CHAPTER 16 ANSWERS TO QUESTIONS

- **Q16.1** The copper's temperature drops and the water temperature rises until both temperatures are the same. Then the metal and the water are in thermal equilibrium.
- **Q16.2** The astronaut is referring to the temperature of the lunar surface, specifically a 400°F difference. A thermometer would register the temperature of the thermometer liquid. Since there is no atmosphere in the moon, the thermometer will not read a realistic temperature unless it is placed into the lunar soil.
- **Q16.3** All dimensions of the heated metal piece increase, including the size of the hole.



- **Q16.4** Thermal expansion of the glass container occurs first (since it is in contact with the hot water). Then the mercury heats up, and it expands.
- **Q16.5** The measurements made with the heated steel tape will be too short but only by a factor of 5×10^{-5} of the measured length.
- **Q16.6** The volume of the balloon will decrease. The pressure of the atmosphere remains the same, so from PV = nRT, volume must decrease with temperature.
- **Q16.7** The ideal gas law, PV = nRT predicts zero volume at absolute zero. This is incorrect because the ideal gas law cannot work all the way down to or below the temperature at which gas turns to liquid, or in the case of CO₂, a solid.
- **Q16.8** Suppose the balloon rises into air uniform in temperature. The air cannot be uniform in pressure because the lower layers support the weight of all the air above them. The rubber in a typical balloon is easy to stretch and stretches or contracts until interior and exterior pressures are nearly equal. So as the balloon rises it expands. This is an isothermal expansion with *P* decreasing as *V* increases by the same factor in PV = nRT. If the rubber wall is very strong it will eventually contain the helium at higher pressure than the air outside but at the same density, so that the balloon will stop rising. More likely, the rubber will stretch and break, releasing the helium to keep rising and "boil out" of the Earth's atmosphere.
- **Q16.9** Call the process isobaric cooling or isobaric contraction. The rubber wall is easy to stretch. The air inside is nearly at atmospheric pressure originally and stays at atmospheric pressure as the wall moves in, just maintaining equality of pressure outside and inside. The air is nearly an ideal gas to start with, but PV = nRT soon fails. Volume will drop by a larger factor than temperature as the water vapor liquefies and then freezes, as the carbon dioxide turns to snow, as the argon turns to slush, and as the oxygen liquefies. From the outside, you see contraction to a small fraction of the original volume.
- **Q16.10** Cylinder A must be at lower pressure. If the gas is thin, it will be at one-third the absolute pressure of B.
- **Q16.11** At high temperature and pressure, the steam inside exerts large forces on the pot and cover. Strong latches hold them together, but they would explode apart if you tried to open the hot cooker.

- **Q16.12** (a) The water level in the cave rises by a smaller distance than the water outside, as the trapped air is compressed. Air can escape from the cave if the rock is not completely airtight, and also by dissolving in the water.
 - (b) The ideal cave stays completely full of water at low tide. The water in the cave is supported by atmospheric pressure on the free water surface outside.



- **Q16.13** The alcohol evaporates, absorbing energy from the skin to lower the skin temperature.
- **Q16.14** Refer to equations 16.15 and 16.19.

(a) 3 (b) $\sqrt{3}$

Now think of the first steps in the kinetic-theory account of how a gas exerts pressure. (c) $\sqrt{3}$ (d) $\sqrt{3}$ (e) 3

Q16.15 Absolute zero is a natural choice for the zero of a temperature scale. If an alien race had bodies that were mostly liquid water — or if they just liked its taste or its cleaning properties — it is conceivable that they might place one hundred degrees between its freezing and boiling points. It is very unlikely, on the other hand, that these would be our familiar "normal" ice and steam points, because atmospheric pressure would surely be different where the aliens come from.

PROBLEM SOLUTIONS

16.1 Since we have a linear graph, the pressure is related to the temperature as P = A + BT, where *A* and *B* are constants. To find *A* and *B*, we use the data

$$0.900 \text{ atm} = A + (-80.0^{\circ}\text{C})B \tag{1}$$

$$1.635 \text{ atm} = A + (78.0^{\circ}\text{C})B \tag{2}$$

Solving (1) and (2) simultaneously,

we find
$$A = 1.272$$
 atmand $B = 4.652 \times 10^{-3}$ atm/°CTherefore, $P = 1.272$ atm + $(4.652 \times 10^{-3} \text{ atm/°C})T$ (a)At absolute zero $P = 0 = 1.272$ atm + $(4.652 \times 10^{-3} \text{ atm/°C})T$ which gives $T = -274^{\circ}C$ (b)At the freezing point of water $P = 1.272$ atm + $0 = 1.272$ atm + $0 = 1.272$ atm(c)And at the boiling point $P = 1.272$ atm + $(4.652 \times 10^{-3} \text{ atm/°C})(100^{\circ}C) = 1.74$ atm

16.2

$$P_1V = nRT_1$$

and $P_2V = nRT_2$

imply that $\frac{P_2}{P_1} = \frac{T_2}{T_1}$ (a) $P_2 = \frac{P_1 T_2}{T_1} = \frac{(0.980 \text{ atm})(273 \text{ K} + 45.0 \text{ K})}{(273 + 20.0) \text{ K}} = \boxed{1.06 \text{ atm}}$ (b) $T_3 = \frac{T_1 P_3}{P_1} = \frac{(293 \text{ K})(0.500 \text{ atm})}{0.980 \text{ atm}} = 149 \text{ K} = \boxed{-124 \text{ °C}}$



16.3 (a)
$$T_F = \frac{9}{5}T_C + 32.0 \text{ °F} = \frac{9}{5}(-195.81) + 32.0 = \boxed{-320 \text{ °F}}$$

(b) $T = T_C + 273.15 = -195.81 + 273.15 = \boxed{77.3 \text{ K}}$

- 16.4 (a) To convert from Fahrenheit to Celsius, we use and the Kelvin temperature is found as
 - (b) In a fashion identical to that used in (a), we find and

$$T_{\rm C} = \frac{5}{9} (T_F - 32.0) = \frac{5}{9} (98.6 - 32.0) = 37.0^{\circ}{\rm C}$$
$$T = T_{\rm C} + 273 = 310 {\rm K}$$
$$T_{\rm C} = -20.6 {\rm ^{\circ}{\rm C}}$$
$$T = 253 {\rm K}$$

16.5 (a)
$$\Delta T = 450^{\circ}\text{C} = 450^{\circ}\text{C} \left(\frac{212^{\circ}\text{F} - 32.0^{\circ}\text{F}}{100^{\circ}\text{C} - 0.00^{\circ}\text{C}}\right) = 810^{\circ}\text{F}$$

(b) $\Delta T = 450^{\circ}\text{C} = 450 \text{ K}$

*16.6
$$\alpha = 1.10 \times 10^{-5} \, {}^{\circ}\text{C}^{-1}$$
 for steel
 $\Delta L = 518 \, \text{m} (1.10 \times 10^{-5} \, {}^{\circ}\text{C}^{-1}) [35.0^{\circ}\text{C} - (-20.0^{\circ}\text{C})] = \boxed{0.313 \, \text{m}}$

*16.7 (a)
$$\Delta L = \alpha L_i \Delta T = (24.0 \times 10^{-6} \, ^{\circ}\text{C}^{-1})(3.0000 \, \text{m})(80.0^{\circ}\text{C}) = 0.00576 \, \text{m}$$

 $L_f = \boxed{3.0058 \, \text{m}}$
(b) $\Delta L = \alpha L_i \Delta T = (24.0 \times 10^{-6} \, ^{\circ}\text{C}^{-1})(3.0000 \, \text{m})(-20.0^{\circ}\text{C}) = -0.0014 \, \text{m}$
 $L_f = \boxed{2.9986 \, \text{m}}$

16.8 For the dimensions to increase,
$$\Delta L = \alpha L_i \Delta T$$

 $1.00 \times 10^{-2} \text{ cm} = 1.30 \times 10^{-4} \text{ °C}^{-1}(2.20 \text{ cm})(T - 20.0 \text{ °C})$
 $T = 55.0 \text{ °C}$

16.9 (a)
$$\Delta L = \alpha L_i \Delta T = 9.00 \times 10^{-6} \, ^{\circ}\mathrm{C}^{-1}(30.0 \, \mathrm{cm})(65.0 \, ^{\circ}\mathrm{C}) = \boxed{0.176 \, \mathrm{mm}}$$

(b) $\Delta L = \alpha L_i \Delta T = 9.00 \times 10^{-6} \, ^{\circ}\mathrm{C}^{-1}(1.50 \, \mathrm{cm})(65.0 \, ^{\circ}\mathrm{C}) = \boxed{8.78 \times 10^{-4} \, \mathrm{cm}}$
(c) $\Delta V = 3\alpha V_i \Delta T = 3(9.00 \times 10^{-6} \, ^{\circ}\mathrm{C}^{-1})\left(\frac{30.0(\pi)(1.50)^2}{4} \, \mathrm{cm}^3\right)(65.0 \, ^{\circ}\mathrm{C}) = \boxed{0.0930 \, \mathrm{cm}^3}$

16.10 (a)
$$\Delta A = 2\alpha A_i \Delta T$$
: $\Delta A = 2(17.0 \times 10^{-6} \text{ °C}^{-1})(0.0800 \text{ m})^2(50.0 \text{ °C})$
 $\Delta A = 1.09 \times 10^{-5} \text{ m}^2 = \boxed{0.109 \text{ cm}^2}$

(b) The length of each side of the hole has increased. Thus, this represents an increase in the area of the hole.

16.11
$$\Delta V = (\beta - 3\alpha)V_i \Delta T = (5.81 \times 10^{-4} - 3(11.0 \times 10^{-6}))(50.0 \text{ gal})(20.0) = 0.548 \text{ gal}$$

*16.12 (a)
$$L = L_i(1 + \alpha \Delta T)$$
: 5.050 cm = 5.000 cm $[1 + 24.0 \times 10^{-6} \circ C^{-1}(T - 20.0 \circ C)]$
 $T = 437 \circ C$
(b) We must get $L_{A1} = L_{Brass}$ for some ΔT , or
 $L_{i,A1}(1 + \alpha_{A1}\Delta T) = L_{i,Brass}(1 + \alpha_{Brass}\Delta T)$
5.000 cm $[1 + (24.0 \times 10^{-6} \circ C^{-1})\Delta T] = 5.050$ cm $[1 + (19.0 \times 10^{-6} \circ C^{-1})\Delta T]$
Solving for ΔT , $\Delta T = 2080 \circ C$,
so $T = 3000 \circ C$
This will not work because aluminum melts at 660°C.

*16.13 (a)
$$\Delta V = V_t \beta_t \Delta T - V_{Al} \beta_{Al} \Delta T = (\beta_t - 3\alpha_{Al}) V_t \Delta T = (9.00 \times 10^{-4} - 0.720 \times 10^{-4}) \,^{\circ}\text{C}^{-1}(2000 \,\text{cm}^3)(60.0\,^{\circ}\text{C})$$

 $\Delta V = \boxed{99.4 \,\text{cm}^3} \text{ overflows}$

(b) The whole new volume of turpentine is

so the fraction lost is $\frac{99.4 \text{ cm}^3}{2108 \text{ cm}^3} = 4.71 \times 10^{-2}$

and this fraction of the cylinder's depth will be empty upon cooling:

$$4.71 \times 10^{-2} (20.0 \text{ cm}) = 0.943 \text{ cm}$$

*16.14 The area of the chip decreases according to
$$\Delta A = \gamma A_1 \Delta T = A_f - A_i$$

 $A_f = A_i (1 + \gamma \Delta T) = A_i (1 + 2\alpha \Delta T)$

The star images are scattered uniformly, so the number N of stars that fit is proportional to the area.

Then
$$N_f = N_i (1 + 2\alpha \Delta T) = 5342 [1 + 2(4.68 \times 10^{-6} \text{ °C}^{-1})(-100^{\circ}\text{C} - 20^{\circ}\text{C})] = 5336 \text{ star images}$$

*16.15 The horizontal section expands according to $\Delta L = \alpha L_i \Delta T$

$$\Delta x = (17 \times 10^{-6} \text{ °C}^{-1})(28.0 \text{ cm})(46.5 \text{ °C} - 18.0 \text{ °C}) = 1.36 \times 10^{-2} \text{ cm}$$

The vertical section expands similarly by

$$\Delta y = (17 \times 10^{-6} \text{ °C}^{-1})(134 \text{ cm})(28.5 \text{ °C}) = 6.49 \times 10^{-2} \text{ cm}$$

The vector displacement of the pipe elbow has magnitude

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(0.136 \text{ mm})^2 + (0.649 \text{ mm})^2} = 0.663 \text{ mm}$$

and is directed to the right below the horizontal at angle

$$\theta = \tan^{-1} \left(\frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left(\frac{0.649 \text{ mm}}{0.136 \text{ mm}} \right) = 78.2^{\circ}$$

$$\Delta \mathbf{r} = 0.663 \text{ mm to the right at } 78.2^{\circ} \text{ below the horizontal}$$

16.16 (a) Initially,
$$P_iV_i = n_iRT_i$$
 $(1.00 \text{ atm})V_i = n_iR(10.0 + 273.15) \text{ K}$

 Finally, $P_fV_f = n_fRT_f$
 $P_f(0.280V_i) = n_iR(40.0 + 273.15) \text{ K}$

 Dividing these equations,
 $\frac{0.280P_f}{1.00 \text{ atm}} = \frac{313.15 \text{ K}}{283.15 \text{ K}}$

 giving
 $P_f = 3.95 \text{ atm}$

 or
 $P_f = \boxed{4.00 \times 10^5 \text{ Pa}(\text{abs.})}$

 (b) After being driven
 $P_d(1.02)(0.280V_i) = n_iR(85.0 + 273.15) \text{ K}$



16.17 The equation of state of an ideal gas is PV = nRT so we need to solve for the number of moles to find N.

$$n = \frac{PV}{RT} = \frac{(1.01 \times 10^5 \text{ N/m}^2)[(10.0 \text{ m})(20.0 \text{ m})(30.0 \text{ m})]}{(8.315 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 2.49 \times 10^5 \text{ mol}$$
$$N = nN_A = 2.49 \times 10^5 \text{ mol}(6.022 \times 10^{23} \text{ molecules/mol}) = 1.50 \times 10^{29} \text{ molecules}$$

16.18
$$PV = NP'V' = \frac{4}{3}\pi r^3 NP'$$
: $N = \frac{3PV}{4\pi r^3 P'} = \frac{3(150)(0.100)}{4\pi (0.150)^3 (1.20)} = \boxed{884 \text{ balloons}}$

16.19
$$\Sigma F_y = 0$$
: $\rho_{out}gV - \rho_{in}gV - (200 \text{ kg})g = 0$
 $(\rho_{out} - \rho_{in})(400 \text{ m}^3) = 200 \text{ kg}$

The density of the air outside is 1.25 kg/m^3 .

From PV = nRT, $\frac{n}{V} = \frac{P}{RT}$

Then

The density is inversely proportional to the temperature, and the density of the hot air is

/

$$\rho_{in} = \left(1.25 \text{ kg/m}^3\right) \left(\frac{283 \text{ K}}{T_{in}}\right)$$

$$\left(1.25 \text{ kg/m}^3\right) \left(1 - \frac{283 \text{ K}}{T_{in}}\right) (400 \text{ m}^3) = 200 \text{ kg} \qquad 1 - \frac{283 \text{ K}}{T_{in}} = 0.400$$

$$0.600 = \frac{283 \text{ K}}{T_{in}} \qquad T_{in} = \boxed{472 \text{ K}}$$

*16.20 (a)
$$PV = nRT$$
 $n = \frac{PV}{RT}$
 $m = nM = \frac{PVM}{RT} = \frac{1.013 \times 10^5 \text{ Pa}(0.100 \text{ m})^3 (28.9 \times 10^{-3} \text{ kg/mol})}{(8.315 \text{ J/mol} \cdot \text{K})(300 \text{ K})}$
 $m = \boxed{1.17 \times 10^{-3} \text{ kg}}$
(b) $F_g = mg = 1.17 \times 10^{-3} \text{ kg}(9.80 \text{ m/s}^2) = \boxed{11.5 \text{ mN}}$
(c) $F = PA = (1.013 \times 10^5 \text{ N/m}^2)(0.100 \text{ m})^2 = \boxed{1.01 \text{ kN}}$

(d) The molecules must be moving very fast to hit the walls hard.



16.21 At depth,
$$P = P_0 + \rho gh$$
 and $PV_i = nRT_i$
At the surface, $P_0V_f = nRT_f$: $\frac{P_0V_f}{(P_0 + \rho gh)V_i} = \frac{T_f}{T_i}$
Therefore $V_f = V_i \left(\frac{T_f}{T_i}\right) \left(\frac{P_0 + \rho gh}{P_0}\right)$
 $V_f = 1.00 \text{ cm}^3 \left(\frac{293 \text{ K}}{278 \text{ K}}\right) \left(\frac{1.013 \times 10^5 \text{ Pa} + (1025 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(25.0 \text{ m})}{1.013 \times 10^5 \text{ Pa}}\right)$
 $V_f = 3.67 \text{ cm}^3$

My bedroom is 4 m long, 4 m wide, and 2.4 m high, enclosing air at 100 kPa and 20 $^\circ C$ = 293 K. Think of the air as 80.0% N_2 and 20.0% $O_2.$ 16.22

Avogadro's number of molecules has mass

(0.800)(28.0 g/mol) + (0.200)(32.0 g/mol) = 0.0288 kg/molPV = nRT = (m/M)RTThen

gives
$$m = \frac{PVM}{RT} = \frac{(1.00 \times 10^5 \text{ N/m}^2)(38.4 \text{ m}^3)(0.0288 \text{ kg/mol})}{(8.315 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 45.4 \text{ kg} \boxed{\sim 10^2 \text{ kg}}$$

*16.23
$$PV = nRT$$
: $\frac{m_f}{m_i} = \frac{n_f}{n_i} = \frac{P_f V_f}{RT_f} \frac{RT_f}{P_i V_f}$

$$\frac{m_f}{m_i} = \frac{n_f}{n_i} = \frac{P_f V_f}{RT_f} \frac{RT_i}{P_i V_i} = \frac{P_f}{P_i}$$

so

$$m_f = m_i \left(\frac{P_f}{P_i}\right)$$
$$|\Delta m| = m_i - m_f = m_i \left(\frac{P_i - P_f}{P_i}\right) = 12.0 \text{ kg} \left(\frac{41.0 \text{ atm} - 26.0 \text{ atm}}{41.0 \text{ atm}}\right) = 4.39 \text{ kg}$$

*16.24 The CO₂ is far from liquefaction, so after it comes out of solution it behaves as an ideal gas. Its molar mass is M = 12.0 g/mol + 2(16.0 g/mol) = 44.0 g/mol. The quantity of gas in the cylinder is $n = m_{\text{sample}} / M = 6.50 \text{ g} / (44.0 \text{ g} / \text{mol}) = 0.148 \text{ mol}$

> Then pV = nRT

gives
$$V = \frac{nRT}{p} = \frac{0.148 \text{ mol}(8.315 \text{ J/mol} \cdot \text{K})(273 \text{ K} + 20 \text{ K})}{1.013 \times 10^5 \text{ N/m}^2} \left(\frac{1 \text{ N} \cdot \text{m}}{1 \text{ J}}\right) \left(\frac{10^3 \text{ L}}{1 \text{ m}^3}\right) = 3.55 \text{ L}$$

*16.25
$$N = \frac{PVN_A}{RT} = \frac{(10^{-9} \text{ Pa})(1.00 \text{ m}^3)(6.02 \times 10^{23} \text{ molecule/mol})}{(8.315 \text{ J/K} \cdot \text{mol})(300 \text{ K})} = 2.41 \times 10^{11} \text{ molecules}$$

16.26 $P_0 V = n_1 R T_1 = (m_1 / M) R T_1$ $P_0 V = n_2 R T_2 = (m_2 / M) R T_2$ $m_1 - m_2 = \frac{P_0 V M}{R} \left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

*16.27 Consider the *x* axis to be perpendicular to the plane of the window. Then, the average force exerted on the window by the hail stones is

$$\overline{F} = Nm\left(\frac{\Delta v}{\Delta t}\right) = Nm\left(\frac{v_{xf} - v_{xi}}{t}\right) = Nm\left(\frac{v\sin\theta - (-v\sin\theta)}{t}\right) = \boxed{Nm\left(\frac{2v\sin\theta}{t}\right)}$$
Thus, the pressure on the window pane is
$$P = \frac{\overline{F}}{A} = \boxed{Nm\left(\frac{2v\sin\theta}{At}\right)}$$

16.28

 $\overline{F} = \frac{(5.00 \times 10^{23}) [2(4.68 \times 10^{-26} \text{ kg})(300 \text{ m/s})]}{1.00 \text{ s}} = 14.0 \text{ N}$ $P = \frac{\overline{F}}{A} = \frac{14.0 \text{ N}}{8.00 \times 10^{-4} \text{ m}^2} = \boxed{17.6 \text{ kPa}}$

and

*16.29 (a)
$$PV = Nk_BT$$
:

$$N = \frac{PV}{k_BT} = \frac{1.013 \times 10^5 \text{ Pa}\left[\frac{4}{3}\pi (0.150 \text{ m})^3\right]}{(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})} = \boxed{3.54 \times 10^{23} \text{ atoms}}$$

(b)
$$\overline{K} = \frac{3}{2}k_BT = \frac{3}{2}(1.38 \times 10^{-23})(293) \text{ J} = \boxed{6.07 \times 10^{-21} \text{ J}}$$

(c) For helium, the atomic mass is
$$m = \frac{4.00 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 6.64 \times 10^{-24} \text{ g/molecule}$$

 $m = 6.64 \times 10^{-27} \text{ kg/molecule}$
 $\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$: $\therefore v_{rms} = \sqrt{\frac{3k_BT}{m}} = \boxed{1.35 \text{ km/s}}$

16.30 One mole of helium contains Avogadro's number of molecules and has a mass of 4.00 g. Let us call *m* the mass of one atom, and we have

or $M_A m = 4.00 \text{ g/mol}$ $m = \frac{4.00 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 6.64 \times 10^{-24} \text{ g/molecule}$ $m = \boxed{6.64 \times 10^{-27} \text{ kg}}$

16.31 (a)
$$\overline{K} = \frac{3}{2}k_BT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(423 \text{ K}) = 8.76 \times 10^{-21} \text{ J}$$

(b)
$$\overline{K} = \frac{1}{2}mv_{rms}^2 = 8.76 \times 10^{-21} \text{ J}$$

so

$$v_{rms} = \sqrt{\frac{1.75 \times 10^{-20} \text{ J}}{m}}$$
(1)

For helium,

$$m = \frac{4.00 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 6.64 \times 10^{-24} \text{ g/molecule}$$
$$m = 6.64 \times 10^{-27} \text{ kg/molecule}$$

Similarly for argon,

$$m = \frac{39.9 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 6.63 \times 10^{-23} \text{ g/molecule}$$

$$m = 6.63 \times 10^{-26} \text{ kg/molecule}$$

Substituting in (1) above,

we find for helium,
$$v_{rms} = 1.62 \text{ km/s}$$

and for argon, $v_{rms} = 514 \text{ m/s}$

***16.32** (a)
$$PV = nRT = \frac{Nmv^2}{3}$$

The total translational kinetic energy is $\frac{Nmv^2}{2} = E_{\text{trans}}$: $E_{\text{trans}} = \frac{3}{2}PV = \frac{3}{2}(3.00 \times 1.013 \times 10^5)(5.00 \times 10^{-3}) = 2.28 \text{ kJ}$ (b) $\frac{mv^2}{2} = \frac{3k_BT}{2} = \frac{3RT}{2N_A} = \frac{3(8.315)(300)}{2(6.02 \times 10^{23})} = 6.22 \times 10^{-21} \text{ J}$

***16.33** (a)
$$v_{av} = \frac{\sum n_i v_i}{N} = \frac{1}{15} [1(2) + 2(3) + 3(5) + 4(7) + 3(9) + 2(12)] = 6.80 \text{ m/s}$$

(b)
$$\left(v^2\right)_{av} = \frac{\Sigma n_i v_i^2}{N} = 54.9 \text{ m}^2/\text{s}^2$$

so

$$v_{rms} = \sqrt{\left(v^2\right)_{av}} = \sqrt{54.9} = \boxed{7.41 \text{ m/s}}$$

(c)
$$v_{mp} = 7.00 \text{ m/s}$$

*16.34 Following Equation 16.23,
$$v_{mp} = \sqrt{\frac{2k_BT}{m}} = \sqrt{\frac{2(1.38 \times 10^{-23} \text{ J/K})(4.20 \text{ K})}{6.64 \times 10^{-27} \text{ kg}}} = 132 \text{ m/s}$$

*16.35 Use Equation 16.20; take
$$\frac{dN_v}{dv} = 0$$
 $4\pi N \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right) \left(2v - \frac{2mv^3}{2k_B T}\right) = 0$

and solve for v_{mp} to get Equation 16.23.

Reject as solutions
$$v = 0$$
 and $v = \infty$ Retain only $2 - \frac{mv^2}{k_BT} = 0$

Then
$$v_{mp} = \sqrt{\frac{2k_BT}{m}}$$

*16.36 (a) From
$$v_{av} = \sqrt{\frac{8k_BT}{\pi m}}$$

(b)

$$T = \frac{\pi \left(6.64 \times 10^{-27} \text{ kg}\right) \left(1.12 \times 10^4 \text{ m/s}\right)^2}{8 \left(1.38 \times 10^{-23} \text{ J/mol} \cdot \text{K}\right)} = \boxed{2.37 \times 10^4 \text{ K}}$$

$$T = \frac{\pi \left(6.64 \times 10^{-27} \text{ kg}\right) \left(2.37 \times 10^3 \text{ m/s}\right)^2}{8 \left(1.38 \times 10^{-23} \text{ J/mol} \cdot \text{K}\right)} = \boxed{1.06 \times 10^3 \text{ K}}$$

*16.37 For a uniform lapse rate, the identity
$$\frac{\Delta T}{\Delta y} = \frac{T_f - T_i}{\Delta y}$$

implies
$$T_f = T_i + \frac{\Delta T}{\Delta y} \Delta y = 30^{\circ}\text{C} - (6.7^{\circ}\text{C}/\text{km})(3.66 \text{ km}) = 5.5^{\circ}\text{C}$$

*16.38 (a)
$$\frac{dT}{dy} = -\frac{\gamma - 1}{\gamma} \frac{gM}{R} = -\frac{0.40}{1.40} \frac{(9.8 \text{ m/s}^2)(28.9 \text{ g/mol})}{(8.315 \text{ J/mol} \cdot \text{K})} \left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) \left(\frac{1 \text{ J}}{1 \text{ kg} \cdot \text{m}^2 / \text{s}^2}\right)$$
$$= -9.73 \times 10^{-3} \text{ K/m} = \boxed{-9.73^{\circ}\text{C/km}}$$

- (b) Air contains water vapor. Air does not behave as an ideal gas. As a parcel of air rises in the atmosphere and its temperature drops, its ability to contain water vapor decreases, so water will likely condense out as liquid drops or as ice crystals. (The condensate may or may not be visible as clouds.) The condensate releases its heat of vaporization, raising the air temperature above the value that would be expected according to part (a).
- (b) For an object of mass *m* on Mars,

weight = force of planet's gravity:

$$mg = \frac{GM_{\text{Mars}}m}{r_{\text{Mars}}^2} \quad \text{or} \quad g = \frac{GM_{\text{Mars}}}{r_{\text{Mars}}^2}$$

$$g = \frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(6.42 \times 10^{23} \text{ kg}\right)}{\left(3.37 \times 10^6 \text{ m}\right)^2} = 3.77 \text{ m/s}^2$$

$$\frac{dT}{dy} = -\frac{\gamma - 1}{\gamma} \frac{gM}{R} = -\frac{0.30}{1.30} \frac{(3.77 \text{ m/s}^2)(0.0440 \text{ kg/mol})}{(8.315 \text{ J/mol} \cdot \text{K})} = -4.60 \times 10^{-3} \text{ K/m} = -4.60 \text{ K/m}$$

(d)
$$\frac{\Delta T}{\Delta y} = \frac{dT}{dy}:$$

$$\Delta y = \frac{\Delta T}{dT/dy} = \frac{-60^{\circ}\text{C} - (-40^{\circ}\text{C})}{-4.60^{\circ}\text{C}/\text{km}} = \boxed{4.34 \text{ km}}$$

(e) The dust in the atmosphere absorbs and scatters energy from the electromagnetic radiation coming through the atmosphere from the sun. The dust contributes energy to the gas molecules high in the atmosphere, resulting in an increase in the internal energy of the atmosphere aloft and a smaller decrease in temperature with height, than in the case where there is no absorption of sunlight. The larger the amount of dust, the more the lapse rate will deviate from the theoretical value in part (c). Thus it was dustier during the *Mariner* flights in 1969.

16.39 The excess expansion of the brass is
$$\Delta L_{\rm rod} - \Delta L_{\rm tape} = (\alpha_{\rm brass} - \alpha_{\rm steel})L_i\Delta T$$

 $\Delta(\Delta L) = (19.0 - 11.0) \times 10^{-6} (^{\circ}{\rm C})^{-1}(0.950 \text{ m})(35.0^{\circ}{\rm C})$
 $\Delta(\Delta L) = 2.66 \times 10^{-4} \text{ m}$
(a) The rod contracts more than tape to

a length reading 0.9500 m - 0.000266 m = 0.9497 m(b) 0.9500 m + 0.000266 m = 0.9503 m **16.40** At 0°C, 10.0 gallons of gasoline has mass,

from $\rho = m/V$

$$m = \rho V = (730 \text{ kg/m}^3)(10.0 \text{ gal}) \left(\frac{0.00380 \text{ m}^3}{1.00 \text{ gal}}\right) = 27.7 \text{ kg}$$

The gasoline will expand in volume by

$$\Delta V = \beta V_i \Delta T = 9.60 \times 10^{-4} \text{ °C}^{-1} (10.0 \text{ gal}) (20.0^{\circ}\text{C} - 0.0^{\circ}\text{C}) = 0.192 \text{ gal}$$

At 20.0°C, 10.192 gal = 27.7 kg

10.0 gal = 27.7 kg
$$\left(\frac{10.0 \text{ gal}}{10.192 \text{ gal}}\right)$$
= 27.2 kg

The extra mass contained in 10.0 gallons at 0.0°C is

$$27.7 \text{ kg} - 27.2 \text{ kg} = 0.523 \text{ kg}$$

16.41 Neglecting the expansion of the glass,

$$\Delta h = \frac{V}{A} \beta \Delta T$$
$$\Delta h = \frac{\frac{4}{3} \pi \left(\frac{0.250 \text{ cm}}{2}\right)^3}{\pi \left(2.00 \times 10^{-3} \text{ cm}\right)^2} (1.82 \times 10^{-4} \text{ °C}^{-1})(30.0^{\circ}\text{C}) = \boxed{3.55 \text{ cm}}$$



16.42 (a) The volume of the liquid increases as $\Delta V_{\ell} = V_i \beta \Delta T$. The volume of the flask increases as $\Delta V_g = 3\alpha V_i \Delta T$. Therefore, the overflow in the capillary is $V_c = V_i \Delta T (\beta - 3\alpha)$; and in the capillary $V_c = A \Delta h$.

Therefore, $\Delta h = \frac{V_i}{A} (\beta - 3\alpha) \Delta T$

or

(b) For a mercury thermometer β (Hg) = $1.82 \times 10^{-4} \circ C^{-1}$ and for glass, $3\alpha = 3 \times 3.20 \times 10^{-6} \circ C^{-1}$ Thus $\beta - 3\alpha \cong \beta$

 $\alpha << \beta$

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16.43 (a)
$$\rho = \frac{m}{V}$$
 and $d\rho = -\frac{m}{V^2}dV$

For very small changes in *V* and ρ , this can be expressed as

$$\Delta \rho = -\frac{m}{V} \frac{\Delta V}{V} = -\rho \,\beta \,\Delta T$$

The negative sign means that any increase in temperature causes the density to decrease and vice versa.

(b) For water we have
$$\beta = \left| \frac{\Delta \rho}{\rho \Delta T} \right| = \left| \frac{1.0000 \text{ g/cm}^3 - 0.9997 \text{ g/cm}^3}{(1.0000 \text{ g/cm}^3)(10.0 \text{ }^\circ\text{C} - 4.0 \text{ }^\circ\text{C})} \right| = 5 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$$

***16.44** The astronauts exhale this much CO₂:

$$n = \frac{m_{\text{sample}}}{M} = \frac{1.09 \text{ kg}}{\text{astronaut} \cdot \text{day}} \left(\frac{1000 \text{ g}}{1 \text{ kg}}\right) (3 \text{ astronauts}) (7 \text{ days}) \left(\frac{1 \text{ mol}}{44.0 \text{ g}}\right) = 520 \text{ mol}$$

Then 520 mol of methane is generated. It is far from liquefaction and behaves as an ideal gas.

$$P = \frac{nRT}{V} = \frac{520 \text{ mol}(8.315 \text{ J/mol} \cdot \text{K})(273 \text{ K} - 45 \text{ K})}{150 \times 10^{-3} \text{ m}^3} = 6.58 \times 10^6 \text{ Pa}$$

16.45 (a) We assume that air at atmospheric pressure is above the piston.

 $P_{gas} = \frac{mg}{A} + P_0$

 $\frac{nRT}{hA} = \frac{mg}{A} + P_0$

In equilibrium

Therefore,

or

$$h = \frac{nRT}{mg + P_0A}$$

where we have used V = hA as the volume of the gas.

(b) From the data given,



$$h = \frac{0.200 \text{ mol}(8.315 \text{ J/K} \cdot \text{mol})(400 \text{ K})}{20.0 \text{ kg}(9.80 \text{ m/s}^2) + (1.013 \times 10^5 \text{ N/m}^2)(0.00800 \text{ m}^2)} = \boxed{0.661 \text{ m}}$$

*16.46 The angle of bending θ , between tangents to the two ends of the strip, is equal to the angle the strip subtends at its center of curvature. (The angles are equal because their sides are perpendicular, right side to the right side and left side to left side.)

and



(a) The definition of radian measure gives

By subtraction,

(b) In the expression from part (a), θ is directly proportional to ΔT and also to $(\alpha_2 - \alpha_1)$. Therefore θ is zero when either of these quantities becomes zero.

 $L_i + \Delta L_1 = \theta r_1$

 $L_i + \Delta L_2 = \theta r_2$

 $\Delta L_2 - \Delta L_1 = \theta (r_2 - r_1)$

 $\theta = \frac{(\alpha_2 - \alpha_1)L_i\Delta T}{\Delta r}$

 $\alpha_2 L_i \Delta T - \alpha_1 L_i \Delta T = \theta \, \Delta r$

(c) The material that expands more when heated contracts more when cooled, so the bimetallic strip bends the other way. It is fun to demonstrate this with liquid nitrogen.

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(d)
$$\theta = \frac{2(\alpha_2 - \alpha_1)L_i\Delta T}{2\Delta r} = \frac{2((19 \times 10^{-6} - 0.9 \times 10^{-6})^{\circ}C^{-1})(200 \text{ mm})(1^{\circ}C)}{0.500 \text{ mm}}$$

= $1.45 \times 10^{-2} = 1.45 \times 10^{-2} \text{ rad} \left(\frac{180^{\circ}}{\pi \text{ rad}}\right) = \boxed{0.830^{\circ}}$

...

*16.47 (a)
$$T_i = 2\pi \sqrt{\frac{L_i}{g}}$$
 so $L_i = \frac{T_i^2 g}{4\pi^2} = \frac{(1.000 \text{ s})^2 (9.80 \text{ m/s}^2)}{4\pi^2} = 0.2482 \text{ m}$
 $\Delta L = \alpha L_i \Delta T = 19.0 \times 10^{-6} \text{ °C}^{-1} (0.2843 \text{ m}) (10.0^{\circ} \text{C}) = 4.72 \times 10^{-5} \text{ m}$
 $T_f = 2\pi \sqrt{\frac{L_i + \Delta L}{g}} = 2\pi \sqrt{\frac{0.2483 \text{ m}}{9.80 \text{ m/s}^2}} = 1.0000949 \text{ s}$
 $\Delta T = \boxed{9.49 \times 10^{-5} \text{ s}}$
(b) In one week, the time lost is time lost = 1 week(9.49×10^{-5} s lost per second)

time lost =
$$(7.00 \text{ d/week})\left(\frac{86400 \text{ s}}{1.00 \text{ d}}\right)\left(9.49 \times 10^{-5} \frac{\text{s lost}}{\text{s}}\right)$$

time lost = 57.4 s lost

and

 $\Delta \ell = \ell \, \alpha \, \Delta T,$

16.48 From the diagram we see that the change in area is

$$\Delta A = \ell \, \Delta w + w \Delta \ell + \Delta w \Delta \ell$$

 $\Delta A = \ell w \alpha \, \Delta T + \ell w \alpha \, \Delta T$

 $\Delta w = w \alpha \, \Delta T$

Since $\Delta \ell$ and Δw are each small quantities, the product $\Delta w \Delta \ell$ will be very small. Therefore, we assume $\Delta w \Delta \ell \cong 0$.

 $w = \int_{T_i}^{T_i} T_i$ $w + \Delta w = \int_{T_i}^{T_i} T_i + \Delta T$

we then have

Since

and since $A = \ell w$, $\Delta A = 2\alpha A \Delta T$

The approximation assumes $\Delta w \Delta l \cong 0$, or $\alpha \Delta T \cong 0$. Another way of stating this is $\alpha \Delta T \ll 1$.

*16.49		$I = \int r^2 dm$	and since	$r(T) = r(T_i)(1 + \alpha \Delta T)$
		for $\alpha \Delta T \ll 1$ we find		$\frac{I(T)}{I(T_i)} = (1 + \alpha \Delta T)^2$
		thus		$\frac{I(T) - I(T_i)}{I(T_i)} \cong 2\alpha \Delta T$
	(a)	With $\alpha = 17.0 \times 10$	$^{-6} \circ C^{-1}$ and	$\Delta T = 100^{\circ} \text{C}$
		we find for Cu:		$\frac{\Delta I}{I} = 2(17.0 \times 10^{-6} \text{ °C}^{-1})(100^{\circ}\text{C}) = \boxed{0.340\%}$
	(b)	With		$\alpha = 24.0 \times 10^{-6} {}^{\circ}\mathrm{C}^{-1}$
		and		$\Delta T = 100^{\circ} \text{C}$
		we find for Al:		$\frac{\Delta I}{I} = 2(24.0 \times 10^{-6} \text{ °C}^{-1})(100^{\circ}\text{C}) = \boxed{0.480\%}$

16.50 (a) Let *m* represent the sample mass. The number of moles is n = m/M and the density is $\rho = m/V$

So PV = nRT becomes $PV = \frac{m}{M}RT$ or $PM = \frac{m}{V}RT$ Then, $\rho = \frac{m}{V} = \boxed{\frac{PM}{RT}}$

(b)
$$\rho = \frac{PM}{RT} = \frac{(1.013 \times 10^5 \text{ N/m}^2)(0.0320 \text{ kg/mol})}{(8.315 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 1.33 \text{ kg/m}^3$$

*16.51 After expansion, the length of one of the spans is

$$L_f = L_i(1 + \alpha \Delta T) = 125 \text{ m}[1 + 12 \times 10^{-6} \text{ °C}^{-1}(20.0 \text{ C}^{\circ})] = 125.03 \text{ m}$$

 L_f , y, and the original 125 m length of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives:

 $(125.03 \text{ m})^2 = y^2 + (125 \text{ m})^2$ y = 2.74 m

*16.52 After expansion, the length of one of the spans is $L_f = L(1 + \alpha \Delta T)$. L_f , y, and the original length L of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives

$$L_f^2 = L^2 + y^2$$
, or $y = \sqrt{L_f^2 - L^2} = L\sqrt{(1 + \alpha \Delta T)^2 - 1} = L\sqrt{2\alpha \Delta T + (\alpha \Delta T)^2}$
Since $\alpha \Delta T \ll 1$, $y \equiv L\sqrt{2\alpha \Delta T}$

 $\frac{dV}{dT} = \frac{nR}{P} = \frac{V}{T}$

 $\beta \equiv \left(\frac{1}{V}\right) \frac{dV}{dT} = \left(\frac{1}{V}\right) \frac{V}{T}, \quad \text{or} \quad \beta = \boxed{\frac{1}{T}}$

*16.53 (a) From PV = nRT, the volume is: $V = \left(\frac{nR}{P}\right)T$

Therefore, when pressure is held constant,

Thus,

yielding

(b) At
$$T = 0^{\circ}C = 273$$
 K, this predicts $\beta = \frac{1}{273} = 3.66 \times 10^{-3} \text{ K}^{-1}$
Experimental values are: $\beta_{\text{He}} = 3.665 \times 10^{-3} \text{ K}^{-1}$ and $\beta_{\text{air}} = 3.67 \times 10^{-3} \text{ K}^{-1}$

They agree within 0.06% and 0.2%, respectively.

16.54 For $\Delta L = L_s - L_c$ to be constant, the rods must expand by equal amounts: $\alpha_c L_c \Delta T = \alpha_s L_s \Delta T$

$$L_{s} = \frac{\alpha_{c}L_{c}}{\alpha_{s}} \qquad \text{and} \qquad \Delta L = \frac{\alpha_{c}L_{c}}{\alpha_{s}} - L_{c}$$
$$\therefore \quad L_{c} = \frac{\Delta L\alpha_{s}}{\alpha_{c} - \alpha_{s}} = \frac{5.00 \text{ cm}(11.0 \times 10^{-6} \text{ °C}^{-1})}{(17.0 \times 10^{-6} \text{ °C}^{-1} - 11.0 \times 10^{-6} \text{ °C}^{-1})} = 9.17 \text{ cm}$$
and
$$L_{s} = \frac{\Delta L\alpha_{c}}{\alpha_{c} - \alpha_{s}} = 5.00 \text{ cm}\left(\frac{17.0}{6.00}\right) = 14.2 \text{ cm}$$

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T = constant, so $PV = P_0V_0$

***16.55** (a) With piston alone:

or
$$P(Ah_i) = P_0(Ah_0)$$

With A = constant,

But,

where m_p is the mass of the piston.

Thus,

$$P_0 + \frac{m_p g}{A} = P_0 \bigg(\frac{h_0}{h_i} \bigg)$$

 $P = P_0 \left(\frac{h_0}{h_i}\right)$

 $P = P_0 + \frac{m_p g}{A}$

which reduces to

$$h_i = \frac{h_0}{1 + \frac{m_p g}{P_0 A}} = \frac{50.0 \text{ cm}}{1 + \frac{20.0 \text{ kg}(9.80 \text{ m/s}^2)}{1.013 \times 10^5 \text{ Pa}[\pi (0.400 \text{ m})^2]}} = 49.81 \text{ cm}$$

With the man of mass *M* on the piston, a very similar calculation (replacing m_p by $m_p + M$) gives:

$$h' = \frac{h_0}{1 + \frac{(m_p + M)g}{P_0 A}} = \frac{50.0 \text{ cm}}{1 + \frac{95.0 \text{ kg}(9.80 \text{ m/s}^2)}{1.013 \times 10^5 \text{ Pa}[\pi (0.400 \text{ m})^2]}} = 49.10 \text{ cm}$$

Thus, when the man steps on the piston, it moves downward by

(b)
$$P = \text{const}$$
, so $\frac{V}{T} = \frac{V'}{T_i}$ or $\frac{Ah_i}{T} = \frac{Ah'}{T_i}$
giving $T = T_i \left(\frac{h_i}{h'}\right) = 293 \text{ K} \left(\frac{49.81}{49.10}\right) = \boxed{297 \text{ K}}$ (or 24°C)

*16.56 (a)
$$\frac{dL}{L} = \alpha dT$$
: $\int_{T_i}^{T_i} \alpha dT = \int_{L_i}^{L_i} \frac{dL}{L} \Rightarrow \ln\left(\frac{L_f}{L_i}\right) = \alpha \Delta T \Rightarrow L_f = L_i e^{\alpha \Delta T}$

(b)
$$L_f = (1.00 \text{ m})e^{\left[2.00 \times 10^{-5} \text{ °C}^{-1}(100^{\circ}\text{C})\right]} = 1.002002 \text{ m}$$

 $L'_f = 1.00 \text{ m}\left[1 + 2.00 \times 10^{-5} \text{ °C}^{-1}(100^{\circ}\text{C})\right] = 1.002000 \text{ m}; \qquad \frac{L_f - L'_f}{L_f} = 2.00 \times 10^{-6} = 2.00 \times 10^{-4\%}$
 $L_f = (1.00 \text{ m})e^{\left[2.00 \times 10^{-2} \text{ °C}^{-1}(100^{\circ}\text{C})\right]} = 7.389 \text{ m}$
 $L'_f = 1.00 \text{ m}\left[1 + 0.0200^{\circ}\text{C}^{-1}(100^{\circ}\text{C})\right] = 3.000 \text{ m}; \qquad \frac{L_f - L'_f}{L_f} = 59.4\%$



*16.57 Some gas will pass through the porous plug from the reaction chamber 1 to the reservoir 2 as the reaction chamber is heated, but the net quantity of gas stays constant according to

$$n_{i1} + n_{i2} = n_{f1} + n_{f2}$$

Assuming the gas is ideal, we apply n = PV / RT to each term:

$$\frac{P_i V_0}{(300 \text{ K})R} + \frac{P_i (4V_0)}{(300 \text{ K})R} = \frac{P_f V_0}{(673 \text{ K})R} + \frac{P_f (4V_0)}{(300 \text{ K})R}$$

$$1 \operatorname{atm} \left(\frac{5}{300 \text{ K}}\right) = P_f \left(\frac{1}{673 \text{ K}} + \frac{4}{300 \text{ K}}\right)$$

$$P_f = 1.12 \text{ atm}$$

*16.58 The pressure of the gas in the lungs of the diver must be the same as the absolute pressure of the water at this depth of 50.0 meters. This is:

or
$$P = P_0 + \rho gh = 1.00 \text{ atm} + (1.03 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(50.0 \text{ m})$$
$$P = 1.00 \text{ atm} + 5.05 \times 10^5 \text{ Pa} \left(\frac{1.00 \text{ atm}}{1.013 \times 10^5 \text{ Pa}}\right) = 5.98 \text{ atm}$$

If the partial pressure due to the oxygen in the gas mixture is to be 1.00 atmosphere (or the fraction 1/5.98 of the total pressure) oxygen molecules should make up only 1/5.98 of the total number of molecules. This will be true if 1.00 mole of oxygen is used for every 4.98 mole of helium. The ratio by weight is then

$$\frac{(4.98 \text{ mole He})(4.003 \text{ g/mole He})g}{(1.00 \text{ mole } O_2)(2 \times 15.999 \text{ g/mole } O_2)g} = \boxed{0.623}$$

*16.59 Let 2θ represent the angle the curved rail subtends. We have

$$L_i + \Delta L = 2\theta R = L_i (1 + \alpha \Delta T)$$

and

Thus, $\theta = \frac{L_i}{2R} (1 + \alpha \Delta T) = (1 + \alpha \Delta T) \sin \theta$

 $\sin\theta = \frac{L_i/2}{R} = \frac{L_i}{2R}$



and we must solve the transcendental equation

 $\theta = (1 + \alpha \Delta T) \sin \theta = (1.0000055) \sin \theta$

 $\theta = 0.01816 \text{ rad} = 1.0405^{\circ}$

 $h = R - R\cos\theta = \frac{L_i(1 - \cos\theta)}{2\sin\theta}$

Homing in on the non-zero solution gives, to four digits,

This yields h = 4.54 m, a remarkably large value compared to $\Delta L = 5.50 \text{ cm}$.

 $N = 1.00 \times 10^4$, $m = \frac{M}{N_A} = \frac{0.032 \text{ kg}}{6.02 \times 10^{23}} = 5.32 \times 10^{-26} \text{ kg}$

*16.60 (a) Maxwell's speed distribution function is

$$N_{v} = 4\pi N \left(\frac{m}{2\pi k_{B}T}\right)^{3/2} v^{2} e^{-mv^{2}/2k_{B}T}$$

With

 $T = 500 {
m K}$

and

$$k_B = 1.38 \times 10^{-23} \text{ J/molecule} \cdot \text{K}$$

this becomes $N_v = (1.71 \times 10^{-4}) v^2 e^{-(3.85 \times 10^{-6}) v^2}$

To the right is a plot of this function for the range $0 \le v \le 1500 \text{ m/s}$.

(b) The most probable speed occurs where N_v is a maximum.

From the graph, $v_{mp} \cong 510 \text{ m/s}$

(c)
$$v_{av} = \sqrt{\frac{8k_BT}{\pi m}} = \sqrt{\frac{8(1.38 \times 10^{-23})(500)}{\pi (5.32 \times 10^{-26})}} = 575 \text{ m/s}$$

Also,

$$v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23})(500)}{5.32 \times 10^{-26}}} = 624 \text{ m/s}$$

(d) The fraction of particles in the range

 $300 \text{ m/s} \le v \le 600 \text{ m/s}$

 $N = 10^4$

$$rac{\int_{300}^{600} N_v dv}{N}$$

where

is

and the integral of N_v is read from the graph as the area under the curve.

This is approximately 4400 and the fraction is 0.44 or 44%



*16.61
$$N_{v}(v) = 4\pi N \left(\frac{m}{2\pi k_{B}T}\right)^{3/2} v^{3} \exp\left(-mv^{2}/2k_{B}T\right)$$

Note that $v_{mp} = (2k_BT / m)^{1/2}$

 $v = v_{mp} / 50$

Thus,

$$N_{v}(v) = 4\pi N \left(\frac{m}{2\pi k_{B}T}\right)^{3/2} v^{2} e^{\left(-v^{2}/v_{mp}^{2}\right)}$$

And

$$\frac{N_v(v)}{N_v(v_{mp})} = \left(\frac{v}{v_{mp}}\right)^2 e^{\left(1 - v^2 / v_{mp}^2\right)}$$

For

$$\frac{N_v(v)}{N_v(v_{mp})} = \left(\frac{1}{50}\right)^2 e^{\left[1 - (1/50)^2\right]} = 1.09 \times 10^{-3}$$

The other values are computed similarly, with the following results:

	$N_v(v)$
v_{mp}	$N_v(v_{mp})$
1/50	1.09×10^{-3}
1/10	2.69×10^{-2}
1/2	0.529
1	1.00
2	0.199
10	1.01×10^{-41}
50	1.25×10^{-1082}

To find the last value, note:

$$(50)^2 e^{1-2500} = 2500 e^{-2499}$$

 $10^{\log 2500} e^{(\ln 10)(-2499 / \ln 10)} = 10^{\log 2500} 10^{-2499 / \ln 10} = 10^{\log 2500 - 2499 / \ln 10} = 10^{-1081.904}$

ANSWERS TO EVEN NUMBERED PROBLEMS

2.	(a) 1.06 atm	(b)	–124°C
4.	(a) $37.0 \circ C = 310 \text{ K}$	(b)	−20.6 °C = 253 K
6.	0.313 m		
8.	55.0 °C		
10.	(a) 0.109 cm^2	(b)	increase
12.	(a) 437 °C	(b)	3 000 °C. No; aluminum melts at 660 °C.
14.	5 336 star images		
16.	(a) 400 kPa	(b)	449 kPa
18.	884 balloons		
20.	(a) 1.17 g (c) 1.01 kN	(b) (d)	11.5 mN The molecules are moving very fast.
22.	between 10 and 100 kg		
24.	3.55 L		
26.	$m_1 - m_2 = \frac{P_0 V M}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$		
28.	17.6 kPa		
30.	$6.64 \times 10^{-27} \text{ kg}$		
32.	(a) 2.28 kJ	(b)	$6.22 \times 10^{-21} \text{ J}$
34.	132 m/s		

36.	(a) 2.37×10^4 K	(b)	$1.06 \times 10^3 \text{ K}$
38.	(a) -9.73 °C/km (d) 4.34 km	(b) (e)	See the solution. (c) -4.60 °C/km See the solution; the <i>Mariner</i> flights.
40.	0.523 kg		
42.	(a) See the solution.	(b)	$\alpha_{ m glass} << \beta_{ m Hg}$
44.	6.58 MPa		
46.	(a) $\theta = L_i(\alpha_2 - \alpha_1) \frac{\Delta T}{(r_2 - r_1)}$ (c) It bends the opposite way.	(b) (d)	$\theta \rightarrow 0 \text{ as } \Delta T \rightarrow 0 \text{ and as } \alpha_1 \rightarrow \alpha_2$ 0.830°
48.	See the solution. We assume that $\alpha \Delta$	ΔT is	much less than 1.
50.	(a) See the solution.	(b)	1.33 kg/m^3
52.	$y = L \sqrt{2\alpha \Delta T}$		
54.	$L_{\rm C} = 9.17$ cm, $L_{\rm S} = 14.2$ cm		
56.	(a) $L_f = L_i e^{\alpha \Delta T}$	(b)	2.00×10^{-4} %; 59.4%
58.	0.623		
60.	 (a) See the solution (c) 575 m/s, 624 m/s 	(b) (d)	about 510 m/s 44%