CHAPTER 12

ANSWERS TO QUESTIONS

Q12.1 No; Yes. The pendulum can hang at rest with respect to the car, accelerating along with the car at $g\sin\theta$, if its string makes angle θ behind the vertical. The tension in the card is $mg\cos\theta$. An observer in the car can displace the pendulum slightly from this position to see it oscillate with period $2\pi\sqrt{L/(g\cos\theta)}$, somewhat larger than the period on the level.

Q12.2 When the spring with two masses is set into oscillation in space, the coil in the exact center of the spring does not move. Thus, we can imagine clamping the center coil in place without affecting the motion. If we do this, we have two separate oscillating systems, one on each side of the clamp. The half-spring on each side of the clamp has twice the spring constant of the full spring, as shown by the following argument. The force exerted by a spring is proportional to the separation of the coils as the spring is extended. Imagine that we extend a spring by a given distance and measure the distance between coils. We then cut the spring in half. If one of the half-springs is now extended by the same distance, the coils will be twice as far apart as they were for the complete spring. Thus, it takes twice as much force to stretch the half-string, from which we conclude the half-spring has a spring constant that is twice that of the complete spring. Our clamped system of masses on two half-springs, therefore, will vibrate with a frequency that is higher than *f* by a factor of $\sqrt{2}$.

Q12.3 Higher frequency. When it supports your weight, the center of the diving board flexes down less than the end does when it supports your weight. Thus the stiffness constant describing the center of the board is greater than the stiffness constant describing the end. And then $f = (1/2\pi)\sqrt{k/m}$ is greater for you bouncing on the center of the board.

Q12.4 You can take $\phi = \pi$, or equally well, $\phi = -\pi$. At $t = 0$, the particle is at its turning point on the negative side of equilibrium, at $x = -A$.

Q12.5 The two will be equal if and only if the origin of coordinates is the position of the particle at time zero.

Q12.6
(a) In simple harmonic motion, one-half of the time, the velocity is in the same direction as the displacement away from equilibrium.
(b) Velocity and acceleration are in the same direction half the time.
(c) Acceleration is always opposite to displacement, never in the same direction.

Q12.7 No. It is necessary to know both the position and velocity at time zero.

Q12.8 Kinetic energy is $mv^2/2$, and potential energy is $kx^2/2$, both always positive.

Q12.9 No; Kinetic, Yes; Potential, No. For constant amplitude, the energy $\frac{1}{2}kA^2$ stays constant. The kinetic energy $\frac{1}{2}mv^2$ would increase for larger mass if the speed were constant, but here the greater mass causes a decrease in frequency and in the average and maximum speed, so that the kinetic and potential energies at every point are unchanged.

Q12.10 We have $T_i = \sqrt{L_i/g}$ and $T_f = \sqrt{L_f/g} = \sqrt{2L_i/g} = \sqrt{2}\,T_i$. The period gets larger by $\sqrt{2}$ times. Changing the mass has no effect on the period of a simple pendulum.

Q12.11 The motion will be periodic—that is, it will repeat. The period is nearly constant as the angular amplitude increases through small values; then the period becomes noticeably larger as θ increases farther.

Q12.12 No. If the resistive force is greater than the restoring force of the spring (in particular, if $b^2 > 4mk$), the system will be overdamped and will not oscillate.

Q12.13 Yes. An oscillator with damping can vibrate at resonance with amplitude that remains constant in time. Without damping, the amplitude would increase without limit at resonance.

Q12.14 Shorten the pendulum to decrease the period between ticks.

PROBLEM SOLUTIONS

12.1 (a) $|F| = kx$, so $k = \frac{|F|}{x} = \frac{230 \text{ N}}{0.4 \text{ m}} = \boxed{575 \text{ N/m}}$

(b) $U_s = \frac{1}{2}kx^2 = \frac{1}{2}(575 \text{ N/m})(0.4 \text{ m})^2 = \boxed{46.0 \text{ J}}$

***12.2** (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the $\boxed{\text{motion is periodic}}$

(b) To determine the period, we use: $x = \frac{1}{2}gt^2$

The time for the ball to hit the ground is $t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00 \text{ m})}{9.80 \text{ m/s}^2}} = 0.909 \text{ s}$

This equals one-half the period, so $T = 2(0.909 \text{ s}) = \boxed{1.82 \text{ s}}$

(c) $\boxed{\text{No}}$. The net force acting on the ball is a constant given by $F = -mg$ (except when it is in contact with the ground), which is not in the form of Hooke's law.

12.3 $x = (4.00 \text{ m})\cos(3.00\pi t + \pi)$ Compare this with $x = A\cos(\omega t + \phi)$ to find

(a) $\omega = 2\pi f = 3.00\pi$

or $\boxed{f = 1.50 \text{ Hz}}$ $T = \frac{1}{f} = \boxed{0.667 \text{ s}}$

(b) $A = \boxed{4.00 \text{ m}}$

(c) $\phi = \boxed{\pi \text{ rad}}$

(d) $x(t = 0.250 \text{ s}) = (4.00 \text{ m})\cos(1.75\pi) = \boxed{2.83 \text{ m}}$

12.4 (a) $\boxed{20.0 \text{ cm}}$

(b) $v_{max} = \omega A = 2\pi fA = \boxed{94.2 \text{ cm/s}}$

This occurs as the particle passes through equilibrium.

(c) $a_{max} = \omega^2 A = (2\pi f)^2 A = \boxed{17.8 \text{ m/s}^2}$

This occurs at maximum excursion from equilibrium.

12.5 (a) $x = (5.00\text{ cm})\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $x = (5.00\text{ cm})\cos\left(\frac{\pi}{6}\right) = \boxed{4.33\text{ cm}}$

(b) $v = \frac{dx}{dt} = -(10.0\text{ cm/s})\sin\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $v = \boxed{-5.00\text{ cm/s}}$

(c) $a = \frac{dv}{dt} = -\left(20.0\text{ cm/s}^2\right)\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $a = \boxed{-17.3\text{ cm/s}^2}$

(d) $A = \boxed{5.00\text{ cm}}$ and $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \boxed{3.14\text{ s}}$

***12.6** $k = \frac{|F|}{x} = \frac{10.0\times10^{-3}\text{ kg}\left(9.80\text{ m/s}^2\right)}{3.90\times10^{-2}\text{ m}} = 2.51\text{ N/m}$ and $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{25.0\times10^{-3}\text{ kg}}{2.51\text{ N/m}}} = \boxed{0.627\text{ s}}$

12.7 (a) At $t = 0$, $x = 0$ and v is positive (to the right). Therefore, this situation corresponds to $x = A\sin\omega t$

and $v = v_i\cos\omega t$

Since $f = 1.50$ Hz, $\omega = 2\pi f = 3.00\pi$

Also, $A = 2.00$ cm, so that $\boxed{x = (2.00\text{ cm})\sin 3.00\pi t}$

(b) $v_{max} = v_i = A\omega = 2.00(3.00\pi) = \boxed{6.00\pi\text{ cm/s}}$

The particle has this speed at $t = 0$ and next at $t = \frac{T}{2} = \boxed{\frac{1}{3}\text{ s}}$

(c) $a_{max} = A\omega^2 = 2.00(3.00\pi)^2 = \boxed{18.0\pi^2\text{ cm/s}^2}$

This positive value of acceleration first occurs at $t = \frac{3}{4}T = \boxed{0.500\text{ s}}$

(d) Since $T = \frac{2}{3}$ s and $A = 2.00$ cm, the particle will travel 8.00 cm in this time.

Hence, in 1.00 s$\left(= \frac{3}{2}T\right)$, the particle will travel $8.00\text{ cm} + 4.00\text{ cm} = \boxed{12.0\text{ cm}}$

***12.8** (a) $T = \frac{12.0\text{ s}}{5} = \boxed{2.40\text{ s}}$

(b) $f = \frac{1}{T} = \frac{1}{2.40} = \boxed{0.417\text{ Hz}}$

(c) $\omega = 2\pi f = 2\pi(0.417) = \boxed{2.62\text{ rad/s}}$

***12.9** $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ or $T = \dfrac{1}{f} = 2\pi\sqrt{\dfrac{m}{k}}$

Solving for k, $k = \dfrac{4\pi^2 m}{T^2} = \dfrac{4\pi^2(7.00\text{ kg})}{(2.60\text{ s})^2} = \boxed{40.9\text{ N/m}}$

12.10 The proposed solution $x(t) = x_i\cos\omega t + \left(\dfrac{v_i}{\omega}\right)\sin\omega t$

implies velocity $v = \dfrac{dx}{dt} = -x_i\omega\sin\omega t + v_i\cos\omega t$

and acceleration $a = \dfrac{dv}{dt} = -x_i\omega^2\cos\omega t - v_i\omega\sin\omega t = -\omega^2\left(x_i\cos\omega t + \left(\dfrac{v_i}{\omega}\right)\sin\omega t\right) = -\omega^2 x$

(a) The acceleration being a negative constant times position means we do have SHM, and its angular frequency is ω. At $t = 0$ the equations reduce to $x = x_i$ and $v = v_i$ so they satisfy all the requirements.

(b) $v^2 - ax = (-x_i\omega\sin\omega t + v_i\cos\omega t)^2 - (-x_i\omega^2\cos\omega t - v_i\sin\omega t)\left(x_i\cos\omega t + \left(\dfrac{v_i}{\omega}\right)\sin\omega t\right)$

$v^2 - ax = x_i^2\omega^2\sin^2\omega t - 2x_iv_i\omega\sin\omega t\cos\omega t + v_i^2\cos^2\omega t$

$+x_i^2\omega^2\cos^2\omega t + x_iv_i\omega\cos\omega t\sin\omega t + x_iv_i\omega\sin\omega t\cos\omega t + v_i^2\sin^2\omega t = x_i^2\omega^2 + v_i^2$

So this expression is constant in time. On one hand, it must keep its original value $v_i^2 - a_ix_i$. On the other hand, if we evaluate it at a turning point where $v = 0$ and $x = A$, it is $A^2\omega^2 + 0^2 = A^2\omega^2$. Thus it is proved.

12.11 (a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{8.00\text{ N/m}}{0.500\text{ kg}}} = 4.00\text{ s}^{-1}$ so position is given by $x = 10.0\sin(4.00t)$ cm

From this we find that $v = 40.0\cos(4.00t)$ cm/s $v_{max} = \boxed{40.0\text{ cm/s}}$

$a = -160\sin(4.00t)$ cm/s^2 $a_{max} = \boxed{160\text{ cm/s}^2}$

(b) $t = \left(\dfrac{1}{4.00}\right)\sin^{-1}\left(\dfrac{x}{10.0}\right)$ and when $x = 6.00$ cm, $t = 0.161$ s

We find $v = 40.0\cos[4.00(0.161)] = \boxed{32.0\text{ cm/s}}$

$a = -160\sin[4.00(0.161)] = \boxed{-96.0\text{ cm/s}^2}$

(c) Using $t = \left(\dfrac{1}{4.00}\right)\sin^{-1}\left(\dfrac{x}{10.0}\right)$

when $x = 0$, $t = 0$ and when $x = 8.00$ cm, $t = 0.232$ s

Therefore, $\Delta t = \boxed{0.232\text{ s}}$

12.12 $x = A\cos\omega t$ $A = 0.05\text{ m}$ $v = -A\omega\sin\omega t$ $a = -A\omega^2\cos\omega t$

If $f = 3600\text{ rev/min} = 60\text{ Hz}$, then $\omega = 120\pi\text{ s}^{-1}$

$v_{max} = 0.05(120\pi)\text{ m/s} = \boxed{18.8\text{ m/s}}$ $a_{max} = 0.05(120\pi)^2\text{ m/s}^2 = \boxed{7.11\text{ km/s}^2}$

12..13 $\omega = \sqrt{\dfrac{k}{m}} = \dfrac{2\pi}{T}$

(a) $k = \omega^2 m = \boxed{\dfrac{4\pi^2 m}{T^2}}$

(b) $m' = \dfrac{k(T')^2}{4\pi^2} = \boxed{m\left(\dfrac{T'}{T}\right)^2}$

12.14 $m = 1.00\text{ kg}$, $k = 25.0\text{ N/m}$, and $A = 3.00\text{ cm}$. At $t = 0$, $x = -3.00\text{ cm}$

(a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{25.0}{1.00}} = 5.00\text{ rad/s}$

so that, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{5.00} = \boxed{1.26\text{ s}}$

(b) $v_{max} = A\omega = 3.00\times10^{-2}\text{ m}(5.00\text{ rad/s}) = \boxed{0.150\text{ m/s}}$

$a_{max} = A\omega^2 = 3.00\times10^{-2}\text{ m}(5.00\text{ rad/s})^2 = \boxed{0.750\text{ m/s}^2}$

(c) Because $x = -3.00\text{ cm}$ and $v = 0$ at $t = 0$, the required solution is $x = -A\cos\omega t$

or $\boxed{x = -3.00\cos(5.00t)\text{ cm}}$

$v = \dfrac{dx}{dt} = \boxed{15.0\sin(5.00t)\text{ cm/s}}$

$a = \dfrac{dv}{dt} = \boxed{75.0\cos(5.00t)\text{ cm/s}^2}$

12.15 Choose the car with its shock-absorbing bumper as the system; by conservation of energy,

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$: $v = x\sqrt{\dfrac{k}{m}} = \left(3.16\times10^{-2}\text{ m}\right)\sqrt{\dfrac{5.00\times10^6}{10^3}} = \boxed{2.23\text{ m/s}}$

12.16 (a) Energy is conserved for the block-spring system between the maximum-displacement and the half-maximum points:

$$(K+U)_i = (K+U)_f \qquad 0+\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

$$\tfrac{1}{2}(6.50\text{ N/m})(0.100\text{ m})^2 = \tfrac{1}{2}m(0.300\text{ m/s})^2 + \tfrac{1}{2}(6.50\text{ N/m})\left(5.00\times10^{-2}\text{ m}\right)^2$$

$$32.5\text{ mJ} = \tfrac{1}{2}m(0.300\text{ m/s})^2 + 8.12\text{ mJ} \qquad m = \frac{2(24.4\text{ mJ})}{9.00\times10^{-2}\text{ m}^2/\text{s}^2} = \boxed{0.542\text{ kg}}$$

(b) $$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6.50\text{ N/m}}{0.542\text{ kg}}} = 3.46\text{ rad/s} \qquad \therefore\ T = \frac{2\pi}{\omega} = \frac{2\pi\text{ rad}}{3.46\text{ rad/s}} = \boxed{1.81\text{ s}}$$

(c) $$a_{max} = A\omega^2 = 0.100\text{ m}(3.46\text{ rad/s})^2 = \boxed{1.20\text{ m/s}^2}$$

12.17 $$m = 200\text{ g},\ T = 0.250\text{ s},\ E = 2.00\text{ J};\ \omega = \frac{2\pi}{T} = \frac{2\pi}{0.250} = 25.1\text{ rad/s}$$

(a) $$k = m\omega^2 = 0.200\text{ kg}(25.1\text{ rad/s})^2 = \boxed{126\text{ N/m}}$$

(b) $$E = \frac{kA^2}{2} \Rightarrow A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(2.00)}{126}} = \boxed{0.178\text{ m}}$$

12.18 (a) $$E = \frac{kA^2}{2} = \frac{250\text{ N/m}\left(3.50\times10^{-2}\text{ m}\right)^2}{2} = \boxed{0.153\text{ J}}$$

(b) $$v_{max} = A\omega \qquad \text{where} \qquad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{250}{0.500}} = 22.4\text{ s}^{-1}$$

$$v_{max} = \boxed{0.784\text{ m/s}}$$

(c) $$a_{max} = A\omega^2 = 3.50\times10^{-2}\text{ m}\left(22.4\text{ s}^{-1}\right)^2 = \boxed{17.5\text{ m/s}^2}$$

12.19 (a) $$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(35.0\text{ N/m})\left(4.00\times10^{-2}\text{ m}\right)^2 = \boxed{28.0\text{ mJ}}$$

(b) $$|v| = \omega\sqrt{A^2-x^2} = \sqrt{\frac{k}{m}}\sqrt{A^2-x^2} \qquad |v| = \sqrt{\frac{35.0}{50.0\times10^{-3}}}\sqrt{\left(4.00\times10^{-2}\right)^2 - \left(1.00\times10^{-2}\right)^2} = \boxed{1.02\text{ m/s}}$$

(c) $$\tfrac{1}{2}mv^2 = \tfrac{1}{2}kA^2 - \tfrac{1}{2}kx^2 = \tfrac{1}{2}(35.0)\left[\left(4.00\times10^{-2}\right)^2 - \left(3.00\times10^{-2}\right)^2\right] = \boxed{12.2\text{ mJ}}$$

(d) $$\tfrac{1}{2}kx^2 = E - \tfrac{1}{2}mv^2 = \boxed{15.8\text{ mJ}}$$

12.20 (a) $k = \frac{|F|}{x} = \frac{20.0\text{ N}}{0.200\text{ m}} = \boxed{100\text{ N/m}}$

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{50.0}\text{ rad/s}$ so $f = \frac{\omega}{2\pi} = \boxed{1.13\text{ Hz}}$

(c) $v_{max} = \omega A = \sqrt{50.0}(0.200) = \boxed{1.41\text{ m/s}}$ at $x = 0$

(d) $a_{max} = \omega^2 A = 50.0(0.200) = \boxed{10.0\text{ m/s}^2}$ at $x = \pm A$

(e) $E = \frac{1}{2}kA^2 = \frac{1}{2}(100)(0.200)^2 = \boxed{2.00\text{ J}}$

(f) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{50.0}\sqrt{\frac{8}{9}(0.200)^2} = \boxed{1.33\text{ m/s}}$

(g) $|a| = \omega^2 x = 50.0\left(\frac{0.200}{3}\right) = \boxed{3.33\text{ m/s}^2}$

***12.21** Model the oscillator as a block-spring system.

From energy considerations, $v^2 + \omega^2 x^2 = \omega^2 A^2$

$v_{max} = \omega A$ and $v = \frac{\omega A}{2}$ so $\left(\frac{\omega A}{2}\right)^2 + \omega^2 x^2 = \omega^2 A^2$

From this we find $x^2 = \frac{3}{4}A^2$ and $x = \frac{\sqrt{3}}{2}A = \boxed{\pm 2.60\text{ cm}}$ where $A = 3.00\text{ cm}$

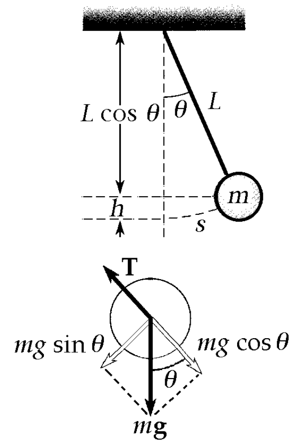
12.22 The period in Tokyo is $T_T = 2\pi\sqrt{\frac{L_T}{g_T}}$

and the period in Cambridge is $T_C = 2\pi\sqrt{\frac{L_C}{g_C}}$

We know $T_T = T_C = 2.00\text{ s}$

For which, we see $\frac{L_T}{g_T} = \frac{L_C}{g_C}$

or $\frac{g_C}{g_T} = \frac{L_C}{L_T} = \frac{0.9942}{0.9927} = \boxed{1.0015}$



12.23 (a) $mgh = \frac{1}{2}mv^2$ and $h = L(1-\cos\theta)$

$\therefore\ v_{max} = \sqrt{2gL(1-\cos\theta)} = \boxed{0.817\ \text{m/s}}$

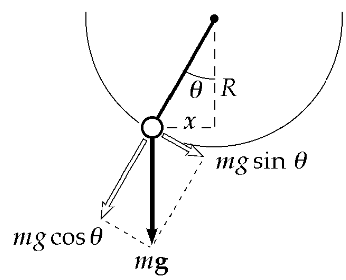
(b) $I\alpha = mgL\sin\theta$

$\alpha_{max} = \dfrac{mgL\sin\theta}{mL^2} = \dfrac{g}{L}\sin\theta_i = \boxed{2.54\ \text{rad/s}^2}$

(c) $F_{max} = mg\sin\theta_i = 0.250(9.80)(\sin 15.0°) = \boxed{0.634\ \text{N}}$

12.24 $\omega = \dfrac{2\pi}{T}$: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4.43} = \boxed{1.42\ \text{s}}$

$\omega = \sqrt{\dfrac{g}{L}}$: $L = \dfrac{g}{\omega^2} = \dfrac{9.80}{(4.43)^2} = \boxed{0.499\ \text{m}}$



12.25 Referring to the sketch we have

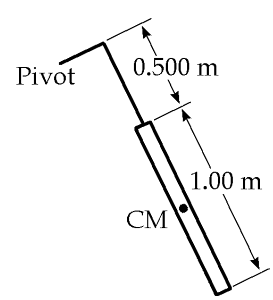
$F = -mg\sin\theta$ and $\tan\theta = \dfrac{x}{R}$

For small displacements, $\tan\theta \cong \sin\theta$

and $F = -\dfrac{mg}{R}x = -kx$

Since the restoring force is proportional to the displacement from equilibrium, the motion is simple harmonic motion.

Comparing to $F = -m\omega^2 x$ shows $\boxed{\omega = \sqrt{k/m} = \sqrt{g/R}}$



12.26 (a) The parallel-axis theorem:

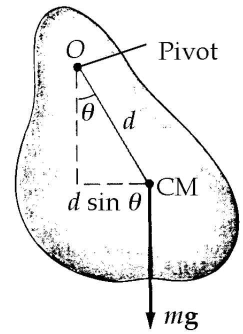
$I = I_{CM} + Md^2 = \frac{1}{12}ML^2 + Md^2 = \frac{1}{12}M(1.00\ \text{m})^2 + M(1.00\ \text{m})^2 = M\left(\frac{13}{12}\ \text{m}^2\right)$

$T = 2\pi\sqrt{\dfrac{I}{Mgd}} = 2\pi\sqrt{\dfrac{M(13\ \text{m}^2)}{12Mg(1.00\ \text{m})}} = 2\pi\sqrt{\dfrac{13\ \text{m}}{12(9.80\ \text{m/s}^2)}} = \boxed{2.09\ \text{s}}$

(b) For the simple pendulum

$T = 2\pi\sqrt{\dfrac{1.00\ \text{m}}{9.80\ \text{m/s}^2}} = 2.01\ \text{s}$ difference $= \dfrac{2.09\ \text{s} - 2.01\ \text{s}}{2.01\ \text{s}} = \boxed{4.08\%}$

***12.27** $f = 0.450\text{ Hz},\ d = 0.350\text{ m},\ \text{and}\ m = 2.20\text{ kg}$



$$T = \frac{1}{f};$$

$$T = 2\pi\sqrt{\frac{I}{mgd}}; \qquad T^2 = \frac{4\pi^2 I}{mgd}$$

$$I = T^2\frac{mgd}{4\pi^2} = \left(\frac{1}{f}\right)^2\frac{mgd}{4\pi^2} = \frac{2.20(9.80)(0.350)}{4\pi^2\left(0.450\text{ s}^{-1}\right)^2} = \boxed{0.944\text{ kg}\cdot\text{m}^2}$$

12..28 The total energy is $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

Taking the time-derivative, $\frac{dE}{dt} = mv\frac{d^2x}{dt^2} + kxv$

Use Equation 12.28: $\frac{md^2x}{dt^2} = -kx - bv$

$$\frac{dE}{dt} = v(-kx - bv) + kvx$$

Thus, $\boxed{\frac{dE}{dt} = -bv^2 < 0}$

12.29 $\theta_i = 15.0°$ $\qquad \theta(t = 1000) = 5.50°$

$x = Ae^{-bt/2m}$ $\qquad \frac{x_{1000}}{x_i} = \frac{Ae^{-bt/2m}}{A} = \frac{5.50}{15.0} = e^{-b(1000)/2m}$

$$\ln(5.50/15.0) = -1.00 = -b(1000)/2m$$

$$\therefore\ \frac{b}{2m} = \boxed{1.00\times10^{-3}\text{ s}^{-1}}$$

12.30 Show that $$x = Ae^{-bt/2m}\cos(\omega t + \phi)$$

is a solution of $$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2} \tag{1}$$

where $$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}. \tag{2}$$

$$x = Ae^{-bt/2m}\cos(\omega t + \phi) \tag{3}$$

$$\frac{dx}{dt} = Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi) \tag{4}$$

$$\frac{d^2x}{dt^2} = -\frac{b}{2m}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi)\right]$$
$$-\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\omega\sin(\omega t + \phi) + Ae^{-bt/2m}\omega^2\cos(\omega t + \phi)\right] \tag{5}$$

Substitute (3), (4) into the left side of (1) and (5) into the right side of (1);

$$-kAe^{-bt/2m}\cos(\omega t + \phi) + \frac{b^2}{2m}Ae^{-bt/2m}\cos(\omega t + \phi) + b\omega Ae^{-bt/2m}\sin(\omega t + \phi)$$
$$= -\frac{b}{2}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi)\right]$$
$$+\frac{b}{2}Ae^{-bt/2m}\omega\sin(\omega t + \phi) - m\omega^2 Ae^{-bt/2m}\cos(\omega t + \phi)$$

Compare the coefficients of $Ae^{-bt/2m}\cos(\omega t + \phi)$ and $Ae^{-bt/2m}\sin(\omega t + \phi)$:

cosine-term: $-k + \frac{b^2}{2m} = -\frac{b}{2}\left(-\frac{b}{2m}\right) - m\omega^2 = \frac{b^2}{4m} - m\left(\frac{k}{m} - \frac{b^2}{4m^2}\right) = -k + \frac{b^2}{2m}$

sine-term: $b\omega = +\frac{b}{2}(\omega) + \frac{b}{2}(\omega) = b\omega$

Since the coefficients are equal, $x = Ae^{-bt/2m}\cos(\omega t + \phi)$ is a solution of the equation.

***12.31** (a) For resonance, her frequency must match

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.30\times10^3\ \text{N/m}}{12.5\ \text{kg}}} = \boxed{2.95\ \text{Hz}}$$

(b) From $x = A\cos\omega t$, $v = dx/dt = -A\omega\sin\omega t$, and $a = dv/dt = -A\omega^2\cos\omega t$, the maximum acceleration is $A\omega^2$. When this becomes equal to the acceleration due to gravity, the normal force exerted on her by the mattress will drop to zero at one point in the cycle:

$$A\omega^2 = g \qquad \text{or} \qquad A = \frac{g}{\omega^2} = \frac{g}{k/m} = \frac{gm}{k} \qquad A = \frac{9.80\ \text{m/s}^2(12.5\ \text{kg})}{4.30\times10^3\ \text{N/m}} = \boxed{2.85\ \text{cm}}$$

12.32 $F = 3.00\cos(2\pi t)\ \text{N}$ and $k = 20.0\ \text{N/m}$

(a) $\omega = \frac{2\pi}{T} = 2\pi\ \text{rad/s}$ so $T = \boxed{1.00\ \text{s}}$

(b) In this case, $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{20.0}{2.00}} = 3.16\ \text{rad/s}$

Taking $b = 0$ in Equation 12.33 gives $A = \left(\frac{F_0}{m}\right)\left(\omega^2 - \omega_0^2\right)^{-1} = \frac{3}{2}\left[4\pi^2 - (3.16)^2\right]^{-1}$

Thus $A = 0.0509\ \text{m} = \boxed{5.09\ \text{cm}}$

12.33 $F_0\cos\omega t - kx = m\frac{d^2x}{dt^2}$ $\qquad \omega_0 = \sqrt{\frac{k}{m}}$ (1)

$x = A\cos(\omega t + \phi)$ (2)

$\frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$ (3)

$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi)$ (4)

Substitute (2) and (4) into (1): $F_0\cos\omega t - kA\cos(\omega t + \phi) = m\left(-A\omega^2\right)\cos(\omega t + \phi)$

Solve for the amplitude: $\left(kA - mA\omega^2\right)\cos(\omega t + \phi) = F_0\cos\omega t$

These will be equal, provided only that ϕ must be zero and $kA - mA\omega^2 = F_0$

Thus, $A = \frac{F_0/m}{(k/m) - \omega^2}$

***12.34** $$A = \frac{F_{ext}/m}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + (b\omega/m)^2}}$$

With $b = 0$, $$A = \frac{F_{ext}/m}{\sqrt{(\omega^2 - \omega_0^2)^2}} = \frac{F_{ext}/m}{\pm(\omega^2 + \omega_0^2)} = \pm\frac{F_{ext}/m}{\omega^2 - \omega_0^2}$$

Thus, $$\omega^2 = \omega_0^2 \pm \frac{F_{ext}/m}{A} = \frac{k}{m} \pm \frac{F_{ext}}{mA} = \frac{6.30\text{ N/m}}{0.150\text{ kg}} \pm \frac{1.70\text{ N}}{(0.150\text{ kg})(0.440\text{ m})}$$

This yields $\omega = 8.23\text{ rad/s}$ or $\omega = 4.03\text{ rad/s}$

Then, $f = \dfrac{\omega}{2\pi}$ gives either $f = \boxed{1.31\text{ Hz}}$ or $f = \boxed{0.641\text{ Hz}}$

***12.35** The beeper must resonate at the frequency of a simple pendulum of length 8.21 cm:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{0.0821\text{ m}}} = \boxed{1.74\text{ Hz}}$$

***12.36** For the resonance vibration with the occupants in the car, we have for the spring constant of the suspension

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$: $$k = 4\pi^2 f^2 m = 4\pi^2\left(1.8\text{ s}^{-1}\right)^2\left(1130\text{ kg} + 4(72.4\text{ kg})\right) = 1.82\times10^5\text{ kg/s}^2$$

Now as the occupants exit $$x = \frac{F}{k} = \frac{4(72.4\text{ kg})\left(9.8\text{ m/s}^2\right)}{1.82\times10^5\text{ kg/s}^2} = \boxed{1.56\times10^{-2}\text{ m}}$$

12.37 Suppose a 100-kg biker compresses the suspension 2.00 cm.

Then, $$k = \frac{F}{x} = \frac{980\text{ N}}{2.00\times10^{-2}\text{ m}} = 4.90\times10^4\text{ N/m}$$

If total mass of motorcycle and biker is 500 kg, the frequency of free vibration is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.90\times10^4\text{ N/m}}{500\text{ kg}}} = 1.58\text{ Hz}$$

If he encounters washboard bumps at the same frequency, resonance will make the motorcycle bounce a lot. Assuming a speed of 20.0 m/s, we find these ridges are separated by

$$\frac{20.0\text{ m/s}}{1.58\text{ s}^{-1}} = 12.7\text{ m} \; \boxed{\sim10^1\text{ m}}$$

In addition to this vibration mode of bouncing up and down as one unit, the motorcycle can also vibrate at higher frequencies by rocking back and forth between front and rear wheels, by having just the front wheel bounce inside its fork, or by doing other things. Other spacings of bumps will excite all of these other resonances.

***12.38** (a) Total energy $= \frac{1}{2}kA^2 = \frac{1}{2}(100 \text{ N/m})(0.200 \text{ m})^2 = 2.00 \text{ J}$

At equilibrium, the total energy is: $\frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}(16.0 \text{ kg})v^2 = (8.00 \text{ kg})v^2$

Therefore, $(8.00 \text{ kg})v^2 = 2.00 \text{ J}$, and $v = \boxed{0.500 \text{ m/s}}$

This is the speed of m_1 and m_2 at the equilibrium point. Beyond this point, the mass m_2 moves with the constant speed of 0.500 m/s while mass m_1 starts to slow down due to the restoring force of the spring.

(b) The energy of the m_1–spring system at equilibrium is:

$$\frac{1}{2}m_1v^2 = \frac{1}{2}(9.00 \text{ kg})(0.500 \text{ m/s})^2 = 1.125 \text{ J}$$

This is also equal to $\frac{1}{2}k(A')^2$, where A' is the amplitude of the m_1–spring system.

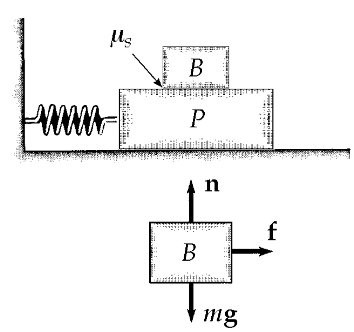
Therefore, $\frac{1}{2}(100)(A')^2 = 1.125$ or $A' = 0.150 \text{ m}$

The period of the m_1–spring system is: $T = 2\pi\sqrt{\frac{m_1}{k}} = 1.885 \text{ s}$

and it takes $\frac{1}{4}T = 0.471 \text{ s}$ after it passes the equilibrium point for the spring to become fully stretched the first time. The distance separating m_1 and m_2 at this time is:

$$D = v\left(\frac{T}{4}\right) - A' = 0.500 \text{ m/s}(0.471 \text{ s}) - 0.150 \text{ m} = 0.0856 = \boxed{8.56 \text{ cm}}$$

12.39 $\left(\frac{d^2x}{dt^2}\right)_{\max} = A\omega^2$



$f_{\max} = \mu_s n = \mu_s mg = mA\omega^2$

$A = \frac{\mu_s g}{\omega^2} = \boxed{6.62 \text{ cm}}$

***12.40** (a) In $x = A\cos(\omega t + \phi)$, $v = -\omega A\sin(\omega t + \phi)$

we have at $t = 0$ $v = -\omega A\sin\phi = -v_{max}$

This requires $\phi = 90°$, so $x = A\cos(\omega t + 90°)$

And this is equivalent to $x = -A\sin\omega t$.

Numerically we have $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50\text{ N/m}}{0.5\text{ kg}}} = 10\text{ s}^{-1}$

and $v_{max} = \omega A$ $20\text{ m/s} = (10\text{ s}^{-1})A$ $A = 2\text{ m}$

So $\boxed{x = (-2\text{ m})\sin[(10\text{ s}^{-1})t]}$

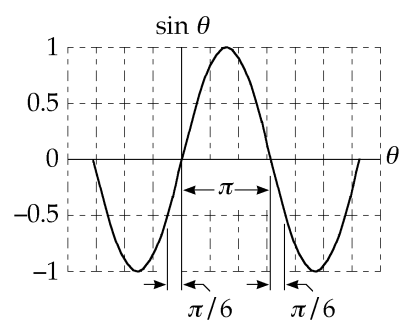
(b) In $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$, $\frac{1}{2}kx^2 = 3\left(\frac{1}{2}mv^2\right)$

implies $\frac{1}{3}\frac{1}{2}kx^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ $\frac{4}{3}x^2 = A^2$

$x = \pm\sqrt{\frac{3}{4}}A = \pm 0.866A = \boxed{\pm 1.73\text{ m}}$

(c) $\omega = \sqrt{g/L}$ $L = g/\omega^2 = \frac{9.8\text{ m/s}^2}{(10\text{ s}^{-1})^2} = \boxed{0.0980\text{ m}}$

(d) In $x = (-2\text{ m})\sin[(10\text{ s}^{-1})t]$



the particle is at $x = 0$ at $t = 0$, at $10t = \pi s$, and so on.

The particle is at $x = 1\text{ m}$

when $-\frac{1}{2} = \sin[(10\text{ s}^{-1})t]$

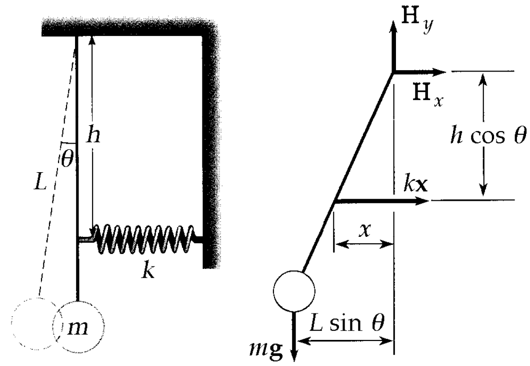
with solutions $(10\text{ s}^{-1})t = -\pi/6$,

$(10\text{ s}^{-1})t = \pi + \pi/6$, and so on.

The minimum time for the motion is Δt in $10\Delta t = (\pi/6)$ s

$\Delta t = (\pi/60)\text{ s} = \boxed{0.0524\text{ s}}$

12.41 We draw a free-body diagram of the pendulum. The force **H** exerted by the hinge causes no torque about the axis of rotation.



$\tau = I\alpha$ and $\frac{d^2\theta}{dt^2} = -\alpha$

$$\tau = MgL\sin\theta + kxh\cos\theta = -I\frac{d^2\theta}{dt^2}$$

For small amplitude vibrations, use the approximations: $\sin\theta \cong \theta$, $\cos\theta \cong 1$, and $x \cong s = h\theta$

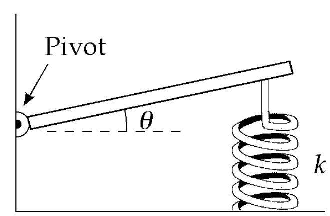
Therefore, $\frac{d^2\theta}{dt^2} = -\left(\frac{MgL + kh^2}{I}\right)\theta = -\omega^2\theta$ $\qquad \omega = \sqrt{\frac{MgL + kh^2}{ML^2}} = 2\pi f$

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{MgL + kh^2}{ML^2}}}$$

***12.42** Deuterium is the isotope of the element hydrogen with atoms having nuclei consisting of one proton and one neutron. For brevity we refer to the molecule formed by two deuterium atoms as *D* and to the diatomic molecule of hydrogen-1 as *H*.

$M_D = 2M_H$ $\qquad \frac{\omega_D}{\omega_H} = \frac{\sqrt{k/M_D}}{\sqrt{k/M_H}} = \sqrt{\frac{M_H}{M_D}} = \sqrt{\frac{1}{2}}$ $\qquad f_D = \frac{f_H}{\sqrt{2}} = \boxed{0.919 \times 10^{14}\ \text{Hz}}$

12.43 (a) At equilibrium, we have



$$\Sigma\tau = 0 = -mg\left(\frac{L}{2}\right) + kx_0L$$

where x_0 is the equilibrium compression.
After displacement by a small angle,

$$\Sigma\tau = -mg\left(\frac{L}{2}\right) + kxL = -mg\left(\frac{L}{2}\right) + k(x_0 - L\theta)L = -k\theta L^2$$

But, $\Sigma\tau = I\alpha = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$. $\qquad$ So $\frac{d^2\theta}{dt^2} = -\frac{3k}{m}\theta$

The angular acceleration is opposite in direction and proportional to the displacement, so we have simple harmonic motion with $\boxed{\omega^2 = \frac{3k}{m}}$.

(b) $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}} = \frac{1}{2\pi}\sqrt{\frac{3(100\ \text{N/m})}{5.00\ \text{kg}}} = \boxed{1.23\ \text{Hz}}$

***12.44** As it passes through equilibrium, the 4-kg object has speed

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}}A = \sqrt{\frac{100\text{ N/m}}{4\text{ kg}}}2\text{ m} = 10.0\text{ m/s}$$

In the completely inelastic collision momentum of the two-object system is conserved. So the new 10-kg object starts its oscillation with speed given by

$$4\text{ kg}(10\text{ m/s}) + (6\text{ kg})0 = (10\text{ kg})v_{\max}$$

$$v_{\max} = 4.00\text{ m/s}$$

(a) The new amplitude is given by $\frac{1}{2}mv_{\max}{}^2 = \frac{1}{2}kA^2$

$$10\text{ kg}(4\text{ m/s})^2 = (100\text{ N/m})A^2$$

$$A = 1.26\text{ m}$$

Thus the amplitude has $\boxed{\text{decreased by}}$ $2.00\text{ m} - 1.26\text{ m} = \boxed{0.735\text{ m}}$

(b) The old period was $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\text{ kg}}{100\text{ N/m}}} = 1.26\text{ s}$

The new period is $T = 2\pi\sqrt{\frac{10}{100}\text{ s}^2} = 1.99\text{ s}$

The period has $\boxed{\text{increased by}}$ $1.99\text{ m} - 1.26\text{ m} = \boxed{0.730\text{ s}}$

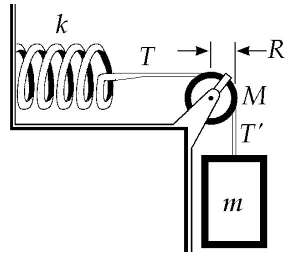
(c) The old energy was $\frac{1}{2}mv_{\max}{}^2 = \frac{1}{2}(4\text{ kg})(10\text{ m/s})^2 = 200\text{ J}$

The new mechanical energy is $\frac{1}{2}(10\text{ kg})(4\text{ m/s})^2 = 80\text{ J}$

The energy has $\boxed{\text{decreased by 120 J}}$

(d) The missing mechanical energy has turned into internal energy in the completely inelastic collision.

***12.45** One can write the following equations of motion:



$T - kx = 0$ (describes the spring)

$mg - T' = ma = m\dfrac{d^2x}{dt^2}$ (for the hanging object)

$R(T' - T) = I\dfrac{d^2\theta}{dt^2} = \dfrac{I}{R}\dfrac{d^2x}{dt^2}$ (for the pulley)

with $I = \frac{1}{2}MR^2$

Combining these equations gives the equation of motion

$$\left(m + \tfrac{1}{2}M\right)\frac{d^2x}{dt^2} + kx = mg$$

The solution is $x(t) = A\sin\omega t + \dfrac{mg}{k}$ (where mg/k arises because of the extension of the spring due to the weight of the hanging object), with frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m + \frac{1}{2}M}} = \frac{1}{2\pi}\sqrt{\frac{100\text{ N/m}}{0.200\text{ kg} + \frac{1}{2}M}}$$

(a) For $M = 0$ $f = \boxed{3.56\text{ Hz}}$

(b) For $M = 0.250$ kg $f = \boxed{2.79\text{ Hz}}$

(c) For $M = 0.750$ kg $f = \boxed{2.10\text{ Hz}}$

12.46 (a) $\omega_0 = \sqrt{\dfrac{k}{m}} = \boxed{15.8\text{ rad/s}}$

(b) $F_s - mg = ma = m\left(\frac{1}{3}g\right)$ $F_s = \frac{4}{3}mg = 26.1$ N

$x_s = \dfrac{F_s}{k} = \boxed{5.23\text{ cm}}$

(c) When the acceleration of the car is zero, the new equilibrium position can be found as follows:

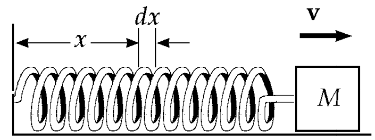
$F_s' = mg = 19.6\text{ N} = kx_s'$ $x_s' = 3.92$ cm

Thus, $A = |x_s' - x_s| = \boxed{1.31\text{ cm}}$

The phase constant is $\boxed{\pi\text{ rad}}$

12.47 (a) For each segment of the spring

$$dK = \tfrac{1}{2}(dm)v_x^{\,2}$$



Also, $v_x = \frac{x}{\ell}v$ and $dm = \frac{m}{\ell}dx$

Therefore, the total kinetic energy of the block-spring system is

$$K = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\int_0^\ell \left(\frac{x^2v^2}{\ell^2}\right)\frac{m}{\ell}dx = \boxed{\tfrac{1}{2}\left(M + \frac{m}{3}\right)v^2}$$

(b) $\omega = \sqrt{\dfrac{k}{m_{eff}}}$ and $\tfrac{1}{2}m_{eff}v^2 = \tfrac{1}{2}\left(M + \frac{m}{3}\right)v^2$

Therefore, $T = \dfrac{2\pi}{\omega} = \boxed{2\pi\sqrt{\dfrac{M + \frac{m}{3}}{k}}}$

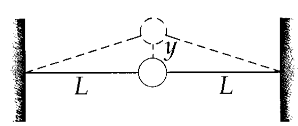
12.48 (a) $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{L}{g}} = \boxed{3.00 \text{ s}}$

(b) $E = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(6.74)(2.06)^2 = \boxed{14.3 \text{ J}}$

(c) At maximum angular displacement $mgh = \tfrac{1}{2}mv^2$ $h = \dfrac{v^2}{2g} = 0.217 \text{ m}$

$h = L - L\cos\theta = L(1 - \cos\theta)$ $\cos\theta = 1 - \dfrac{h}{L}$ $\boxed{\theta = 25.5°}$

12.49 (a) $\Sigma\mathbf{F} = -2T\sin\theta\,\mathbf{j}$ where $\theta = \tan^{-1}\left(\dfrac{y}{L}\right)$



Therefore, for a small displacement

$\sin\theta \cong \tan\theta = \dfrac{y}{L}$ and $\boxed{\Sigma\mathbf{F} = \dfrac{-2Ty}{L}\,\mathbf{j}}$

(b) The total force exerted on the ball is opposite in direction and proportional to its displacement from equilibrium, so the ball moves with simple harmonic motion. For a spring system,

$\Sigma\mathbf{F} = -k\mathbf{x}$ becomes here $\Sigma\mathbf{F} = -\dfrac{2T}{L}\mathbf{y}$

Therefore, the effective spring constant is $2T/L$ and $\boxed{\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{2T}{mL}}}$

12.50 (a) Assuming a Hooke's Law type spring,

$$F = Mg = kx$$

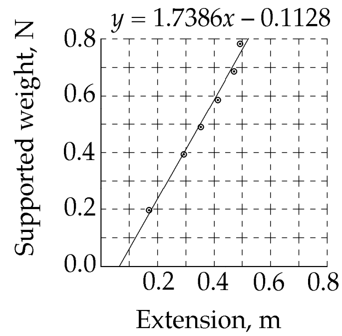
and empirically

$$Mg = 1.74x - 0.113$$

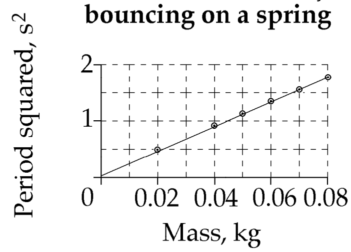
so $k \cong \boxed{1.74 \text{ N/m} \pm 6\%}$

M, kg	x, m	Mg, N
0.0200	0.17	0.196
0.0400	0.293	0.392
0.0500	0.353	0.49
0.0600	0.413	0.588
0.0700	0.471	0.686
0.0800	0.493	0.784

Static stretching of a spring



Squared period as a function of the mass of an object bouncing on a spring



(b) We may write the equation as theoretically

$$T^2 = \frac{4\pi^2}{k}M + \frac{4\pi^2}{3k}m_s$$

and empirically

$$T^2 = 21.7M + 0.0589$$

so $k = \dfrac{4\pi^2}{21.7} \cong \boxed{1.82 \text{ N/m} \pm 3\%}$

Time, s	T, s	M, kg	T^2, s^2
7.03	0.703	0.0200	0.494
9.62	0.962	0.0400	0.925
10.67	1.067	0.0500	1.138
11.67	1.167	0.0600	1.362
12.52	1.252	0.0700	1.568
13.41	1.341	0.0800	1.798

The k values $1.74 \text{ N/m} \pm 6\%$

and $1.82 \text{ N/m} \pm 3\%$ differ by 4%

so $\boxed{\text{they agree.}}$

(c) Utilizing the axis-crossing point, $m_s = 3\left(\dfrac{0.0589}{21.7}\right) \text{kg} \cong \boxed{8 \text{ grams} \pm 12\%}$

$\boxed{\text{in agreement}}$ with 7.4 grams.

12.51 (a) Newton's law of universal gravitation is $F = -\frac{GMm}{r^2} = -\frac{Gm}{r^2}\left(\frac{4}{3}\pi r^3\right)\rho$

Thus, $F = -\left(\frac{4}{3}\pi \rho Gm\right)r$

Which is of Hooke's law form with $k = \frac{4}{3}\pi \rho Gm$

(b) The sack of mail moves without friction according to $-\left(\frac{4}{3}\right)\pi \rho Gmr = ma$

$$a = -\left(\frac{4}{3}\right)\pi \rho Gr = -\omega^2 r$$

Since acceleration is a negative constant times excursion from equilibrium, it executes SHM with

$\omega = \sqrt{\frac{4\pi \rho G}{3}}$ and period $T = \frac{2\pi}{\omega} = \sqrt{\frac{3\pi}{\rho G}}$

The time for a one-way trip through the earth is $\frac{T}{2} = \sqrt{\frac{3\pi}{4\rho G}}$

We have also $g = \frac{GM_e}{R_e^2} = \frac{G4\pi R_e^3\rho}{3R_e^2} = \frac{4}{3}\pi \rho GR_e$

so $\frac{4\rho G}{3} = \frac{g}{(\pi R_e)}$ and $\frac{T}{2} = \pi\sqrt{\frac{R_e}{g}} = \pi\sqrt{\frac{6.37\times 10^6\text{ m}}{9.8\text{ m/s}^2}} = 2.53\times 10^3\text{ s} = \boxed{42.2\text{ min}}$

12.52 (a) We require $Ae^{-bt/2m} = \frac{A}{2}$ $e^{+bt/2m} = 2$

or $\frac{bt}{2m} = \ln 2$ or $\frac{0.100\text{ kg/s}}{2(0.375\text{ kg})}t = 0.693$ $\therefore\ t = \boxed{5.20\text{ s}}$

The spring constant is irrelevant.

(b) We can evaluate the energy at successive turning points, where

$\cos(\omega t + \phi) = \pm 1$ and the energy is $\frac{1}{2}kx^2 = \frac{1}{2}kA^2e^{-bt/m}$ We require $\frac{1}{2}kA^2e^{-bt/m} = \frac{1}{2}\left(\frac{1}{2}kA^2\right)$

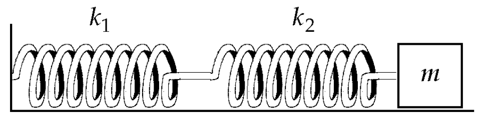
or $e^{+bt/m} = 2$ $\therefore\ t = \frac{m\ln 2}{b} = \frac{0.375\text{ kg}(0.693)}{0.100\text{ kg/s}} = \boxed{2.60\text{ s}}$

(c) From $E = \frac{1}{2}kA^2$, the fractional rate of change of energy over time is

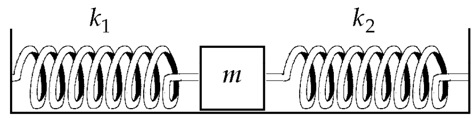
$$\frac{dE/dt}{E} = \frac{\frac{d}{dt}\left(\frac{1}{2}kA^2\right)}{\frac{1}{2}kA^2} = \frac{\frac{1}{2}k(2A)\frac{dA}{dt}}{\frac{1}{2}kA^2} = 2\frac{dA/dt}{A}$$

two times faster than the fractional rate of change in amplitude.

12.53 (a) When the mass is displaced a distance x from equilibrium, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2.



(a)



(b)

By Newton's third law, we expect

$$k_1 x_1 = k_2 x_2.$$

When this is combined with the requirement that

$$x = x_1 + x_2,$$

we find $$x_1 = \left[\frac{k_2}{k_1 + k_2}\right]x$$

The force on either spring is given by $$F_1 = \left[\frac{k_1 k_2}{k_1 + k_2}\right]x = ma$$

where a is the acceleration of the mass m.

This is in the form $$F = k_{eff} x = ma$$

and $$T = 2\pi\sqrt{\frac{m}{k_{eff}}} = \boxed{2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}}$$

(b) In this case each spring is distorted by the distance x which the mass is displaced. Therefore, the restoring force is

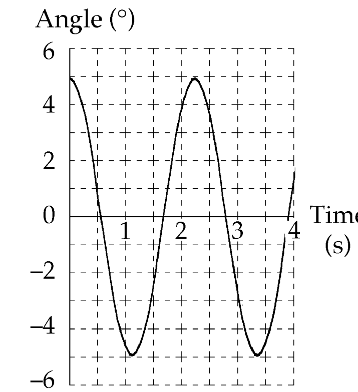
$$F = -(k_1 + k_2)x \qquad \text{and} \qquad k_{eff} = k_1 + k_2$$

so that $$T = \boxed{2\pi\sqrt{\frac{m}{(k_1 + k_2)}}}$$

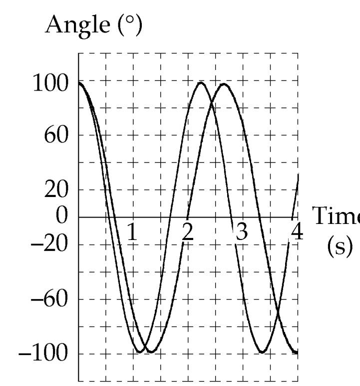
12.54 For $\theta_{max} = 5.00°$, the motion calculated by the Euler method agrees quite precisely with the prediction of $\theta_{max} \cos \omega t$. The period is $T = 2.20$ s.

Time, t (s)	Angle, θ (°)	Ang. speed (°/s)	Ang. Accel. $\left(°/s^2\right)$	$\theta_{max} \cos \omega t$
0.000	5.0000	0.0000	–40.7815	5.0000
0.004	4.9993	–0.1631	–40.7762	4.9997
0.008	4.9980	–0.3262	–40.7656	4.9987
…				
0.544	0.0560	–14.2823	–0.4576	0.0810
0.548	–0.0011	–14.2842	0.0090	0.0239
0.552	–0.0582	–14.2841	0.4756	–0.0333
…				
1.092	–4.9994	–0.3199	40.7765	–4.9989
1.096	–5.0000	–0.1568	40.7816	–4.9998
1.100	–5.0000	0.0063	40.7814	–5.0000
1.104	–4.9993	0.1694	40.7759	–4.9996
…				
1.644	–0.0638	14.2824	0.4397	–0.0716
1.648	0.0033	14.2842	–0.0270	–0.0145
1.652	0.0604	14.2841	–0.4936	0.0427
…				
2.192	4.9994	0.3137	–40.7768	4.9991
2.196	5.0000	0.1506	–40.7817	4.9999
2.200	5.0000	–0.0126	–40.7813	5.0000
2.204	4.9993	–0.1757	–40.7756	4.9994

Motion of a Simple Pendulum



Small amplitude



Large amplitude

For $\theta_{max} = 100°$, the simple harmonic motion approximation $\theta_{max} \cos \omega t$ diverges greatly from the Euler calculation. The period is $T = 2.71$ s, larger than the small–angle period by 23%.

Time, t (s)	Angle, θ (°)	Ang. speed (°/s)	Ang. Accel. $\left(°/s^2\right)$	$\theta_{max} \cos \omega t$
0.000	100.0000	0.0000	–460.6066	100.0000
0.004	99.9926	–1.8432	–460.8173	99.9935
0.008	99.9776	–3.6865	–460.8382	99.9739
…				
1.096	–84.7449	–120.1910	465.9488	–99.9954
1.100	–85.2182	–118.3272	466.2869	–99.9998
1.104	–85.6840	–116.4620	466.5886	–99.9911
…				
1.348	–99.9960	–3.0533	460.8125	–75.7979
1.352	–100.0008	–1.2100	460.8057	–75.0474
1.356	–99.9983	0.6332	460.8093	–74.2870
…				
2.196	40.1509	224.8677	–301.7132	99.9971
2.200	41.0455	223.6609	–307.2607	99.9993
2.204	41.9353	222.4318	–312.7035	99.9885
…				
2.704	99.9985	2.4200	–460.8090	12.6422
2.708	100.0008	0.5768	–460.8057	11.5075
2.712	99.9957	–1.2664	–460.8129	10.3712

***12.55** (a) The block moves with the board in what we take as the positive x direction, stretching the spring until the spring force $-kx$ is equal in magnitude to the maximum force of static friction $\mu_s n = \mu_s mg$. This occurs at $x = \mu_s mg / k$.

(b) Since v is small, the block is nearly at the rest at this break point. It starts almost immediately to move back to the left, the forces on it being $-kx$ and $+\mu_k mg$. While it is sliding the net force exerted on it can be written as

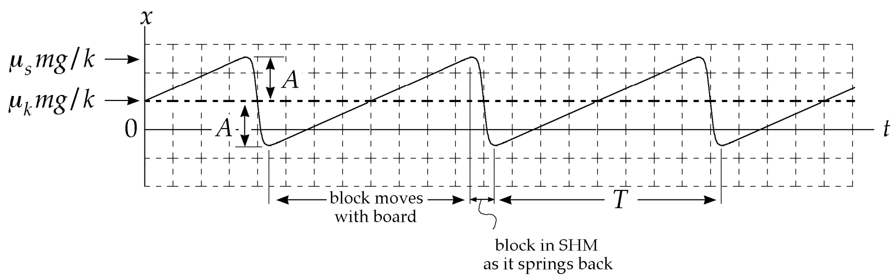
$$-kx + \mu_k mg = -kx + \frac{k\mu_k mg}{k} = -k\left(x - \frac{\mu_k mg}{k}\right) = -kx_{rel}$$

where x_{rel} is the excursion of the block away from the point $\mu_k mg / k$.

Conclusion: the block goes into simple harmonic motion centered about the equilibrium position where the spring is stretched by $\mu_k mg / k$.

(d) The amplitude of its motion is its original displacement, $A = \mu_s mg / k - \mu_k mg / k$. It first comes to rest at spring extension $\mu_k mg / k - A = (2\mu_k - \mu_s)mg / k$. Almost immediately at this point it latches onto the slowly-moving board to move with the board. The board exerts a force of static friction on the block, and the cycle continues.

(c) The graph of the motion looks like this:



(e) The time during each cycle when the block is moving with the board is $2A / v = 2(\mu_s - \mu_k)mg / kv$. The time for which the block is springing back is one half a cycle of simple harmonic motion, $\frac{1}{2}\left(2\pi\sqrt{m/k}\right) = \pi\sqrt{m/k}$. We ignore the times at the end points of the motion when the speed of the block changes from v to 0 and from 0 to v. Since v is small compared to $2A / \pi\sqrt{m/k}$, these times are negligible. Then the period is

$$T = \frac{2(\mu_s - \mu_k)mg}{kv} + \pi\sqrt{\frac{m}{k}}$$

(f) $$T = \frac{2(0.4 - 0.25)(0.3 \text{ kg})(9.8 \text{ m/s}^2)}{0.024 \text{ m/s}(12 \text{ N/m})} + \pi\sqrt{\frac{0.3 \text{ kg}}{12 \text{ N/m}}} = 3.06 \text{ s} + 0.497 \text{ s} = 3.56 \text{ s}$$

Then $f = \frac{1}{T} = \boxed{0.281 \text{ Hz}}$

(g) $T = \frac{2(\mu_s - \mu_k)mg}{kv} + \pi\sqrt{\frac{m}{k}}$ increases as m increases, so the frequency $\boxed{\text{decreases}}$.

(h) As k increase, T decreases and f $\boxed{\text{increases}}$.

(i) As v increases, T decreases and f $\boxed{\text{increases}}$.

(j) As $(\mu_s - \mu_k)$ increases, T increases and f $\boxed{\text{decreases}}$.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. (a) See the solution (b) 1.82 s
(c) No, the force is not in the form of Hooke's law

4. (a) 20.0 cm (b) 94.2 cm/s as the particle passes through equilibrium
(c) 17.8 m/s^2 at maximum excursion from equilibrium

6. 0.627 s

8. (a) 2.40 s (b) 0.417 Hz (c) 2.62 rad/s

10. See the solution

12. 18.8 m/s, 7.11 km/s^2

14. (a) 1.26 s (b) 0.150 m/s, 0.750 m/s^2
(c) $x = -(3.00\text{ cm})\cos(5.00t)$, $v = (15.0\text{ cm/s})\sin(5.00t)$, $a = (75.0\text{ cm/s}^2)\cos(5.00t)$

16. (a) 0.542 kg (b) 1.81 s (c) 1.20 m/s^2

18. (a) 0.153 J (b) 0.784 m/s (c) 17.5 m/s^2

20. (a) 100 N/m (b) 1.13 Hz
(c) 1.41 m/s as the block passes through equilibrium
(d) 10.0 m/s^2 at maximum excursion from equilibrium (e) 2.00 J
(f) 1.33 m/s (g) 3.33 m/s^2

22. $\frac{g_{\text{Cambridge}}}{g_{\text{Tokyo}}} = 1.0015$

24. 1.42 s, 0.499 m

26. (a) 2.09 s (b) 4.08%

28. See the solution

30. See the solution

32. (a) 1.00 s (b) 5.09 cm

34. Either 1.31 Hz or 0.641 Hz

36. 1.56 cm

38. (a) 0.500 m/s (b) 8.56 cm

40. (a) $x = -(2\text{ m})\sin(10\,t)$ (b) at $x = \pm 1.73$ m
(c) 98.0 mm (d) 52.4 ms

42. 9.19×10^{13} Hz

44. (a) The amplitude is reduced by 0.735 m
(b) The period increases by 0.730 s
(c) The energy decreases by 120 J
(d) See the solution

46. (a) 15.8 rad/s (b) 5.23 cm (c) 1.31 cm, π rad

48. (a) 3.00 s (b) 14.3 J (c) 25.5°

50. See the solution. (a) $k = 1.74$ N/m $\pm$ 6%
(b) $k = 1.82$ N/m $\pm$ 3% showing agreement
(c) 8 grams $\pm$ 12% showing agreement

52. (a) 5.20 s (b) 2.60 s (c) See the solution

54. See the solution. For $\theta_{max} = 5.00°$ there is precise agreement. For $\theta_{max} = 100°$ there are large differences, and the period is 23% greater than small-angle period.