CHAPTER 11

ANSWERS TO QUESTIONS

Q11.1 Because g is the same for all objects near the Earth's surface. The larger mass needs a larger force to give it just the same acceleration.

Q11.2 To a good first approximation, your bathroom scale reading is unaffected because you, Earth, and the scale are in free fall in the Sun's gravitational field, in orbit around the Sun. To a precise second approximation, you weigh slightly less at noon and at midnight than you do at sunrise or sunset. The Sun's gravitational field is a little weaker at the center of the Earth than at the surface subsolar point, and a little weaker still on the far side of the planet. When the Sun is high in your sky, its gravity pulls up on you a little more strongly than on the Earth as a whole. At midnight the Sun pulls down on you a little less strongly than it does on the Earth below you. So you can have another doughnut with lunch, and your bedsprings will still last a little longer.

Q11.3 Because both the Earth and Moon are moving in orbit about the Sun. As described by $F_{\text{gravitational}} = ma_{\text{centripetal}}$, the gravitational force of the Sun merely keeps the Moon (and Earth) in a nearly circular orbit of radius 150 million kilometers. Because of its velocity, the Moon is kept in its orbit about the Earth by the gravitational force of the Earth. There is no imbalance of these forces, at new moon or full moon.

Q11.4 Since the escape speed from the Earth is 11.2 km/s and that from the Moon is 2.3 km/s, smaller by a factor of 5, the energy required—and fuel—would go as v^2, or 25 times more fuel required to leave the Earth versus leaving the Moon.

Q11.5 In a circular orbit each increment of displacement is perpendicular to the force applied. The work done by the gravitational force on a planet in an elliptical orbit speeds up the planet at closest approach, but negative work is done by gravity and the planet slows as it sweeps out to its farthest distance from the Sun. Therefore, net work in one complete orbit is zero.

Q11.6 Speed is maximum at closest approach. Speed is minimum at farthest distance.

Q11.7 For a satellite in orbit, one focus of an elliptical orbit or the center of a circular orbit must be located at the center of the Earth. If the satellite is over the northern hemisphere for half of its orbit, it must be over the southern hemisphere for the other half.

Q11.8 Air resistance causes a decrease in the energy of the satellite-Earth system. This reduces the diameter of the orbit, bringing the satellite closer to the surface of the Earth. A satellite in a smaller orbit, however, must travel faster. Thus, the effect of air resistance is to speed up the satellite!

Q11.9 Cavendish determined G. Then from $g = \frac{GM}{R^2}$, one may determine the mass of the Earth.

Q11.10 The gravitational force of the Earth on an extra particle at its center must be zero, not infinite as one interpretation of Equation 11.1 would suggest. All the bits of matter that make up the Earth will pull in different directions on the extra particle.

Q11.11 The gravitational force is conservative. An encounter with a stationary mass cannot permanently speed up a spacecraft. Jupiter is moving. A spacecraft flying across its orbit just behind the planet will gain kinetic energy as the planet's gravity does net positive work on it.

Q11.12 The spacecraft did not have enough fuel to stop dead in its high-speed course for the Moon.

Q11.13 Kepler's third law, which applies to all planets, tells us that the period of a planet is proportional to $r^{3/2}$. Because Saturn and Jupiter are farther from the Sun than Earth, they have longer periods. The Sun's gravitational field is much weaker at a distant Jovian planet. Thus, an outer planet experiences much smaller centripetal acceleration than Earth and has a correspondingly longer period.

Q11.14 From Equations 11.17, 11.18, and 11.19, we have

$$-|E| = -\frac{k_e e^2}{2r} = +\frac{k_e e^2}{2r} - \frac{k_e e^2}{r} = K + U_e$$

Then $K = |E|$ and $U_e = -2|E|$.

Q11.15 One assumption is natural from the standpoint of classical physics: The electron feels an electric force of attraction to the nucleus, causing the centripetal acceleration to hold it in orbit. The other assumptions are in sharp contrast to the behavior of ordinary-size objects: The electron's angular momentum must be one of a set of certain special allowed values. During the time when it is in one of these quantized orbits, the electron emits no electromagnetic radiation. The atom radiates a photon when the electron makes a quantum jump from one orbit to a lower one.

Q11.16 An atomic electron does not possess enough kinetic energy to escape from its electrical attraction to the nucleus. Positive ionization energy must be injected to pull the electron out to a very large separation from the nucleus, a condition for which we define the energy of the atom to be zero. The atom is a bound system. All this is summarized by saying that the total energy of an atom is negative.

Q11.17 If an electron moved like a hockey puck, it could have any arbitrary frequency of revolution around an atomic nucleus. If it behaved like a charge in a radio antenna, it would radiate light with frequency equal to its own frequency of oscillation. Thus, the electron in hydrogen atoms would emit a continuous spectrum, electromagnetic waves of all frequencies smeared together.

PROBLEM SOLUTIONS

11.1 $F = m_1 g = \frac{G m_1 m_2}{r^2}$

$$g = \frac{G m_2}{r^2} = \frac{\left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(4.00 \times 10^4 \times 10^3\ \text{kg}\right)}{(100\ \text{m})^2} = \boxed{2.67 \times 10^{-7}\ \text{m/s}^2}$$

***11.2** (a) At the midpoint between the two objects, the forces exerted by the 200-kg and 500-kg objects are oppositely directed,

and from $F_g = \frac{G m_1 m_2}{r^2}$

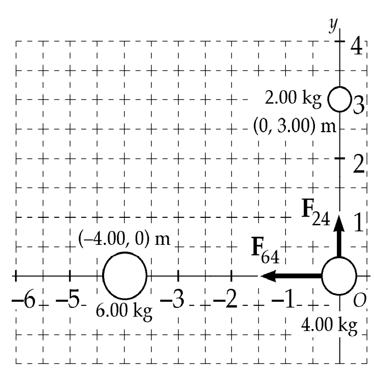
we have $\sum F = \frac{G(50.0\ \text{kg})(500\ \text{kg} - 200\ \text{kg})}{(0.200\ \text{m})^2} = \boxed{2.50 \times 10^{-5}\ \text{N}}$ toward the 500-kg object

(b) At a point between the two objects at a distance d from the 500-kg objects, the net force on the 50.0-kg object will be zero when

$$\frac{G(50.0\ \text{kg})(200\ \text{kg})}{(0.400\ \text{m} - d)^2} = \frac{G(50.0\ \text{kg})(500\ \text{kg})}{d^2}$$

or $d = \boxed{0.245\ \text{m}}$

***11.3** The force exerted on the 4.00-kg mass by the 2.00-kg mass is directed upward and given by



$$\mathbf{F}_{24} = G\frac{m_4 m_2}{r_{24}^2}\mathbf{j} = \left(6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(4.00\ \text{kg})(2.00\ \text{kg})}{(3.00\ \text{m})^2}\mathbf{j}$$

$$= 5.93 \times 10^{-11}\ \mathbf{j}\ \text{N}$$

The force exerted on the 4.00-kg mass by the 6.00-kg mass is directed to the left

$$\mathbf{F}_{64} = G\frac{m_4 m_6}{r_{64}^2}(-\mathbf{i}) = \left(-6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(4.00\ \text{kg})(6.00\ \text{kg})}{(4.00\ \text{m})^2}\mathbf{i} = -10.0 \times 10^{-11}\ \mathbf{i}\ \text{N}$$

Therefore, the resultant force on the 4.00-kg mass is $\mathbf{F}_4 = \mathbf{F}_{24} + \mathbf{F}_{64} = \boxed{(-10.0\mathbf{i} + 5.93\mathbf{j}) \times 10^{-11}\ \text{N}}$

***11.4** (a) The Sun-Earth distance is 1.496×10^{11} m, and the Earth-Moon distance is 3.84×10^{8} m, so the distance from the Sun to the Moon during a solar eclipse is

$$1.496 \times 10^{11} \text{ m} - 3.84 \times 10^{8} \text{ m} = 1.492 \times 10^{11} \text{ m}$$

The mass of the Sun, Earth , and Moon are $M_S = 1.99 \times 10^{30}$ kg

$M_E = 5.98 \times 10^{24}$ kg

and $M_M = 7.36 \times 10^{22}$ kg

We have $$F_{SM} = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67 \times 10^{-11}\right)\left(1.99 \times 10^{30}\right)\left(7.36 \times 10^{22}\right)}{\left(1.492 \times 10^{11}\right)^2} = \boxed{4.39 \times 10^{20} \text{ N}}$$

(b) $$F_{EM} = \frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24}\right)\left(7.36 \times 10^{22}\right)}{\left(3.84 \times 10^{8}\right)^2} = \boxed{1.99 \times 10^{20} \text{ N}}$$

(c) $$F_{SE} = \frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(1.99 \times 10^{30}\right)\left(5.98 \times 10^{24}\right)}{\left(1.496 \times 10^{11}\right)^2} = \boxed{3.55 \times 10^{22} \text{ N}}$$

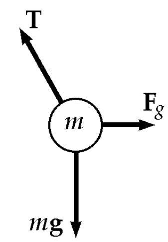
Note that the force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon. In a sense, the Moon orbits the Sun more than it orbits the Earth. The Moon's path is not precisely as shown in the text's Figure 11.24; instead, it is everywhere concave toward the Sun. Only by subtracting out the solar orbital motion of the Earth-Moon system do we see the Moon orbiting the center of mass of this system.

***11.5** $$g = \frac{GM}{R^2} = \frac{G\rho\left(4\pi R^3/3\right)}{R^2} = \tfrac{4}{3}\pi G\rho R$$

If $$\frac{g_M}{g_E} = \frac{1}{6} = \frac{4\pi G\rho_M R_M/3}{4\pi G\rho_E R_E/3}$$

then $$\frac{\rho_M}{\rho_E} = \left(\frac{g_M}{g_E}\right)\left(\frac{R_E}{R_M}\right) = \left(\frac{1}{6}\right)(4) = \boxed{\frac{2}{3}}$$

11.6 Let θ represent the angle each cable makes with the vertical, L the cable length, x the distance each ball scrunches in, and $d = 1\text{ m}$ the original distance between them. Then $r = d - 2x$ is the separation of the balls. We have



$\Sigma F_y = 0$: $\quad T\cos\theta - mg = 0$

$\Sigma F_x = 0$: $\quad T\sin\theta - \dfrac{Gmm}{r^2} = 0$

Then $\quad \tan\theta = \dfrac{Gmm}{r^2 mg} \qquad \dfrac{x}{\sqrt{L^2 - x^2}} = \dfrac{Gm}{g(d-2x)^2} \qquad x(d-2x)^2 = \dfrac{Gm}{g}\sqrt{L^2 - x^2}$

The factor Gm/g is numerically small. There are two possibilities: either x is small or else $d - 2x$ is small.

Possibility one: We can ignore x in comparison to d and L, obtaining

$$x(1\text{ m})^2 = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m/kg}^2\right)(100\text{ kg})}{\left(9.8\text{ m/s}^2\right)}45\text{ m} \qquad x = 3.06\times10^{-8}\text{ m}$$

The separation distance is $\quad r = 1\text{ m} - 2\left(3.06\times10^{-8}\text{ m}\right) = \boxed{1.000\text{ m} - 61.3\text{ nm}}$

Possibility two: If $d - 2x$ is small, $x \cong 0.5\text{ m}$ and the equation becomes

$$(0.5\text{ m})r^2 = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)(100\text{ kg})}{(9.8\text{ N/kg})}\sqrt{(45\text{ m})^2 - (0.5\text{ m})^2} \qquad r = \boxed{2.74\times10^{-4}\text{ m}}$$

For this answer to apply, the spheres would have to be compressed to a density like that of the nucleus of atom.

11.7 (a) At the zero-total field point, $\quad \dfrac{GmM_E}{r_E^{\,2}} = \dfrac{GmM_M}{r_M^{\,2}}$

so $\quad r_M = r_E\sqrt{\dfrac{M_M}{M_E}} = r_E\sqrt{\dfrac{7.36\times10^{22}}{5.98\times10^{24}}} = \dfrac{r_E}{9.01}$

$$r_E + r_M = 3.84\times10^8\text{ m} = r_E + \frac{r_E}{9.01}$$

$$r_E = \frac{3.84\times10^8\text{ m}}{1.11} = \boxed{3.46\times10^8\text{ m}}$$

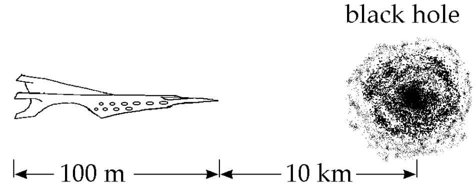
(b) At this distance the acceleration due to the Earth's gravity is

$$g_E = \frac{GM_E}{r_E^{\,2}} = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}{\left(3.46\times10^8\text{ m}\right)^2}$$

$$g_E = \boxed{3.34\times10^{-3}\text{ m/s}^2\text{ directed toward the Earth}}$$

11.8 (a) $F = \dfrac{GMm}{r^2} = \dfrac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left[100\left(1.99\times10^{30}\ \text{kg}\right)\left(10^3\ \text{kg}\right)\right]}{\left(1.00\times10^4\ \text{m}+50.0\ \text{m}\right)^2} = \boxed{1.31\times10^{17}\ \text{N}}$

(b) $\Delta F = \dfrac{GMm}{r_{\text{front}}^{\,2}} - \dfrac{GMm}{r_{\text{back}}^{\,2}}$

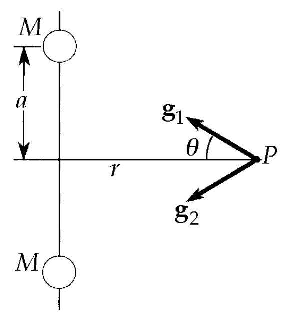


$$\Delta g = \frac{\Delta F}{m} = \frac{GM\left(r_{\text{back}}^{\,2} - r_{\text{front}}^{\,2}\right)}{r_{\text{front}}^{\,2}\, r_{\text{back}}^{\,2}}$$

$$\Delta g = \frac{\left(6.67\times10^{-11}\right)\left[100\left(1.99\times10^{30}\right)\right]\left[\left(1.01\times10^4\ \text{m}\right)^2 - \left(1.00\times10^4\ \text{m}\right)^2\right]}{\left(1.00\times10^4\ \text{m}\right)^2\left(1.01\times10^4\ \text{m}\right)^2}$$

$$\Delta g = \boxed{2.62\times10^{12}\ \text{N/kg}}$$

***11.9** $g_1 = g_2 = \dfrac{MG}{r^2 + a^2}$



$g_{1y} = -g_{2y}$ $\qquad g_y = g_{1y} + g_{2y} = 0$

$g_{1x} = g_{2x} = g_2 \cos\theta$ $\qquad \cos\theta = \dfrac{r}{\left(a^2 + r^2\right)^{1/2}}$

$$\mathbf{g} = 2g_{2x}(-\mathbf{i})$$

or $\quad \mathbf{g} = \boxed{\dfrac{2MGr}{\left(r^2 + a^2\right)^{3/2}} \text{ toward the center of mass}}$

11.10 $W = -\Delta U = -\left(\dfrac{-Gm_1 m_2}{r} - 0\right)$

$$W = \frac{\left(+6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.36\times10^{22}\ \text{kg}\right)\left(1.00\times10^3\ \text{kg}\right)}{1.74\times10^6\ \text{m}} = \boxed{2.82\times10^9\ \text{J}}$$

11.11 (a) $\rho = \frac{M_S}{\frac{4}{3}\pi r_E{}^3} = \frac{3\left(1.99\times10^{30}\text{ kg}\right)}{4\pi\left(6.37\times10^{6}\text{ m}\right)^3} = \boxed{1.84\times10^{9}\text{ kg/m}^3}$

(b) $g = \frac{GM_S}{r_E{}^2} = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)}{\left(6.37\times10^{6}\text{ m}\right)^2} = \boxed{3.27\times10^{6}\text{ m/s}^2}$

(c) $U_g = -\frac{GM_S m}{r_E} = -\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)\left(1.00\text{ kg}\right)}{6.37\times10^{6}\text{ m}} = \boxed{-2.08\times10^{13}\text{ J}}$

***11.12** (a) For the geosynchronous satellite,

$\Sigma F_r = ma_r$ becomes $\frac{GmM_E}{r^2} = \frac{mv^2}{r}$

and in turn $\frac{GM_E}{r} = \left(\frac{2\pi r}{T}\right)^2$ or $r^3 = \frac{GM_E T^2}{4\pi^2}$

Thus, the radius of the satellite orbit is

$$r = \left[\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)\left(86\,400\text{ s}\right)^2}{4\pi^2}\right]^{1/3} = \boxed{4.23\times10^{7}\text{ m}}$$

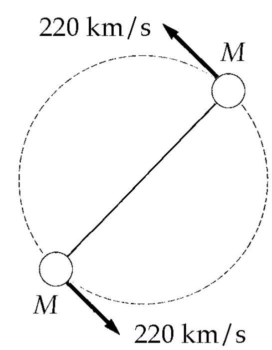
(b) The satellite is so far out that its distance from the north pole,

$$d = \sqrt{\left(6.37\times10^{6}\text{ m}\right)^2 + \left(4.23\times10^{7}\text{ m}\right)^2} = 4.27\times10^{7}\text{ m}$$

is nearly the same as its orbital radius. The travel time for the radio signal is

$$t = \frac{2d}{c} = \frac{2\left(4.27\times10^{7}\text{ m}\right)}{3.00\times10^{8}\text{ m/s}} = \boxed{0.285\text{ s}}$$

11.13 Applying Newton's 2nd Law, $\Sigma F = ma$ yields $F_g = ma_c$ for each star:



$\frac{GMM}{(2r)^2} = \frac{Mv^2}{r}$ or $M = \frac{4v^2 r}{G}$

We can write r in terms of the period, T, by considering the time and distance of one complete cycle. The distance traveled in one orbit is the circumference of the stars' common orbit, so $2\pi r = vT$. Therefore

$$M = \frac{4v^2 r}{G} = \frac{4v^2}{G}\left(\frac{vT}{2\pi}\right)$$

so, $M = \frac{2v^3 T}{\pi G} = \frac{2\left(220\times10^{3}\text{ m/s}\right)^3\left(14.4\text{ d}\right)\left(86\,400\text{ s/d}\right)}{\pi\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)} = \boxed{1.26\times10^{32}\text{ kg} = 63.3\text{ solar masses}}$

11.14 By conservation of angular momentum for the satellite,

$$r_p v_p = r_a v_a \qquad \frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{2289 \text{ km} + 6.37 \times 10^3 \text{ km}}{459 \text{ km} + 6.37 \times 10^3 \text{ km}} = \frac{8659 \text{ km}}{6829 \text{ km}} = \boxed{1.27}$$

We do not need to know the period.

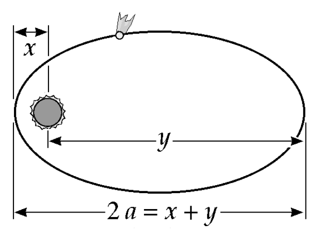
11.15 $T^2 = \dfrac{4\pi^2 a^3}{GM}$ (Kepler's third law with $m << M$)

$$M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2 \left(4.22 \times 10^8 \text{ m}\right)^3}{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(1.77 \times 86\,400 \text{ s}\right)^2} = \boxed{1.90 \times 10^{27} \text{ kg}}$$

(Approximately 316 Earth masses)

11.16 By Kepler's Third Law, $T^2 = ka^3$ (a = semi-major axis)

For any object orbiting the Sun, with T in years and a in A.U., $k = 1.00$. Therefore, for Comet Halley



$$(75.6)^2 = (1.00)\left(\frac{0.570 + y}{2}\right)^3$$

The farthest distance the comet gets from the Sun is

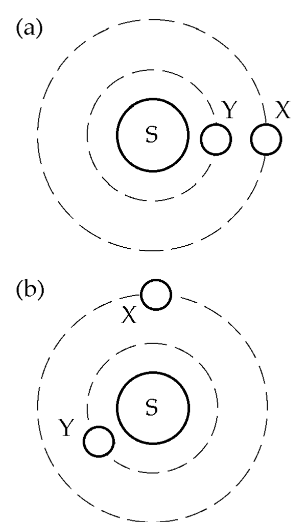
$$y = 2(75.6)^{2/3} - 0.570 = \boxed{35.2 \text{ A.U.}}$$ (out around the orbit of Pluto)

11.17 $\sum F = ma$: $\dfrac{Gm_{\text{planet}} M_{\text{star}}}{r^2} = \dfrac{m_{\text{planet}} v^2}{r}$

$$\frac{GM_{\text{star}}}{r} = v^2 = r^2\omega^2$$

$$GM_{\text{star}} = r^3\omega^3 = r_x{}^3\omega_x{}^2 = r_y{}^3\omega_y{}^2$$

$$\omega_y = \omega_x\left(\frac{r_x}{r_y}\right)^{3/2} \qquad \omega_y = \left(\frac{90.0°}{5.00 \text{ yr}}\right)3^{3/2} = \frac{468°}{5.00 \text{ yr}}$$



So $\boxed{\text{planet Y has turned through 1.30 revolutions}}$

***11.18** The speed of a planet in a circular orbit is given by

$$\Sigma F = ma: \quad \frac{GM_{sun}m}{r^2} = \frac{mv^2}{r} \qquad v = \sqrt{\frac{GM_{sun}}{r}}$$

For Mercury the speed is $$v_M = \sqrt{\frac{(6.67\times10^{-11})(1.99\times10^{30})\ \text{m}^2}{(5.79\times10^{10})\ \text{s}^2}} = 4.79\times10^4\ \text{m/s}$$

and for Pluto, $$v_P = \sqrt{\frac{(6.67\times10^{-11})(1.99\times10^{30})\ \text{m}^2}{(5.91\times10^{12})\ \text{s}^2}} = 4.74\times10^3\ \text{m/s}$$

With greater speed, Mercury will eventually move farther from the Sun than Pluto. With original distances r_P and r_M perpendicular to their lines of motion, they will be equally far from Sun after time t where

$$\sqrt{r_P{}^2 + v_P{}^2t^2} = \sqrt{r_M{}^2 + v_M{}^2t^2}$$

$$r_P{}^2 - r_M{}^2 = \left(v_M{}^2 - v_P{}^2\right)t^2$$

$$t = \sqrt{\frac{(5.91\times10^{12}\ \text{m})^2 - (5.79\times10^{10}\ \text{m})^2}{(4.79\times10^4\ \text{m/s})^2 - (4.74\times10^3\ \text{m/s})^2}} = \sqrt{\frac{3.49\times10^{25}\ \text{m}^2}{2.27\times10^9\ \text{m}^2/\text{s}^2}} = 1.24\times10^8\ \text{s} = \boxed{393\ \text{yr}}$$

***11.19** For the Earth, $\Sigma F = ma$: $$\frac{GM_sm}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$$

Then $$GM_sT^2 = 4\pi^2r^3$$

Also the angular momentum $L = mvr = m\dfrac{2\pi r}{T}r$ is a constant for the Earth.

We eliminate $r = \sqrt{\dfrac{LT}{2\pi m}}$ between the equations:

$$GM_sT^2 = 4\pi^2\left(\frac{LT}{2\pi m}\right)^{3/2} \qquad GM_sT^{1/2} = 4\pi^2\left(\frac{L}{2\pi m}\right)^{3/2}$$

Now the rate of change is described by

$$GM_s\left(\frac{1}{2}T^{-1/2}\frac{dT}{dt}\right) + G\left(1\frac{dM_s}{dt}T^{1/2}\right) = 0 \qquad \frac{dT}{dt} = -\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) \cong \frac{\Delta T}{\Delta t}$$

$$\Delta T \cong -\Delta t\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) = -5000\ \text{yr}\left(\frac{3.16\times10^7\ \text{s}}{1\ \text{yr}}\right)\left(-3.64\times10^9\ \text{kg/s}\right)\left(2\frac{1\ \text{yr}}{1.991\times10^{30}\ \text{kg}}\right)$$

$$\Delta T = \boxed{1.82\times10^{-2}\ \text{s}}$$

***11.20** $$\frac{v_i^2}{R_E+h}=\frac{GM_E}{(R_E+h)^2}$$

$$K_i=\frac{1}{2}mv_i^2=\frac{1}{2}\left(\frac{GM_Em}{R_E+h}\right)=\frac{1}{2}\left[\frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)(500\ \mathrm{kg})}{\left(6.37\times10^6\ \mathrm{m}\right)+\left(0.500\times10^6\ \mathrm{m}\right)}\right]=1.45\times10^{10}\ \mathrm{J}$$

The change in gravitational potential energy of the satellite-Earth system is

$$\Delta U=\frac{GM_Em}{R_i}-\frac{GM_Em}{R_f}=GM_Em\left(\frac{1}{R_i}-\frac{1}{R_f}\right)$$

$$=\left(6.67\times10^{-11}\ \mathrm{N\cdot m/kg^2}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)(500\ \mathrm{kg})\left(-1.14\times10^{-8}\ \mathrm{m^{-1}}\right)=-2.27\times10^9\ \mathrm{J}$$

Also, $K_f=\frac{1}{2}mv_f^2=\frac{1}{2}(500\ \mathrm{kg})\left(2.00\times10^3\ \mathrm{m/s}\right)^2=1.00\times10^9\ \mathrm{J}$

The energy transformed due to friction is

$$\Delta E_{\mathrm{int}}=K_i-K_f-\Delta U=(14.5-1.00+2.27)\times10^9\ \mathrm{J}=\boxed{1.58\times10^{10}\ \mathrm{J}}$$

11.21 To obtain the orbital velocity, we use $\sum F=\frac{mMG}{R^2}=\frac{mv^2}{R}$

or $v=\sqrt{\frac{MG}{R}}$

We can obtain the escape velocity from $\frac{1}{2}mv_{\mathrm{esc}}{}^2=\frac{mMG}{R}$

or $v_{\mathrm{esc}}=\sqrt{\frac{2MG}{R}}=\boxed{\sqrt{2}v}$

***11.22** The gravitational force supplies the needed centripetal acceleration.

Thus, $\dfrac{GM_E m}{(R_E+h)^2} = \dfrac{mv^2}{(R_E+h)}$ or $v^2 = \dfrac{GM_E}{R_E+h}$

(a) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(R_E+h)}{\sqrt{GM_E/(R_E+h)}}$ $T = \boxed{2\pi\sqrt{\dfrac{(R_E+h)^3}{GM_E}}}$

(b) $v = \boxed{\sqrt{\dfrac{GM_E}{R_E+h}}}$

(c) Minimum energy input is $\Delta E_{\min} = \left(K_f + U_{gf}\right) - \left(K_i - U_{gi}\right)$

It is simplest to $\boxed{\text{launch the satellite from a location on the equator, and launch it toward the east.}}$

This choice has the object starting with energy $K_i = \frac{1}{2}mv_i^2$

with $v_i = \dfrac{2\pi R_E}{1.00 \text{ day}} = \dfrac{2\pi R_E}{86\,400 \text{ s}}$ and $U_{gi} = -\dfrac{GM_E m}{R_E}$

Thus, $\Delta E_{\min} = \frac{1}{2}m\left(\dfrac{GM_E}{R_E+h}\right) - \dfrac{GM_E m}{R_E+h} - \frac{1}{2}m\left[\dfrac{4\pi^2 R_E^2}{(86\,400\text{ s})^2}\right] + \dfrac{GM_E m}{R_E}$

or $\Delta E_{\min} = \boxed{GM_E m\left[\dfrac{R_E+2h}{2R_E(R_E+h)}\right] - \dfrac{2\pi^2 R_E^2 m}{(86\,400\text{ s})^2}}$

11.23 $g_E = \dfrac{Gm_E}{r_E^2}$ $g_U = \dfrac{Gm_U}{r_U^2}$

(a) $\dfrac{g_U}{g_E} = \dfrac{m_U r_E^2}{m_E r_U^2} = 14.0\left(\dfrac{1}{3.70}\right)^2 = 1.02$ $g_U = (1.02)(9.80\text{ m/s}^2) = \boxed{10.0\text{ m/s}^2}$

(b) $v_{esc,E} = \sqrt{\dfrac{2Gm_E}{r_E}}$; $v_{esc,U} = \sqrt{\dfrac{2Gm_U}{r_U}}$: $\dfrac{v_{esc,E}}{v_{esc,U}} = \sqrt{\dfrac{m_U r_E}{m_E r_U}} = \sqrt{\dfrac{14.0}{3.70}} = 1.95$

For the Earth, (from Table 11.2) $v_{esc,E} = 11.2\text{ km/s}$

$\therefore v_{esc,U} = (1.95)(11.2\text{ km/s}) = \boxed{21.8\text{ km/s}}$

***11.24** For a satellite in an orbit of radius r around the Earth, the total energy of the satellite-Earth system is $E = -\dfrac{GM_E}{2r}$. Thus, in changing from a circular orbit of radius $r = 2R_E$ to one of radius $r = 3R_E$, the required work is

$$W = \Delta E = -\frac{GM_E m}{2r_f} + \frac{GM_E m}{2r_i} = GM_E m\left[\frac{1}{4R_E} - \frac{1}{6R_E}\right] = \boxed{\frac{GM_E m}{12R_E}}$$

***11.25** (a) The major axis of the orbit is $2a = 50.5\ \text{AU}$ so $a = 25.25\ \text{AU}$

Further, in Figure 11.7, $a + c = 50\ \text{AU}$ so $c = 24.75\ \text{AU}$

Then $e = c/a = 24.75/25.25 = \boxed{0.980}$

(b) In $T^2 = K_s a^3$ for objects in solar orbit, the Earth gives us

$$(1\ \text{yr})^2 = K_s(1\ \text{AU})^3 \qquad K_s = (1\ \text{yr})^2/(1\ \text{AU})^3.$$

Then $T^2 = \dfrac{(1\ \text{yr})^2}{(1\ \text{AU})^3}(25.25\ \text{AU})^3$ $\qquad T = \boxed{127\ \text{yr}}$

(c) $$U = -\frac{GMm}{r} = -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\ \text{kg}\right)\left(1.2\times10^{10}\ \text{kg}\right)}{50\left(1.496\times10^{11}\ \text{m}\right)} = \boxed{-2.13\times10^{17}\ \text{J}}$$

11.26 (a) The energy of the photon is found as $E = E_i - E_f = \dfrac{-13.606\ \text{eV}}{n_i^2} - \dfrac{(-13.606\ \text{eV})}{n_f^2}$

$$E = 13.606\ \text{eV}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Thus, for $n = 3$ to $n = 2$ transition $\qquad E = 13.606\ \text{eV}\left(\dfrac{1}{4} - \dfrac{1}{9}\right) = \boxed{1.89\ \text{eV}}$

(b) $E = \dfrac{hc}{\lambda}$ and $\lambda = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}\left(2.998\times10^{8}\ \text{m/s}\right)}{1.89\ \text{eV}\left(1.602\times10^{-19}\ \text{J/eV}\right)} = \boxed{656\ \text{nm}}$

(c) $f = \dfrac{c}{\lambda}$ $\qquad f = \dfrac{3\times10^{8}\ \text{m/s}}{6.56\times10^{-7}\ \text{m}} = \boxed{4.57\times10^{14}\ \text{Hz}}$

***11.27** (a) Lyman series $\frac{1}{\lambda} = R\left(1 - \frac{1}{n_i^2}\right)$ $n_i = 2, 3, 4, \ldots$

$$\frac{1}{\lambda} = \frac{1}{94.96 \times 10^{-9}} = \left(1.097 \times 10^7\right)\left(1 - \frac{1}{n_i^2}\right) \qquad \boxed{n_i = 5}$$

(b) Paschen series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$ $n_i = 4, 5, 6, \ldots$

The shortest wavelength for this series corresponds to $n_i = \infty$ for ionization

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{9} - \frac{1}{n_i^2}\right)$$

For $n_i = \infty$, this gives $\lambda = 820$ nm

This is larger than 94.96 nm, so this wave length $\boxed{\text{cannot be associated with the Paschen series}}$

Balmer series: $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$ $n_i = 3, 4, 5, \ldots$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{n_i^2}\right) \qquad n_i = \infty \text{ for ionization } \lambda_{\min} = 365 \text{ nm}$$

Once again this wavelength $\boxed{\text{cannot be associated with the Balmer series}}$

11.28 (a) $v_1 = \sqrt{\frac{k_e e^2}{m_e r_1}}$

where $r_1 = (1)^2 a_0 = 0.00529 \text{ nm} = 5.29 \times 10^{-11} \text{ m}$

$$v_1 = \sqrt{\frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{\left(9.11 \times 10^{-31} \text{ kg}\right)\left(5.29 \times 10^{-11} \text{ m}\right)}} = \boxed{2.19 \times 10^6 \text{ m/s}}$$

(b) $K_1 = \frac{1}{2} m_e v_1^2 = \frac{1}{2}\left(9.11 \times 10^{-31} \text{ kg}\right)\left(2.19 \times 10^6 \text{ m/s}\right)^2 = 2.18 \times 10^{-18} \text{ J} = \boxed{13.6 \text{ eV}}$

(c) $U_1 = -\frac{k_e e^2}{r_1} = -\frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{5.29 \times 10^{-11} \text{ m}} = -4.35 \times 10^{-18} \text{ J} = \boxed{-27.2 \text{ eV}}$

11.29 (a) $r_2^2 = (0.0529\text{ nm})(2)^2 = \boxed{0.212\text{ nm}}$

(b) $m_e v_2 = \sqrt{\dfrac{m_e k_e e^2}{r_2}} = \sqrt{\dfrac{(9.11\times10^{-31}\text{ kg})(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{0.212\times10^{-9}\text{ m}}}$

$m_e v_2 = \boxed{9.95\times10^{-25}\text{ kg}\cdot\text{m/s}}$

(c) $L_2 = m_e v_2 r_2 = (9.95\times10^{-25}\text{ kg}\cdot\text{m/s})(0.212\times10^{-9}\text{ m}) = \boxed{2.11\times10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}}$

(d) $K_2 = \frac{1}{2}m_e v_2^2 = \dfrac{(m_e v_2)^2}{2m_e} = \dfrac{(9.95\times10^{-25}\text{ kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\text{ kg})} = 5.43\times10^{-19}\text{ J} = \boxed{3.40\text{ eV}}$

(e) $U_2 = -\dfrac{k_e e^2}{r_2} = -\dfrac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{0.212\times10^{-9}\text{ m}} = -1.09\times10^{-18}\text{ J} = \boxed{-6.80\text{ eV}}$

(f) $E_2 = K_2 + U_2 = 3.40\text{ eV} - 6.80\text{ eV} = \boxed{-3.40\text{ eV}}$

11.30 $\Delta E = (13.6\text{ eV})\left(\dfrac{1}{n_i^2} - \dfrac{1}{n_f^2}\right)$

Where for $\Delta E > 0$ we have absorption and for $\Delta E < 0$ we have emission.

(i) for $n_i = 2$ and $n_f = 5$, $\Delta E = 2.86$ eV (absorption)

(ii) for $n_i = 5$ and $n_f = 3$, $\Delta E = -0.967$ eV (emission)

(iii) for $n_i = 7$ and $n_f = 4$, $\Delta E = -0.572$ eV (emission)

(iv) for $n_i = 4$ and $n_f = 7$, $\Delta E = 0.572$ eV (absorption)

(a) $E = \dfrac{hc}{\lambda}$ so the shortest wavelength is emitted in transition $\boxed{\text{ii}}$.

(b) The atom gains most energy in transition $\boxed{\text{i}}$.

(c) The atom loses energy in transitions $\boxed{\text{ii and iii}}$.

11.31 We use $$E_n = \frac{-13.6 \text{ eV}}{n^2}$$

To ionize the atom when the electron is in the n^{th} level,

it is necessary to add an amount of energy given by $$E = -E_n = \frac{13.6 \text{ eV}}{n^2}$$

(a) Thus, in the ground state where $n = 1$, we have $\boxed{E = 13.6 \text{ eV}}$

(b) In the $n = 3$ level, $$E = \frac{13.6 \text{ eV}}{9} = \boxed{1.51 \text{ eV}}$$

11.32 Starting with $$\tfrac{1}{2} m_e v^2 = \frac{k_e e^2}{2r}$$

we have $$v^2 = \frac{k_e e^2}{m_e r}$$

and using $$r_n = \frac{n^2 \hbar^2}{m_e k_e e^2}$$

gives $$v_n{}^2 = \frac{k_e e^2}{m_e \dfrac{n^2 \hbar^2}{m_e k_e e^2}}$$

or $$v_n = \frac{k_e e^2}{n\hbar}$$

11.33 Each atom gives up its kinetic energy in emitting a photon,

so $$\frac{1}{2} m v^2 = \frac{hc}{\lambda} = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{\left(1.216 \times 10^{-7} \text{ m}\right)} = 1.63 \times 10^{-18} \text{ J}$$

$$v = \boxed{4.42 \times 10^4 \text{ m/s}}$$

***11.34** The original orbit radius is

$$r = a = 6.37 \times 10^6 \text{ m} + 500 \times 10^3 \text{ m} = 6.87 \times 10^6 \text{ m}$$

The original energy is

$$E_i = -\frac{GMm}{2a} = -\frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)\left(10^4 \text{ kg}\right)}{2\left(6.87 \times 10^6 \text{ m}\right)} = -2.90 \times 10^{11} \text{ J}$$

We assume that the perigee distance in the new orbit is 6.87×10^6 m. Then the major axis is

$$2a = 6.87 \times 10^6 \text{ m} + 2.00 \times 10^7 \text{ m} = 2.69 \times 10^7 \text{ m}$$

and the final energy is

$$E_f = -\frac{GMm}{2a} = -\frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)\left(10^4 \text{ kg}\right)}{2.69 \times 10^7 \text{ m}} = -1.48 \times 10^{11} \text{ J}$$

The energy input required from the engine is

$$E_f - E_i = -1.48 \times 10^{11} \text{ J} - \left(-2.90 \times 10^{11} \text{ J}\right) = \boxed{1.42 \times 10^{11} \text{ J}}$$

***11.35** (a) Energy of the spacecraft-Mars system is conserved as the spacecraft moves between a very distant point and the point of closest approach:

$$0 + 0 = \frac{1}{2} m v_r^2 - \frac{GM_{\text{Mars}} m}{r}$$

$$v_r = \sqrt{\frac{2GM_{\text{Mars}}}{r}}$$

After the engine burn, for a circular orbit we have

$$\Sigma F = ma: \qquad \frac{GM_{\text{Mars}} m}{r^2} = \frac{m v_0^2}{r}$$

$$v_0 = \sqrt{\frac{GM_{\text{Mars}}}{r}}$$

The percentage reduction from the original speed is

$$\frac{v_r - v_0}{v_r} = \frac{\sqrt{2} v_0 - v_0}{\sqrt{2} v_0} = \frac{\sqrt{2} - 1}{\sqrt{2}} \times 100\% = \boxed{29.3\%}$$

(b) The answer to part (a) applies with $\boxed{\text{no changes}}$, as the solution to part (a) shows.

***11.36** Let m represent the mass of the spacecraft, r_E the radius of the Earth's orbit, and x the distance from Earth to the spacecraft.

The Sun exerts on the spacecraft a radial inward force of $$F_s = \frac{GM_s m}{(r_E - x)^2}$$

while the Earth exerts on it a radial outward force of $$F_E = \frac{GM_E m}{x^2}$$

The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year.

Thus, $$F_S - F_E = \frac{GM_S m}{(r_E - x)^2} - \frac{GM_E m}{x^2} = \frac{mv^2}{(r_E - x)} = \frac{m}{(r_E - x)}\left[\frac{2\pi(r_E - x)}{T}\right]^2$$

which reduces to $$\frac{GM_S}{(r_E - x)^2} - \frac{GM_E}{x^2} = \frac{4\pi^2(r_E - x)}{T^2} \qquad (1)$$

Cleared of fractions, this equation would contain powers of x ranging from the fifth to the zeroth. We do not solve it algebraically. We may test the assertion that x is between 1.47×10^9 m and 1.48×10^9 m by substituting both of these as trial solutions, along with the following data: $M_S = 1.991 \times 10^{30}$ kg, $M_E = 5.983 \times 10^{24}$ kg, $r_E = 1.496 \times 10^{11}$ m, and $T = 1.000 \text{ yr} = 3.156 \times 10^7$ s.

With $x = 1.47 \times 10^9$ m substituted into equation (1), we obtain

$$6.052 \times 10^{-3} \text{ m/s}^2 - 1.85 \times 10^{-3} \text{ m/s}^2 \cong 5.871 \times 10^{-3} \text{ m/s}^2$$

or $$5.868 \times 10^{-3} \text{ m/s}^2 \cong 5.871 \times 10^{-3} \text{ m/s}^2$$

With $x = 1.48 \times 10^9$ m substituted into the same equation, the result is

$$6.053 \times 10^{-3} \text{ m/s}^2 - 1.82 \times 10^{-3} \text{ m/s}^2 \cong 5.8708 \times 10^{-3} \text{ m/s}^2$$

or $$5.8709 \times 10^{-3} \text{ m/s}^2 \cong 5.8708 \times 10^{-3} \text{ m/s}^2$$

Since the first trial solution makes the left-hand side of equation (1) slightly less than the right hand side, and the second trial solution does the opposite, the true solution is determined as between the trial values. To three-digit precision, it is 1.48×10^9 m.

As an equation of fifth degree, equation (1) has five roots. The Sun-Earth system has five Lagrange points, all revolving around the Sun synchronously with the Earth. The SOHO and ACE satellites are at one. Another is beyond the far side of the Sun. Another is beyond the night side of the Earth. Two more are on the Earth's orbit, ahead of the planet and behind it by 60°. Plans are under way to gain perspective on the Sun by placing a spacecraft at one of these two co-orbital Lagrange points. The Greek and Trojan asteroids are at the co-orbital Lagrange points of the Jupiter-Sun system.

***11.37** Let r represent the distance between the electron and the positron. The two move in a circle of radius $r/2$ around their center of mass with opposite velocities. The total angular momentum of the electron-positron system is quantized to according to

$$L_n = \frac{mvr}{2} + \frac{mvr}{2} = n\hbar$$

where $n = 1, 2, 3, \ldots$

For each particle, $\Sigma F = ma$ expands to $\dfrac{k_e e^2}{r^2} = \dfrac{mv^2}{r/2}$

We can eliminate $v = \dfrac{n\hbar}{mr}$ to find $\dfrac{k_e e^2}{r} = \dfrac{2mn^2\hbar}{m^2 r^2}$

So the separation distances are $r = \dfrac{2n^2\hbar^2}{mk_e e^2} = 2a_0 n^2 = \boxed{\left(1.06\times10^{-10}\ \text{m}\right)n^2}$

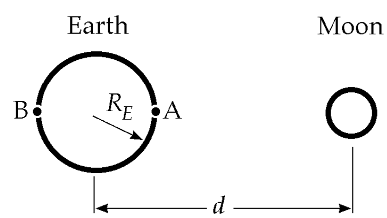
The orbital radii are $r/2 = a_0 n^2$, the same as for the electron in hydrogen.

The energy can be calculated from $E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 - \dfrac{k_e e^2}{r}$

Since $mv^2 = \dfrac{k_e e^2}{2r}$, $E = \dfrac{k_e e^2}{2r} - \dfrac{k_e e^2}{r} = -\dfrac{k_e e^2}{2r} = \dfrac{-k_e e^2}{4a_0 n^2} = \boxed{-\dfrac{6.80\ \text{eV}}{n^2}}$

***11.38** The acceleration of an object at the center of the Earth due to the gravitational force of the Moon is given by

$$a = G\frac{M_{\text{Moon}}}{d^2}$$



At the point A nearest the Moon, $a_+ = G\dfrac{M_M}{(d-r)^2}$

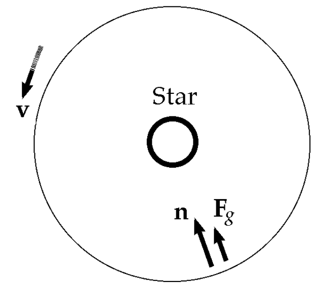
At the point B farthest from the Moon, $a_- = G\dfrac{M_M}{(d+r)^2}$

$$\Delta a = a_+ - a = GM_M\left[\frac{1}{(d-r)^2} - \frac{1}{d^2}\right]$$

For $d >> r$, $\Delta a = \dfrac{2GM_M r}{d^3} = 1.11\times10^{-6}\ \text{m/s}^2$

Across the planet, $\dfrac{\Delta g}{g} = \dfrac{2\Delta a}{g} = \dfrac{2.22\times10^{-6}\ \text{m/s}^2}{9.80\ \text{m/s}^2} = \boxed{2.26\times10^{-7}}$

***11.39** (a) $a_c = \dfrac{v^2}{r}$ $\quad a_c = \dfrac{\left(1.25\times10^6\ \text{m/s}\right)^2}{1.53\times10^{11}\ \text{m}} = \boxed{10.2\ \text{m/s}^2}$



(b) $\text{diff} = 10.2 - 9.90 = 0.312\ \text{m/s}^2 = \dfrac{GM}{r^2}$

$$M = \frac{\left(0.312\ \text{m/s}^2\right)\left(1.53\times10^{11}\ \text{m}\right)^2}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = \boxed{1.10\times10^{32}\ \text{kg}}$$

***11.40** (a) The free-fall acceleration produced by the Earth is $g = \dfrac{GM_E}{r^2} = GM_E r^{-2}$ (directed downward)

Its rate of change is $\dfrac{dg}{dr} = G\,M_E(-2)\,r^{-3} = -2G\,M_E\,r^{-3}$

The minus sign indicates that g decreases with increasing height.

At the Earth's surface, $\boxed{\dfrac{dg}{dr} = -\dfrac{2GM_E}{R_E^{\,3}}}$

(b) For small differences,

$\dfrac{|\Delta g|}{\Delta r} = \dfrac{|\Delta g|}{h} = \dfrac{2GM_E}{R_E^{\,3}}$ $\quad$ Thus, $\boxed{|\Delta g| = \dfrac{2GM_E h}{R_E^{\,3}}}$

(c) $$|\Delta g| = \frac{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(6.00\ \text{m})}{\left(6.37\times10^6\ \text{m}\right)^3} = \boxed{1.85\times10^{-5}\ \text{m/s}^2}$$

11.41 To approximate the height of the sulfur, set

$\dfrac{mv^2}{2} = mg_{Io}h$ $\quad h = 70\,000\ \text{m}$ $\quad g_{Io} = \dfrac{GM}{r^2} = 1.79\ \text{m/s}^2$

$v = \sqrt{2g_{Io}\,h}$ $\quad v = \sqrt{2(1.79)(70\,000)} \cong 500\ \text{m/s}$ (over 1000 mi/h)

A more precise answer is given by

$$\tfrac{1}{2}mv^2 - \frac{GMm}{r_1} = -\frac{GMm}{r_2}$$

$$\tfrac{1}{2}v^2 = \left(6.67\times10^{-11}\right)\left(8.90\times10^{22}\right)\left(\frac{1}{1.82\times10^6} - \frac{1}{1.89\times10^6}\right) \qquad v = \boxed{492\ \text{m/s}}$$

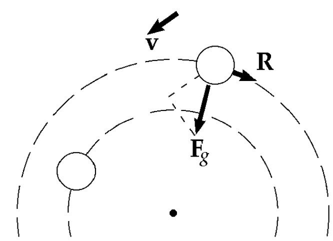
11.42 From the walk, $2\pi r = 25\,000$ m. Thus, the radius of the planet is $r = \dfrac{25000\text{ m}}{2\pi} = 3.98 \times 10^3\text{ m}$

From the drop: $\Delta y = \frac{1}{2}gt^2 = \frac{1}{2}g(29.2\text{ s})^2 = 1.40\text{ m}$

so, $g = \dfrac{2(1.40\text{ m})}{(29.2\text{ s})^2} = 3.28 \times 10^{-3}\text{ m/s}^2 = \dfrac{MG}{r^2}$ $\quad\therefore\quad M = \boxed{7.79 \times 10^{14}\text{ kg}}$

11.43 $$F = \frac{GMm}{r^2} = \left(6.67 \times 10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(1.50\text{ kg})(15.0 \times 10^{-3}\text{ kg})}{(4.50 \times 10^{-2}\text{ m})^2} = \boxed{7.41 \times 10^{-10}\text{ N}}$$

***11.44** For both circular orbits,



$\Sigma F = ma$: $$\frac{GM_Em}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

(a) The original speed is $$v_i = \sqrt{\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})}{(6.37 \times 10^6\text{ m} + 2 \times 10^5\text{ m})}} = \boxed{7.79 \times 10^3\text{ m/s}}$$

(b) The final speed is $$v_i = \sqrt{\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})}{(6.47 \times 10^6\text{ m})}} = \boxed{7.85 \times 10^3\text{ m/s}}$$

The energy of the satellite-Earth system is

$$K + U_g = \frac{1}{2}mv^2 - \frac{GM_Em}{r} = \frac{1}{2}m\frac{GM_E}{r} - \frac{GM_E}{r} = -\frac{GM_Em}{2r}$$

(c) originally $$E_i = -\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})(100\text{ kg})}{2(6.57 \times 10^6\text{ m})} = \boxed{-3.04 \times 10^9\text{ J}}$$

(d) finally $$E_f = -\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\text{ kg})(100\text{ kg})}{2(6.47 \times 10^6\text{ m})} = \boxed{-3.08 \times 10^9\text{ J}}$$

(e) Thus the object speeds up as it spirals down to the planet. The loss of gravitational energy is so large that the total energy decreases by

$$E_i - E_f = -3.04 \times 10^9\text{ J} - (-3.08 \times 10^9\text{ J}) = \boxed{4.69 \times 10^7\text{ J}}$$

(f) The only forces on the object are the backward force of air resistance R, comparatively very small in magnitude, and the force of gravity. Because the spiral path of the satellite is not perpendicular to the gravitational force, $\boxed{\text{one component of the gravitational force}}$ pulls forward on the satellite to do positive work and make its speed increase.

11.45 (a) At infinite separation $U = 0$ and at rest $K = 0$. Since energy of the two-planet system is conserved we have,

$$0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} \qquad (1)$$

The initial momentum of the system is zero and momentum is conserved.

Therefore, $$0 = m_1v_1 - m_2v_2 \qquad (2)$$

Combine equations (1) and (2) : $\boxed{v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}}$ and $\boxed{v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}}$

Relative velocity $v_r = v_1 - (-v_2) = \boxed{\sqrt{\frac{2G(m_1+m_2)}{d}}}$

(b) Substitute given numerical values into the equation found for v_1 and v_2 in part (a) to find

$v_1 = 1.03 \times 10^4$ m/s and $v_2 = 2.58 \times 10^3$ m/s

Therefore, $K_1 = \frac{1}{2}m_1v_1^2 = \boxed{1.07 \times 10^{32}\text{ J}}$ and $K_2 = \frac{1}{2}m_2v_2^2 = \boxed{2.67 \times 10^{31}\text{ J}}$

11.46 (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum of the Earth is conserved;

$mr_av_a = mr_pv_p$ and $v_a = v_p\left(\frac{r_p}{r_a}\right) = \left(3.027 \times 10^4\text{ m/s}\right)\left(\frac{1.471}{1.521}\right) = \boxed{2.93 \times 10^4\text{ m/s}}$

(b) $K_p = \frac{1}{2}mv_p^2 = \frac{1}{2}\left(5.98 \times 10^{24}\right)\left(3.027 \times 10^4\right)^2 = \boxed{2.74 \times 10^{33}\text{ J}}$

$$U_p = -\frac{GmM}{r_p} = -\frac{\left(6.673 \times 10^{-11}\right)\left(5.98 \times 10^{24}\right)\left(1.99 \times 10^{30}\right)}{1.471 \times 10^{11}} = \boxed{-5.40 \times 10^{33}\text{ J}}$$

(c) Using the same form as in part (b), $K_a = \boxed{2.57 \times 10^{33}\text{ J}}$ and $U_a = \boxed{-5.22 \times 10^{33}\text{ J}}$

Compare to find that $K_p + U_p = \boxed{-2.66 \times 10^{33}\text{ J}}$ and $K_a + U_a = \boxed{-2.65 \times 10^{33}\text{ J}}$ They agree.

11.47 (a) $T = \frac{2\pi r}{v} = \frac{2\pi\left(30\,000 \times 9.46 \times 10^{15}\text{ m}\right)}{2.50 \times 10^5\text{ m/s}} = 7\ \times 10^{15}\text{ s} = \boxed{2 \times 10^8\text{ yr}}$

(b) $$M = \frac{4\pi^2a^3}{GT^2} = \frac{4\pi^2\left(30\,000 \times 9.46 \times 10^{15}\text{ m}\right)^3}{\left(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.13 \times 10^{15}\text{ s}\right)^2} = 2.66 \times 10^{41}\text{ kg}$$

$M = 1.34 \times 10^{11}$ solar masses $\boxed{\sim 10^{11}\text{ solar masses}}$

The number of stars is $\boxed{\text{on the order of } 10^{11}}$

***11.48** (a) From the data about perigee, the energy of the satellite-Earth system is

$$E = \frac{1}{2}mv_p^2 - \frac{GM_E m}{r_p} = \frac{1}{2}(1.60)\left(8.23\times10^3\right)^2 - \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1.60)}{7.02\times10^6}$$

or $$E = \boxed{-3.67\times10^7 \text{ J}}$$

(b) $L = mvr\sin\theta = m\,v_p r_p \sin 90.0° = (1.60\text{ kg})(8.23\times10^3\text{ m/s})(7.02\times10^6\text{ m}) = \boxed{9.24\times10^{10}\text{ kg}\cdot\text{m}^2/\text{s}}$

(c) Since both the energy of the satellite-Earth system and the angular momentum of the Earth are conserved,

at apogee we must have $\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = E$

and $m\,v_a r_a \sin 90.0° = L$

Thus, $$\frac{1}{2}(1.60)v_a^2 - \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1.60)}{r_s} = -3.67\times10^7\text{ J}$$

and $$(1.60\text{ kg})v_a r_a = 9.24\times10^{10}\text{ kg}\cdot\text{m}^2/\text{s}$$

Solving simultaneously, $$\frac{1}{2}(1.60)v_a^2 - \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1.60)(1.60)v_a}{9.24\times10^{10}} = -3.67\times10^7$$

which reduces to $$0.800\,v_a^2 - 11046v_a + 3.6723\times10^7 = 0$$

so $$v_a = \frac{11046 \pm \sqrt{(11046)^2 - 4(0.800)\left(3.6723\times10^7\right)}}{2(0.800)}$$

This gives $v_a = 8230\text{ m/s}$ or $\boxed{5580\text{ m/s}}$. The smaller answer refers to the velocity at the apogee while the larger refers to perigee.

Thus, $$r_a = \frac{L}{mv_a} = \frac{9.24\times10^{10}\text{ kg}\cdot\text{m}^2/\text{s}}{(1.60\text{ kg})\left(5.58\times10^3\text{ m/s}\right)} = \boxed{1.04\times10^7\text{ m}}$$

(d) The major axis is $2a = r_p + r_a$, so the semi-major axis is

$$a = \frac{1}{2}\left(7.02\times10^6\text{ m} + 1.04\times10^7\text{ m}\right) = \boxed{8.69\times10^6\text{ m}}$$

(e) $$T = \sqrt{\frac{4\pi^2a^3}{GM_E}} = \sqrt{\frac{4\pi^2\left(8.69\times10^6\text{ m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}}$$

$$T = 8060\text{ s} = \boxed{134\text{ min}}$$

11.49 $v_i = 2\sqrt{Rg}$ $\qquad g = \dfrac{MG}{R^2}$

Utilizing conservation of energy for the system of the Earth and the coasting rocket,

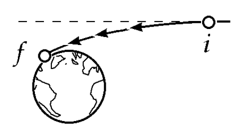
$$\frac{mv^2}{2} - \frac{mGM}{r} = \frac{mv_i^2}{2} - \frac{mGM}{R} \qquad \frac{mv^2}{2} = \frac{mv_i^2}{2} - \frac{mGM}{R} + \frac{mGM}{r}$$

$$v^2 = v_i^2 - 2MG\left(\frac{1}{R} - \frac{1}{r}\right) \qquad v = \sqrt{v_i^2 + 2MG\left(\frac{1}{r} - \frac{1}{R}\right)}$$

$$v = \sqrt{4Rg + 2MG\left(\frac{1}{r} - \frac{1}{R}\right)} \qquad v = \sqrt{\frac{4MG}{R} - \frac{2MG}{R} + \frac{2MG}{r}}$$

$$v = \boxed{\sqrt{2MG\left(\frac{1}{R} + \frac{1}{r}\right)} = \sqrt{2R^2 g\left(\frac{1}{R} + \frac{1}{r}\right)}}$$

***11.50** Let m represent the mass of the meteoroid and v_i its speed when far away. No torque acts on the meteoroid, so its angular momentum is conserved as it moves between the distant point and the point where it grazes the Earth, moving perpendicular to the radius:



$L_i = L_f$: $\qquad m\mathbf{r}_i \times \mathbf{v}_i = m\mathbf{r}_f \times \mathbf{v}_f$

$$m(3R_E v_i) = mR_E v_f$$

$$v_f = 3v_i$$

Now energy of the meteoroid-Earth system is also conserved:

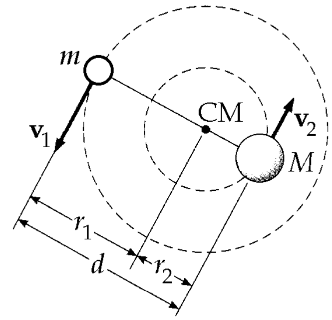
$(K + U_g)_i = (K + U_g)_f$: $\qquad \frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 - \dfrac{GM_E m}{R_E}$

$$\frac{1}{2}v_i^2 = \frac{1}{2}(9v_i^2) - \frac{GM_E}{R_E}$$

$\dfrac{GM_E}{R_E} = 4v_i^2$: $\qquad \boxed{v_i = \sqrt{\dfrac{GM_E}{4R_E}}}$

11.51 If we choose the coordinate of the center of mass at the origin, then

$$0 = \frac{(Mr_2 - mr_1)}{M+m} \qquad \text{and} \qquad M r_2 = m r_1$$



(Note: this is equivalent to saying that the net torque must be zero and the two experience no angular acceleration.) For each mass $F = ma$ so

$$mr_1\omega_1{}^2 = \frac{MGm}{d^2} \qquad \text{and} \qquad Mr_2\omega_2{}^2 = \frac{MGm}{d^2}$$

Combining these two equations and using $d = r_1 + r_2$ gives $(r_1 + r_2)\,\omega^2 = \frac{(M+m)G}{d^2}$

with $\omega_1 = \omega_2 = \omega$

and $T = \frac{2\pi}{\omega}$

we find $\boxed{T^2 = \frac{4\pi^2 d^3}{G(M+m)}}$

***11.52** From Kepler's third law, minimum period means minimum orbit size. The "treetop satellite" in Figure P11.21 has minimum period. The radius of the satellite's circular orbit is essentially equal to the radius R of the planet.

$\Sigma F = ma$: $$\frac{GMm}{R^2} = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

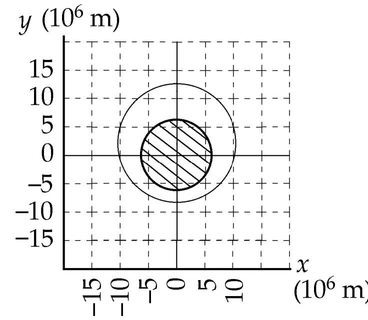
$$G\rho V = \frac{R^2\left(4\pi^2 R^2\right)}{RT^2}$$

$$G\rho\left(\tfrac{4}{3}\pi R^3\right) = \frac{4\pi^2 R^3}{T^2}$$

The radius divides out: $T^2 G\rho = 3\pi$ $\boxed{T = \sqrt{\frac{3\pi}{G\rho}}}$

11.53 Initial Conditions and Constants:

Mass of planet:	5.98×10^{24} kg
Radius of planet:	6.37×10^{6} m
Initial x:	0.0 planet radii
Initial y:	2.0 planet radii
Initial v_x:	+5000 m/s
Initial v_y:	0.0 m/s
Time interval:	10.9 s



t (s)	x (m)	y (m)	r (m)	v_x (m/s)	v_y (m/s)	a_x (m/s^2)	a_y (m/s^2)
0.0	0.0	12,740,000.0	12,740,000.0	5,000.0	0.0	0.0000	–2.4575
10.9	54,315.3	12,740,000.0	12,740,115.8	4,999.9	–26.7	–0.0100	–2.4574
21.7	108,629.4	12,739,710.0	12,740,173.1	4,999.7	–53.4	–0.0210	–2.4573
32.6	162,941.1	12,739,130.0	12,740,172.1	4,999.3	–80.1	–0.0310	–2.4572
…							
5,431.6	112,843.8	–8,466,816.0	8,467,567.9	–7,523.0	–39.9	–0.0740	5.5625
5,442.4	31,121.4	–8,467,249.7	8,467,306.9	–7,523.2	20.5	–0.0200	5.5633
5,453.3	–50,603.4	–8,467,026.9	8,467,178.2	–7,522.8	80.9	0.0330	5.5634
5,464.1	–132,324.3	–8,466,147.7	8,467,181.7	–7,521.9	141.4	0.0870	5.5628
…							
10,841.3	–108,629.0	12,739,134.4	12,739,597.5	4,999.9	53.3	0.0210	–2.4575
10,852.2	–54,314.9	12,739,713.4	12,739,829.2	5,000.0	26.6	0.0100	–2.4575
10,863.1	0.4	12,740,002.4	12,740,002.4	5,000.0	–0.1	0.0000	–2.4575

The $\boxed{\text{object does not hit the Earth}}$; its minimum radius is $\boxed{1.33 R_E}$.

Its period is $\boxed{1.09 \times 10^4 \text{ s}}$. A circular orbit would require a speed of $\boxed{5.60 \text{ km/s}}$.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. (a) 2.50×10^{-5} N toward the 500-kg mass
(b) between the masses and 0.245 m from the 500 kg mass

4. (a) 4.39×10^{20} N toward the Sun (b) 1.99×10^{20} N away from the Sun.
(c) 3.55×10^{22} N

6. See the solution. Either $1.000\text{ m} - 61.3\text{ nm}$ or 2.47×10^{-4} m

8. (a) 1.31×10^{17} N toward the black hole (b) 2.62×10^{12} N/kg

10. 2.82×10^{9} J

12. (a) 4.23×10^{7} m (b) 0.285 s

14. 1.27

16. 35.2 AU

18. After 393 yr Mercury would be farther from the Sun than Pluto

20. 1.58×10^{10} J

22. (a) $2\pi\sqrt{\dfrac{(R_E + h)^3}{GM_E}}$ (b) $\sqrt{\dfrac{GM_E}{R_E + h}}$
(c) $GM_E m\left[\dfrac{R_E + 2h}{2R_E(R_E + h)}\right] - \dfrac{2\pi^2 R_E{}^2 m}{(86400\text{ s})^2}$ The satellite should be launched from a point on the equator, toward the east.

24. $GM_E m / 12R_E$

26. (a) 1.89 eV (b) 656 nm (c) 4.57×10^{14} Hz

28. (a) 2.19×10^{6} m/s (b) 13.6 eV (c) –27.2 eV

30. (a) ii (b) i (c) ii and iii

32. See the solution

34. 1.42×10^{11} J

36. See the solution

38. 2.26×10^{-7}

40. (a) and (b) see the solution (c) $1.85 \times 10^{-5}\ \mathrm{m/s^2}$

42. 7.79×10^{14} kg

44. (a) 7.79 km/s (b) 7.85 km/s (c) –3.04 GJ
(d) –3.08 GJ (e) loss = 46.9 MJ
(f) A component of the Earth's gravity pulls forward on the satellite on its downward-banking trajectory.

46. (a) 2.93×10^{4} m/s (b) $K = 2.74 \times 10^{33}$ J, $U = -5.40 \times 10^{33}$ J,
(c) $K = 2.57 \times 10^{33}$ J, $U = -5.22 \times 10^{33}$ J. Total energy is constant.

48. (a) –36.7 MJ (b) $9.24 \times 10^{10}\ \mathrm{kg \cdot m^2/s}$ (c) 10.4 Mm, 5.58 km/s
(d) 8.69 Mm (e) 134 min

50. $\left(GM_E / 4R_E\right)^{1/2}$

52. See the solution