CHAPTER 14

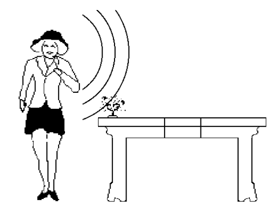
ANSWERS TO QUESTIONS

Q14.1 No. The total energy of each wave remains the same.

Q14.2 No. Waves with other waveforms may also combine amplitudes.

Q14.3 When the bottles are struck, standing wave vibrations are established in the vibrating material of the bottles. The frequencies of those vibrations are determined by the stiffness of the glass and the mass of the vibrating material. As the water level rises, there is more mass of water vibrating, because the glass is in contact with the water. This increased mass decreases the frequency. On the other hand, blowing across a bottle establishes a standing wave vibration in the air cavity above the water. As the water level rises, the length of this cavity decreases, and the frequency rises.

Q14.4 Resonance, where maintaining the same frequency causes the amplitude of vibrations to increase in time.

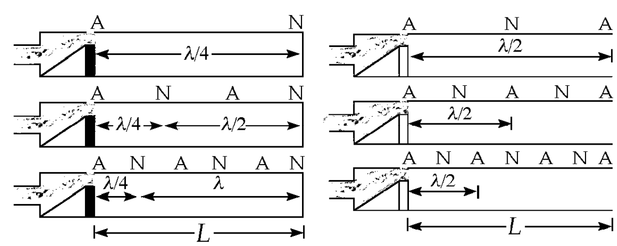


Q14.5 Damping, and non–linear effects in the vibration.

Q14.6 At the center of the string, there is a node for the second harmonic, as well as for every even-numbered harmonic. By placing the finger at the center and plucking, the guitarist is eliminating any harmonic that does not have a node at that point, which is any of the odd harmonics. The even harmonics can vibrate relatively freely with the finger at the center because they exhibit no displacement at that point. The result is a sound with a mixture of frequencies that are integer multiples of the second harmonic, which is one octave higher than the fundamental.

Q14.7 The bow string is pulled away from equilibrium and released, similar to the way that a guitar string is pulled and released when it is plucked. Thus, standing waves will be excited in the bow string. If the arrow leaves from the exact center of the string, then a series of odd harmonics will be excited. Even harmonies will not be excited because they have a node at the point where the string exhibits its maximum displacement.

Q14.8 The vibration of the air must have zero amplitude at the closed end. For air in a pipe closed at one end, the diagrams show how resonance vibrations have NA distances that are odd integer submultiples of the NA distance in the fundamental vibration. If the pipe is open, resonance vibrations have NA distances that are all integer submultiples of the NA distance in the fundamental.



Q14.9 Stick a bit of chewing gum to one tine of the second fork. If the beat frequency is then faster than 4 beats per second, the second has a lower frequency than the standard fork. If the beats have slowed down, the second fork has a higher frequency than the standard. Remove the gum, add or subtract 4 Hz according to what you found, and your answer will be the frequency of the second fork.

Q14.10 Beat frequency. The propellers are rotating at slightly different frequencies.

Q14.11 In a classical guitar, vibrations of the strings are transferred to the wooden body through the bridge. Because of its large area, the guitar body is a much more efficient radiator of sound than an individual guitar string. Thus, energy associated with the vibration is transferred to the air relatively rapidly by the guitar body, resulting in a more intense sound.

Q14.12 Instead of just radiating sound very softly into the surrounding air, the tuning fork makes the chalkboard vibrate. With its large area this stiff sounding board radiates sound into the air with higher power. So it drains away the fork's energy of vibration faster and the fork stops vibrating sooner. This process exemplifies conservation of energy, as the energy of vibration of the fork is transferred through the blackboard into energy of vibration of the air.

Q14.13 A clarinet in the center of the orchestra could be recorded with identical signals in the right and left channels. If one speaker is connected red-black, black-red, instead of red-red, black-black, then it will produce sound reversed in phase. On the perpendicular bisector of the line joining the speakers, destructive interference could cancel out the sound of the clarinet. In practice, reflections from the walls and the distance between your two ears will let you hear some sound, but it may be much too soft. The loss of fidelity is most pronounced for bass notes. With low frequency and long wavelength, they can have a geometrically wide region of destructive interference in front of the speakers.

Q14.14 Air blowing fast by a rim of the pipe creates a "shshshsh" sound called edgetone noise, a mixture of all frequencies, as the air turbulently switches between flowing on one side of the edge and the other. The air column inside the pipe finds one or more of its resonance frequencies in the noise. The air column starts vibrating with large amplitude in a standing wave vibration mode. It radiates sound into the surrounding air (and also locks the flapping airstream at the edge to its own frequency, making the noise disappear after just a few cycles).

Q14.15 The difference between static and kinetic friction makes your finger alternately slip and stick as it slides over the glass. Your finger produces a noisy vibration, a mixture of different frequencies, like new sneakers on a gymnasium floor. The glass finds one of its resonance frequencies in the noise. The thin stiff wall of the cup starts vibrating with large amplitude in a standing wave vibration mode. A typical possibility is shown in Figure Q14.4(a). It radiates sound into the surrounding air, and also can lock your squeaking finger to its own frequency, making the noise disappear after just a few cycles. Get a lot of different thin–walled glasses of fine crystal and try them out. Each will generally produce a different note. You can tune them by adding wine.

Q14.16 A typical standing–wave vibration possibility for a bell is similar to that for the glass shown in Figure Q14.4(a). Here six node-to-node distances fit around the circumference of the rim. The circumference is equal to three times the wavelength of the transverse wave of in-and-out bending of the material. In other states the circumference is two, four, five, or higher integers times the wavelengths of the higher–frequency vibrations. (The circumference being equal to the wavelength would describe the bell moving from side to side without bending, which it can do without producing any sound.) A tuned bell is cast and shaped so that some of these vibrations will have their frequencies constitute higher harmonics of a musical note, the strike tone. This tuning is lost if a crack develops in the bell. The sides of the crack vibrate as antinodes. Energy of vibration may be rapidly converted into internal energy at the end of the crack, so the bell may not ring for so long a time.

Q14.17 Walking makes the person's hand vibrate a little. If the frequency of this motion equals the natural frequency of coffee sloshing from side to side in the cup, then a large–amplitude vibration of the coffee will build up in resonance. To get off resonance and back to the normal case of a small-amplitude disturbance producing a small–amplitude result, the person can walk faster, walk slower, or get a larger or smaller cup. Alternatively, even at resonance he can reduce the amplitude by adding damping, as by stirring high–fiber quick–cooking oatmeal into the hot coffee.

Q14.18 The air in the shower stall can vibrate in standing wave patterns to intensify those frequencies in your voice which correspond to its free vibrations. The hard walls of the bathroom reflect sound very well to make your voice more intense at all frequencies, giving the room a longer reverberation time. The reverberant sound may help you to stay on key.

Q14.19 The trombone slide and trumpet valves change change the length of the air column inside the instrument, to change its resonant frequencies.

Chapter 14

PROBLEM SOLUTIONS

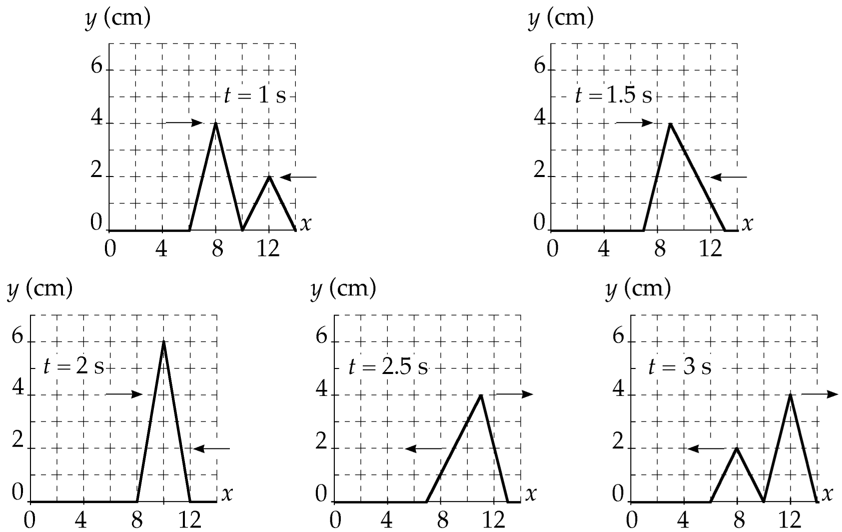
***14.1** $y = y_1 + y_2 = 3.00 \cos(4.00x - 1.60t) + 4.00 \sin(5.00x - 2.00t)$ evaluated at the given x values.

(a) $x = 1.00$, $t = 1.00$ $\quad y = 3.00 \cos(2.40 \text{ rad}) + 4.00 \sin(+3.00 \text{ rad}) = \boxed{-1.65 \text{ cm}}$

(b) $x = 1.00$, $t = 0.500$ $\quad y = 3.00 \cos(+3.20 \text{ rad}) + 4.00 \sin(+4.00 \text{ rad}) = \boxed{-6.02 \text{ cm}}$

(c) $x = 0.500$, $t = 0$ $\quad y = 3.00 \cos(+2.00 \text{ rad}) + 4.00 \sin(+2.50 \text{ rad}) = \boxed{1.15 \text{ cm}}$

14.2



14.3 (a) $y_1 = f(x - vt)$, so wave 1 travels in the $\boxed{+x \text{ direction}}$

$y_2 = f(x + vt)$, so wave 2 travels in the $\boxed{-x \text{ direction}}$

(b) To cancel, $y_1 + y_2 = 0$:

$$\frac{5}{(3x - 4t)^2 + 2} = \frac{+5}{(3x + 4t - 6)^2 + 2}$$

$$(3x - 4t)^2 = (3x + 4t - 6)^2$$

$$3x - 4t = \pm(3x + 4t - 6)$$

for the positive root, $\quad 8t = 6 \quad \boxed{t = 0.750 \text{ s}}$

(at $t = 0.750$ s, the waves cancel everywhere)

(c) for the negative root, $\quad 6x = 6 \quad \boxed{x = 1.00 \text{ m}}$

(at x=1.00 m, the waves cancel always)

14.4 Suppose the waves are sinusoidal.

The sum is $(4.00 \text{ cm}) \sin(kx - \omega t) + (4.00 \text{ cm}) \sin(kx - \omega t + 90.0°)$

$2(4.00 \text{ cm}) \sin(kx - \omega t + 45.0°) \cos 45.0°$

So the amplitude is $(8.00 \text{ cm}) \cos 45.0° = \boxed{5.66 \text{ cm}}$

14.5 The resultant wave function has the form

$$y = 2A_0 \cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

(a) $A = 2A_0 \cos\left(\frac{\phi}{2}\right) = 2(5.00)\cos\left[\frac{-\pi/4}{2}\right] = \boxed{9.24 \text{ m}}$

(b) $f = \frac{\omega}{2\pi} = \frac{1200\pi}{2\pi} = \boxed{600 \text{ Hz}}$

14.6 $2A_0 \cos\left(\frac{\phi}{2}\right) = A_0$ so $\frac{\phi}{2} = \cos^{-1}\left(\frac{1}{2}\right) = 60.0° = \frac{\pi}{3}$

Thus, the phase difference is $\phi = 120° = \frac{2\pi}{3}$

This phase difference results if the time delay is $\frac{T}{3} = \frac{1}{3f} = \frac{\lambda}{3v}$

$$\text{Time delay} = \frac{3.00 \text{ m}}{3(2.00 \text{ m/s})} = \boxed{0.500 \text{ s}}$$

14.7 Waves reflecting from the near end travel 28.0 m (14.0 m down and 14.0 m back), while waves reflecting from the far end travel 66.0 m. The path difference for the two waves is:

$$\Delta r = 66.0 \text{ m} - 28.0 \text{ m} = 38.0 \text{ m}$$

Since $\lambda = \frac{v}{f}$

Then $\frac{\Delta r}{\lambda} = \frac{(\Delta r)f}{v} = \frac{(38.0 \text{ m})(246 \text{ Hz})}{343 \text{ m/s}} = 27.254$

or, $\Delta r = 27.254\lambda$

The phase difference between the two reflected waves is then

$$\phi = 0.254(1 \text{ cycle}) = 0.254(2\pi \text{ rad}) = \boxed{91.3°}$$

14.8 (a) $\Delta x = \sqrt{9.00 + 4.00} - 3.00 = \sqrt{13} - 3.00 = 0.606 \text{ m}$

The wavelength is $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{300 \text{ Hz}} = 1.14 \text{ m}$

Thus, $\frac{\Delta x}{\lambda} = \frac{0.606}{1.14} = 0.530$ of a wave,

or $\Delta\phi = 2\pi(0.530) = \boxed{3.33 \text{ rad}}$

(b) For destructive interference, we want $\frac{\Delta x}{\lambda} = 0.500 = f\frac{\Delta x}{v}$

where Δx is a constant in this set up. $f = \frac{v}{2\Delta x} = \frac{343}{2(0.606)} = \boxed{283 \text{ Hz}}$

***14.9** Suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\Delta r = \sqrt{L^2 + d^2} - L$.

He hears a minimum when $\Delta r = (2n-1)\left(\frac{\lambda}{2}\right)$ with $n = 1, 2, 3, \ldots$

Then,

$$\sqrt{L^2 + d^2} - L = (n - 1/2)(v/f)$$

$$\sqrt{L^2 + d^2} = (n - 1/2)(v/f) + L$$

$$L^2 + d^2 = (n - 1/2)^2 (v/f)^2 + 2(n - 1/2)(v/f)L + L^2$$

$$d^2 - \left(n - \tfrac{1}{2}\right)^2 (v/f)^2 = 2\left(n - \tfrac{1}{2}\right)(v/f)L \qquad (1)$$

Equation 1 gives the distances from the lower speaker at which the man will hear a minimum. The path difference Δr starts from nearly zero when the man is very far away and increases to d when $L = 0$.

(a) The number of minima he hears is the greatest integer value for which $L \geq 0$. This is the same as the greatest integer solution to $d \geq (n - 1/2)(v/f)$, or

$$\boxed{\text{number of minima heard} = n_{max} = \text{ greatest integer } \leq d(f/v) + 1/2}$$

(b) From equation 1, the distances at which minima occur are given by

$$\boxed{L_n = \frac{d^2 - (n - 1/2)^2 (v/f)^2}{2(n - 1/2)(v/f)} \text{ where } n = 1, 2, \ldots, n_{max}}$$

14.10 $y = (1.50\text{ m})\sin(0.400x)\cos(200t) = 2A_0 \sin kx \cos\omega t$

Therefore, $k = \frac{2\pi}{\lambda} = 0.400\text{ rad/m}$ $\lambda = \frac{2\pi}{0.400\text{ rad/m}} = \boxed{15.7\text{ m}}$

and $\omega = 2\pi f$ so $f = \frac{\omega}{2\pi} = \frac{200\text{ rad/s}}{2\pi\text{ rad}} = \boxed{31.8\text{ Hz}}$

The speed of waves in the medium is $v = \lambda f = \frac{\lambda}{2\pi}2\pi f = \frac{\omega}{k} = \frac{200\text{ rad/s}}{0.400\text{ rad/m}} = \boxed{500\text{ m/s}}$

14.11 The facing speakers produce a standing wave in the space between them, with the spacing between nodes being

$$d_{\text{NN}} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\text{ m/s}}{2(800\text{ s}^{-1})} = 0.214\text{ m}$$

If the speakers vibrate in phase, the point halfway between them is an antinode of pressure at a distance from either speaker of

$$\frac{1.25\text{ m}}{2} = 0.625$$

Then there is a node at $0.625 - \frac{0.214}{2} = \boxed{0.518\text{ m}}$

a node at $0.518\text{ m} - 0.214\text{ m} = \boxed{0.303\text{ m}}$

a node at $0.303\text{ m} - 0.214\text{ m} = \boxed{0.0891\text{ m}}$

a node at $0.518\text{ m} + 0.214\text{ m} = \boxed{0.732\text{ m}}$

a node at $0.732\text{ m} + 0.214\text{ m} = \boxed{0.947\text{ m}}$

and a node at $0.947\text{ m} + 0.214\text{ m} = \boxed{1.16\text{ m}}$ from either speaker

14.12 (a) The resultant wave is $y = 2A\sin\left(kx + \frac{\phi}{2}\right)\cos\left(\omega t - \frac{\phi}{2}\right)$

The nodes are located at $kx + \frac{\phi}{2} = n\pi$

so $x = \frac{n\pi}{k} - \frac{\phi}{2k}$

which means that each node is shifted $\phi/2k$ to the left.

(b) The separation of nodes is $\Delta x = \left[(n+1)\frac{\pi}{k} - \frac{\phi}{2k}\right] - \left[\frac{n\pi}{k} - \frac{\phi}{2k}\right]$ $\Delta x = \frac{\pi}{k} = \frac{\lambda}{2}$

The nodes are still separated by half a wavelength.

***14.13** $y_1 = 3.00 \sin [\pi(x + 0.600t)]$ cm $\qquad y_2 = 3.00 \sin [\pi(x - 0.600t)]$ cm

$y = y_1 + y_2 = [3.00 \sin (\pi x) \cos (0.600\,\pi t) + 3.00 \sin (\pi x) \cos (0.600\pi t)]$ cm

$y = (6.00 \text{ cm}) \sin (\pi x) \cos (0.600\pi t)$

(a) We can take $\cos(0.600\pi t) = 1$ to get the maximum y.

At $x = 0.250$ cm, $\qquad y_{max} = (6.00 \text{ cm}) \sin(0.250\pi) = \boxed{4.24 \text{ cm}}$

(b) At $x = 0.500$ cm, $\qquad y_{max} = (6.00 \text{ cm}) \sin(0.500\pi) = \boxed{6.00 \text{ cm}}$

(c) Now take $\cos (0.600\pi t) = -1$ to get y_{max}:

At $x = 1.50$ cm, $\qquad y_{max} = (6.00 \text{ cm}) \sin(1.50\pi)(-1) = \boxed{6.00 \text{ cm}}$

(d) The antinodes occur when $\qquad x = n\lambda/4 \quad (n = 1, 3, 5, \ldots)$.

But $\quad k = 2\pi/\lambda = \pi, \qquad$ so $\qquad \lambda = 2.00$ cm

and $\qquad x_1 = \lambda/4 = \boxed{0.500 \text{ cm}}$ as in (b)

$x_2 = 3\lambda/4 = \boxed{1.50 \text{ cm}}$ as in (c)

$x_3 = 5\lambda/4 = \boxed{2.50 \text{ cm}}$

***14.14** $y = 2A_0 \sin kx \cos \omega t$

$$\frac{\partial^2 y}{\partial x^2} = -2A_0 k^2 \sin kx \cos \omega t \qquad \frac{\partial^2 y}{\partial t^2} = -2A_0 \omega^2 \sin kx \cos \omega t$$

Substitution into the wave equation gives $\qquad -2A_0 k^2 \sin kx \cos \omega t = \left(\frac{1}{v^2}\right)\left(-2A_0\omega^2 \sin kx \cos \omega t\right)$

This is satisfied, provided that $\qquad v = \omega / k$

14.15 The lowest frequency will have the longest wavelength. It will be the simplest mode, with nodes at both ends and one antinode in the center. The node–to–node distance is

$$d_{\text{N to N}} = \frac{\lambda}{2} = 1.8 \text{ m}$$

so $\qquad \lambda = 3.6$ m

and $\qquad f = \frac{v}{\lambda} = \frac{540 \text{ m/s}}{3.6 \text{ m}} = \boxed{150 \text{ Hz}}$

14.16 The tension in the string is $T = (4\ \text{kg})(9.8\ \text{m/s}^2) = 39.2\ \text{N}$

Its linear density is $\mu = \dfrac{m}{L} = \dfrac{8 \times 10^{-3}\ \text{kg}}{5\ \text{m}} = 1.6 \times 10^{-3}\ \text{kg/m}$

and the wave speed on the string is $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{39.2\ \text{N}}{1.6 \times 10^{-3}\ \text{kg/m}}} = 156.5\ \text{m/s}$

In its fundamental mode of vibration, we have $\lambda = 2L = 2(5\ \text{m}) = 10\ \text{m}$

Thus, $f = \dfrac{v}{\lambda} = \dfrac{156.5\ \text{m/s}}{10\ \text{m}} = \boxed{15.7\ \text{Hz}}$

14.17 $L = 30.0\ \text{m}$ $\quad \mu = 9.00 \times 10^{-3}\ \text{kg/m}$ $\quad T = 20.0\ \text{N}$ $\quad f_1 = \dfrac{v}{2L}$

where $v = \left(\dfrac{T}{\mu}\right)^{1/2} = 47.1\ \text{m/s}$

so $f_1 = \dfrac{47.1}{60.0} = \boxed{0.786\ \text{Hz}}$ $\qquad f_2 = 2f_1 = \boxed{1.57\ \text{Hz}}$

$f_3 = 3f_1 = \boxed{2.36\ \text{Hz}}$ $\qquad f_4 = 4f_1 = \boxed{3.14\ \text{Hz}}$

***14.18** $f_0 = \dfrac{v_0}{\lambda} = \sqrt{\dfrac{T_0}{\mu}}\dfrac{1}{2L}$. We assume the string does not stretch, so μ and L are constant.

(a) Then $f_a = \sqrt{\dfrac{T_a}{\mu}}\dfrac{1}{2L}$ and $\dfrac{f_a}{f_0} = \sqrt{\dfrac{T_a}{T_0}}$

$$\frac{T_a}{T_0} = \left(\frac{f_a}{f_0}\right)^2 = \left(\frac{500\ \text{Hz}}{800\ \text{Hz}}\right)^2 = \boxed{0.391}$$

(b) $f_b = \sqrt{\dfrac{T_b}{\mu}}\dfrac{1}{2L}$ $\qquad \dfrac{f_b}{f_0} = \sqrt{\dfrac{T_b}{T_0}} = \sqrt{\dfrac{4T_0}{T_0}} = 2$

$$f_b = 2f_0 = 2(800\ \text{Hz}) = \boxed{1.60\ \text{kHz}}$$

14.19 $L = 120$ cm $\qquad f = 120$ Hz

(a) For four segments, $L = 2\lambda$ $\qquad$ or $\qquad \lambda = 60.0 \text{ cm} = \boxed{0.600 \text{ m}}$

(b) $v = \lambda f = 72.0 \text{ m/s}$ $\qquad f_1 = \dfrac{v}{2L} = \dfrac{72.0}{2(1.20)} = \boxed{30.0 \text{ Hz}}$

***14.20** (a) Let n be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then $n + 1$ is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = 2L/n$, and the frequency is $f = v/\lambda$.

Thus, $$f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}}$$

and also $$f = \frac{n+1}{2L}\sqrt{\frac{T_{n+1}}{\mu}}$$

Thus, $$\frac{n+1}{n} = \sqrt{\frac{T_n}{T_{n+1}}} = \sqrt{\frac{(25.0 \text{ kg})g}{(16.0 \text{ kg})g}} = \frac{5}{4}$$

Therefore, $4n + 4 = 5n$, $\qquad$ or $\qquad n = 4$

Then, $$f = \frac{4}{2(2.00 \text{ m})}\sqrt{\frac{(25.0 \text{ kg})(9.80 \text{ m/s}^2)}{0.00200 \text{ kg/m}}} = \boxed{350 \text{ Hz}}$$

(b) The largest mass will correspond to a standing wave of 1 loop

$(n = 1)$ $\qquad$ so $$350 \text{ Hz} = \frac{1}{2(2.00 \text{ m})}\sqrt{\frac{m(9.80 \text{ m/s}^2)}{0.00200 \text{ kg/m}}}$$

yielding $\qquad m = \boxed{400 \text{ kg}}$

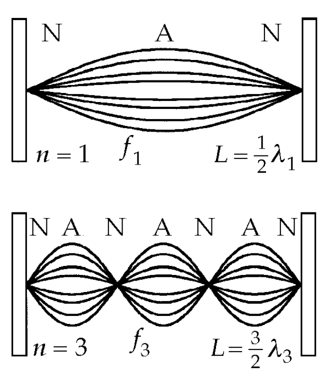
14.21 $d_{NN} = 0.700$ m

$\lambda = 1.40$ m

$$f\lambda = v = 308 \text{ m/s} = \sqrt{\frac{T}{(1.20\times 10^{-3})/(0.700)}}$$

(a) $T = \boxed{163 \text{ N}}$

(b) $f_3 = \boxed{660 \text{ Hz}}$



14.22 $\lambda_G = 2(0.350 \text{ m}) = \dfrac{v}{f_G}$ $\qquad$ $\lambda_A = 2L_A = \dfrac{v}{f_A}$

$$L_G - L_A = L_G - \left(\frac{f_G}{f_A}\right)L_G = L_G\left(1 - \frac{f_G}{f_A}\right) = (0.350 \text{ m})\left(1 - \frac{392}{440}\right) = 0.0382 \text{ m}$$

Thus, $L_A = L_G - 0.0382 \text{ m} = 0.350 \text{ m} - 0.0382 \text{ m} = 0.312 \text{ m}$,

or the finger should be placed $\boxed{\text{31.2 cm from the bridge}}$.

$L_A = \dfrac{v}{2f_A} = \dfrac{1}{2f_A}\sqrt{\dfrac{T}{\mu}}$ $\qquad$ $dL_A = \dfrac{dT}{4f_A\sqrt{T\mu}}$ $\qquad$ $\dfrac{dL_A}{L_A} = \frac{1}{2}\dfrac{dT}{T}$

$$\frac{dT}{T} = 2\frac{dL_A}{L_A} = 2\frac{0.600 \text{ cm}}{(35.0 - 3.82) \text{ cm}} = \boxed{3.84\ \%}$$

***14.23** Comparing $\qquad y = (0.002 \text{ m})\sin((\pi \text{ rad/m})x)\cos((100\pi \text{ rad/s})t)$

with $\qquad y = 2A\sin kx\cos\omega t$

we find $\qquad k = \dfrac{2\pi}{\lambda} = \pi \text{ m}^{-1}$, $\quad \lambda = 2.00 \text{ m}$, and $\quad \omega = 2\pi f = 100\pi \text{ s}^{-1}$: $\quad f = 50.0 \text{ Hz}$

(a) Then the distance between adjacent nodes is $\qquad d_{NN} = \dfrac{\lambda}{2} = 1.00 \text{ m}$

and on the string are $\qquad \dfrac{L}{d_{NN}} = \dfrac{3.00 \text{ m}}{1.00 \text{ m}} = \boxed{3 \text{ loops}}$

For the speed we have $\qquad v = f\lambda = (50 \text{ s}^{-1})2 \text{ m} = 100 \text{ m/s}$

(b) In the simplest standing wave vibration, $d_{NN} = 3.00 \text{ m} = \dfrac{\lambda_b}{2}$, $\lambda_b = 6.00 \text{ m}$,

and $\qquad f_b = \dfrac{v_a}{\lambda_b} = \dfrac{100 \text{ m/s}}{6.00 \text{ m}} = \boxed{16.7 \text{ Hz}}$

(c) In $v_0 = \sqrt{T_0/\mu}$, if the tension increases to $T_c = 9T_0$ and the string does not stretch, the speed increases to

$$v_c = \sqrt{\frac{9T_0}{\mu}} = 3\sqrt{\frac{T_0}{\mu}} = 3v_0 = 3(100 \text{ m/s}) = 300 \text{ m/s}$$

Then $\qquad \lambda_c = \dfrac{v_c}{f_a} = \dfrac{300 \text{ m/s}}{50 \text{ s}^{-1}} = 6.00 \text{ m}$ $\qquad d_{NN} = \dfrac{\lambda_c}{2} = 3.00 \text{ m}$,

and $\boxed{\text{one}}$ loop fits onto the string.

14.24 $\dfrac{\lambda}{2} = d_{AA} = \dfrac{L}{n}$ or $L = \dfrac{n\lambda}{2}$ for $n = 1, 2, 3, \ldots$

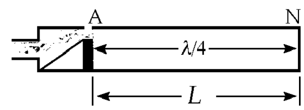
Since $\lambda = \dfrac{v}{f}$, $L = n\left(\dfrac{v}{2f}\right)$ for $n = 1, 2, 3, \ldots$

With $v = 343 \text{ m/s}$ and $f = 680 \text{ Hz}$,

$$L = n\left(\frac{343 \text{ m/s}}{2(680 \text{ Hz})}\right) = n(0.252 \text{ m}) \quad \text{for} \quad n = 1, 2, 3, \ldots$$

Possible lengths for resonance are: $L = \boxed{0.252 \text{ m}, 0.504 \text{ m}, 0.757 \text{ m}, \ldots, n(0.252) \text{ m}}$

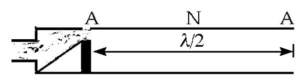
14.25 (a) For the fundamental mode in a closed pipe, $\lambda = 4L$. (see Figure 14.9b)



But $v = f\lambda$, therefore $L = \dfrac{v}{4f}$

So, $L = \dfrac{343 \text{ m/s}}{4(240 \text{ s}^{-1})} = \boxed{0.357 \text{ m}}$

(b) For an open pipe, $\lambda = 2L$. (see Figure 14.9a)



So, $L = \dfrac{v}{2f} = \dfrac{343 \text{ m/s}}{2(240 \text{ s}^{-1})} = \boxed{0.715 \text{ m}}$

14.26 $d_{AA} = 0.320 \text{ m}$ $\lambda = 0.640 \text{ m}$

(a) $f = \dfrac{v}{\lambda} = \boxed{531 \text{ Hz}}$

(b) $\lambda = 0.0850 \text{ m}$ $d_{AA} = \boxed{42.5 \text{ mm}}$

14.27 The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{343 \text{ m/s}}{261.6/\text{s}} = 1.31 \text{ m}$

so the length of the open pipe vibrating in its simplest (A-N-A) mode is

$$d_{\text{A to A}} = \tfrac{1}{2}\lambda = \boxed{0.656 \text{ m}}$$

A closed pipe has (N-A) for its simplest resonance,
(N-A-N-A) for the second,
and (N-A-N-A-N-A) for the third.

Here, the pipe length is $5d_{\text{N to A}} = \dfrac{5\lambda}{4} = \dfrac{5}{4}(1.31 \text{ m}) = \boxed{1.64 \text{ m}}$

14.28 The air in the auditory canal, about 3 cm long, can vibrate with a node at the closed end and antinode at the open end,

with $d_{\text{N to A}} = 3\text{ cm} = \dfrac{\lambda}{4}$

so $\lambda = 0.12\text{ m}$

and $f = \dfrac{v}{\lambda} = \dfrac{343\text{ m/s}}{0.12\text{ m}} \cong \boxed{3\text{ kHz}}$

> A small-amplitude external excitation at this frequency can, over time, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

14.29 $\lambda = \dfrac{v}{f} = \dfrac{333\text{ m/s}}{300\text{ Hz}} = 1.11\text{ m}$

(a) $L = \dfrac{\lambda}{2}$ for the fundamental $\boxed{0.555\text{ m}}$

(b) For the second harmonic $f = \dfrac{v}{\lambda} = \dfrac{344\text{ m/s}}{0.555\text{ m}} = \boxed{620\text{ Hz}}$

14.30 The wavelength of sound is $\lambda = \dfrac{v}{f}$

The distance between water levels at resonance is $d = \dfrac{v}{2f}$ $\quad\therefore\ Rt = \pi r^2 d = \dfrac{\pi r^2 v}{2f}$

and $t = \boxed{\dfrac{\pi r^2 v}{2Rf}}$

14.31 For a closed box, the resonant frequencies will have nodes at both sides, so the permitted wavelengths will be $L = \frac{1}{2} n\lambda$, $\quad (n = 1, 2, 3, \ldots)$.

i.e., $L = \dfrac{n\lambda}{2} = \dfrac{nv}{2f}$ and $f = \dfrac{nv}{2L}$

Therefore, with $L = 0.860$ m and $L' = 2.10$ m, the resonant frequencies are

$f_n = \boxed{n(206\text{ Hz})}$ for $L = 0.860$ m for each n from 1 to 9

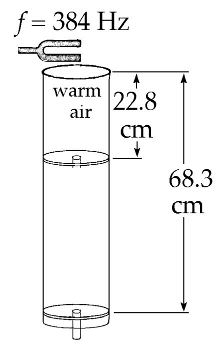
and $f'_n = \boxed{n(84.5\text{ Hz})}$ for $L' = 2.10$ m for each n from 2 to 23

***14.32** The length corresponding to the fundamental satisfies $f = \frac{v}{4L}$: $L_1 = \frac{v}{4f} = \frac{343}{4(512)} = 0.167\text{ m}$

Since $L > 20.0$ cm, the *next* two modes will be observed, corresponding to $f = \frac{3v}{4L_2}$ and $f = \frac{5v}{4L_3}$

or $L_2 = \frac{3v}{4f} = \boxed{0.502\text{ m}}$ and $L_3 = \frac{5v}{4f} = \boxed{0.837\text{ m}}$

***14.33** For resonance in a tube open at one end,



$$f = n\frac{v}{4L}\,(n = 1,\ 3,\ 5,\ldots) \qquad \text{Equation 14.12}$$

(a) Assuming $n = 1$ and $n = 3$,

$$384 = \frac{v}{4(0.228)} \qquad \text{and} \qquad 384 = \frac{3v}{4(0.683)}$$

In either case, $v = \boxed{350\text{ m/s}}$

(b) For the next resonance $n = 5$, and $L = \frac{5v}{4f} = \frac{5(350\text{ m/s})}{4\left(384\text{ s}^{-1}\right)} = \boxed{1.14\text{ m}}$

***14.34** Call L the depth of the well and v the speed of sound.

Then for some integer n $$L = (2n-1)\frac{\lambda_1}{4} = (2n-1)\frac{v}{4f_1} = \frac{(2n-1)(343\text{ m/s})}{4\left(51.5\text{ s}^{-1}\right)}$$

and for the next resonance $$L = \left[2(n+1)-1\right]\frac{\lambda_2}{4} = (2n+1)\frac{v}{4f_2} = \frac{(2n+1)(343\text{ m/s})}{4\left(60.0\text{ s}^{-1}\right)}$$

Thus, $$\frac{(2n-1)(343\text{ m/s})}{4\left(51.5\text{ s}^{-1}\right)} = \frac{(2n+1)(343\text{ m/s})}{4\left(60.0\text{ s}^{-1}\right)}$$

and we require an *integer* solution to $$\frac{2n+1}{60.0} = \frac{2n-1}{51.5}$$

The equation gives $n = \frac{111.5}{17} = 6.56$, so the best fitting integer is $n = 7$.

Then $$L = \frac{\left[2(7)-1\right](343\text{ m/s})}{4\left(51.5\text{ s}^{-1}\right)} = 21.6\text{ m}$$

and $$L = \frac{\left[2(7)+1\right](343\text{ m/s})}{4\left(60.0\text{ s}^{-1}\right)} = 21.4\text{ m}$$

suggest the best value for the depth of the well is $\boxed{21.5\text{ m}}$.

14.35 $f \propto v \propto \sqrt{T}$ $\qquad f_{\text{new}} = 110\sqrt{\dfrac{540}{600}} = 104.4\ \text{Hz}$

$\Delta f = \boxed{5.64\ \text{beats/s}}$

***14.36** (a) The string could be tuned to either $\boxed{521\ \text{Hz or } 525\ \text{Hz}}$ from this evidence.

(b) Tightening the string raises the wave speed and frequency. If the frequency were originally 521 Hz, the beats would slow down.

Instead, the frequency must have started at 525 Hz to become $\boxed{526\ \text{Hz}}$.

(c) From $\qquad f = \dfrac{v}{\lambda} = \dfrac{\sqrt{T/\mu}}{2L} = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \qquad \text{and} \qquad T_2 = \left(\frac{f_2}{f_1}\right)^2 T_1 = \left(\frac{523\ \text{Hz}}{526\ \text{Hz}}\right)^2 T_1 = 0.989\,T_1$$

The fractional change that should be made in the tension is then

$$\text{fractional change} = \frac{T_1 - T_2}{T_1} = 1 - 0.989 = 0.0114 = 1.14\%\ \text{lower}$$

The tension should be $\boxed{\text{reduced by } 1.14\%}$.

14.37 For an echo $f' = f\dfrac{(v+v_s)}{(v-v_s)}$ the beat frequency is $f_b = |f' - f|$

Solving for f_b

gives $f_b = f\dfrac{(2v_s)}{(v-v_s)}$ when approaching wall.

(a) $$f_b = (256)\frac{2(1.33)}{(343-1.33)} = \boxed{1.99\ \text{Hz}}\ \text{beat frequency}$$

(b) When he is moving away from the wall, v_s changes sign. Solving for v_s gives

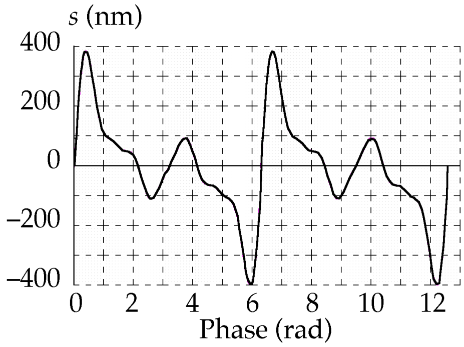
$$v_s = \frac{f_b v}{2f - f_b} = \frac{(5)(343)}{(2)(256) - 5} = \boxed{3.38\ \text{m/s}}$$

***14.38** We evaluate

$$s = 100\sin\theta + 157\sin 2\theta + 62.9\sin 3\theta + 105\sin 4\theta$$

$$+ 51.9\sin 5\theta + 29.5\sin 6\theta + 25.3\sin 7\theta$$

where s represents particle displacement in nanometers and θ represents the phase of the wave in radians. As θ advances by 2π, time advances by (1/523) s. Here is the result:



***14.39** (a) The wave speed is $v = \dfrac{9.15\text{ m}}{2.50\text{ s}} = \boxed{3.66\text{ m/s}}$

(b) From Figure P14.39, there are antinodes at both ends, so the distance between adjacent antinodes

is $d_{AA} = \dfrac{\lambda}{2} = 9.15\text{ m},$

and the wavelength is $\lambda = 18.3\text{ m}$

The frequency is then $f = \dfrac{v}{\lambda} = \dfrac{3.66\text{ m/s}}{18.3\text{ m}} = \boxed{0.200\text{ Hz}}$

We have assumed the wave speed is the same for all wavelengths.

***14.40** The wave speed is $v = \sqrt{gd} = \sqrt{(9.80\text{ m/s}^2)(36.1\text{ m})} = 18.8\text{ m/s}$

The bay has one end open and one closed. Its simplest resonance is with a node of horizontal velocity, which is also an antinode of vertical displacement, at the head of the bay and an antinode of velocity, which is a node of displacement, at the mouth. The vibration of the water in the bay is like that in one half of the pond shown in Figure P14.39.

Then, $d_{NA} = 210 \times 10^3\text{ m} = \dfrac{\lambda}{4}$

and $\lambda = 840 \times 10^3\text{ m}$

Therefore, the period is $T = \dfrac{1}{f} = \dfrac{\lambda}{v} = \dfrac{840 \times 10^3\text{ m}}{18.8\text{ m/s}} = 4.47 \times 10^4\text{ s} = \boxed{12\text{ h }24\text{ min}}$

$\boxed{\text{This agrees precisely with the period of the lunar excitation}}$, so we identify the extra-high tides as amplified by resonance.

***14.41** (a) First we calculate the wavelength: $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{21.5 \text{ Hz}} = 16.0 \text{ m}$

Then we note that the path difference equals $9.00 \text{ m} - 1.00 \text{ m} = \boxed{\frac{1}{2}\lambda}$

Therefore, the receiver will record a minimum in sound intensity.

(b) We choose the origin at the midpoint between the speakers. If the receiver is located at point (x, y), then we must solve:

$$\sqrt{(x+5.00)^2 + y^2} - \sqrt{(x-5.00)^2 + y^2} = \frac{1}{2}\lambda$$

Then, $$\sqrt{(x+5.00)^2 + y^2} = \sqrt{(x-5.00)^2 + y^2} + \frac{1}{2}\lambda$$

Square both sides and simplify to get: $$20.0x - \frac{\lambda^2}{4} = \lambda\sqrt{(x-5.00)^2 + y^2}$$

Upon squaring again, this reduces to: $$400x^2 - 10.0\lambda^2 x + \frac{\lambda^4}{16.0} = \lambda^2 (x-5.00)^2 + \lambda^2 y^2$$

Substituting $\lambda = 16.0$ m, and reducing, $$\boxed{9.00x^2 - 16.0y^2 = 144}$$

or $$\frac{x^2}{16.0} - \frac{y^2}{9.00} = 1$$

(When plotted this yields a curve called a hyperbola.)

14.42 (a) $L = \frac{v}{4f}$ so $\frac{L'}{L} = \frac{f}{f'}$

Letting the longest L be 1, the ratio is $1 : \frac{4}{5} : \frac{2}{3} : \frac{1}{2}$ or in integers $\boxed{30 : 24 : 20 : 15}$

(b) $L = \boxed{\frac{343}{(4)(256)} = 33.5 \text{ cm}}$

This is the longest pipe, so using the ratios, the lengths are: $\boxed{33.5,\ 26.8,\ 22.3,\ 16.7 \text{ cm}}$

(c) The frequencies are, using the ratios, $\boxed{256,\ 320,\ 384, \text{ and } 512 \text{ Hz}}$

These represent notes C, E, G, and C' on the just musical scale in physical tuning.

14.43 The distance between adjacent nodes is one-quarter of the circumference.

$$d_{NN} = d_{AA} = \frac{\lambda}{2} = \frac{20.0 \text{ cm}}{4} = 5.00 \text{ cm}$$

so $\lambda = 10.0$ cm and $f = \frac{v}{\lambda} = \frac{900 \text{ m/s}}{0.100 \text{ m}} = 9000 \text{ Hz} = \boxed{9.00 \text{ kHz}}$

The singer must match this frequency quite precisely for some interval of time to feed enough energy into the glass to crack it.

14.44 We suppose these are the lowest resonances of the enclosed air columns.

For one, $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{256 \text{ s}^{-1}} = 1.34 \text{ m}$ length $= d_{AA} = \frac{\lambda}{2} = 0.670 \text{ m}$

For the other, $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{440 \text{ s}^{-1}} = 0.780 \text{ m}$ length $= 0.390$ m

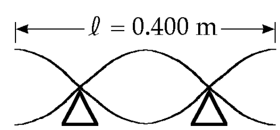
So,

(b) original length $= \boxed{1.06 \text{ m}}$

$\lambda = 2 d_{AA} = 2.12 \text{ m}$

(a) $f = \frac{343 \text{ m/s}}{2.12 \text{ m}} = \boxed{162 \text{ Hz}}$

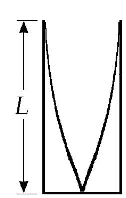
14.45 $f = 87.0$ Hz



speed of sound in air: $v_a = 340 \text{ m/s}$

(a) $\lambda_b = \ell$ $v = f\lambda_b = (87.0 \text{ s}^{-1})(0.400 \text{ m})$

$v = \boxed{34.8 \text{ m/s}}$



(b) $\left.\begin{array}{l}\lambda_a = 4L \\ v_a = \lambda_a f\end{array}\right\}$ $L = \frac{v_a}{4f} = \frac{340 \text{ m/s}}{4(87.0 \text{ s}^{-1})} = \boxed{0.977 \text{ m}}$

14.46 Moving away from station, frequency is depressed:

$f' = 180 - 2.00 = 178$ Hz: $\quad 178 = 180\dfrac{343}{343-(-v)}$

Solving for v gives $\quad v = \dfrac{(2.00)(343)}{178}$

Therefore, $\quad v = \boxed{3.85 \text{ m/s away from station}}$

Moving toward the station, the frequency is enhanced:

$f' = 180 + 2.00 = 182$ Hz: $\quad 182 = 180\dfrac{343}{343-v}$

Solving for v gives $\quad 4 = \dfrac{(2.00)(343)}{182}$

Therefore, $\quad v = \boxed{3.77 \text{ m/s toward the station}}$

14.47 $v = \sqrt{\dfrac{(48.0)(2.00)}{4.80\times 10^{-3}}} = 141 \text{ m/s}$

$d_{NN} = 1.00$ m $\quad \lambda = 2.00$ m $\quad f = \dfrac{v}{\lambda} = 70.7$ Hz

$\lambda_a = \dfrac{v_a}{f} = \dfrac{343 \text{ m/s}}{70.7 \text{ Hz}} = \boxed{4.85 \text{ m}}$

***14.48** The second standing wave mode of the air in the pipe reads ANAN, with $d_{NA} = \dfrac{\lambda}{4} = \dfrac{1.75 \text{ m}}{3}$

so $\quad \lambda = 2.33$ m

and $\quad f = \dfrac{v}{\lambda} = \dfrac{343 \text{ m/s}}{2.33 \text{ m}} = 147$ Hz

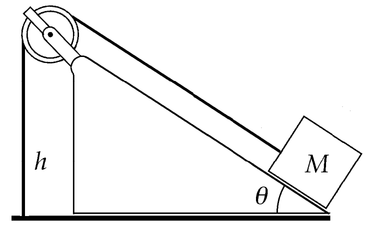
For the string, λ and v are different but f is the same. $\quad \dfrac{\lambda}{2} = d_{NN} = \dfrac{0.400 \text{ m}}{2}$

so $\quad \lambda = 0.400$ m

$$v = \lambda f = (0.400 \text{ m})(147 \text{ Hz}) = 58.8 \text{ m/s} = \sqrt{T/\mu}$$

$$T = \mu v^2 = (9.00\times 10^{-3} \text{ kg/m})(58.8 \text{ m/s})^2 = \boxed{31.1 \text{ N}}$$

***14.49** (a) For the block:



$$\Sigma F_x = T - Mg\sin 30.0° = 0$$

so $T = Mg\sin 30.0° = \boxed{\frac{1}{2}Mg}$

(b) The length of the section of string parallel to the incline is $h/\sin 30.0° = 2h$. The total length of the string is then $\boxed{3h}$.

(c) The mass per unit length of the string is $\mu = \boxed{m/3h}$

(d) The speed of waves in the string is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\left(\frac{Mg}{2}\right)\left(\frac{3h}{m}\right)} = \boxed{\sqrt{\frac{3Mgh}{2m}}}$

(e) In the fundamental mode, the segment of length h vibrates as one loop. The distance between adjacent nodes is then $d_{NN} = \lambda/2 = h$, so the wavelength is $\lambda = 2h$.

The frequency is $f = \frac{v}{\lambda} = \frac{1}{2h}\sqrt{\frac{3Mgh}{2m}} = \boxed{\sqrt{\frac{3Mg}{8mh}}}$

(g) When the vertical segment of string vibrates with 2 loops (i.e., 3 nodes), then $h = 2\left(\frac{\lambda}{2}\right)$ and the wavelength is $\lambda = \boxed{h}$

(f) The period of the standing wave of 3 nodes (or two loops) is

$$T = \frac{1}{f} = \frac{\lambda}{v} = h\sqrt{\frac{2m}{3Mgh}} = \boxed{\sqrt{\frac{2mh}{3Mg}}}$$

(h) $f_b = 1.02f - f = (2.00\times 10^{-2})f = \boxed{\left(2.00\times 10^{-2}\right)\sqrt{\frac{3Mg}{8mh}}}$

14.50 (a) Since the first node is at the weld, the wavelength in the thin wire is $2L$ or 80.0 cm. The frequency and tension are the same in both sections, so

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(0.400)}\sqrt{\frac{4.60}{2.00\times 10^{-3}}} = \boxed{59.9\text{ Hz}}$$

(b) As the thick wire is twice the diameter, the linear density is 4 times that of the thin wire.

$\mu' = 8.00$ g/m

so $L' = \frac{1}{2f}\sqrt{\frac{T}{\mu'}}$ $\quad L' = \left[\frac{1}{(2)(59.9)}\right]\sqrt{\frac{4.60}{8.00\times 10^{-3}}} = \boxed{20.0\text{ cm}}$ half the length of the thin wire

14.51 (a) $$f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

so $$\frac{f'}{f} = \frac{L}{L'} = \frac{L}{2L} = \frac{1}{2}$$

The $\boxed{\text{frequency should be halved}}$ to get the same number of antinodes for twice the length.

(b) $$\frac{n'}{n} = \sqrt{\frac{T}{T'}}$$ so $$\frac{T'}{T} = \left(\frac{n}{n'}\right)^2 = \left[\frac{n}{n+1}\right]^2$$

The tension must be $$T' = \boxed{\left[\frac{n}{n+1}\right]^2 T}$$

(c) $$\frac{f'}{f} = \frac{n'L}{nL'}\sqrt{\frac{T'}{T}}$$ so $$\frac{T'}{T} = \left(\frac{nf'L'}{n'fL}\right)^2$$

$$\frac{T'}{T} = \left(\frac{3}{2\cdot 2}\right)^2$$ $\boxed{\frac{T'}{T} = \frac{9}{16}}$ to get twice as many antinodes.

14.52 $f_B = f_A$ $\qquad$ $\lambda_B = \frac{1}{3}\lambda_A$

$v_B = \frac{1}{3}v_A$ $\qquad$ $v_B{}^2 = \frac{1}{9}v_A{}^2$

$v = \sqrt{\frac{T}{\mu}}$ $\qquad$ $\frac{T_B}{T_A} = \frac{v_B{}^2}{v_A{}^2} = \boxed{0.111}$

14.53 The odd-numbered harmonics of the organ-pipe vibration are:

650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, 50.0 Hz

Closed $\qquad f_1 = \boxed{50.0\ \text{Hz}}$

$\lambda = 6.80\ \text{m}$ $\qquad$ $\boxed{L = 1.70\ \text{m}}$

14.54 For the wire, $\mu = \dfrac{0.0100\text{ kg}}{2.00\text{ m}} = 5.00\times10^{-3}\text{ kg/m}$: $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{(200\text{ kg}\cdot\text{m/s}^2)}{5.00\times10^{-3}\text{ kg/m}}}$

$v = 200\text{ m/s}$

If it vibrates in its simplest state, $d_{NN} = 2.00\text{ m} = \dfrac{\lambda}{2}$: $f = \dfrac{v}{\lambda} = \dfrac{(200\text{ m/s})}{4.00\text{ m}} = 50.0\text{ Hz}$

(a) The tuning fork can have frequencies $\boxed{45.0\text{ Hz or }55.0\text{ Hz}}$

(b) If $f = 45.0$ Hz, $v = f\lambda = (45.0/\text{s})\,4.00\text{ m} = 180\text{ m/s}$

Then, $T = v^2\mu = (180\text{ m/s})^2\left(5.00\times10^{-3}\text{ kg/m}\right) = \boxed{162\text{ N}}$

$\boxed{\text{or}}$ if $f = 55.0$ Hz, $T = v^2\mu = f^2\lambda^2\mu = (55.0/\text{s})^2\,(4.00\text{ m})^2\,(5.00\times10^{-3}\text{ kg/m}) = \boxed{242\text{ N}}$

***14.55** We look for a solution of the form

$5.00\sin(2.00x - 10.0t) + 10.0\cos(2.00x - 10.0t) = A\sin(2.00x - 10.0t + \phi)$

$= A\sin(2.00x - 10.0t)\cos\phi + A\cos(2.00x - 10.0t)\sin\phi$

This will be true if both $5.00 = A\cos\phi$ and $10.0 = A\sin\phi$,

requiring $(5.00)^2 + (10.0)^2 = A^2$

$A = 11.2$ and $\phi = 63.4°$

The resultant wave $\boxed{11.2\sin(2.00x - 10.0t + 63.4°)}$ is sinusoidal.

14.56 (a) With $k = \dfrac{2\pi}{\lambda}$ and $\omega = 2\pi f = \dfrac{2\pi v}{\lambda}$: $y(x, t) = 2A\sin kx\cos\omega t = \boxed{2A\sin\left(\dfrac{2\pi x}{\lambda}\right)\cos\left(\dfrac{2\pi vt}{\lambda}\right)}$

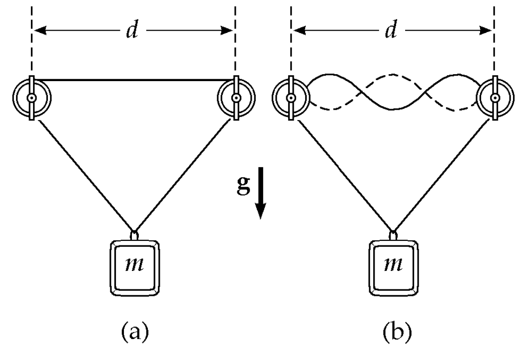
(b) For the fundamental vibration, $\lambda_1 = 2L$

so $y_1(x, t) = \boxed{2A\sin\left(\dfrac{\pi x}{L}\right)\cos\left(\dfrac{\pi vt}{L}\right)}$

(c) For the second harmonic $\lambda_2 = L$ and $y_2(x, t) = \boxed{2A\sin\left(\dfrac{2\pi x}{L}\right)\cos\left(\dfrac{2\pi vt}{L}\right)}$

(d) In general, $\lambda_n = \dfrac{2L}{n}$ and $y_n(x, t) = \boxed{2A\sin\left(\dfrac{n\pi x}{L}\right)\cos\left(\dfrac{n\pi vt}{L}\right)}$

***14.57** (a) Let θ represent the angle each slanted rope makes with the vertical.



In the diagram, observe that: $\sin\theta = \frac{1.00\text{ m}}{1.50\text{ m}} = \frac{2}{3}$

or $\theta = 41.8°$

Considering the mass,

$\Sigma F_y = 0$: $2T\cos\theta = mg$

or $T = \frac{(12.0\text{ kg})(9.80\text{ m/s}^2)}{2\cos 41.8°} = \boxed{78.9\text{ N}}$

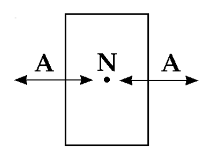
(b) The speed of transverse waves in the string is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{78.9\text{ N}}{0.00100\text{ kg/m}}} = 281\text{ m/s}$

For the standing wave pattern shown (3 loops), $d = \frac{3}{2}\lambda$

or $\lambda = \frac{2(2.00\text{ m})}{3} = 1.33\text{ m}$

Thus, the required frequency is $f = \frac{v}{\lambda} = \frac{281\text{ m/s}}{1.33\text{ m}} = \boxed{211\text{ Hz}}$

***14.58** $d_{AA} = \frac{\lambda}{2} = 7.05\times10^{-3}\text{ m}$ is the distance between antinodes.



Then $\lambda = 14.1\times10^{-3}\text{ m}$

and $f = \frac{v}{\lambda} = \frac{3.70\times10^{3}\text{ m/s}}{14.1\times10^{-3}\text{ m}} = \boxed{2.62\times10^{5}\text{ Hz}}$

The crystal can be tuned to vibrate at 2^{18} Hz, so that binary counters can derive from it a signal at precisely 1 Hz.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. See the solution

4. 5.66 cm

6. 0.500 s

8. (a) 3.33 rad (b) 283 Hz

10. 15.7 m, 31.8 Hz, 500 m/s

12. See the solution

14. See the solution

16. 15.7 Hz

18. (a) 0.391 (b) 1.60 kHz

20. (a) 350 Hz (b) 400 kg

22. 31.2 cm from the bridge; 3.84%

24. $n(0.252\text{ m})$ with $n = 1, 2, 3, \ldots$

26. (a) 531 Hz (b) 42.5 mm

28. Around 3 kHz. A small-amplitude incoming sound at this frequency can, over time, excite a larger-amplitude oscillation of the air in the ear canal, to make the sound audible.

30. $\dfrac{\pi r^2 v}{2Rf}$

32. 0.502 m and 0.837 m

34. 21.5 m

36. (a) 521 Hz or 525 Hz (b) 526 Hz (c) reduced by 1.14%

38. See the solution

40. The condition for resonance is satisfied, because the 12 hr 24 min period of free oscillation agrees precisely with the period of the lunar excitation.

42. (a) 30:24:20:15 (b) 33.5, 26.8, 22.3, 16.7 cm (c) 256, 320, 384, 512 Hz

44. (a) 162 Hz (b) 1.06 m

46. 3.85 m/s away from the station or 3.77 m/s toward the station

48. 31.1 N

50. (a) 59.9 Hz (b) 20.0 cm

52. 0.111

54. (a) 45.0 or 55.0 Hz (b) 162 or 242 N

56. (a) $2A\sin(2\pi x/\lambda)\cos(2\pi vt/\lambda)$ (b) $2A\sin(\pi x/L)\cos(\pi vt/L)$
(c) $2A\sin(2\pi x/L)\cos(2\pi vt/L)$ (d) $2A\sin(n\pi x/L)\cos(n\pi vt/L)$

58. 262 kHz