CHAPTER 9

ANSWERS TO QUESTIONS

Q9.1 The speed of light, c, and the speed, v, of their relative motion.

Q9.2 No, because this is still less than the free – space speed of light, c.

Q9.3 The clock in orbit runs slower. No, although they both run at the same rate on return, there is a time difference between the two clocks.

Q9.4 (a) Yours does. (b) His does.
(c) If the velocity of relative motion is constant, both observers have equally valid views.

Q9.5 An ellipsoid. The dimension in the direction of motion would be measured to be "scrunched in".

Q9.6 Otherwise, we are making a *time measurement* as well.

Q9.7 Nothing physically unusual. However, the moving clock has been accelerated.

Q9.8 As the object approaches the speed of light, its kinetic energy grows without limit. It would take an infinite investment of work to accelerate the object to the speed of light.

Q9.9 Get a *Mr. Tompkins* book by George Gamow for a wonderful fictional exploration of this question. Driving home in a hurry, you push on the gas pedal not to increase your speed by very much, but rather to make the blocks get shorter. Big Doppler shifts in wave frequencies make red lights look green as you approach them and make car horns and car radios useless. High-speed transportation is very expensive, requiring huge fuel purchases. And it is dangerous, as a speeding car can knock down a building. Having had breakfast at home, you return hungry for lunch, but you find you have missed dinner. There is a five-day delay in transmission when you watch the Olympics in Australia on live television. It takes ninety-five years for sunlight to reach Earth. We cannot see the Milky Way; the fireball of the Big Bang surrounds us at the distance of Rigel or Deneb.

Q9.10 You see no change in your reflection at any speed you can attain. You cannot attain the speed of light, for that would take an infinite amount of energy.

Q9.11 Quasar light moves at three hundred million meters per second, just like the light from a firefly at rest.

Q9.12 A photon transports energy. The relativistic equivalence of mass and energy means that is enough to give it momentum.

Q9.13 Any physical theory must agree with experimental measurements within some domain. Newtonian mechanics agrees with experiment for objects moving slowly compared to the speed of light. Relativistic mechanics agrees with experiment for objects at all speeds. Thus the two theories must agree with each other for ordinary nonrelativistic objects.

Q9.14 According to $\mathbf{p} = \gamma m \mathbf{u}$, doubling the speed u will make the momentum of an object increase by the factor $2[(c^2 - u^2)/(c^2 - 4u^2)]^{1/2}$.

Q9.15 There is no upper limit on the momentum of an electron. As energy E is fed into the object, without limit, its speed approaches the speed of light and its momentum approaches E/c.

PROBLEM SOLUTIONS

9.1 In the rest frame,

$$p_i = m_1 v_{1i} + m_2 v_{2i} = (2000\ \text{kg})(20.0\ \text{m/s}) + (1500\ \text{kg})(0\ \text{m/s}) = 4.00 \times 10^4\ \text{kg} \cdot \text{m/s}$$

$$p_f = (m_1 + m_2)\, v_f = (2000\ \text{kg} + 1500\ \text{kg})\ v_f$$

Since $p_i = p_f$,
$$v_f = \frac{4.00 \times 10^4\ \text{kg} \cdot \text{m/s}}{2000\ \text{kg} + 1500\ \text{kg}} = 11.429\ \text{m/s}$$

In the moving frame, these velocities are all reduced by +10.0 m/s.

$$v'_{1i} = v_{1i} - v' = 20.0\ \text{m/s} - (+10.0\ \text{m/s}) = 10.0\ \text{m/s}$$

$$v'_{2i} = v_{2i} - v' = 0\ \text{m/s} - (+10.0\ \text{m/s}) = -10.0\ \text{m/s}$$

$$v'_f = 11.429\ \text{m/s} - (+10.0\ \text{m/s}) = 1.429\ \text{m/s}$$

Our initial momentum is then

$$p'_i = m_1 v'_{1i} + m_2 v'_{2i} = (2000\ \text{kg})(10.0\ \text{m/s}) + (1500\ \text{kg})(-10.0\ \text{m/s}) = 5000\ \text{kg} \cdot \text{m/s}$$

and our final momentum is

$$p'_f = (2000\ \text{kg} + 1500\ \text{kg})\, v'_f = (3500\ \text{kg})(1.429\ \text{m/s}) = 5000\ \text{kg} \cdot \text{m/s}$$

9.2 (a) $v = v_T + v_B = \boxed{60.0\ \text{m/s}}$

(b) $v = v_T - v_B = \boxed{20\ \text{m/s}}$

(c) $v = \sqrt{{v_T}^2 + {v_B}^2} = \sqrt{20^2 + 40^2} = \boxed{44.7\ \text{m/s}}$

9.3 The first observer watches some object accelerate under applied forces. Call the instantaneous velocity of the object $\mathbf{v}_1$. The second observer has constant velocity $\mathbf{v}_{21}$ relative to the first, and measures the object to have velocity $\mathbf{v}_2 = \mathbf{v}_1 - \mathbf{v}_{21}$.

The second observer measures an acceleration of
$$\mathbf{a}_2 = \frac{d\mathbf{v}_2}{dt} = \frac{d\mathbf{v}_1}{dt}$$

This is the same as that measured by the first observer. In this nonrelativistic case, they measure the same forces as well. Thus, the second observer also confirms that $\Sigma\mathbf{F} = m\mathbf{a}$.

9.4 The laboratory observer notes Newton's second law to hold: $\mathbf{F}_1 = m\mathbf{a}_1$

(where the subscript 1 implies the measurement was made in the laboratory frame of reference). The observer in the accelerating frame measures the acceleration of the mass as $\mathbf{a}_2 = \mathbf{a}_1 - \mathbf{a}'$

(where the subscript 2 implies the measurement was made in the accelerating frame of reference, and the primed acceleration term is the acceleration of the accelerated frame with respect to the laboratory frame of reference). If Newton's second law held for the accelerating frame, that observer would then find valid the relation

$$\mathbf{F}_2 = m\mathbf{a}_2 \qquad \text{or} \qquad \mathbf{F}_1 = m\mathbf{a}_2$$

(since $\mathbf{F}_1 = \mathbf{F}_2$ and the mass is unchanged in each). But, instead, the accelerating frame observer will find that $\mathbf{F}_2 = m\mathbf{a}_2 - m\mathbf{a}'$ which is *not* Newton's second law.

9.5 $L = L_p\sqrt{1 - v^2/c^2}$ $\qquad v = c\sqrt{1 - (L/L_p)^2}$

Taking $L = L_p/2$ where $L_p = 1.00$ m gives $\quad v = c\sqrt{1 - \left(\dfrac{L_p/2}{L_p}\right)^2} = c\sqrt{1 - \dfrac{1}{4}} = \boxed{0.866\ c}$

9.6 $\Delta t = \dfrac{\Delta t_p}{\left[1 - (v/c)^2\right]^{1/2}}$ $\qquad$ so $\qquad v = c\left[1 - \left(\dfrac{\Delta t_p}{\Delta t}\right)^2\right]^{1/2}$

For $\quad \Delta t = 2\Delta t_p$ $\qquad v = c\left[1 - \left(\dfrac{\Delta t_p}{2\Delta t_p}\right)^2\right]^{1/2} = c\left[1 - \dfrac{1}{4}\right]^{1/2} = \boxed{0.866\ c}$

9.7 (a) $\gamma = \dfrac{1}{\sqrt{1 - (v/c)^2}} = \dfrac{1}{\sqrt{1 - (0.500)^2}} = \dfrac{2}{\sqrt{3}}$

The time interval between pulses as measured by the Earth observer is

$$\Delta t = \gamma\,\Delta t_p = \frac{2}{\sqrt{3}}\left(\frac{60.0\ \text{s}}{75.0}\right) = 0.924\ \text{s}$$

Thus, the Earth observer records a pulse rate of $\dfrac{60.0\ \text{s/min}}{0.924\ \text{s}} = \boxed{64.9/\text{min}}$

(b) At a relative speed $v = 0.990c$, the relativistic factor γ increases to 7.09 and the pulse rate recorded by the Earth observer decreases to $\boxed{10.6/\text{min}}$. That is, the life span of the astronaut (reckoned by the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

***9.8** (a) The $0.8\,c$ and the 20 ly are measured in the Earth frame,

so in this frame, $$\Delta t = \frac{x}{v} = \frac{20\text{ ly}}{0.8c} = \frac{20\text{ ly}}{0.8c}\frac{1c}{1\text{ ly/yr}} = \boxed{25.0\text{ yr}}$$

(b) We see a clock on the meteoroid moving, so we do not measure proper time; that clock measures proper time.

$\Delta t = \gamma \Delta t_p$: $$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{25.0\text{ yr}}{1/\sqrt{1-v^2/c^2}} = 25.0\text{ yr}\sqrt{1-0.8^2} = 25.0\text{ yr}(0.6) = \boxed{15.0\text{ yr}}$$

(c) Method one: We measure the 20 ly on a stick stationary in our frame, so it is proper length. The tourist measures it to be contracted to

$$L = \frac{L_p}{\gamma} = \frac{20\text{ ly}}{1/\sqrt{1-0.8^2}} = \frac{20\text{ ly}}{1.667} = \boxed{12.0\text{ ly}}$$

Method two: The tourist sees the Earth approaching at $0.8\,c$ and covering the distance in 15 yr. So the distance is

$$(0.8\text{ ly/yr})(15\text{ yr}) = \boxed{12.0\text{ ly}}$$

Not only do distances and times differ between Earth and meteoroid reference frames, but within the Earth frame apparent distances differ from actual distances. As we have interpreted it, the 20-lightyear actual distance from the Earth to the meteoroid at the time of discovery must be a calculated result, different from the distance measured directly. Because of the finite maximum speed of information transfer, the astronomer sees the meteoroid as it was years previously, when it was much farther away. Call its apparent distance d. The time required for light to reach us from the newly-visible meteoroid is the lookback time $t = d/c$. The astronomer calculates that the meteoroid has approached to be 20 ly away as it moved with constant velocity throughout the lookback time. We can work backwards to reconstruct her calculation:

$$d = 20\text{ ly} + 0.8ct = 20\text{ ly} = 0.8cd/c$$

$$0.2d = 20\text{ ly}$$

$$d = 100\text{ ly}$$

Thus in terms of direct observation, the meteoroid we see covers 100 ly in only 25 years. Such an apparent superluminal velocity is actually observed for some jets of material emanating from quasars, because they happen to be pointed nearly toward the Earth. If we can watch events unfold on the meteoroid, we see them slowed by relativistic time dilation, but also greatly speeded up by the Doppler effect.

9.9 $$\Delta t = \gamma\,\Delta t_p = \frac{\Delta t_p}{\sqrt{1-v^2/c^2}} \quad \text{so} \quad \Delta t_p = \left(\sqrt{1-v^2/c^2}\right)\Delta t \cong \left(1-\frac{v^2}{2c^2}\right)\Delta t$$

and $$\Delta t - \Delta t_p = \left(\frac{v^2}{2c^2}\right)\Delta t$$

If $$v = 1000\text{ km/h} = \frac{1.00\times10^6\text{ m}}{3600\text{ s}} = 277.8\text{ m/s}$$

then $$\frac{v}{c} = 9.26\times10^{-7}$$

and $$\left(\Delta t - \Delta t_p\right) = (4.28\times10^{-13})(3600\text{ s}) = 1.54\times10^{-9}\text{ s} = \boxed{1.54\text{ ns}}$$

9.10 For $\dfrac{v}{c} = 0.990$, $\gamma = 7.09$

(a) The muon's lifetime as measured in the Earth's rest frame is

$$\Delta t = \frac{4.60 \text{ km}}{0.990\,c}$$

and the lifetime measured in the muon's rest frame is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09}\left[\frac{4.60\times 10^3 \text{ m}}{0.990\left(3.00\times 10^8 \text{ m/s}\right)}\right] = \boxed{2.18\ \mu\text{s}}$$

(b) $$L = L_p\sqrt{1-(v/c)^2} = \frac{L_p}{\gamma} = \frac{4.60\times 10^3 \text{ m}}{7.09} = \boxed{649 \text{ m}}$$

9.11 The spaceship is measured by the Earth observer to be length-contracted to

$$L = L_p\sqrt{1 - v^2/c^2} \qquad \text{or} \qquad L^2 = L_p{}^2\left(1 - v^2/c^2\right)$$

Also, the contracted length is related to the time required to pass overhead by:

$$L = vt \qquad \text{or} \qquad L^2 = v^2t^2 = \frac{v^2}{c^2}(ct)^2$$

Equating these two expressions gives $\quad L_p{}^2 - L_p{}^2\dfrac{v^2}{c^2} = (ct)^2\dfrac{v^2}{c^2}$

or $\quad \left[L_p{}^2 + (ct)^2\right]\dfrac{v^2}{c^2} = L_p{}^2$

Using the given values: $\quad L_p = 300 \text{ m} \quad$ and $\quad t = 7.50\times 10^{-7} \text{ s}$

this becomes $\quad (1.41\times 10^5 \text{ m}^2)\dfrac{v^2}{c^2} = 9.00\times 10^4 \text{ m}^2$

giving $\quad v = \boxed{0.800\ c}$

***9.12** (a) The spaceship is measured by Earth observers to be of length L, where

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{and} \qquad L = v\Delta t$$

$$v\Delta t = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{and} \qquad v^2\Delta t^2 = L_p{}^2\left(1-\frac{v^2}{c^2}\right)$$

Solving for v, $$v^2\left(\Delta t^2+\frac{L_p{}^2}{c^2}\right) = L_p{}^2 \qquad \boxed{v = \frac{c\,L_p}{\sqrt{c^2\Delta t^2+L_p{}^2}}}$$

(b) The tanks move nonrelativistically, so we have $v = \dfrac{300\text{ m}}{75\text{ s}} = \boxed{4.00\text{ m/s}}$

(c) For the data in problem 11,

$$v = \frac{c(300\text{ m})}{\sqrt{\left(3\times10^8\text{ m/s}\right)^2\left(0.75\times10^{-6}\text{ s}\right)^2+(300\text{ m})^2}} = \frac{c(300\text{ m})}{\sqrt{225^2+300^2}\text{ m}} = 0.800c$$

in agreement with problem 11. For the data in part (b),

$$v = \frac{c(300\text{ m})}{\sqrt{\left(3\times10^8\text{ m/s}\right)^2(75\text{ s})^2+(300\text{ m})^2}} = \frac{c(300\text{ m})}{\sqrt{\left(2.25\times10^{10}\right)^2+300^2}\text{ m}} = 1.33\times10^{-8}c = 4.00\text{ m/s}$$

in agreement with part (b).

9.13 $$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1.01 \qquad \text{so} \qquad \boxed{v = 0.140\,c}$$

9.14 (a) Since your ship is identical to his, and you are at rest with respect to your own ship, its length is $\boxed{20.0\text{ m}}$

(b) His ship is in motion relative to you, so you measure its length contracted to $\boxed{19.0\text{ m}}$

(c) We have $L = L_p\sqrt{1-v^2/c^2}$

from which $\dfrac{L}{L_p} = \dfrac{19.0\text{ m}}{20.0\text{ m}} = 0.950 = \sqrt{1-\dfrac{v^2}{c^2}}$ and $\boxed{v = 0.312\,c}$

***9.15** (a) $\Delta t = \gamma \Delta t_p = \dfrac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \dfrac{15.0\text{ yr}}{\sqrt{1-(0.700)^2}} = \boxed{21.0\text{ yr}}$

(b) $d = v(\Delta t) = [0.700c](21.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](21.0\text{ yr}) = \boxed{14.7\text{ ly}}$

(c) The astronauts see Earth flying out the back window at $0.700c$:

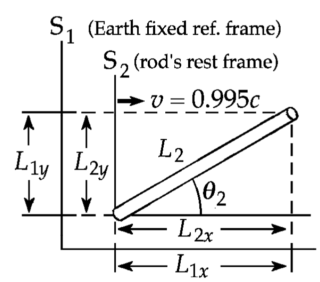
$$d = v(\Delta t_p) = [0.700c](15.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](15.0\text{ yr}) = \boxed{10.5\text{ ly}}$$

(d) Mission control gets signals for 21.0 yr while the battery is operating, and then for 14.7 years after the battery stops powering the transmitter, 14.7 ly away:

$$21.0\text{ yr} + 14.7\text{ yr} = \boxed{35.7\text{ yr}}$$

9.16

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.995^2}} = 10.0$$



We are also given: $L_1 = 2.00$ m, and $\theta = 30.0°$ (both measured in a reference frame moving relative to the rod).

Thus, $L_{1x} = L_1 \cos\theta_1 = (2.00\text{ m})(0.867) = 1.73\text{ m}$

and $L_{1y} = L_1 \sin\theta_1 = (2.00\text{ m})(0.500) = 1.00\text{ m}$

L_{2x} is a proper length, related to L_{1x} by $L_{1x} = L_{2x}/\gamma$

Therefore, $L_{2x} = 10.0 L_{1x} = 17.3\text{ m}$

and $L_{2y} = L_{1y} = 1.00\text{ m}$

(Lengths perpendicular to the motion are unchanged).

(a) $L_2 = \sqrt{(L_{2x})^2 + (L_{2y})^2}$ gives $\boxed{L_2 = 17.4\text{ m}}$

(b) $\theta_2 = \tan^{-1}\dfrac{L_{2y}}{L_{2x}}$ gives $\boxed{\theta_2 = 3.30°}$

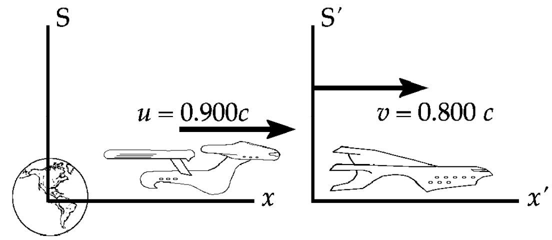
9.17 $u_x' = \dfrac{u_x - v}{1 - u_x v/c^2} = \dfrac{0.950c - 0.750c}{1 - 0.950 \times 0.750} = \boxed{0.696\ c}$

***9.18** $u_x =$ Enterprise velocity

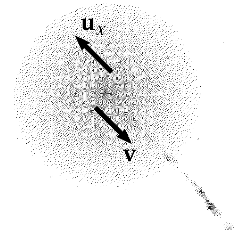
$v =$ Klingon velocity

From Equation 9.11

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{0.900c - 0.800c}{1 - (0.900)(0.800)} = \boxed{0.357c}$$



9.19 $$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{-0.750c - 0.750c}{1 - (-0.750)(0.750)} = \boxed{-0.960\ c}$$



***9.20** Einstein's reasoning about lightning striking the ends of a train shows that the moving observer sees the event toward which she is moving, $\boxed{\text{event B}}$, as occurring first. The S-frame coordinates of the events we may take as ($x = 0,\ y = 0,\ z = 0,\ t = 0$) and ($x = 100$ m, $y = 0,\ z = 0,\ t = 0$). Then the coordinates in S' are given by Equations 9.8. Event A is at ($x' = 0,\ y' = 0,\ z' = 0,\ t' = 0$). The time of event B is

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \frac{1}{\sqrt{1 - 0.8^2}}\left(0 - \frac{0.8c}{c^2}(100\text{ m})\right) = 1.667\left(-\frac{80\text{ m}}{3 \times 10^8\text{ m/s}}\right) = -4.44 \times 10^{-7}\text{ s.}$$

The time elapsing before A occurs is $\boxed{444\text{ ns}}$

***9.21** (a) From Equations 9.8 the separations between the blue–light and red–light events are described by

$$\Delta x' = \gamma(\Delta x - v\Delta t) \qquad 0 = \gamma\left[2.00\text{ m} - v\left(8.00 \times 10^{-9}\text{ s}\right)\right]$$

$$v = \frac{2.00\text{ m}}{8.00 \times 10^{-9}\text{ s}} = \boxed{2.50 \times 10^8\text{ m/s}} \qquad \gamma = \frac{1}{\sqrt{1 - \left(2.50 \times 10^8\text{ m/s}\right)^2 / \left(3.00 \times 10^8\text{ m/s}\right)^2}} = 1.81$$

(b) Again from Equation 9.8, $x' = \gamma(x - vt)$:

$$x' = 1.81\left[3.00\text{ m} - \left(2.50 \times 10^8\text{ m/s}\right)\left(1.00 \times 10^{-9}\text{ s}\right)\right]$$

$$x' = \boxed{4.97\text{ m}}$$

(c) $t' = \gamma\left(t - \frac{v}{c^2}x\right)$:

$$t' = 1.81\left[1.00 \times 10^{-9}\text{ s} - \frac{\left(2.50 \times 10^8\text{ m/s}\right)}{\left(3.00 \times 10^8\text{ m/s}\right)^2}(3.00\text{ m})\right]$$

$$t' = \boxed{-1.33 \times 10^{-8}\text{ s}}$$

***9.22** Let frame S be the Earth frame of reference. Then $v = -0.7c$

The components of the velocity of the first spacecraft are $u_x = (0.6c)\cos 50° = 0.386c$

and $u_y = (0.6c)\sin 50° = 0.459c$

As measured from the S' frame of the second spacecraft,

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{0.386c - (-0.7c)}{1 - \frac{(0.386c)(-0.7c)}{c^2}} = \frac{1.086c}{1.27} = 0.855c$$

$$u'_y = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)} = \frac{0.459c\sqrt{1-(0.7)^2}}{1-(0.386)(-0.7)} = \frac{0.459c(0.714)}{1.27} = 0.258c$$

The magnitude of **u′** is $\sqrt{(0.855c)^2 + (0.285c)^2} = \boxed{0.893c}$

and its direction is at $\tan^{-1}\frac{0.258c}{0.855c} = \boxed{16.8° \text{ above the } x'\text{-axis}}$

9.23 (a) $p = \gamma mu$; for an electron moving at $0.0100c$, $\gamma = \frac{1}{\sqrt{1-(u/c)^2}} = \frac{1}{\sqrt{1-(0.0100)^2}} = 1.00005 \cong 1.00$

Thus, $p = 1.00\left(9.11\times10^{-31}\text{ kg}\right)(0.0100)\left(3.00\times10^{8}\text{ m/s}\right)$

$p = \boxed{2.73\times10^{-24}\text{ kg}\cdot\text{m/s}}$

(b) Following the same steps as used in part (a),

we find at $0.500c$, $\gamma = 1.15$ and $p = \boxed{1.58\times10^{-22}\text{ kg}\cdot\text{m/s}}$

(c) At $0.900c$, $\gamma = 2.29$ and $p = \boxed{5.64\times10^{-22}\text{ kg}\cdot\text{m/s}}$

9.24 Using the relativistic form, $p = \frac{mu}{\sqrt{1-(u/c)^2}} = \gamma mu$

we find the difference Δp from the classical momentum, mu: $\Delta p = \gamma mu - mu = (\gamma - 1)mu$

(a) The difference is 1.00% when $(\gamma-1)mu = 0.0100\,\gamma mu$: $\gamma = \frac{1}{0.990} = \frac{1}{\sqrt{1-(u/c)^2}}$

thus $1-(u/c)^2 = (0.990)^2$, and $u = \boxed{0.141\,c}$

(b) The difference is 10.0% when $(\gamma-1)mu = 0.100\,\gamma mu$: $\gamma = \frac{1}{0.900} = \frac{1}{\sqrt{1-(u/c)^2}}$

thus $1-(u/c)^2 = (0.900)^2$ and $u = \boxed{0.436\,c}$

9.25

$$\frac{p-mu}{mu}=\frac{\gamma mu-mu}{mu}=\gamma-1: \qquad \gamma-1=\frac{1}{\sqrt{1-(u/c)^2}}-1\approx 1+\frac{1}{2}\left(\frac{u}{c}\right)^2-1=\frac{1}{2}\left(\frac{u}{c}\right)^2$$

$$\frac{p-mu}{mu}=\frac{1}{2}\left(\frac{90.0\ \text{m/s}}{3.00\times10^{8}\ \text{m/s}}\right)^2=\boxed{4.50\times10^{-14}}$$

9.26

$$p=\frac{mu}{\sqrt{1-(u/c)^2}} \quad \text{becomes} \quad 1-\frac{u^2}{c^2}=\frac{m^2u^2}{p^2}$$

which gives: $$1=u^2\left(\frac{m^2}{p^2}+\frac{1}{c^2}\right)$$

or $$c^2=u^2\left(\frac{m^2c^2}{p^2}+1\right)$$ and $$\boxed{u=\frac{c}{\sqrt{(m^2c^2/p^2)+1}}}$$

9.27 Relativistic momentum of the system of fragments must be conserved. For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2=p_1$

or $$\gamma_2m_2u_2=\gamma_1m_1u_1=\frac{2.50\times10^{-28}\ \text{kg}}{\sqrt{1-(0.893)^2}}\times(0.893c)$$

or $$\frac{(1.67\times10^{-27}\ \text{kg})u_2}{\sqrt{1-(u_2/c)^2}}=(4.960\times10^{-28}\ \text{kg})c$$

Proceeding to solve, we find $$\left(\frac{1.67\times10^{-27}}{4.960\times10^{-28}}\frac{u_2}{c}\right)^2=1-\frac{u_2^2}{c^2}$$

$$12.3\frac{u_2^2}{c^2}=1 \quad \text{and} \quad u_2=\boxed{0.285c}$$

9.28 $$\Delta E=(\gamma_1-\gamma_2)mc^2$$

For an electron, $mc^2=0.511$ MeV

(a) $$\Delta E=\left(\sqrt{\frac{1}{(1-0.810)}}-\sqrt{\frac{1}{(1-0.250)}}\right)mc^2=\boxed{0.582\ \text{MeV}}$$

(b) $$\Delta E=\left(\sqrt{\frac{1}{1-(0.990)^2}}-\sqrt{\frac{1}{1-0.810}}\right)mc^2=\boxed{2.45\ \text{MeV}}$$

***9.29** $E = \gamma mc^2 = 2mc^2$ or $\gamma = 2$

Thus, $\dfrac{u}{c} = \sqrt{1-(1/\gamma)^2} = \dfrac{\sqrt{3}}{2}$ or $u = \dfrac{c\sqrt{3}}{2}$

The momentum is then $p = \gamma mu = 2m\left(\dfrac{c\sqrt{3}}{2}\right) = \left(\dfrac{mc^2}{c}\right)\sqrt{3}$

$$p = \left(\frac{938.3\ \text{MeV}}{c}\right)\sqrt{3} = \boxed{1.63\times10^3\ \frac{\text{MeV}}{c}}$$

9.30 The relativistic kinetic energy of an object of mass m and speed u is $K_r = \left(\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2$

For $u = 0.100c$, $K_r = \left(\dfrac{1}{\sqrt{1-0.0100}} - 1\right)mc^2 = 0.005038\,mc^2$

The classical equation $K_c = \frac{1}{2}mu^2$ gives $K_c = \frac{1}{2}m(0.100\,c)^2 = 0.005000\,mc^2$

different by $\dfrac{0.005038 - 0.005000}{0.005038} = 0.751\%$

For still smaller speeds the agreement will be still better.

9.31 (a) $E_R = mc^2 = (1.67\times10^{-27}\ \text{kg})(2.998\times10^8\ \text{m/s})^2 = 1.50\times10^{-10}\ \text{J} = \boxed{938\ \text{MeV}}$

(b) $E = \gamma mc^2 = \dfrac{1.50\times10^{-10}\ \text{J}}{[1-(0.95c/c)^2]^{1/2}} = 4.81\times10^{-10}\ \text{J} = \boxed{3.00\times10^3\ \text{MeV}}$

(c) $K = E - mc^2 = 4.81\times10^{-10}\ \text{J} - 1.50\times10^{-10}\ \text{J} = 3.31\times10^{-10}\ \text{J} = \boxed{2.07\times10^3\ \text{MeV}}$

9.32 (a) $K = E - E_R = 5E_R$

$E = 6E_R = 6\left(9.11\times10^{-31}\ \text{kg}\right)\left(3.00\times10^8\ \text{m/s}\right)^2 = 4.92\times10^{-13}\ \text{J} = \boxed{3.07\ \text{MeV}}$

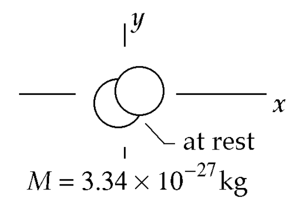
(b) $E = \gamma mc^2 = \gamma E_R$

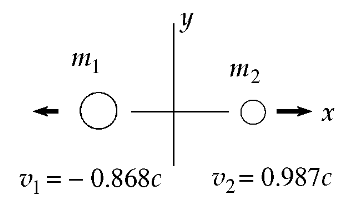
Thus $\gamma = \dfrac{E}{E_R} = 6 = \dfrac{1}{\sqrt{1-u^2/c^2}}$ which yields $\boxed{u = 0.986c}$

9.33 The relativistic density is

$$\frac{E_R}{c^2 V} = \frac{mc^2}{c^2 V} = \frac{m}{V} = \frac{m}{(L_p)(L_p)\left[L_p\sqrt{1-(u/c)^2}\right]} = \frac{8.00\text{ g}}{(1.00\text{ cm})^3\sqrt{1-(0.900)^2}} = \boxed{18.4\text{ g/cm}^3}$$

9.34 We must conserve both mass-energy and relativistic momentum of the system of fragments. With subscript 1 referring to the $0.868c$ particle and subscript 2 to the $0.987c$ particle,





$$\gamma_1 = \frac{1}{\sqrt{1-(0.868)^2}} = 2.01 \quad \text{and} \quad \gamma_2 = \frac{1}{\sqrt{1-(0.987)^2}} = 6.22$$

Conservation of mass-energy gives $E_1 + E_2 = E_{\text{total}}$

which is $\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = m_{\text{total}} c^2$

or $2.01 m_1 + 6.22 m_2 = 3.34 \times 10^{-27}$ kg

This reduces to: $m_1 + 3.09 m_2 = 1.66 \times 10^{-27}$ kg (1)

Since the final momentum of the system must equal zero, $p_1 = p_2$

gives $\gamma_1 m_1 u_1 = \gamma_2 m_2 u_2$

or $(2.01)(0.868c) m_1 = (6.22)(0.987c) m_2$

which becomes $m_1 = 3.52 m_2$ (2)

Solving (1) and (2) simultaneously, $m_1 = \boxed{8.84 \times 10^{-28}\text{ kg}}$ and $m_2 = \boxed{2.51 \times 10^{-28}\text{ kg}}$

9.35 $E = \gamma mc^2$ $\qquad p = \gamma mu$

$E^2 = (\gamma mc^2)^2$ $\qquad p^2 = (\gamma mu)^2$

$$E^2 - p^2c^2 = (\gamma mc^2)^2 - (\gamma mu)^2 c^2 = \gamma^2\left((mc^2)^2 - (mc)^2 u^2\right) = (mc^2)^2\left(1 - \frac{u^2}{c^2}\right)\left(1 - \frac{u^2}{c^2}\right)^{-1} = (mc^2)^2$$

Q.E.D.

***9.36** (a) Using the classical equation, $K = \frac{1}{2}mv^2 = \frac{1}{2}(78.0 \text{ kg})\left(1.06\times10^5 \text{ m/s}\right)^2 = \boxed{4.38\times10^{11} \text{ J}}$.

(b) Using the relativistic equation, $K = \left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right)mc^2$

$$K = \left(\frac{1}{\sqrt{1-\left(1.06\times10^5/2.998\times10^8\right)^2}} - 1\right)(78.0 \text{ kg})\left(2.998\times10^8 \text{ m/s}\right)^2 = \boxed{4.38\times10^{11} \text{ J}}$$

When $(v/c) << 1$, the binomial series expansion gives $\left[1-(v/c)^2\right]^{-1/2} \cong 1+\frac{1}{2}(v/c)^2$

Thus, $\left[1-(v/c)^2\right]^{-1/2} - 1 \cong \frac{1}{2}(v/c)^2$

and the relativistic expression for kinetic energy becomes $K \cong \frac{1}{2}(v/c)^2 mc^2 = \frac{1}{2}mv^2$. That is, in the limit of speeds much smaller than the speed of light, the relativistic and classical expressions yield the same results.

***9.37** Conserving total momentum of the decaying particle system, $p_{\text{before decay}} = p_{\text{after decay}} = 0$:

$$p_\nu = p_\mu = \gamma\, m_\mu u = \gamma\left(207 m_e\right)u$$

Conservation of mass-energy for the system gives $E_\mu + E_\nu = E_\pi$: $\gamma m_\mu c^2 + p_\nu c = m_\pi c^2$

$$\gamma\left(207 m_e\right) + \frac{p_\nu}{c} = 270 m_e$$

Substituting from the momentum equation above, $\gamma\left(207 m_e\right) + \gamma\left(207 m_e\right)\frac{u}{c} = 270 m_e$

or $\gamma\left(1+\frac{u}{c}\right) = \frac{270}{207} = 1.31$: $\frac{u}{c} = 0.260$

Then, $K_\mu = (\gamma-1)m_\mu c^2 = (\gamma-1)207\left(m_e c^2\right)$: $K_\mu = \left(\frac{1}{\sqrt{1-(0.260)^2}} - 1\right)207(0.511 \text{ MeV})$

$$K_\mu = \boxed{3.76 \text{ MeV}}$$

Also, $E_\nu = E_\pi - E_\mu$: $E_\nu = m_\pi c^2 - \gamma m_\mu c^2 = (270 - 207\gamma)m_e c^2$

$$E_\nu = \left(270 - \frac{207}{\sqrt{1-(0.260)^2}}\right)(0.511 \text{ MeV})$$

$$E_\nu = \boxed{28.4 \text{ MeV}}$$

9.38 $E = 2.86\times10^5$ J. Also, the mass-energy relation says that $E = mc^2$.

Therefore, $$m = \frac{E}{c^2} = \frac{2.86\times10^5\ \text{J}}{\left(3.00\times10^8\ \text{m/s}\right)^2} = \boxed{3.18\times10^{-12}\ \text{kg}}$$

No, a mass loss of this magnitude (out of a total of 9.00 g) $\boxed{\text{could not be detected}}$

***9.39** $$\mathcal{P} = \frac{dE}{dt} = \frac{d\left(mc^2\right)}{dt} = c^2\frac{dm}{dt} = 3.77\times10^{26}\ \text{W}$$

Thus, $$\frac{dm}{dt} = \frac{3.77\times10^{26}\ \text{J/s}}{\left(3.00\times10^8\ \text{m/s}\right)^2} = \boxed{4.19\times10^9\ \text{kg/s}}$$

***9.40** $$\Delta m = \frac{E}{c^2} = \frac{\mathcal{P}t}{c^2} = \frac{0.800\left(1.00\times10^9\ \text{J/s}\right)\left(3.00\ \text{yr}\right)\left(3.16\times10^7\ \text{s/yr}\right)}{\left(3.00\times10^8\ \text{m/s}\right)^2} = \boxed{0.842\ \text{kg}}$$

***9.41** $2m_ec^2 = 1.02$ MeV: $E_\gamma \geq \boxed{1.02\ \text{MeV}}$

***9.42** We find Carpenter's speed: $$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Solving, $$v = \left[\frac{GM}{(R+h)}\right]^{1/2} = \left[\frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.37\times10^6 + 0.160\times10^6)}\right]^{1/2} = 7.82\ \text{km/s}$$

Then the time period of one orbit, $$T = \frac{2\pi(R+h)}{v} = \frac{2\pi(6.53\times10^6)}{7.82\times10^3} = 5.25\times10^3\ \text{s}$$

(a) The time difference for 22 orbits is $$\Delta t - \Delta t_p = (\gamma-1)\Delta t_p = \left[\left(1 - v^2/c^2\right)^{-1/2} - 1\right](22T)$$

$$\Delta t - \Delta t_p \cong \left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right)(22T) = \frac{1}{2}\left(\frac{7.82\times10^3\ \text{m/s}}{3\times10^8\ \text{m/s}}\right)^2 22\left(5.25\times10^3\ \text{s}\right) = \boxed{39.2\ \mu\text{s}}$$

(b) For one orbit, $\Delta t - \Delta t_p = \dfrac{39.2\ \mu\text{s}}{22} = 1.78\ \mu\text{s}$. The press report is $\boxed{\text{accurate to one digit}}$.

***9.43** (a) $d_{\text{earth}} = vt_{\text{earth}} = v\gamma t_{\text{astro}}$ so $\left(2.00\times 10^{6}\text{ yr}\right)c = v\dfrac{1}{\sqrt{1-v^2/c^2}}30.0\text{ yr}$

$$\sqrt{1-v^2/c^2} = (v/c)\left(1.50\times 10^{-5}\right) \qquad 1-\frac{v^2}{c^2} = \frac{v^2\left(2.25\times 10^{-10}\right)}{c^2}$$

$$1 = \frac{v^2}{c^2}\left(1+2.25\times 10^{-10}\right) \quad \text{so} \quad \frac{v}{c} = \left(1+2.25\times 10^{-10}\right)^{-1/2} = 1-\tfrac{1}{2}\left(2.25\times 10^{-10}\right)$$

$$\boxed{\frac{v}{c} = 1-1.12\times 10^{-10}}$$

(b) $$K = \left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2 = \left(\frac{2.00\times 10^{6}\text{ yr}}{30\text{ yr}}-1\right)1000(1000\text{ kg})\left(3\times 10^{8}\text{ m/s}\right)^2 = \boxed{6.00\times 10^{27}\text{ J}}$$

(c) $$6.00\times 10^{27}\text{ J} = 6.00\times 10^{27}\text{ J}\left(\frac{13\text{ ¢}}{\text{kWh}}\right)\left(\frac{\text{k}}{10^3}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3600\text{ s}}\right) = \boxed{\$\ 2.17\times 10^{20}}$$

9.44 (a) When $K_e = K_p$, $m_ec^2\left(\gamma_e-1\right) = m_pc^2\left(\gamma_p-1\right)$

In this case, $m_ec^2 = 0.511\text{ MeV}$, $m_pc^2 = 938\text{ MeV}$

and $\gamma_e = \left[1-(0.750)^2\right]^{-1/2} = 1.5119$

Substituting, $$\gamma_p = 1+\frac{m_ec^2(\gamma_e-1)}{m_pc^2} = 1+\frac{(0.511\text{ MeV})(1.5119-1)}{938\text{ MeV}} = 1.000279$$

but $$\gamma_p = \frac{1}{\left[1-\left(u_p/c\right)^2\right]^{1/2}}.$$

Therefore, $u_p = c\sqrt{1-\gamma_p^{-2}} = \boxed{0.0236c}$

(b) When $p_e = p_p$, $\gamma_pm_pu_p = \gamma_em_eu_e$ or $\gamma_pu_p = \dfrac{\gamma_em_eu_e}{m_p}$.

Thus, $$\gamma_pu_p = \frac{(1.5119)\left(0.511\text{ MeV}/c^2\right)(0.750c)}{938\text{ MeV}/c^2} = 6.1772\times 10^{-4}c$$

and $$\frac{u_p}{c} = 6.1772\times 10^{-4}\sqrt{1-\left(\frac{u_p}{c}\right)^2}$$

which yields $u_p = \boxed{6.18\times 10^{-4}\ c} = 185\text{ km/s}$

9.45 (a) $10^{13}\text{ MeV} = (\gamma - 1)m_p c^2$ so $\gamma = 10^{10}$

$v_p \cong c$ $t' = \dfrac{t}{\gamma} = \dfrac{10^5\text{ yr}}{10^{10}} = 10^{-5}\text{ yr} \sim \boxed{10^2\text{ s}}$

(b) $d' = ct'$ $\boxed{\sim 10^8\text{ km}}$

9.46 $$\frac{\Delta mc^2}{mc^2} = \frac{4(938.78\text{ MeV}) - 3728.4\text{ MeV}}{4(938.78\text{ MeV})} \times 100\% = \boxed{0.712\%}$$

***9.47** (a) The charged battery stores energy $E = \mathcal{P}t = (1.20\text{ J/s})(50\text{ min})(60\text{ s/min}) = 3600\text{ J}$

so its mass excess is $\Delta m = \dfrac{E}{c^2} = \dfrac{3600\text{ J}}{\left(3\times10^8\text{ m/s}\right)^2} = \boxed{4.00\times10^{-14}\text{ kg}}$

(b) $\dfrac{\Delta m}{m} = \dfrac{4.00\times10^{-14}\text{ kg}}{25\times10^{-3}\text{ kg}} = \boxed{1.60\times10^{-12}}$ too small to measure.

***9.48** (a) Take the spaceship as the primed frame, moving toward the right at $v = +0.600c$.

Then $u_x' = +0.800c$, and $u_x = \dfrac{u_x' + v}{1 + (u_x' v)/c^2} = \dfrac{0.800c + 0.600c}{1 + (0.800)(0.600)} = \boxed{0.946c}$

(b) $L = \dfrac{L_p}{\gamma}$: $L = (0.200\text{ ly})\sqrt{1 - (0.600)^2} = \boxed{0.160\text{ ly}}$

(c) The aliens observe the 0.160-ly distance closing because the probe nibbles into it from one end at $0.800c$ and the Earth reduces it at the other end at $0.600c$.

Thus, $\text{time} = \dfrac{0.160\text{ ly}}{0.800c + 0.600c} = \boxed{0.114\text{ yr}}$

(d) $K = \left(\dfrac{1}{\sqrt{1 - u^2/c^2}} - 1\right)mc^2$: $K = \left(\dfrac{1}{\sqrt{1 - (0.946)^2}} - 1\right)\left(4.00\times10^5\text{ kg}\right)\left(3.00\times10^8\text{ m/s}\right)^2$

$K = \boxed{7.50\times10^{22}\text{ J}}$

9.49 In this case, the proper time is T_0 (the time measured by the students on a clock at rest relative to them). The dilated time measured by the professor is: $\Delta t = \gamma T_0$

where $\Delta t = T + t$. Here T is the time she waits before sending a signal and t is the time required for the signal to reach the students.

Thus, we have: $T + t = \gamma T_0$ (1)

To determine the travel time t, realize that the distance the students will have moved beyond the professor before the signal reaches them is: $d = v(T + t)$

The time required for the signal to travel this distance is: $t = \frac{d}{c} = \left(\frac{v}{c}\right)(T + t)$

Solving for t gives: $t = \frac{(v/c)T}{1-(v/c)}$

Substituting this into equation (1) yields: $T + \frac{(v/c)T}{1-(v/c)} = \gamma T_0$

or $T/(1 - v/c) = \gamma T_0$

Then $$T = T_0 \frac{1-(v/c)}{\sqrt{1-(v^2/c^2)}} = T_0 \frac{1-(v/c)}{\sqrt{[1+(v/c)][1-(v/c)]}} = \boxed{T_0\sqrt{\frac{1-(v/c)}{1+(v/c)}}}$$

9.50 The energy which arrives in one year is

$$E = \mathcal{P}t = (1.79 \times 10^{17}\ \text{J/s})(3.16 \times 10^{7}\ \text{s}) = 5.66 \times 10^{24}\ \text{J}$$

Thus, $$m = \frac{E}{c^2} = \frac{5.66 \times 10^{24}\ \text{J}}{(3.00 \times 10^{8}\ \text{m/s})^2} = \boxed{6.28 \times 10^{7}\ \text{kg}}$$

9.51 The observer measures the proper length of the tunnel, 50.0 m, but measures the train contracted to length

$$L = L_p\sqrt{1 - v^2/c^2} = 100\ \text{m}\sqrt{1-(0.950)^2} = 31.2\ \text{m}$$

shorter than the tunnel by $50.0 - 31.2 = \boxed{18.8\ \text{m}}$ so $\boxed{\text{it is completely within the tunnel.}}$

9.52 If the energy required to remove a mass m from the surface is equal to its mass energy mc^2,

then $$\frac{GM_s m}{R_g} = mc^2$$

and $$R_g = \frac{GM_s}{c^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(3.00\times10^8\ \text{m/s})^2}$$

$$R_g = 1.47\times10^3\ \text{m} = \boxed{1.47\ \text{km}}$$

9.53 (a) At any speed, the momentum of the particle is given by

$$p = \gamma mu = \frac{mu}{\sqrt{1-(u/c)^2}}$$

Since $F = qE = \dfrac{dp}{dt}$: $$qE = \frac{d}{dt}\left[mu\left(1-u^2/c^2\right)^{-1/2}\right]$$

$$qE = m\left(1-u^2/c^2\right)^{-1/2}\frac{du}{dt} + \tfrac{1}{2}mu\left(1-u^2/c^2\right)^{-3/2}\left(2u/c^2\right)\frac{du}{dt}$$

So $$\frac{qE}{m} = \frac{du}{dt}\left[\frac{1-u^2/c^2+u^2/c^2}{\left(1-u^2/c^2\right)^{3/2}}\right]$$

and $$\boxed{a = \frac{du}{dt} = \frac{qE}{m}\left(1-\frac{u^2}{c^2}\right)^{3/2}}$$

(b) As $u \to c$, $\boxed{a \to 0}$

(c) $$\int_0^v \frac{du}{\left(1-u^2/c^2\right)^{3/2}} = \int_{t=0}^t \frac{qE}{m}dt \qquad \boxed{u = \frac{qEct}{\sqrt{m^2c^2+q^2E^2t^2}}}$$

$$x = \int_0^t u\,dt = qEc\int_0^t \frac{t\,dt}{\sqrt{m^2c^2+q^2E^2t^2}} \qquad \boxed{x = \frac{c}{qE}\left(\sqrt{m^2c^2+q^2E^2t^2} - mc\right)}$$

***9.54** (a) An observer at rest relative to the mirror sees the light travel a distance $D = 2d - x$, where $x = vt_S$ is the distance the ship moves toward the mirror in time t_S. Since this observer agrees that the speed of light is c, the time for it to travel distance D is

$$t_S = \frac{D}{c} = \frac{2d - vt_S}{c} \qquad t_S = \boxed{\frac{2d}{c+v}}$$

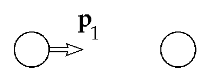
(b) The observer in the rocket measures a length-contracted initial distance to the mirror of

$$L = d\sqrt{1 - \left(v^2/c^2\right)}$$

and the mirror moving toward the ship at speed v. Thus, he measures the distance the light travels as $D = 2(L - y)$ where $y = vt/2$ is the distance the mirror moves toward the ship before the light reflects off it. This observer also measures the speed of light to be c, so the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2}{c}\left(d\sqrt{1 - \frac{v^2}{c^2}} - \frac{vt}{2}\right) \quad \text{so} \quad (c+v)t = \frac{2d}{c}\sqrt{(c+v)(c-v)} \quad \text{or} \quad t = \boxed{\frac{2d}{c}\sqrt{\frac{c-v}{c+v}}}$$

***9.55** Take the two colliding protons as the system

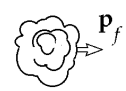


initial

$$E_1 = K + mc^2 \qquad E_2 = mc^2$$

$$E_1^2 = p_1^2c^2 + m^2c^4 \qquad p_2 = 0$$

In the final state, $E_f = K_f + Mc^2$: $\quad E_f^2 = p_f^2c^2 + M^2c^4$



final

By energy conservation, $E_1 + E_2 = E_f$, so $\quad E_1^2 + 2E_1E_2 + E_2^2 = E_f^2$

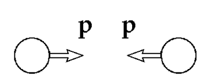
$$p_1^2c^2 + m^2c^4 + 2\left(K + mc^2\right)mc^2 + m^2c^4 = p_f^2c^2 + M^2c^4$$

By conservation of momentum, $\quad p_1 = p_f$

Then $\quad M^2c^4 = 2Kmc^2 + 4m^2c^4 = \dfrac{4Km^2c^4}{2mc^2} + 4m^2c^4$

$$Mc^2 = 2mc^2\sqrt{1 + \frac{K}{2mc^2}}$$

By contrast, for colliding beams we have



initial (beams)

In the original state, $\quad E_1 = K + mc^2$

$$E_2 = K + mc^2$$

In the final state, $\quad E_f = Mc^2$

final (beams)

$E_1 + E_2 = E_f$: $\quad K + mc^2 + K + mc^2 = Mc^2$

$$Mc^2 = 2mc^2\left(1 + \frac{K}{2mc^2}\right)$$

***9.56** (a) Since Mary is in the same reference frame, S′, as Ted, she measures the ball to have the same speed Ted observes, namely $|u'_x| = \boxed{0.800c}$.

(b) $$\Delta t' = \frac{L_p}{|u'_x|} = \frac{1.80\times 10^{12}\text{ m}}{0.800\left(3.00\times 10^{8}\text{ m/s}\right)} = \boxed{7.50\times 10^{3}\text{ s}}$$

(c) $$L = L_p\sqrt{1-\frac{v^2}{c^2}} = \left(1.80\times 10^{12}\text{ m}\right)\sqrt{1-\frac{(0.600c)^2}{c^2}} = \boxed{1.44\times 10^{12}\text{ m}}$$

Since $v = 0.600c$ and $u'_x = -0.800c$, the velocity Jim measures for the ball is

$$u_x = \frac{u'_x + v}{1+u'_x v/c^2} = \frac{(-0.800c)+(0.600c)}{1+(-0.800)(0.600)} = \boxed{-0.385c}$$

(d) Jim measures the ball and Mary to be initially separated by 1.44×10^{12} m. Mary's motion at $0.600c$ and the ball's motion at $0.385c$ nibble into this distance from both ends. The gap closes at the rate $0.600c + 0.385c = 0.985c$, so the ball and catcher meet after a time

$$\Delta t = \frac{1.44\times 10^{12}\text{ m}}{0.985\left(3.00\times 10^{18}\text{ m/s}\right)} = \boxed{4.88\times 10^{3}\text{ s}}$$

***9.57** (b) Consider a hermit who lives on an asteroid halfway between the Sun and Tau Ceti, stationary with respect to both. Just as our spaceship is passing him, he also sees the blast waves from both explosions. Judging both stars to be stationary, this observer concludes that $\boxed{\text{the two stars blew up simultaneously}}$.

(a) We in the spaceship moving past the hermit do not calculate the explosions to be simultaneous. We measure the distance we have traveled from the Sun as

$$L = L_p\sqrt{1-(v/c)^2} = (6.00\text{ ly})\sqrt{1-(0.800)^2} = 3.60\text{ ly}$$

We see the Sun flying away from us at $0.800c$ while the light from the Sun approaches at $1.00c$. Thus, the gap between the Sun and its blast wave has opened at $1.80c$, and the time we calculate to have elapsed since the Sun exploded is

$3.60\text{ ly}/1.80c = 2.00\text{ yr}$.

We see Tau Ceti as moving toward us at $0.800c$, while its light approaches at $1.00c$, only $0.200c$ faster. We measure the gap between that star and its blast wave as 3.60 ly and growing at $0.200c$. We calculate that it must have been opening for

$3.60\text{ ly}/0.200c = 18.0\text{ yr}$

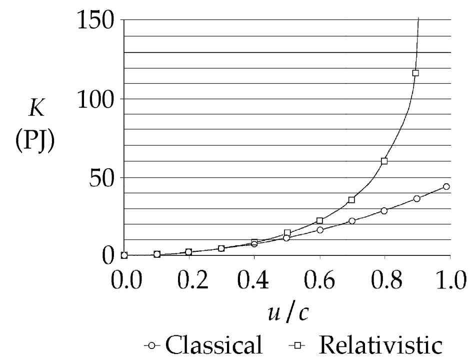
and conclude that $\boxed{\text{Tau Ceti exploded 16.0 years before the Sun}}$.

***9.58** Take $m = 1.00$ kg.

The classical kinetic energy is $K_c = \frac{1}{2}mu^2 = \frac{1}{2}mc^2\left(\frac{u}{c}\right)^2 = \left(4.50\times10^{16}\text{ J}\right)\left(\frac{u}{c}\right)^2$

and the actual kinetic energy is $K_r = \left(\frac{1}{\sqrt{1-(u/c)^2}} - 1\right)mc^2 = \left(9.00\times10^{16}\text{ J}\right)\left(\frac{1}{\sqrt{1-(u/c)^2}} - 1\right)$

u/c	K_c (J)	K_r (J)
0.000	0.000	0.000
0.100	0.045×10^{16}	0.0453×10^{16}
0.200	0.180×10^{16}	0.186×10^{16}
0.300	0.405×10^{16}	0.435×10^{16}
0.400	0.720×10^{16}	0.820×10^{16}
0.500	1.13×10^{16}	1.39×10^{16}
0.600	1.62×10^{16}	2.25×10^{16}
0.700	2.21×10^{16}	3.60×10^{16}
0.800	2.88×10^{16}	6.00×10^{16}
0.900	3.65×10^{16}	11.6×10^{16}
0.990	4.41×10^{16}	54.8×10^{16}



$K_c = 0.990K_r$ when $\frac{1}{2}(u/c)^2 = 0.990\left[\frac{1}{\sqrt{1-(u/c)^2}} - 1\right]$, yielding $u = \boxed{0.115c}$

Similarly, $K_c = 0.950K_r$ when $u = \boxed{0.257c}$

and $K_c = 0.500K_r$ when $u = \boxed{0.786c}$

ANSWERS TO EVEN NUMBERED PROBLEMS

2.	(a) 60.0 m/s	(b) 20.0 m/s	(c) 44.7 m/s
4.	See the solution		
6.	0.866 c		
8.	(a) 25.0 yr	(b) 15.0 yr	(c) 12.0 ly
10.	(a) 2.18 μs	(b) 649 m	
12.	(a) $v = cL_p\left(c^2\Delta t^2 + L_p{}^2\right)^{-1/2}$	(b) 4.00 m/s	(c) See the solution
14.	(a) 20.0 m	(b) 19.0 m	(c) 0.312 c
16.	(a) 17.4 m	(b) 3.30°	
18.	0.357 c		
20.	B occurred 4.44×10^{-7} s before A		
22.	$0.893c$ at $16.8°$ above the x' axis		
24.	(a) $0.141c$	(b) $0.436c$	
26.	See the solution		
28.	(a) 0.582 MeV	(b) 2.45 MeV	
30.	See the solution		
32.	(a) 3.07 MeV	(b) 0.986 c	
34.	8.84×10^{-28} kg and 2.51×10^{-28} kg		
36.	(a) 4.38×10^{11} J	(b) 4.38×10^{11} J	

38. 3.18×10^{-12} kg, not detectable

40. 0.842 kg

42. (a) 39.2 μs (b) Accurate to one digit

44. (a) 0.023 6 c (b) $6.18 \times 10^{-4}\ c$

46. 0.712%

48. (a) 0.946c (b) 0.160 ly
(c) 0.114 yr (d) 7.50×10^{22} J

50. 6.28×10^{7} kg

52. 1.47 km

54. (a) $\dfrac{2d}{c+v}$ (b) $\dfrac{2d}{c}\sqrt{\dfrac{c-v}{c+v}}$

56. (a) 0.800 c (b) 7.50 ks
(c) 1.44 Tm, –0.385 c (d) 4.88 ks

58. See the solution for the graph. K_c = 0.990 K_r when u = 0.115 c, K_c =0.950 K_r when u = 0.257 c, K_c =0.500 K_r when u = 0.786 c.