CHAPTER 13

ANSWERS TO QUESTIONS

Q13.1 Wind can change a Doppler shift but cannot cause one. Both v_o and v_s in our equations must be interpreted as speeds of observer and source relative to the air. If source and observer are moving relative to each other, the observer will hear one shifted frequency in still air and a different shifted frequency if wind is blowing. If the distance between source and observer is constant, there will never be a Doppler shift.

Q13.2 To use a slinky to create a longitudinal wave, pull a few coils back and release. For a transverse wave, jostle the end coil side to side.

Q13.3 $v = \sqrt{T/\mu}$. Therefore, increase tension by a factor of 4.

Q13.4 It depends on what the wave reflects from. If reflecting from a less dense string, the reflected part of the wave will be right-side-up.

Q13.5 No. A wave is not a solid object, but a chain of disturbance. As described by the principle of superposition, the waves move through each other.

Q13.6 Yes. $v_{\text{max}} = \omega A = 2\pi f A$ where $f = v_{\text{wave}}/\lambda$.

Q13.7 Amplitude is increased by a factor of $\sqrt{2}$. No, the wave speed does not depend on the amplitude.

Q13.8 The section of rope moves up and down in SHM. The wave continues on, setting in motion, up and down, further sections of the rope.

Q13.9 Each element of the rope must support the weight of the rope below it. The tension increases with height. (It increases linearly, if the rope does not stretch.) Then the wave speed $v = \sqrt{T/\mu}$ increases with height.

Q13.10 As the source frequency is doubled, the speed of waves on the string stays constant and the wavelength is reduced by one half.

Q13.11 As the source frequency is doubled, the speed of waves on the string stays constant.

Q13.12 Slower. Wave speed is inversely proportional to the square root of linear density.

Q13.13 Higher tension makes wave speed higher. Greater linear density makes the wave move more slowly.

Q13.14 The wave speed is independent of the maximum particle speed. The source determines the maximum particle speed, through its frequency and amplitude. The wave speed depends instead on properties of the medium.

Q13.15 A fluid cannot support shear forces.

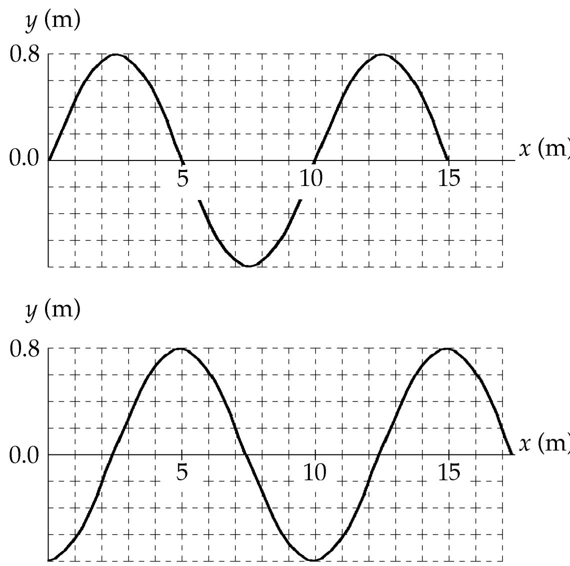
Q13.16 Let $\Delta t = t_s - t_p$ represent the difference in arrival times of the two waves at a station at distance $d = v_s t_s = v_p t_p$ from the epicenter. Then $d = \Delta t(1/v_s - 1/v_p)^{-1}$. Knowing the distance from the first station places the epicenter on a sphere around it. A measurement from a second station limits it to another sphere, which intersects with the first in a circle. Data from a third station will generally limit the possibilities to a point.

PROBLEM SOLUTIONS

13.1 Replace x by $x - vt = x - 4.5t$

to get $$y = \frac{6}{\left[(x-4.5t)^2 + 3\right]}$$

13.2



13.3 $f = \dfrac{40.0 \text{ vibrations}}{30.0 \text{ s}} = \dfrac{4}{3}$ Hz $\qquad v = \dfrac{425 \text{ cm}}{10.0 \text{ s}} = 42.5$ cm/s

$$\lambda = \frac{v}{f} = \frac{42.5 \text{ cm/s}}{\frac{4}{3} \text{ Hz}} = 31.9 \text{ cm} = \boxed{0.319 \text{ m}}$$

***13.4** Using data from the observations, we have $\lambda = 1.20$ m

and $f = \dfrac{8.00}{12.0 \text{ s}}$

Therefore, $v = \lambda f = (1.20 \text{ m})\left(\dfrac{8.00}{12.0 \text{ s}}\right) = \boxed{0.800 \text{ m/s}}$

13.5 (a) Let $u = 10\pi t - 3\pi x + \frac{\pi}{4}$ $\qquad \frac{du}{dt} = 10\pi - 3\pi \frac{dx}{dt} = 0$ at a point of constant phase

$$\frac{dx}{dt} = \frac{10}{3} = \boxed{3.33\ \text{m/s}}$$

The velocity is in the $\boxed{\text{positive } x\text{-direction.}}$

(b) $y(0.100,\ 0) = (0.350\ \text{m})\sin\left(-0.300\pi + \frac{\pi}{4}\right) = -0.0548\ \text{m} = \boxed{-5.48\ \text{cm}}$

(c) $k = \frac{2\pi}{\lambda} = 3\pi$: $\quad \lambda = \boxed{0.667\ \text{m}}$ $\qquad \omega = 2\pi f = 10\pi$: $\quad f = \boxed{5.00\ \text{Hz}}$

(d) $v_y = \frac{\partial y}{\partial t} = (0.350)(10\pi)\cos\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$ $\qquad v_{y,\text{max}} = (10\pi)(0.350) = \boxed{11.0\ \text{m/s}}$

13.6 $y = (0.0200\ \text{m})\sin(2.11x - 3.62t)$ in SI units $\qquad A = \boxed{2.00\ \text{cm}}$

$k = 2.11\ \text{rad/m}$ $\qquad \lambda = \frac{2\pi}{k} = \boxed{2.98\ \text{m}}$

$\omega = 3.62\ \text{rad/s}$ $\qquad f = \frac{\omega}{2\pi} = \boxed{0.576\ \text{Hz}}$

$$v = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{\omega}{k} = \frac{3.62}{2.11} = \boxed{1.72\ \text{m/s}}$$

***13.7** (a) $\omega = 2\pi f = 2\pi\left(5\ \text{s}^{-1}\right) = \boxed{31.4\ \text{rad/s}}$

(b) $\lambda = \frac{v}{f} = \frac{20\ \text{m/s}}{5\ \text{s}^{-1}} = 4.00\ \text{m}$ $\qquad k = \frac{2\pi}{\lambda} = \frac{2\pi}{4\ \text{m}} = \boxed{1.57\ \text{rad/m}}$

(c) In $y = A\sin(kx - \omega t + \phi)$ we take $A = 12\ \text{cm}$

At $x = 0$ and $t = 0$ we have $y = (12\ \text{cm})\sin\phi$

So that $y = 0$ we take $\phi = 0$

Then $\boxed{y = (12.0\ \text{cm})\sin\left((1.57\ \text{rad/m})x - (31.4\ \text{rad/s})t\right)}$

(d) The transverse velocity is $\frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$

Its maximum magnitude is $A\omega = 12\ \text{cm}(31.4\ \text{rad/s}) = \boxed{3.77\ \text{m/s}}$

(e) $$a_y = \frac{\partial v_y}{\partial t} = \frac{\partial}{\partial t}\left(-A\omega\cos(kx - \omega t)\right) = -A\omega^2\sin(kx - \omega t)$$

The maximum value is $A\omega^2 = (0.12\ \text{m})\left(31.4\ \text{s}^{-1}\right)^2 = \boxed{118\ \text{m/s}^2}$

13.8 $y = (0.120 \text{ m}) \sin\left(\frac{\pi}{8}x + 4\pi t\right)$

(a) $v = \frac{dy}{dt}$: $v = (0.120)(4\pi)\cos\left(\frac{\pi}{8}x + 4\pi t\right)$

$v(0.200 \text{ s}, 1.60 \text{ m}) = \boxed{-1.51 \text{ m/s}}$

$a = \frac{dv}{dt}$: $a = (-0.120 \text{ m})(4\pi)^2 \sin\left(\frac{\pi}{8}x + 4\pi t\right)$

$a(0.200 \text{ s}, 1.60 \text{ m}) = \boxed{0}$

(b) $k = \frac{\pi}{8} = \frac{2\pi}{\lambda}$: $\lambda = \boxed{16.0 \text{ m}}$

$\omega = 4\pi = \frac{2\pi}{T}$: $T = \boxed{0.500 \text{ s}}$

$v = \frac{\lambda}{T} = \frac{16.0 \text{ m}}{0.500 \text{ s}} = \boxed{32.0 \text{ m/s}}$

13.9 (a) $A = y_{max} = 8.00 \text{ cm} = 0.0800 \text{ m}$ $k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.800 \text{ m})} = 7.85 \text{ m}^{-1}$

$\omega = 2\pi f = 2\pi(3.00) = 6.00\pi \text{ rad/s}$

Therefore, $y = A \sin(kx + \omega t)$

Or (where $y(0, t) = 0$ at $t = 0$) $\boxed{y = (0.0800)\sin(7.85x + 6\pi t) \text{ m}}$

(b) In general, $y = 0.0800\sin(7.85x + 6\pi t + \phi)$

Assuming $y(x, 0) = 0$ at $x = 0.100 \text{ m}$

then we require that $0 = 0.0800 \sin(0.785 + \phi)$

or $\phi = -0.785$

Therefore, $\boxed{y = 0.0800\sin(7.85x + 6\pi t - 0.785) \text{ m}}$

13.10 (a) Let us write the wave function as $y(x, t) = A \sin(kx + \omega t + \phi)$

$$y(0, 0) = A \sin\phi = 0.0200 \text{ m}$$

$$\left.\frac{dy}{dt}\right|_{0,0} = A\omega \cos\phi = -2.00 \text{ m/s}$$

Also, $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.0250 \text{ s}} = 80.0\ \pi/\text{s}$

$$A^2 = x_i^2 + (v_i/\omega)^2 = (0.0200 \text{ m})^2 + \left(\frac{2.00 \text{ m/s}}{80.0\ \pi/\text{s}}\right)^2$$

$$A = \boxed{0.0215 \text{ m}}$$

(b) $\dfrac{A\sin\phi}{A\cos\phi} = \dfrac{0.0200}{-2/80.0\pi} = -2.51 = \tan\phi$

Your calculator's answer $\tan^{-1}(-2.51) = -1.19$ rad has a negative sine and positive cosine, just the reverse of what is required. You must look beyond your calculator to find

$$\phi = \pi - 1.19 \text{ rad} = \boxed{1.95 \text{ rad}}$$

(c) $v_{y,\max} = A\omega = 0.0215 \text{ m } (80.0\pi/\text{s}) = \boxed{5.41 \text{ m/s}}$

(d) $\lambda = v_x T = (30.0 \text{ m/s})(0.0250 \text{ s}) = 0.750 \text{ m}$

$k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.750 \text{ m}} = 8.38\ /\text{m}$ $\qquad \omega = 80.0\pi\ /\text{s}$

$$\boxed{y(x,t) = (0.0215 \text{ m})\sin\left(8.38x \text{ rad/m} + 80.0\pi t \text{ rad/s} + 1.95 \text{ rad}\right)}$$

***13.11** Equation 13.19 is $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$

If $y = e^{b(x-vt)}$

then $\dfrac{dy}{dt} = -bve^{b(x-vt)}$ and $\dfrac{dy}{dx} = be^{b(x-vt)}$

$\dfrac{d^2y}{dt^2} = b^2v^2e^{b(x-vt)}$ and $\dfrac{d^2y}{dx^2} = b^2e^{b(x-vt)}$

Therefore, $\dfrac{\partial^2 y}{\partial t^2} = v^2\dfrac{\partial^2 y}{\partial x^2}$, demonstrating that $e^{b(x-vt)}$ is a solution

13.12 The down and back distance is 4.00 m + 4.00 m = 8.00 m

The speed is then $v = \frac{d_{total}}{t} = \frac{4(8.00\text{ m})}{0.800\text{ s}} = 40.0\text{ m/s} = \sqrt{T/\mu}$

Now, $\mu = \frac{0.200\text{ kg}}{4.00\text{ m}} = 5.00\times10^{-2}\text{ kg/m}$

So $T = \mu v^2 = (5.00\times10^{-2}\text{ kg/m})(40.0\text{ m/s})^2 = \boxed{80.0\text{ N}}$

13.13 $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1350\text{ kg}\cdot\text{m/s}^2}{5.00\times10^{-3}\text{ kg/m}}} = \boxed{520\text{ m/s}}$

13.14 $T = Mg$ is the tension; $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{(m/L)}} = \sqrt{\frac{MgL}{m}} = \frac{L}{t}$ is the wave speed.

Then, $\frac{MgL}{m} = \frac{L^2}{t^2}$

and $g = \frac{Lm}{Mt^2} = \frac{1.60\text{ m}(4.00\times10^{-3}\text{ kg})}{3.00\text{ kg}(3.61\times10^{-3}\text{ s})^2} = \boxed{1.64\text{ m/s}^2}$

13.15 The total time is the sum of the two times.

In each wire $t = \frac{L}{v} = L\sqrt{\frac{\mu}{T}}$

Let A represent the cross-sectional area of one wire. The mass of one wire can be written both as $m = \rho V = \rho AL$ and also as $m = \mu L$.

Then we have $\mu = \rho A = \frac{\pi\rho d^2}{4}$

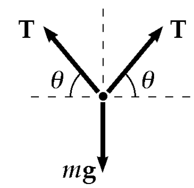
Thus, $t = L\left(\frac{\pi\rho d^2}{4T}\right)^{1/2}$

For copper, $t = (20.0)\left[\frac{(\pi)(8920)(1.00\times10^{-3})^2}{(4)(150)}\right]^{1/2} = 0.137\text{ s}$

For steel, $t = (30.0)\left[\frac{(\pi)(7860)(1.00\times10^{-3})^2}{(4)(150)}\right]^{1/2} = 0.192\text{ s}$

The total time is $0.137 + 0.192 = \boxed{0.329\text{ s}}$

13.16 From the free-body diagram $mg = 2T\sin\theta$



$$T = \frac{mg}{2\sin\theta}$$

The angle θ is found from $\cos\theta = \frac{3L/8}{L/2} = \frac{3}{4}$

$$\therefore\ \theta = 41.4°$$

(a) $v = \sqrt{\frac{T}{\mu}}$ $\quad v = \sqrt{\frac{mg}{2\mu\sin 41.4°}} = \left(\sqrt{\frac{9.80\ \text{m/s}^2}{2\left(8.00\times10^{-3}\ \text{kg/m}\right)\sin 41.4°}}\right)\sqrt{m}$

or $\quad v = \boxed{\left(30.4\frac{\text{m/s}}{\sqrt{\text{kg}}}\right)\sqrt{m}}$

(b) $v = 60.0 = 30.4\sqrt{m}$ and $\boxed{m = 3.89\ \text{kg}}$

***13.17** (a) If the end is fixed, there is inversion of the pulse upon reflection. Thus, when they meet, they cancel and the amplitude is $\boxed{\text{zero}}$.

(b) If the end is free, there is no inversion on reflection. When they meet, the amplitude is $2A = 2(0.150\ \text{m}) = \boxed{0.300\ \text{m}}$.

13.18 $f = \frac{v}{\lambda} = \frac{30.0}{0.500} = 60.0\ \text{Hz}$ $\quad \omega = 2\pi f = 120\pi\ \text{rad/s}$

$$\mathcal{P} = \tfrac{1}{2}\mu\omega^2A^2v = \tfrac{1}{2}\left(\frac{0.180}{3.60}\right)(120\pi)^2(0.100)^2(30.0) = \boxed{1.07\ \text{kW}}$$

13.19 $A = 5.00\times10^{-2}\ \text{m}$ $\quad \mu = 4.00\times10^{-2}\ \text{kg/m}$ $\quad \mathcal{P} = 300\ \text{W}$ $\quad T = 100\ \text{N}$

Therefore, $\quad v = \sqrt{\frac{T}{\mu}} = 50.0\ \text{m/s}$

$\mathcal{P} = \tfrac{1}{2}\mu\omega^2A^2v$: $\quad \omega^2 = \frac{2\mathcal{P}}{\mu A^2 v} = \frac{2(300)}{\left(4.00\times10^{-2}\right)\left(5.00\times10^{-2}\right)^2(50.0)}$

$$\omega = 346\ \text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \boxed{55.1\ \text{Hz}}$$

***13.20** T = constant; $v = \sqrt{\dfrac{T}{\mu}}$; $\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 v$

(a) If L is doubled, v remains constant and $\boxed{\mathcal{P} \text{ is constant}}$.

(b) If A is doubled and ω is halved, $\mathcal{P} \propto \omega^2 A^2$ $\boxed{\text{remains constant}}$.

(c) If λ and A are doubled, the product $\omega^2 A^2 \propto A^2/\lambda^2$ remains constant, so $\boxed{\mathcal{P} \text{ remains constant}}$.

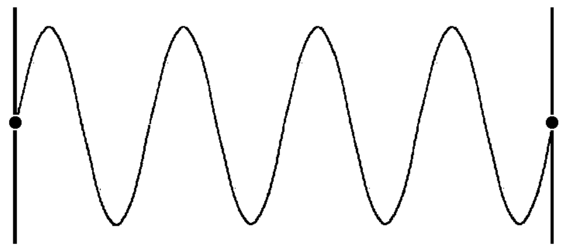
(d) If L and λ are halved, then $\omega^2 \propto 1/\lambda^2$ is quadrupled, so $\boxed{\mathcal{P} \text{ is quadrupled}}$.
(Changing L doesn't affect $\mathcal{P}$).

13.21 $\mu = 30.0 \text{ g/m} = 30.0 \times 10^{-3} \text{ kg/m}$

$\lambda = 1.50 \text{ m}$

$f = 50.0 \text{ Hz}$: $\omega = 2\pi f = 314 \text{ s}^{-1}$

$2A = 0.150 \text{ m}$: $A = 7.50 \times 10^{-2} \text{ m}$



(a) $y = A\sin\left(\dfrac{2\pi}{\lambda}x - \omega t\right)$ $\boxed{y = (7.50 \times 10^{-2})\sin(4.19x - 314t)}$

(b) $\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}(30.0 \times 10^{-3})(314)^2(7.50 \times 10^{-2})^2\left(\dfrac{314}{4.19}\right) \text{ W}$ $\boxed{\mathcal{P} = 625 \text{ W}}$

***13.22** $\lambda = \dfrac{v}{f} = \dfrac{340 \text{ m/s}}{60.0 \times 10^3 \text{ s}^{-1}} = \boxed{5.67 \text{ mm}}$

***13.23** Since $v_{light} >> v_{sound}$: $d \cong (343 \text{ m/s})(16.2 \text{ s}) = \boxed{5.56 \text{ km}}$

13.24 Sound takes this time to reach the man: $\dfrac{(20.0 \text{ m} - 1.75 \text{ m})}{343 \text{ m/s}} = 5.32 \times 10^{-2} \text{ s}$

so the warning should be shouted no later than $0.300 \text{ s} + 5.32 \times 10^{-2} \text{ s} = 0.353 \text{ s}$

before the pot strikes.

Since the whole time of fall is given by $y = \frac{1}{2}gt^2$: $18.25 \text{ m} = \frac{1}{2}(9.80 \text{ m/s}^2)t^2$

$t = 1.93 \text{ s}$

the warning needs to come $1.93 \text{ s} - 0.353 \text{ s} = 1.58 \text{ s}$

into the fall, when the pot has fallen $\frac{1}{2}(9.80 \text{ m/s}^2)(1.58 \text{ s})^2 = 12.2 \text{ m}$

to be above the ground by $20.0 \text{ m} - 12.2 \text{ m} = \boxed{7.82 \text{ m}}$

***13.25** (a) $A = \boxed{2.00\,\mu\text{m}}$

$$\lambda = \frac{2\pi}{15.7} = 0.400\text{ m} = \boxed{40.0\text{ cm}}$$

$$v = \frac{\omega}{k} = \frac{858}{157} = \boxed{54.6\text{ m/s}}$$

(b) $s = 2.00\cos[(15.7)(0.0500) - (858)(3.00\times10^{-3})] = \boxed{-0.433\,\mu\text{m}}$

(c) $v_{max} = A\omega = (2.00\,\mu\text{m})(858\text{ s}^{-1}) = \boxed{1.72\text{ mm/s}}$

13.26 $\Delta P_{max} = \rho\omega v\, s_{max} = (1.20\text{ kg/m}^3)[2\pi(2000\text{ s}^{-1})](343\text{ m/s})(2.00\times10^{-8}\text{ m})$

$\Delta P_{max} = \boxed{0.103\text{ Pa}}$

13.27 $\Delta P_{max} = \rho v\omega s_{max} = \rho v\left(\dfrac{2\pi v}{\lambda}\right)s_{max}$

$$\lambda = \frac{2\pi\rho v^2 s_{max}}{\Delta P_{max}} = \frac{2\pi(1.20)(343)^2\left(5.50\times10^{-6}\right)}{0.840} = \boxed{5.81\text{ m}}$$

13.28 (a) $\Delta P = \Delta P_{max}\sin[kx - \omega t + \phi]$ with $\Delta P_{max} = 4.00\text{ Pa}$

$\Delta P(0, 0) = \Delta P_{max}\sin\phi = 0$ so $\phi = 0$

$\omega = 2\pi f = 2\pi(5000\text{ s}^{-1}) = \pi\times10^4\text{ s}^{-1}$

Therefore, $\Delta P = (4.00\text{ Pa})\sin(kx - \pi\times10^4\, t/\text{s})$

At $x = 0$, $t = 2.00\times10^{-4}\text{ s}$

$\Delta P = (4.00\text{ Pa})\sin(0 - 2.00\pi) = \boxed{0}$

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{v} = \dfrac{\pi\times10^4\text{ s}^{-1}}{343\text{ m/s}} = 91.5\text{ m}^{-1}$

At $x = 0.0200\text{ m}$ $t = 0$ $\Delta P = (4.00\text{ Pa})\sin[(91.5\text{ m}^{-1})(0.0200\text{ m}) - 0]$

$\Delta P = \boxed{3.87\text{ Pa}}$

***13.29** $k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.100\text{ m})} = 62.8\text{ m}^{-1}$

$$\omega = \frac{2\pi v}{\lambda} = \frac{2\pi(343\text{ m/s})}{(0.100\text{ m})} = 2.16\times10^{4}\text{ s}^{-1}$$

Therefore, $\boxed{\Delta P = (0.200\text{ Pa})\sin\left[62.8\, x/\text{m} - 2.16\times10^{4}\, t/\text{s}\right]}$

13.30 (a) $f' = f\frac{v}{v - v_S}$

Approach: $f' = 320\left(\frac{343}{343 - (+40.0)}\right) = 362.2\text{ Hz}$

Receding: $f' = 320\left(\frac{343}{343 - (-40.0)}\right) = 286.5\text{ Hz}$

The *change* in frequency observed is a drop of $362 - 287 = \boxed{75.7\text{ Hz}}$

(b) $\lambda = \frac{v}{f'} = \frac{343\text{ m/s}}{362\text{ Hz}} = \boxed{0.948\text{ m}}$

13.31 Approaching ambulance: $f' = \frac{f}{\left(1 - \frac{v_S}{v}\right)}$

Departing ambulance: $f'' = \frac{f}{\left(1 - (-v_S/v)\right)}$

Since $f' = 560\text{ Hz}$ and $f'' = 480\text{ Hz}$ $\quad 560\left(1 - \frac{v_S}{v}\right) = 480\left(1 + \frac{v_S}{v}\right)$

$$1040\frac{v_S}{v} = 80.0$$

$$v_S = \frac{80.0(343)}{1040}\text{ m/s} = \boxed{26.4\text{ m/s}}$$

13.32 (a) $f' = \frac{f(v + v_o)}{(v - v_s)}$

$$f' = 2500\frac{(343 + 25.0)}{(343 - 40.0)} = \boxed{3.04\text{ kHz}}$$

(b) $f' = 2500\left(\frac{343 + (-25.0)}{343 - (-40.0)}\right) = \boxed{2.08\text{ kHz}}$

(c) $f' = 2500\left(\frac{343 + (-25.0)}{343 - 40.0}\right) = \boxed{2.62\text{ kHz}}$ while police car overtakes

$f' = 2500\left(\frac{343 + 25.0}{343 - (-40.0)}\right) = \boxed{2.40\text{ kHz}}$ after police car passes

13.33 $f' = f\left(\frac{v}{v - v_s}\right)$ $485 = 512\left(\frac{340}{340 - (-9.80\,t_{\text{fall}})}\right)$

$$485(340) + (485)(9.80t_f) = (512)(340)$$

$$t_f = \left(\frac{512 - 485}{485}\right)\frac{340}{9.80} = 1.93 \text{ s}$$

$d_1 = \frac{1}{2}gt_f^2 = 18.3 \text{ m}$: $t_{\text{return}} = \frac{18.3}{340} = 0.0538 \text{ s}$

The fork continues to fall while the sound returns.

$$t_{\text{total fall}} = t_f + t_{\text{return}} = 1.93 \text{ s} + 0.0538 \text{ s} = 1.985 \text{ s}$$

$$d_{\text{total}} = \frac{1}{2}gt_{\text{total fall}}^2 = \boxed{19.3 \text{ m}}$$

***13.34** (a) $\omega = 2\pi f = 2\pi\left(\frac{115/\text{min}}{60.0 \text{ s/min}}\right) = 12.0 \text{ rad/s}$

$$v_{\max} = \omega A = (12.0 \text{ rad/s})(1.80 \times 10^{-3} \text{ m}) = \boxed{0.0217 \text{ m/s}}$$

(b) The heart wall is a moving observer.

$$f' = f\left(\frac{v + v_o}{v}\right) = (2\,000\,000 \text{ Hz})\left(\frac{1500 + 0.0217}{1500}\right) = \boxed{2\,000\,028.9 \text{ Hz}}$$

(c) Now the heart wall is a moving source.

$$f'' = f'\left(\frac{v}{v - v_s}\right) = (2\,000\,029 \text{ Hz})\left(\frac{1500}{1500 - 0.0217}\right) = \boxed{2\,000\,057.8 \text{ Hz}}$$

13.35 The maximum speed of the speaker is described by $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$

$$v_{\max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{20.0 \text{ N/m}}{5.00 \text{ kg}}}(0.500 \text{ m}) = 1.00 \text{ m/s}$$

The frequencies heard by the stationary observer range from

$$f'_{\min} = f\left(\frac{v}{v - (-v_{\max})}\right) \quad \text{to} \quad f'_{\max} = f\left(\frac{v}{v - v_{\max}}\right)$$

where v is the speed of sound. $f'_{\min} = 440 \text{ Hz}\left(\frac{343 \text{ m/s}}{343 \text{ m/s} + 1.00 \text{ m/s}}\right) = \boxed{439 \text{ Hz}}$

$$f'_{\max} = 440 \text{ Hz}\left(\frac{343 \text{ m/s}}{343 \text{ m/s} - 1.00 \text{ m/s}}\right) = \boxed{441 \text{ Hz}}$$

***13.36** (a) $v = (331\text{ m/s}) + 0.6\dfrac{\text{m}}{\text{s}\cdot{}^\circ\text{C}}(-10^\circ\text{C}) = \boxed{325\text{ m/s}}$

(b) Approaching the bell, the athlete hears a frequency of $f' = f\left(\dfrac{v + v_O}{v}\right)$

After passing the bell, she hears a lower frequency of $f'' = f\left(\dfrac{v + (-v_O)}{v}\right)$

The ratio is $\dfrac{f''}{f'} = \dfrac{v - v_O}{v + v_O} = \dfrac{5}{6}$

which gives $6v - 6v_O = 5v + 5v_O$ or $v_O = \dfrac{v}{11} = \dfrac{325\text{ m/s}}{11} = \boxed{29.5\text{ m/s}}$

***13.37** (a) The $\boxed{\text{longitudinal}}$ wave travels a shorter distance and is moving faster, so it will arrive at point *B* first.

(b) The wave that travels through the Earth must travel

a distance of $2R\sin 30.0^\circ = 2(6.37\times10^6\text{ m})\sin 30.0^\circ = 6.37\times10^6\text{ m}$

at a speed of 7800 m/s

Therefore, it takes $\dfrac{6.37\times10^6\text{ m}}{7800\text{ m/s}} = 817\text{ s}$

The wave that travels along the Earth's surface must travel

a distance of $s = R\theta = R\left(\dfrac{\pi}{3}\text{ rad}\right) = 6.67\times10^6\text{ m}$

at a speed of 4500 m/s

Therefore, it takes $\dfrac{6.67\times10^6}{4500} = 1482\text{ s}$

The time difference is $\boxed{665\text{ s}} = 11.1\text{ min}$

***13.38** The distance the waves have traveled is $d = (7.80\text{ km/s})t = (4.50\text{ km/s})(t + 17.3\text{ s})$

where t is the travel time for the faster wave.

Then, $(7.80 - 4.50)(\text{km/s})t = (4.50\text{ km/s})(17.3\text{ s})$

or $t = \dfrac{(4.50\text{ km/s})(17.3\text{ s})}{(7.80 - 4.50)\text{ km/s}} = 23.6\text{ s}$

and the distance is $d = (7.80\text{ km/s})(23.6\text{ s}) = \boxed{184\text{ km}}$

***13.39** Assume a typical distance between adjacent people ~1 m.

Then the wave speed is $v = \frac{\Delta x}{\Delta t} \sim \frac{1\text{ m}}{0.1\text{ s}} \sim 10\text{ m/s}$

Model the stadium as a circle with a radius of order 100 m. Then, the time for one circuit around the stadium is

$$T = \frac{2\pi r}{v} \sim \frac{2\pi(10^2)}{10\text{ m/s}} = 63\text{ s} \quad \boxed{\sim 1\text{ min}}$$

***13.40** (a) $\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{1480\text{ s}^{-1}} = \boxed{0.232\text{ m}}$

(b) $\lambda' = \frac{v}{f'} = \frac{343\text{ m/s}}{1397\text{ s}^{-1}} = 0.246\text{ m}$ $\qquad \Delta\lambda = \lambda' - \lambda = \boxed{13.8\text{ mm}}$

***13.41** Assuming the incline to be frictionless and taking the positive *x*-direction to be up the incline:

$\sum F_x = T - Mg\sin\theta = 0$ or the tension in the string is $T = Mg\sin\theta$

The speed of transverse waves in the string is then $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg\sin\theta}{m/L}} = \sqrt{\frac{MgL\sin\theta}{m}}$

The time interval for a pulse to travel the string's length is $\Delta t = \frac{L}{v} = L\sqrt{\frac{m}{MgL\sin\theta}} = \boxed{\sqrt{\frac{mL}{Mg\sin\theta}}}$

13.42 $Mgx = \frac{1}{2}kx^2$

(a) $T = kx = \boxed{2Mg}$

(b) $L = L_0 + x = \boxed{L_0 + \frac{2Mg}{k}}$

(c) $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}} = \boxed{\sqrt{\frac{2Mg}{m}\left(L_0 + \frac{2Mg}{k}\right)}}$

13.43 Since $\cos^2\theta + \sin^2\theta = 1$, $\quad \sin\theta = \pm\sqrt{1-\cos^2\theta}$ (each sign applying half the time)

$$\Delta P = \Delta P_{max}\sin(kx - \omega t) = \pm\rho v\omega s_{max}\sqrt{1-\cos^2(kx-\omega t)}$$

Therefore $\quad \Delta P = \pm\rho v\omega\sqrt{s_{max}^2 - s_{max}^2\cos^2(kx-\omega t)} = \pm\rho v\omega\sqrt{s_{max}^2 - s^2}$

***13.44** (a) $\mu(x)$ is a linear function, so it is of the form $\mu(x) = mx + b$

To have $\mu(0) = \mu_0$ we require $b = \mu_0$. Then $\mu(L) = \mu_L = mL + \mu_0$

so $m = (\mu_L - \mu_0)/L$

Then $\boxed{\mu(x) = (\mu_L - \mu_0)x/L + \mu_0}$

(b) From $v = dx/dt$, the time required to move from x to $x + dx$ is dx/v. The time required to move from 0 to L is

$$t = \int_0^L \frac{dx}{v} = \int_0^L \frac{dx}{\sqrt{T/\mu}} = \frac{1}{\sqrt{T}}\int_0^L \sqrt{\mu(x)}\,dx$$

$$t = \frac{1}{\sqrt{T}}\int_0^L \left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0\right)^{1/2} \left(\frac{\mu_L - \mu_0}{L}\right)dx\left(\frac{L}{\mu_L - \mu_0}\right)$$

$$t = \frac{1}{\sqrt{T}}\left(\frac{L}{\mu_L - \mu_0}\right)\left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0\right)^{3/2} \frac{1}{3/2}\Bigg|_0^L$$

$$t = \frac{2L}{3\sqrt{T}(\mu_L - \mu_0)}\left(\mu_L{}^{3/2} - \mu_0{}^{3/2}\right)$$

$$t = \frac{2L\left(\sqrt{\mu_L} - \sqrt{\mu_0}\right)\left(\mu_L + \sqrt{\mu_L \mu_0} + \mu_0\right)}{3\sqrt{T}\left(\sqrt{\mu_L} - \sqrt{\mu_0}\right)\left(\sqrt{\mu_L} + \sqrt{\mu_0}\right)}$$

$$T = \frac{2L}{3\sqrt{T}}\left(\frac{\mu_L + \sqrt{\mu_L \mu_0} + \mu_0}{\sqrt{\mu_L} + \sqrt{\mu_0}}\right)$$

13.45 $v = \sqrt{\dfrac{T}{\mu}}$ where $T = \mu x g$, the weight of a length x, of rope.

Therefore, $v = \sqrt{gx}$

But $v = dx/dt$, so that $dt = \dfrac{dx}{\sqrt{gx}}$

and $t = \displaystyle\int_0^L \frac{dx}{\sqrt{gx}} = \frac{1}{\sqrt{g}}\frac{\sqrt{x}}{(1/2)}\Bigg|_0^L = \boxed{2\sqrt{\frac{L}{g}}}$

***13.46** At distance x from the bottom, the tension is $T = (mxg/L) + Mg$, so the wave speed is:

$$v = \sqrt{T/\mu} = \sqrt{TL/m} = \sqrt{xg + (MgL/m)} = \frac{dx}{dt}$$

(a) Then $t = \int_0^t dt = \int_0^L \left[xg + (MgL/m)\right]^{-1/2} dx$ $\qquad t = \frac{1}{g}\frac{\left[xg + (MgL/m)\right]^{1/2}}{\frac{1}{2}}\Bigg|_{x=0}^{x=L}$

$$t = \frac{2}{g}\left[(Lg + MgL/m)^{1/2} - (MgL/m)^{1/2}\right] \qquad \boxed{t = 2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m+M} - \sqrt{M}}{\sqrt{m}}\right)}$$

(b) When $M = 0$, as in Problem 45, $\quad t = 2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m} - 0}{\sqrt{m}}\right) = \boxed{2\sqrt{\frac{L}{g}}}$

(c) As $m \to 0$ we expand $\quad \sqrt{m+M} = \sqrt{M}(1 + m/M)^{1/2} = \sqrt{M}\left(1 + \frac{1}{2}m/M - \frac{1}{8}m^2/M^2 + \ldots\right)$

to obtain $\quad t = 2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{M} + \frac{1}{2}m/\sqrt{M} - \frac{1}{8}m^2/M^{3/2} + \ldots - \sqrt{M}}{\sqrt{m}}\right)$

$$t \cong 2\sqrt{\frac{L}{g}}\left(\frac{1}{2}\sqrt{\frac{m}{M}}\right) = \boxed{\sqrt{\frac{mL}{Mg}}}$$

***13.47** (a) Assume the spring is originally stationary throughout, extended to have a length L much greater than its equilibrium length. We start moving one end forward with the speed v at which a wave propagates on the spring. In this way we create a single pulse of compression that moves down the length of the spring. For an increment of spring with length dx and mass dm, just as the pulse swallows it up, $\Sigma F = ma$

becomes $\quad k\,dx = a\,dm \quad$ or $\quad \frac{k}{dm/dx} = a$

But $\quad \frac{dm}{dx} = \mu \quad$ so $\quad a = \frac{k}{\mu}$

Also, $a = \frac{dv}{dt} = \frac{v}{t}$ when $v_i = 0$. But $L = vt$, $\quad$ so $\quad a = \frac{v^2}{L}$

Equating the two expressions for a, we have $\frac{k}{\mu} = \frac{v^2}{L}$ or $\quad \boxed{v = \sqrt{\frac{kL}{\mu}}}$

(b) Using the expression from part (a) $\quad v = \sqrt{\frac{kL}{\mu}} = \sqrt{\frac{kL^2}{m}} = \sqrt{\frac{(100\text{ N/m})(2.00\text{ m})^2}{0.400\text{ kg}}} = \boxed{31.6\text{ m/s}}$

13.48 (a) $\mathcal{P}(x) = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\mu\omega^2 A_0{}^2 e^{-2bx}\left(\frac{\omega}{k}\right) = \boxed{\frac{\mu\omega^2}{2k} A_0{}^2 e^{-2bx}}$

(b) $\mathcal{P}(0) = \boxed{\frac{\mu\omega^2}{2k} A_0{}^2}$

(c) $\frac{\mathcal{P}(x)}{\mathcal{P}(0)} = \boxed{e^{-2bx}}$

13.49 $v = \frac{4450 \text{ km}}{9.50 \text{ h}} = 468 \text{ km/h} = \boxed{130 \text{ m/s}}$

$\bar{d} = \frac{v^2}{g} = \frac{(130 \text{ m/s})^2}{(9.80 \text{ m/s}^2)} = \boxed{1730 \text{ m}}$

***13.50** The trucks form a train analogous to a wave train of crests with speed $v = 19.7 \text{ m/s}$

and unshifted frequency $f = \frac{2}{3.00 \text{ min}} = 0.667 \text{ min}^{-1}$

(a) The cyclist as observer measures a lower Doppler-shifted frequency:

$f' = f\left(\frac{v + v_o}{v}\right) = \left(0.667 \text{ min}^{-1}\right)\left(\frac{19.7 + (-4.47)}{19.7}\right) = \boxed{0.515/\text{min}}$

(b) $f'' = f\left(\frac{v + v_o'}{v}\right) = \left(0.667 \text{ min}^{-1}\right)\left(\frac{19.7 + (-1.56)}{19.7}\right) = \boxed{0.614/\text{min}}$

The cyclist's speed has decreased very significantly, but there is only a modest increase in the frequency of trucks passing him.

13.51 $v = \frac{2d}{t}$: $d = \frac{vt}{2} = \frac{1}{2}\left(6.50 \times 10^3 \text{ m/s}\right)(1.85 \text{ s}) = \boxed{6.01 \text{ km}}$

13.52 (a) $f' = \frac{fv}{v-u}$ $f'' = \frac{fv}{v-(-u)}$ $f' - f'' = fv\left(\frac{1}{v-u} - \frac{1}{v+u}\right)$

$$\Delta f = \frac{fv(v+u-v+u)}{v^2 - u^2} = \frac{2uvf}{v^2\left(1 - \frac{u^2}{v^2}\right)} = \boxed{\frac{2(u/v)}{1 - \left(u^2/v^2\right)} f}$$

(b) $130 \text{ km/h} = 36.1 \text{ m/s}$ $\therefore \Delta f = \frac{2(36.1)(400)}{340\left[1 - \frac{(36.1)^2}{340^2}\right]} = \boxed{85.9 \text{ Hz}}$

***13.53** (a) Sound moves upwind with speed $(343-15)$ m/s. Crests pass a stationary upwind point at frequency 900 Hz.

Then $$\lambda = \frac{v}{f} = \frac{328\text{ m/s}}{900\text{ s}} = \boxed{0.364\text{ m}}$$

(b) By similar logic, $$\lambda = \frac{v}{f} = \frac{(343+15)\text{ m/s}}{900\text{ s}} = \boxed{0.398\text{ m}}$$

(c) The source is moving through the air at 15 m/s toward the observer. The observer is stationary relative to the air.

$$f' = f\left(\frac{v+v_o}{v-v_s}\right) = 900\text{ Hz}\left(\frac{343+0}{343-(-15)}\right) = \boxed{941\text{ Hz}}$$

(d) The source is moving through the air at 15 m/s away from the downwind firefighter. Her speed relative to the air is 30 m/s toward the source.

$$f' = f\left(\frac{v+v_o}{v-v_s}\right) = 900\text{ Hz}\left(\frac{343+30}{343-(-15)}\right) = 900\text{ Hz}\left(\frac{373}{358}\right) = \boxed{938\text{ Hz}}$$

***13.54** (a) If the source and the observer are moving away from each other, we have: $\theta_S - \theta_0 = 180°$, and since $\cos 180° = -1$, we get Equation 13.30 with negative values for both v_0 and v_S.

(b) If $v_o = 0\text{ m/s}$ then $f' = \dfrac{v}{v - v_S\cos\theta_S} f$

Also, when the train is 40.0 m from the intersection, and the car is 30.0 m from the intersection,

$$\cos\theta_S = \frac{4}{5}$$

so $$f' = \frac{343\text{ m/s}}{343\text{ m/s} - 0.800(25.0\text{ m/s})}(500\text{ Hz})$$

or $$f' = \boxed{531\text{ Hz}}$$

Note that as the train approaches, passes, and departs from the intersection, θ_S varies from 0° to 180° and the frequency heard by the observer varies from:

$$f'_{\max} = \frac{v}{v - v_S\cos 0°} f = \frac{343\text{ m/s}}{343\text{ m/s} - 25.0\text{ m/s}}(500\text{ Hz}) = 539\text{ Hz}$$

$$f'_{\max} = \frac{v}{v - v_S\cos 180°} f = \frac{343\text{ m/s}}{343\text{ m/s} + 25.0\text{ m/s}}(500\text{ Hz}) = 466\text{ Hz}$$

ANSWERS TO EVEN NUMBERED PROBLEMS

2. See the solution

4. 0.800 m/s

6. 2.00 cm, 2.98 m, 0.576 Hz, 1.72 m/s

8. (a) –1.51 m/s, 0 (b) 16.0 m, 0.500 s, 32.0 m/s

10. (a) 0.021 5 m (b) 1.95 rad
(c) 5.41 m/s (d) $y(x, t) = (0.0215 \text{ m})\sin(8.38x \text{ rad/m} + 80.0\pi t \text{ rad/s} + 1.95 \text{ rad})$

12. 80.0 N

14. 1.64 m/s^2

16. (a) $v = \left(30.4 \frac{\text{m/s}}{\sqrt{\text{kg}}}\right)\sqrt{m}$ (b) 3.89 kg

18. 1.07 kW

20. (a) $\mathcal{P}$ remains constant (b) $\mathcal{P}$ remains constant
(c) $\mathcal{P}$ remains constant (d) $\mathcal{P}$ is quadrupled

22. 5.67 mm

24. 7.82 m

26. 0.103 Pa

28. (a) zero (b) 3.87 Pa

30. (a) A drop by 75.7 Hz (b) 0.948 m

32. (a) 3.04 kHz (b) 2.08 kHz (c) 2.62 kHz; 2.40 kHz

34. (a) 0.021 7 m/s (b) 2 000 028.9 Hz (c) 2 000 057.8 Hz

36. (a) 325 m/s (b) 29.5 m/s

38. 184 km

40. (a) 23.2 cm (b) 1.38 cm

42. (a) $2Mg$ (b) $L_0 + \dfrac{2Mg}{k}$ (c) $\sqrt{\dfrac{2Mg}{m}\left(L_0 + \dfrac{2Mg}{k}\right)}$

44. (a) $\mu_0 + (\mu_L - \mu_0)x/L$ (b) See the solution

46. See the solution

48. (a) $\dfrac{\mu\omega^2}{2k}A_0{}^2e^{-2bx}$ (b) $\dfrac{\mu\omega^2}{2k}A_0{}^2$ (c) e^{-2bx}

50. (a) 0.515/min
(b) The noisy, smelly, inefficient, road hogging trucks pass the cyclist at the frequency 0.614/min.

52. (a) See the solution (b) 85.9 Hz

54. (a) See the solution (b) 531 Hz