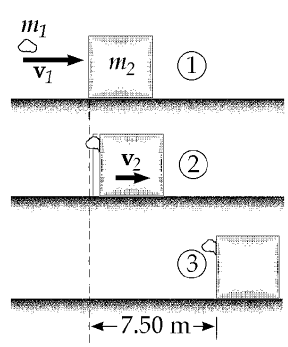
v_{1f}, speed of m_1 at B just after collision.

$$v_{1f} = \frac{m_1-m_2}{m_1+m_2}v_1 = -\tfrac{1}{3}(9.90)\text{ m/s} = -3.30\text{ m/s}$$

At the highest point (after collision)

$$m_1gh_{\max} = \tfrac{1}{2}m_1(-3.30)^2 \qquad h_{\max} = \frac{(-3.30\text{ m/s})^2}{2\left(9.80\text{ m/s}^2\right)} = \boxed{0.556\text{ m}}$$

8.21 At impact, momentum of the clay-block system is conserved, so:



$$mv_1 = (m_1+m_2)v_2$$

After impact, the change in kinetic energy of the clay-block-surface system is equal to the increase in internal energy:

$$\tfrac{1}{2}(m_1+m_2)v_2^2 = f_fd = \mu(m_1+m_2)gd$$

$$\tfrac{1}{2}(0.112\text{ kg})v_2^2 = 0.650(0.112\text{ kg})\left(9.80\text{ m/s}^2\right)(7.50\text{ m})$$

$$v_2^2 = 95.6\text{ m}^2/\text{s}^2 \qquad v_2 = 9.77\text{ m/s}$$

$$\left(12.0\times10^{-3}\text{ kg}\right)v_1 = (0.112\text{ kg})(9.77\text{ m/s}) \qquad v_1 = \boxed{91.2\text{ m/s}}$$

***8.22** For the car-truck-driver-driver system, momentum is conserved:

$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$: $\quad (4000 \text{ kg})(8 \text{ m/s})\mathbf{i} + (800 \text{ kg})(8 \text{ m/s})(-\mathbf{i}) = (4800 \text{ kg})\, v_f\, \mathbf{i}$

$$v_f = \frac{25600 \text{ kg m/s}}{4800 \text{ kg}} = 5.33 \text{ m/s}$$

For the driver of the truck, the impulse-momentum theorem is

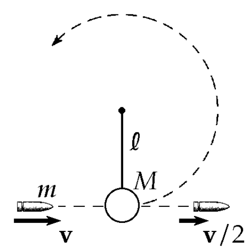
$\mathbf{F}\Delta t = \mathbf{p}_f - \mathbf{p}_i$: $\quad \mathbf{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\mathbf{i} - (80 \text{ kg})(8 \text{ m/s})\mathbf{i}$

$$\mathbf{F} = \boxed{1.78 \times 10^3 \text{ N } (-\mathbf{i}) \text{ on the truck driver}}$$

For the driver of the car, $\quad \mathbf{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\mathbf{i} - (80 \text{ kg})(8 \text{ m/s})(-\mathbf{i})$

$$\mathbf{F} = \boxed{8.89 \times 10^3 \text{ N } \mathbf{i} \text{ on the car driver}}\text{, 5 times larger.}$$

8.23 Energy is conserved for the bob-Earth system between bottom and top of swing:



$K_i + U_i = K_f + U_f$ $\quad \frac{1}{2} M v_b{}^2 + 0 = 0 + Mg2\ell$

$$v_b{}^2 = g4\ell \qquad \text{so} \qquad v_b = 2\sqrt{g\ell}$$

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \qquad \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

8.24 We assume equal firing speeds v and equal forces F required for the two bullets to push wood fibers apart. These equal forces act backward on the two bullets.

For the first, $\quad K_i + \Delta E_{\text{mech}} = K_f \quad \frac{1}{2}(7.00 \times 10^{-3} \text{ kg})v^2 - F(8.00 \times 10^{-2} \text{ m}) = 0$

For the second, $\quad p_i = p_f$: $\quad (7.00 \times 10^{-3} \text{ kg})v = (1.014 \text{ kg})v_f$

$$v_f = \frac{(7.00 \times 10^{-3})v}{1.014}$$

Again, $\quad K_i + \Delta E_{\text{mech}} = K_f$: $\quad \frac{1}{2}(7.00 \times 10^{-3} \text{ kg})v^2 - Fd = \frac{1}{2}(1.014 \text{ kg})v_f^2$

Substituting for v_f, $\quad \frac{1}{2}(7.00 \times 10^{-3} \text{ kg})v^2 - Fd = \frac{1}{2}(1.014 \text{ kg})\left(\frac{7.00 \times 10^{-3} v}{1.014}\right)^2$

$$Fd = \frac{1}{2}(7.00 \times 10^{-3})v^2 - \frac{1}{2}\frac{(7.00 \times 10^{-3})^2}{1.014}v^2$$

Substituting for v, $\quad Fd = F(8.00 \times 10^{-2} \text{ m})\left(1 - \frac{7.00 \times 10^{-3}}{1.014}\right) \qquad d = \boxed{7.94 \text{ cm}}$

8.25 (a) First, we conserve momentum for the system of two football players in the x direction (the direction of travel of the fullback).

$$(90.0 \text{ kg})(5.00 \text{ m/s}) + 0 = (185 \text{ kg})V \cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V\cos\theta = 2.43 \text{ m/s} \qquad (1)$$

Now consider conservation of momentum of the system in the y direction (the direction of travel of the opponent).

$$(95.0 \text{ kg})(3.00 \text{ m/s}) + 0 = (185 \text{ kg})(V\sin\theta)$$

which gives, $V\sin\theta = 1.54 \text{ m/s} \qquad (2)$

Divide equation (2) by (1) $\tan\theta = \dfrac{1.54}{2.43} = 0.633$

From which $\boxed{\theta = 32.3°}$

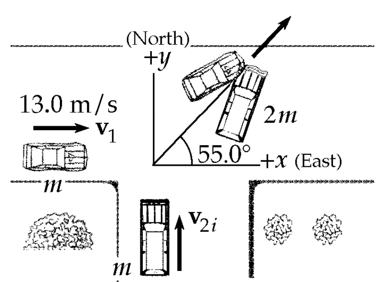
Then, either (1) or (2) gives $V = \boxed{2.88 \text{ m/s}}$

(b) $K_i = \frac{1}{2}(90.0 \text{ kg})(5.00 \text{ m/s})^2 + \frac{1}{2}(95.0 \text{ kg})(3.00 \text{ m/s})^2 = 1.55\times10^3 \text{ J}$

$K_f = \frac{1}{2}(185 \text{ kg})(2.88 \text{ m/s})^2 = 7.67\times10^2 \text{ J}$

Thus, the kinetic energy lost is $\boxed{783 \text{ J into internal energy.}}$

***8.26** We use conservation of momentum for the system of two vehicles for both northward and eastward components.



For the eastward direction:

$$M(13.0 \text{ m/s}) = 2MV_f \cos 55.0°$$

For the northward direction:

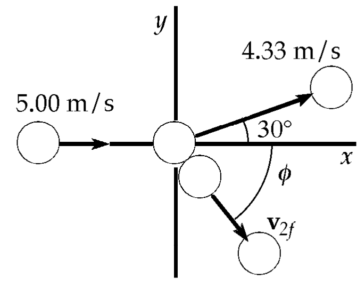
$$Mv_{2i} = 2MV_f \sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0 \text{ m/s})\tan 55.0° = 18.6 \text{ m/s} = \boxed{41.5 \text{ mi/h}}$$

Thus, the driver of the north bound car was untruthful.

8.27 By conservation of momentum for the system of the two billiard balls (with all masses equal),



$$5.00\text{ m/s} + 0 = (4.33\text{ m/s})\cos 30.0° + v_{2fx}$$

$$v_{2fx} = 1.25\text{ m/s}$$

$$0 = (4.33\text{ m/s})\sin 30.0° + v_{2fy}$$

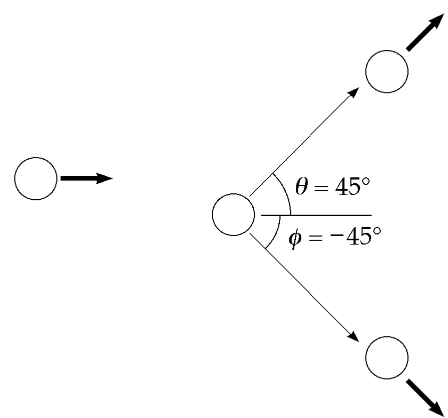
$$v_{2fy} = -2.16\text{ m/s}$$

$$\mathbf{v}_{2f} = \boxed{2.50\text{ m/s at } -60.0°}$$

Note that we did not need to use the fact that the collision is perfectly elastic.

8.28 (a) $\mathbf{p}_i = \mathbf{p}_f$ so $p_{xi} = p_{xf}$

and $p_{yi} = p_{yf}$



$$m v_i = mv\cos\theta + mv\cos\phi \qquad (1)$$

$$0 = mv\sin\theta + mv\sin\phi \qquad (2)$$

From (2), $\sin\theta = -\sin\phi$

so $\theta = -\phi$

Furthermore, energy conservation for the system of two protons requires

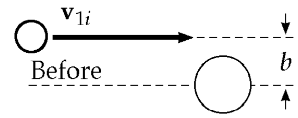
$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 \quad \text{so} \quad \boxed{v = \frac{v_i}{\sqrt{2}}}$$

(b) Hence, (1) gives $v_i = \dfrac{2v_i\cos\theta}{\sqrt{2}}$ $\quad\theta = \boxed{45.0°}$ $\quad\phi = \boxed{-45.0°}$

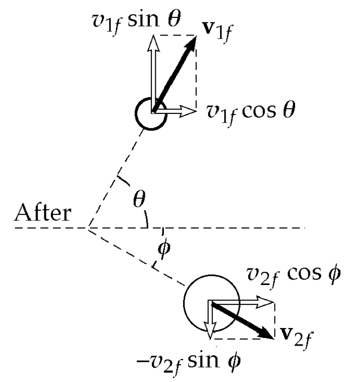
***8.29** $m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$: $\quad 3.00(5.00)\mathbf{i} - 6.00\mathbf{j} = 5.00\mathbf{v}$

$$\mathbf{v} = \boxed{(3.00\mathbf{i} - 1.20\mathbf{j})\text{ m/s}}$$

8.30 (a) Use Equations 8.25 and 8.26 and refer to the figures. Let the puck initially at rest be m_2.



$$m_1 v_{1i} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi: \quad v_{2f}\cos\phi = \frac{m_1 v_{1i} - m_1 v_{1f}\cos\theta}{m_2}$$



$$v_{2f}\cos\phi = \frac{(0.200\text{ kg})(2.00\text{ m/s}) - (0.200\text{ kg})(1.00\text{ m/s})\cos 53.0°}{0.300\text{ kg}}$$

$$0 = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi: \qquad v_{2f}\sin\phi = \frac{m_1 v_{1f}\sin\theta}{m_2}$$

$$0 = (0.200\text{ kg})(1.00\text{ m/s})\sin 53.0° - (0.300\text{ kg})(v_{2f}\sin\phi)$$

From these equations, we find $\tan\phi = \dfrac{\sin\phi}{\cos\phi} = \dfrac{0.160}{0.280} = 0.571$

$$\boxed{\phi = 29.7°}$$

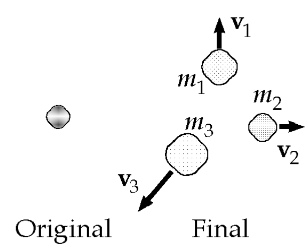
Then $v_{2f} = \dfrac{(0.160\text{ kg}\cdot\text{m/s})}{(0.300\text{ kg})(\sin 29.7°)} = \boxed{1.07\text{ m/s}}$

(b) $f_{\text{lost}} = \dfrac{\Delta K}{K_i} = \dfrac{K_f - K_i}{K_i} = \boxed{-0.318}$

8.31 $m_0 = 17.0\times10^{-27}$ kg $\qquad \mathbf{v}_i = 0$ (the parent nucleus)

$m_1 = 5.00\times10^{-27}$ kg $\qquad \mathbf{v}_1 = 6.00\times10^{6}\mathbf{j}$ m/s

$m_2 = 8.40\times10^{-27}$ kg $\qquad \mathbf{v}_2 = 4.00\times10^{6}\mathbf{i}$ m/s



(a) $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3 = 0$

where $\qquad m_3 = m_0 - m_1 - m_2 = 3.60\times10^{-27}$ kg

$$(5.00\times10^{-27})(6.00\times10^{6}\,\mathbf{j}) + (8.40\times10^{-27})(4.00\times10^{6}\,\mathbf{i}) + (3.60\times10^{-27})\,\mathbf{v}_3 = 0$$

$$\mathbf{v}_3 = \boxed{(-9.33\times10^{6}\mathbf{i} - 8.33\times10^{6}\mathbf{j})\text{ m/s}}$$

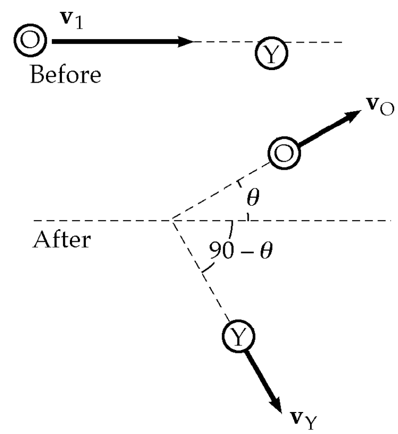
(b) $E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2$

$$E = \tfrac{1}{2}\left[(5.00\times10^{-27})(6.00\times10^{6})^2 + (8.40\times10^{-27})(4.00\times10^{6})^2 + (3.60\times10^{-27})(12.5\times10^{6})^2\right]$$

$$\boxed{E = 4.39\times10^{-13}\text{ J}}$$

***8.32**

$p_{xf} = p_{xi}$: $\quad mv_O \cos\theta + mv_Y \cos(90.0° - \theta) = mv_i$



$$v_O \cos\theta + v_Y \sin\theta = v_i \qquad (1)$$

$p_{yf} = p_{yi}$: $\quad mv_O \sin\theta - mv_Y \sin(90.0° - \theta) = 0$

$$v_O \sin\theta = v_Y \cos\theta \qquad (2)$$

From equation (2),

$$v_O = v_Y(\cos\theta / \sin\theta) \qquad (3)$$

Substituting into equation (1),

$$v_Y\left(\frac{\cos^2\theta}{\sin\theta}\right) + v_Y \sin\theta = v_i$$

so $\quad v_Y(\cos^2\theta + \sin^2\theta) = v_i \sin\theta$, and $\boxed{v_Y = v_i \sin\theta}$

Then, from equation (3), $\boxed{v_O = v_i \cos\theta}$

We did not need to write down an equation expressing conservation of mechanical energy. In the problem situation, the requirement of perpendicular final velocities is equivalent to the condition of elasticity.

8.33 The x-coordinate of the center of mass is

$$x_{CM} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{0 + 0 + 0 + 0}{(2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg})}$$

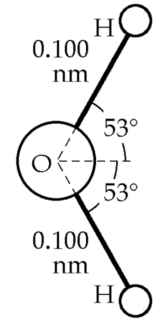
$\boxed{x_{CM} = 0}$

and the y-coordinate of the center of mass is

$$y_{CM} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{(2.00 \text{ kg})(3.00 \text{ m}) + (3.00 \text{ kg})(2.50 \text{ m}) + (2.50 \text{ kg})(0) + (4.00 \text{ kg})(-0.500 \text{ m})}{2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg}}$$

$\boxed{y_{CM} = 1.00 \text{ m}}$

8.34 Take x-axis starting from the oxygen nucleus and pointing toward the middle of the **V**.



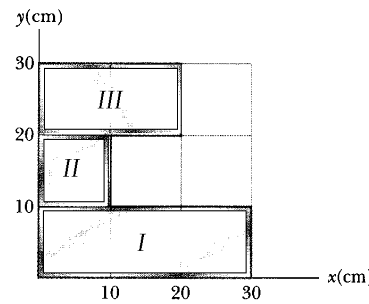
Then $\boxed{y_{CM} = 0}$

and $x_{CM} = \frac{\Sigma m_i x_i}{\Sigma m_i} =$

$$x_{CM} = \frac{0 + 1.008\text{ u}(0.100\text{ nm})\cos 53.0° + 1.008\text{ u}(0.100\text{ nm})\cos 53.0°}{15.999\text{ u} + 1.008\text{ u} + 1.008\text{ u}}$$

$\boxed{x_{CM} = 0.00673\text{ nm from the oxygen nucleus}}$

8.35 Let A_1 represent the area of the bottom row of squares, A_2 the middle square, and A_3 the top pair.



$A = A_1 + A_2 + A_3$

$M = M_1 + M_2 + M_3$

$\frac{M_1}{A_1} = \frac{M}{A}$

$A_1 = 300\text{ cm}^2,\ A_2 = 100\text{ cm}^2,\ A_3 = 200\text{ cm}^2,\ A = 600\text{ cm}^2$

$$M_1 = M\left(\frac{A_1}{A}\right) = \frac{300\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{2}$$

$$M_2 = M\left(\frac{A_2}{A}\right) = \frac{100\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{6}$$

$$M_3 = M\left(\frac{A_3}{A}\right) = \frac{200\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{3}$$

$$x_{CM} = \frac{x_1M_1 + x_2M_2 + x_3M_3}{M} = \frac{15.0\text{ cm}\left(\frac{1}{2}M\right) + 5.00\text{ cm}\left(\frac{1}{6}M\right) + 10.0\text{ cm}\left(\frac{1}{3}M\right)}{M}$$

$x_{CM} = \boxed{11.7\ \text{cm}}$

$$y_{CM} = \frac{\frac{1}{2}M(5.00\text{ cm}) + \frac{1}{6}M(15.0\text{ cm}) + \left(\frac{1}{3}M\right)(25.0\text{ cm})}{M} = 13.3\text{ cm}$$

$y_{CM} = \boxed{13.3\text{ cm}}$

***8.36** (a) $M = \int_0^{0.300\text{ m}} \lambda dx = \int_0^{0.300\text{ m}} \left[50.0\text{ g/m} + 20.0x\text{ g/m}^2\right] dx$

$$M = \left[50.0x\text{ g/m} + 10.0x^2\text{ g/m}^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

(b) $x_{\text{CM}} = \dfrac{\int_{\text{all mass}} x\,dm}{M} = \dfrac{1}{M}\int_0^{0.300\text{ m}} \lambda x dx = \dfrac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x\text{ g/m} + 20.0x^2\text{ g/m}^2\right] dx$

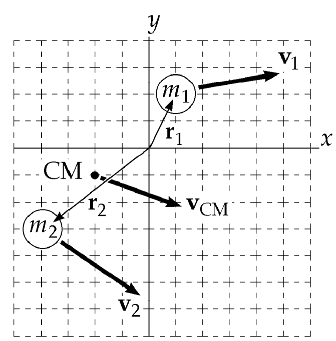
$$x_{\text{CM}} = \frac{1}{15.9\text{ g}}\left[25.0x^2\text{ g/m} + \frac{20x^3\text{ g/m}^2}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

***8.37** (a) $\mathbf{v}_{\text{CM}} = \dfrac{\Sigma m_i \mathbf{v}_i}{M} = \dfrac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{M} = \dfrac{(2.00\text{ kg})(2.00\,\mathbf{i}\text{ m/s} - 3.00\,\mathbf{j}\text{ m/s}) + (3.00\text{ kg})(1.00\,\mathbf{i}\text{ m/s} + 6.00\,\mathbf{j}\text{ m/s})}{5.00\text{ kg}}$

$$\mathbf{v}_{\text{CM}} = \boxed{(1.40\mathbf{i} + 2.40\mathbf{j})\text{ m/s}}$$

(b) $\mathbf{p} = M\mathbf{v}_{\text{CM}} = (5.00\text{ kg})(1.40\mathbf{i} + 2.40\mathbf{j})\text{ m/s} = \boxed{(7.00\mathbf{i} + 12.0\mathbf{j})\text{kg}\cdot\text{m/s}}$

8.38 (a) See figure to the right.



(b) Using the definition of the position vector at the center of mass,

$$\mathbf{r}_{\text{CM}} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

$$\mathbf{r}_{\text{CM}} = \frac{(2.00\text{ kg})(1.00\text{ m, } 2.00\text{ m}) + (3.00\text{ kg})(-4.00\text{ m, } -3.00\text{ m})}{2.00\text{ kg} + 3.00\text{ kg}}$$

$$\mathbf{r}_{\text{CM}} = \boxed{(-2.00\mathbf{i} - 1.00\mathbf{j})\text{ m}}$$

(c) The velocity of the center of mass is

$$\mathbf{v}_{\text{CM}} = \frac{\mathbf{P}}{M} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2} = \frac{(2.00\text{ kg})(3.00\text{ m/s, } 0.50\text{ m/s}) + (3.00\text{ kg})(3.00\text{ m/s, } -2.00\text{ m/s})}{(2.00\text{ kg} + 3.00\text{ kg})}$$

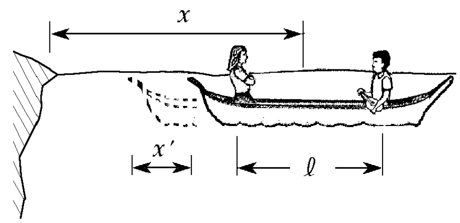
$$\mathbf{v}_{\text{CM}} = \boxed{(3.00\mathbf{i} - 1.00\mathbf{j})\text{ m/s}}$$

(d) The total linear momentum of the system can be calculated as $\mathbf{P} = M\mathbf{v}_{\text{CM}}$

or as $\mathbf{P} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$

Either gives $\mathbf{P} = \boxed{(15.0\mathbf{i} - 5.00\mathbf{j})\text{ kg}\cdot\text{m/s}}$

8.39 Let x = distance from shore to center of boat



ℓ = length of boat

x' = distance boat moves as Juliet moves toward Romeo

The center of mass stays fixed.

Before: $$x_{\text{CM}} = \frac{\left[M_B x + M_J\left(x - \frac{\ell}{2}\right) + M_R\left(x + \frac{\ell}{2}\right)\right]}{\left(M_B + M_J + M_R\right)}$$

After: $$x_{\text{CM}} = \frac{\left[M_B(x - x') + M_J\left(x + \frac{\ell}{2} - x'\right) + M_R\left(x + \frac{\ell}{2} - x'\right)\right]}{\left(M_B + M_J + M_R\right)}$$

$$\ell\left(-\frac{55.0}{2} + \frac{77.0}{2}\right) = x'\ (-80.0 - 55.0 - 77.0) + \frac{\ell}{2}(55.0 + 77.0)$$

$$x' = \frac{55.0\ell}{212} = \frac{55.0(2.70)}{212} = \boxed{0.700\text{ m}}$$

8.40 (a) Conservation of momentum for the two-ball system gives us:

$$2.00\text{ kg}(1.50\text{ m/s}) + 0.300\text{ kg}(-0.400\text{ m/s}) = 0.200\text{ kg } v_{1f} + 0.300\text{ kg } v_{2f}$$

Relative velocity equation:

$$v_{2f} - v_{1f} = 1.90\text{ m/s}$$

Then $$0.300 - 0.120 = 0.200 v_{1f} + 0.300(1.90 + v_{1f})$$

$$v_{1f} = -0.780\text{ m/s} \qquad v_{2f} = 1.12\text{ m/s}$$

$$\boxed{\mathbf{v}_{1f} = -0.780\ \mathbf{i}\text{ m/s}} \qquad \boxed{\mathbf{v}_{2f} = 1.12\ \mathbf{i}\text{ m/s}}$$

(b) Before, $$\mathbf{v}_{\text{CM}} = \frac{(0.200\text{ kg})(1.50\text{ m/s})\mathbf{i} + (0.300\text{ kg})(-0.400\text{ m/s})\mathbf{i}}{0.500\text{ kg}}$$

$$\boxed{\mathbf{v}_{\text{CM}} = (0.360\text{ m/s})\mathbf{i}}$$

Afterwards, the center of mass must move at the same velocity, as momentum of the system is conserved.

8.41 (a) Thrust $= \left| v_e \frac{dM}{dt} \right|$ Thrust $= \left(2.60 \times 10^3 \text{ m/s}\right)\left(1.50 \times 10^4 \text{ kg/s}\right) = \boxed{3.90 \times 10^7 \text{ N}}$

(b) $\Sigma F_y = \text{Thrust} - Mg = Ma$: $3.90 \times 10^7 - \left(3.00 \times 10^6\right)(9.80) = \left(3.00 \times 10^6\right)a$

$a = \boxed{3.20 \text{ m/s}^2}$

***8.42** (a) The fuel burns at a rate $\frac{dM}{dt} = \frac{12.7 \text{ g}}{1.90 \text{ s}} = 6.68 \times 10^{-3} \text{ kg/s}$

Thrust $= v_e \frac{dM}{dt}$: $5.26 \text{ N} = v_e\left(6.68 \times 10^{-3} \text{ kg/s}\right)$

$v_e = \boxed{787 \text{ m/s}}$

(b) $v_f = v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$: $v_f - 0 = (797 \text{ m/s})\ln\left(\frac{53.5 \text{ g} + 25.5 \text{ g}}{53.5 \text{ g} + 25.5 \text{ g} - 12.7 \text{ g}}\right)$

$v_f = \boxed{138 \text{ m/s}}$

8.43 $v = v_e \ln \frac{M_i}{M_f}$

(a) $M_i = e^{v/v_e} M_f$ $M_i = e^5\left(3.00 \times 10^3 \text{ kg}\right) = 4.45 \times 10^5 \text{ kg}$

The mass of fuel and oxidizer is $\Delta M = M_i - M_f = (445 - 3.00) \times 10^3 \text{ kg} = \boxed{442 \text{ metric tons}}$

(b) $\Delta M = e^2(3.00 \text{ metric tons}) - 3.00 \text{ metric tons} = \boxed{19.2 \text{ metric tons}}$

Because of the exponential, a relatively small increase in fuel and/or engine efficiency causes a large change in the amount of fuel and oxidizer required.

***8.44** The thrust acting on the spacecraft is

$\Sigma F = ma$: $\Sigma F = (3500\ \text{kg})(2.50\times10^{-6})(9.80\ \text{m/s}^2) = 8.58\times10^{-2}\ \text{N}$

$\text{thrust} = \left(\frac{dM}{dt}\right)v_e$: $8.58\times10^{-2}\ \text{N} = \left(\frac{\Delta M}{3600\ \text{s}}\right)(70\ \text{m/s})$

$\Delta M = \boxed{4.41\ \text{kg}}$

***8.45** We find the mass from $M = 360\ \text{kg} - (2.50\ \text{kg/s})t$.

We find the acceleration from $a = \frac{\text{Thrust}}{M} = \frac{v_e|dM/dt|}{M} = \frac{(1500\ \text{m/s})(2.50\ \text{kg/s})}{M} = \frac{3750\ \text{N}}{M}$

We find the velocity and position according to Euler,

from $v_{\text{new}} = v_{\text{old}} + a(\Delta t)$

and $x_{\text{new}} = x_{\text{old}} + v(\Delta t)$

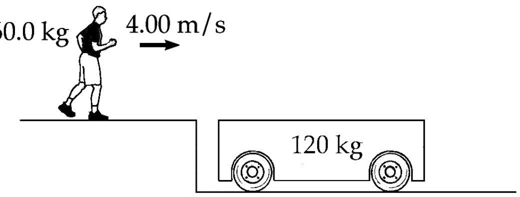
If we take $\Delta t = 0.132\ \text{s}$, a portion of the output looks like this:

Time t(s)	Total mass (kg)	Acceleration a (m/s^2)	Speed, v (m/s)	Position x(m)
0.000	360.00	10.4167	0.0000	0.0000
0.132	359.67	10.4262	1.3750	0.1815
0.264	359.34	10.4358	2.7513	0.54467
...				
65.868	195.330	19.1983	916.54	27191
66.000	195.000	19.2308	919.08	27312
66.132	194.670	19.2634	921.61	27433
...				
131.736	30.660	122.3092	3687.3	152382
131.868	30.330	123.6400	3703.5	152871
132.000	30.000	125.0000	3719.8	153362

(a) The final speed is $v_f = \boxed{3.7\ \text{km/s}}$

(b) The rocket travels $\boxed{153\ \text{km}}$

***8.46** (a) $(60.0 \text{ kg})4.00 \text{ m/s} = (120 + 60.0) \text{ kg } v_f$



$\mathbf{v}_f = \boxed{1.33 \text{ m/s } \mathbf{i}}$

(b) $\sum F_y = 0$: $\quad n - (60.0 \text{ kg})9.80 \text{ m/s}^2 = 0$

$f_k = \mu_k n = 0.400(588 \text{ N}) = 235 \text{ N}$ $\qquad \mathbf{f}_k = \boxed{-235 \text{ N}\,\mathbf{i}}$

(c) For the person, $\quad p_i + I = p_f$ $\qquad mv_i + Ft = mv_f$

$(60.0 \text{ kg})4.00 \text{ m/s} - (235 \text{ N})t = (60.0 \text{ kg})1.33 \text{ m/s}$ $\qquad t = \boxed{0.680 \text{ s}}$

(d) person: $\quad m\mathbf{v}_f - m\mathbf{v}_i = 60.0 \text{ kg}(1.33 - 4.00) \text{ m/s} = \boxed{-160 \text{ N} \cdot \text{s } \mathbf{i}}$

cart: $\quad 120 \text{ kg}(1.33 \text{ m/s}) - 0 = \boxed{+160 \text{ N} \cdot \text{s } \mathbf{i}}$

(e) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}[(4.00 + 1.33) \text{ m/s}]0.680 \text{ s} = \boxed{1.81 \text{ m}}$

(f) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(0 + 1.33 \text{ m/s})0.680 \text{ s} = \boxed{0.454 \text{ m}}$

(g) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}60.0 \text{ kg}(1.33 \text{ m/s})^2 - \frac{1}{2}60.0 \text{ kg}(4.00 \text{ m/s})^2 = \boxed{-427 \text{ J}}$

(h) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}120.0 \text{ kg}(1.33 \text{ m/s})^2 - 0 = \boxed{107 \text{ J}}$

(i) The force exerted by the person on the cart must be equal in magnitude and opposite in direction to the force exerted by the cart on the person. The changes in momentum of the two objects must be equal in magnitude and must add to zero. Their changes in kinetic energy are different in magnitude and do not add to zero. The following represent two ways of thinking about 'why.' The distance the cart moves is different from the distance moved by the point of application of the friction force to the cart. The total change in mechanical energy for both objects together, –320 J, becomes +320 J of additional internal energy in this perfectly inelastic collision.

8.47 We shall use conservation of energy for the Tarzan-Earth system, with the zero level for gravitational potential energy at the bottom of the arc, to find the velocity of Tarzan, v_0, just as he reaches Jane.

$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$: $\qquad 0 + (80.0 \text{ kg})g(3.00 \text{ m}) = \frac{1}{2}(80.0 \text{ kg})v_0^2 + 0$

$v_0 = 7.67 \text{ m/s}$

Now, use conservation of momentum of the Tarzan-Jane system to find the velocity, V, of Tarzan + Jane just after he picks her up.

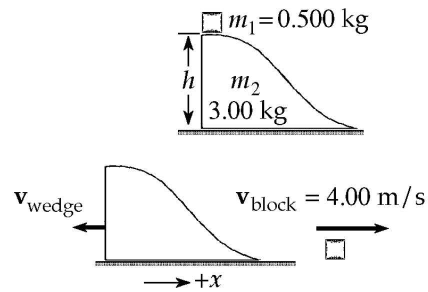
$(80.0 \text{ kg})(7.67 \text{ m/s}) = (140 \text{ kg})V$ $\qquad V = 4.38 \text{ m/s}$

Finally, we use conservation of mechanical energy for the Tarzan-Jane-Earth system from just after he picks her up to the end of their swing to determine the maximum height, H, reached.

$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$: $\qquad \frac{1}{2}(140 \text{ kg})(4.38 \text{ m/s})^2 + 0 = 0 + (140 \text{ kg})gH$

$H = \boxed{0.980 \text{ m}}$

8.48 (a) The initial momentum of the block-wedge system is zero, which remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have



$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or $$(3.00 \text{ kg})v_{\text{wedge}} + (0.500 \text{ kg})(+4.00 \text{ m/s}) = 0$$

so $$v_{\text{wedge}} = \boxed{-0.667 \text{ m/s}}$$

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

or $$\left[0 + m_1 g h\right] + 0 = \left[\tfrac{1}{2} m_1 (4.00)^2 + 0\right] + \tfrac{1}{2} m_2 (-0.667)^2 \text{ which gives } \boxed{h = 0.952 \text{ m}}$$

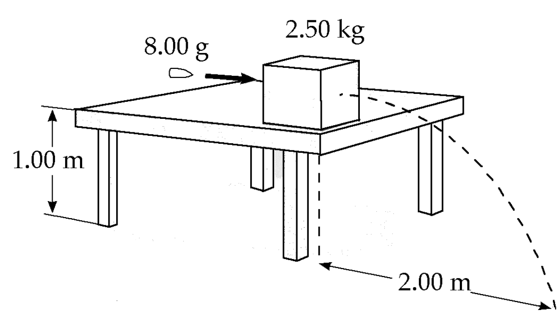
***8.49** $$T = \frac{dm}{dt}(\Delta v_{\text{fuel}}) + \frac{dm}{dt}(\Delta v_{\text{air}}): \qquad T = (3.00 \text{ kg/s})\ (600 \text{ m/s}) + (80.0 \text{ kg/s})(600 \text{ m/s} - 223 \text{ m/s})$$

$$T = 1800 \text{ N} + 3.02 \times 10^4 \text{ N} = \boxed{3.20 \times 10^4 \text{ N}}$$

The delivered power is the force (or thrust) multiplied by the velocity;

$$\mathcal{P} = Tv = (3.20 \times 10^4 \text{ N})(223 \text{ m/s}) = \boxed{7.13 \times 10^6 \text{ W}}$$

8.50 Since no other force acts along the horizontal direction on the bullet-block system before and after impact,



$\Delta p = 0$ or $(8.00 \times 10^{-3})\, v_i = (2.508)\, v_f$

We can find v_f using the kinematic equations:

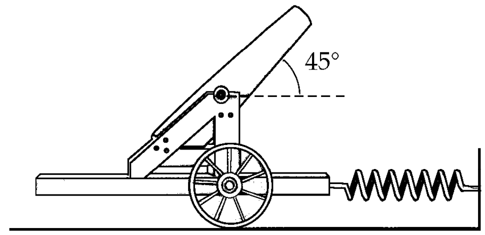
$\Delta x = v_f\, t$ or $2.00 = v_f t$

and $\Delta y = \frac{1}{2} a_y t^2$

or $$-1.00 \text{ m} = \tfrac{1}{2}\left(-9.80 \text{ m/s}^2\right)\left(\frac{2.00}{v_f}\right)^2$$

$$v_f = 4.43 \text{ m/s} \qquad v_i = \frac{2.508(4.43)}{8.00 \times 10^{-3}} = \boxed{1.39 \text{ km/s}}$$

***8.51** (a) Use conservation of the horizontal component of momentum for the system of the shell, the cannon, and the carriage, from just before to just after the cannon firing.



$p_{xf} = p_{xi}$: $\quad m_{\text{shell}} v_{\text{shell}} \cos 45.0° + m_{\text{cannon}} v_{\text{recoil}} = 0$

$$(200)(125)\cos 45.0° + (5000)\, v_{\text{recoil}} = 0$$

or $\quad v_{\text{recoil}} = \boxed{-3.54 \text{ m/s}}$

(b) Use conservation of energy for the system of the cannon, the carriage, and the spring from right after the cannon is fired to the instant when the cannon comes to rest.

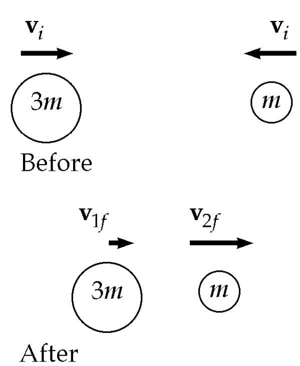
$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$: $\quad 0 + 0 + \frac{1}{2} k x_{\max}^2 = \frac{1}{2} m v_{\text{recoil}}^2 + 0 + 0$

$$x_{\max} = \sqrt{\frac{m v_{\text{recoil}}^2}{k}} = \sqrt{\frac{(5000)(-3.54)^2}{2.00 \times 10^4}} \text{ m} = \boxed{1.77 \text{ m}}$$

(c) $|F_{s,\max}| = k x_{\max}$ $\qquad |F_{s,\max}| = \left(2.00 \times 10^4 \text{ N/m}\right)(1.77 \text{ m}) = \boxed{3.54 \times 10^4 \text{ N}}$

(d) No. The rail exerts a vertical external force (the normal force) on the cannon and prevents it from recoiling vertically. Momentum is not conserved in the vertical direction. The spring does not have time to stretch during the cannon firing. Thus, no external horizontal force is exerted on the system (cannon, carriage, and shell) from just before to just after firing. Momentum of this system is conserved in the horizontal direction during this interval.

8.52 Momentum conservation for the system of the two objects:



$$3m\, v_i - m\, v_i = m\, v_{1f} + 3m\, v_{2f}$$

the relative velocity equation:

$$v_{1i} - v_{2i} = -v_{1f} + v_{2f} \qquad \begin{cases} -v_i - v_i = -v_{1f} + v_{2f} \\ 2v_i = v_{1f} + 3v_{2f} \end{cases}$$

$$0 = 4\, v_{2f}$$

$$v_{1f} = \boxed{2v_i} \qquad v_{2f} = \boxed{0}$$

8.53 x-component of momentum for the system of the two objects:

$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$: $\quad -mv_i + 3mv_i = 0 + 3mv_{2x}$

y-component of momentum of the system: $\quad 0 + 0 = -mv_{1y} + 3mv_{2y}$

by conservation of energy of the system: $\quad +\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$

we have $\quad v_{2x} = \dfrac{2v_i}{3}$

also $\quad v_{1y} = 3v_{2y}$

So the energy equation becomes $\quad 4v_i^2 = 9v_{2y}^2 + \dfrac{4v_i^2}{3} + 3v_{2y}^2$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $\quad v_{2y} = \dfrac{\sqrt{2}v_i}{3}$

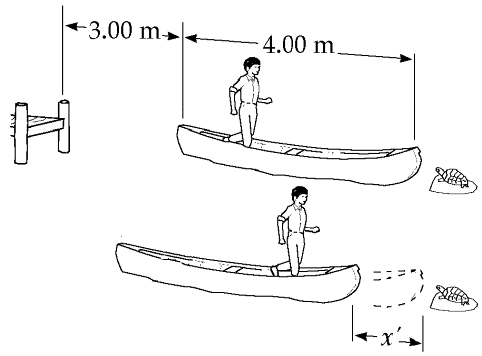
(a) The object of mass m has final speed $\quad v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$

and the object of mass 3 m moves at $\quad \sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\dfrac{4v_i^2}{9} + \dfrac{2v_i^2}{9}}$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta = \tan^{-1}\left(\dfrac{v_{2y}}{v_{2x}}\right)$ $\quad \theta = \tan^{-1}\left(\dfrac{\sqrt{2}v_i}{3}\dfrac{3}{2v_i}\right) = \boxed{35.3°}$

8.54 (a) $\boxed{\text{As the child walks to the right, the boat moves to the left, and the center of mass remains fixed.}}$
Let x = 5.00 m represent the original position of the center of the boat.



(b) Before:
$$x_{CM} = \frac{x m_B + (x - 2.00) m_C}{m_B + m_C}$$

$$x_{CM} = \frac{(5.00)(70.0) + (3.00)(40.0)}{110}$$

Let x' represent the distance the boat drifts back.

After:
$$x_{CM} = \frac{(x - x')m_B + (x - 2.00 + 4.00 - x')m_C}{m_B + m_C} = \frac{(5.00 - x')70.0 + (7.00 - x')40.0}{110}$$

$$5.00(70.0) + 3.00(40.0) = (5.00 - x')70.0 + (7.00 - x')40.0$$

$$470 = 350 - 70.0x' + 280 - 40.0x'$$

$$110x' = 160 \qquad \text{and} \qquad x' = 1.45 \text{ m}$$

Relative to the pier the child is at $x'' = 3.00 + 4.00 - 1.45 = \boxed{5.55 \text{ m}}$

(c) $\boxed{\text{No, the child missed the turtle by 0.45 m.}}$

(The dashed line in the figure shows the original position of the end of the canoe, before the canoe recoils back.)

8.55 (a) $\mathbf{p}_i + \mathbf{F}t = \mathbf{p}_f$: $\quad (3.00 \text{ kg})(7.00 \text{ m/s})\mathbf{j} + (12.0 \text{ N } \mathbf{i})(5.00 \text{ s}) = (3.00 \text{ kg})\mathbf{v}_f$

$$\mathbf{v}_f = \boxed{(20.0\mathbf{i} + 7.00\mathbf{j}) \text{ m/s}}$$

(b) $\mathbf{a} = \dfrac{\mathbf{v}_f - \mathbf{v}_i}{t}$: $\quad \mathbf{a} = \dfrac{(20.0\mathbf{i} + 7.00\mathbf{j} - 7.00\mathbf{j}) \text{ m/s}}{5.00 \text{ s}} = \boxed{4.00\mathbf{i} \text{ m/s}^2}$

(c) $\mathbf{a} = \dfrac{\sum \mathbf{F}}{m}$: $\quad \mathbf{a} = \dfrac{12.0 \text{ N } \mathbf{i}}{3.00 \text{ kg}} = \boxed{4.00\mathbf{i} \text{ m/s}^2}$

(d) $\Delta\mathbf{r} = \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$: $\quad \Delta\mathbf{r} = (7.00 \text{ m/s } \mathbf{j})(5.00 \text{ s}) + \frac{1}{2}(4.00 \text{ m/s}^2 \text{ } \mathbf{i})(5.00 \text{ s})^2$

$$\Delta\mathbf{r} = \boxed{(50.0 \text{ } \mathbf{i} + 35.0 \text{ } \mathbf{j}) \text{ m}}$$

(e) $W = \mathbf{F} \cdot \Delta\mathbf{r}$: $\quad W = (12.0 \text{ N } \mathbf{i}) \cdot (50.0 \text{ m } \mathbf{i} + 35.0 \text{ m } \mathbf{j}) = \boxed{600 \text{ J}}$

(f) $\frac{1}{2}mv_f^2 = \frac{1}{2}(3.00 \text{ kg})(20.0 \text{ } \mathbf{i} + 7.00 \text{ } \mathbf{j}) \cdot (20.0 \text{ } \mathbf{i} + 7.00 \text{ } \mathbf{j}) \text{ m}^2/\text{s}^2$

$$\frac{1}{2}mv_f^2 = (1.50 \text{ kg})(449 \text{ m}^2/\text{s}^2) = \boxed{674 \text{ J}}$$

(g) $\frac{1}{2}mv_i^2 + W = \frac{1}{2}(3.00 \text{ kg})(7.00 \text{ m/s})^2 + 600 \text{ J} = \boxed{674 \text{ J}}$

***8.56** (a) When the spring is fully compressed, each cart moves with same velocity **v**. Apply conservation of momentum for the system of two gliders

$\mathbf{p}_i = \mathbf{p}_f$: $\quad m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}$ $\quad \boxed{\mathbf{v} = \dfrac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}}$

(b) Only conservative forces act, therefore $\Delta E = 0$. $\quad \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$

Substitute for v from (a) and solve for x_m.

$$x_m^2 = \frac{(m_1 + m_2)m_1 v_1^2 + (m_1 + m_2)m_2 v_2^2 - (m_1 v_1)^2 - (m_2 v_2)^2 - 2m_1 m_2 v_1 v_2}{k(m_1 + m_2)}$$

$$x_m = \sqrt{\frac{m_1 m_2\left(v_1^2 + v_2^2 - 2v_1 v_2\right)}{k(m_1 + m_2)}} = \boxed{(v_1 - v_2)\sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}}$$

(c) $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$

Conservation of momentum: $\quad m_1\left(\mathbf{v}_1 - \mathbf{v}_{1f}\right) = m_2\left(\mathbf{v}_{2f} - \mathbf{v}_2\right)$ (1)

Conservation of energy: $\quad \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$

which simplifies to: $\quad m_1\left(v_1^2 - v_{1f}^2\right) = m_2\left(v_2^2 - v_{2f}^2\right)$

Factoring gives $\quad m_1\left(\mathbf{v}_1 - \mathbf{v}_{1f}\right)\cdot\left(\mathbf{v}_1 + \mathbf{v}_{1f}\right) = m_2\left(\mathbf{v}_{2f} + \mathbf{v}_2\right)\cdot\left(\mathbf{v}_{2f} + \mathbf{v}_2\right)$

and with the use of the momentum equation (equation (1)),

this reduces to $\quad \left(\mathbf{v}_1 + \mathbf{v}_{1f}\right) = \left(\mathbf{v}_{2f} + \mathbf{v}_2\right)$

or $\quad \mathbf{v}_{1f} = \mathbf{v}_{2f} + \mathbf{v}_2 - \mathbf{v}_1$ (2)

Substituting equation (2) into equation (1) and simplifying yields:

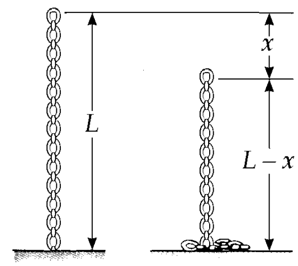
$$\mathbf{v}_{2f} = \boxed{\left(\frac{2m_1}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\mathbf{v}_2}$$

Upon substitution of this expression for $\mathbf{v}_{2f}$ into equation 2, one finds

$$\mathbf{v}_{1f} = \boxed{\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\frac{2m_2}{m_1 + m_2}\right)\mathbf{v}_2}$$

Observe that these results are the same as Equations 8.21 and 8.22, which should have been expected since this is a perfectly elastic collision in one dimension.

***8.57** The force exerted by the table is equal to the change in momentum of each of the links in the chain.



By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}$$

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \qquad \text{and} \qquad m\frac{dv}{dt} = 0$$

Since the mass per unit length is uniform, we can express each link of length dx as having a mass dm :

$$dm = \frac{M}{L}dx$$

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements dm.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance x, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}$$

The links already on the table have a total length x, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}$$

That is, *the total force is three times the weight of the chain on the table at that instant.*

***8.58** A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $\dfrac{\Delta p_x}{\Delta t} = \dfrac{(5.00\text{ kg})(0.750\text{ m/s})}{1.00\text{ s}} = \boxed{3.75\text{ N}}$

(b) The only horizontal force on the sand is belt friction,

so from $p_{xi} + f\,\Delta t = p_{xf}$ this is $f = \dfrac{\Delta p_x}{\Delta t} = \boxed{3.75\text{ N}}$

(c) The belt is in equilibrium:

$\sum F_x = ma_x$: $+F_{\text{ext}} - f = 0$ and $F_{\text{ext}} = \boxed{3.75\text{ N}}$

(d) $W = F\Delta r\cos\theta = 3.75\text{ N}(0.750\text{ m})\cos 0° = \boxed{2.81\text{ J}}$

(e) $\frac{1}{2}(\Delta m)v^2 = \frac{1}{2}5.00\text{ kg}(0.750\text{ m/s})^2 = \boxed{1.41\text{ J}}$

(f) Friction between sand and belt converts half of the input work into extra internal energy

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 6.25 cm/s west

4. (a) 6.00 m/s toward the left (b) 8.40 J

6. 40.5 g

8. 364 kg·m/s forward, 438 N forward

10. (a) 5.40 N·s in the direction of $\mathbf{v}_f$ (b) –27.0 J

12. 15.0 N in the direction of the initial velocity of the exiting water stream

14. 2.66 m/s

16. 15.6 m/s

18. (a) 2.50 m/s (b) 37.5 kJ
(c) Each process is the time-reversal of the other. The same momentum conservation equation describes both.

20. 0.556 m

22. Force on truck driver $= 1.78 \times 10^3$ N; Force on car driver 8.89×10^3 N

24. 7.94 cm

26. No; his speed was 41.5 mi/h.

28. (a) $v_i / \sqrt{2}$ (b) $\pm 45.0°$

30. (a) 1.07 m/s at –29.7° (b) $\Delta K / K_i = -0.318$

32. Orange: $v_i \cos\theta$; yellow: $v_i \sin\theta$

34. 0.006 73 nm from the oxygen nucleus along the bisector of the angle

36. (a) 15.9 g (b) 0.153 m

38. (a) See the solution. (b) $(-2.00\mathbf{i} - 1.00\mathbf{j})$ m
(c) $(3.00\mathbf{i} - 1.00\mathbf{j})$ m/s (d) $(15.0\mathbf{i} - 5.00\mathbf{j})$ kg·m/s

40. (a) $\mathbf{v}_{1f} = -0.780\mathbf{i}$ m/s, $\mathbf{v}_{2f} = 1.12\mathbf{i}$ m/s (b) $0.360\mathbf{i}$ m/s

42. (a) 787 m/s (b) 138 m/s

44. 4.41 kg

46. (a) $(1.33\text{ m/s})\mathbf{i}$ (b) $(-235\text{ N})\mathbf{i}$ (c) 0.680 s
(d) $(-160\text{ N}\cdot\text{s})\mathbf{i}$ and $(+160\text{ N}\cdot\text{s})\mathbf{i}$ (e) 1.81 m (f) 0.454 m
(g) -427 J (h) $+107$ J
(i) The total change in mechanical energy for person and cart together, -320 J, becomes $+320$ J of additional internal energy in this perfectly inelastic collision.

48. (a) -0.667 m/s (b) 0.952 m

50. 1.39 km/s

52. $2v_i$ and 0

54. (a) As the child walks to the right, the boat moves to the left, and the center of mass remains fixed.
(b) 5.55 m from the pier
(c) No, since 6.55 m is less than 7.00 m.

56. (a) $\mathbf{v} = \dfrac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$ (b) $x_m = (v_1 - v_2)\sqrt{\dfrac{m_1 m_2}{k(m_1 + m_2)}}$
(c) $\mathbf{v}_{1f} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\dfrac{2m_2}{m_1 + m_2}\right)\mathbf{v}_2$, $\mathbf{v}_{2f} = \left(\dfrac{2m_1}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)\mathbf{v}_2$

58. (a) $3.75\text{ kg}\cdot\text{m/s}^2$ (b) 3.75 N (c) 3.75 N
(d) 2.81 J (e) 1.41 J
(f) Friction between sand and belt converts half of the input work into internal energy.