CHAPTER 19

ANSWERS TO QUESTIONS

Q19.1 Electrons are less massive and more mobile than protons. Also, they are more easily detached from atoms than protons.

Q19.2 The clothes dryer rubs dissimilar materials together as it tumbles the clothes. Electrons are transferred from one kind of molecule to another. The charges on pieces of cloth, or on nearby objects charged by induction, can produce strong electric fields that promote the ionization process in the surrounding air that is necessary for a spark to occur. Then you hear or see the sparks.

Q19.3 No. The balloon induces polarization of the molecules in the wall, so that a layer of positive charge exists near the balloon. This is just like the situation in Figure 19.6a, except that the signs of the charges are reversed. The attraction between these charges and the negative charges on the balloon is stronger than the repulsion between the negative charges on the balloon and the negative charges in the polarized molecules (because they are farther from the balloon), so that there is a net attractive force toward the wall. Ionization processes in the air surrounding the balloon provide ions to which excess electrons in the balloon can transfer, reducing the charge on the balloon and eventually causing the attractive force to be insufficient to support the weight of the balloon.

Q19.4 When the comb is nearby, molecules in the paper are polarized, similar to the molecules in the wall in Figure 19.6a, and the paper is attracted. After contact, charge from the comb is transferred to the paper so it now has the same charge as the comb, and is thus repelled.

Q19.5 To avoid making a spark. Rubber-soled shoes acquire a charge by friction with the floor and could discharge with a spark, possibly causing an explosion.

Q19.6 No. Life would be no different if electrons were + charged and protons were – charged. Opposite charges would still attract, and like charges would repel.
The naming of + and – charge is merely a convention.

Q19.7 Similarities: A force of gravity is proportional to the product of the intrinsic properties (masses) of two particles, and inversely proportional to the square of the separation distance. An electrical force exhibits the same proportionalities, with charge as the intrinsic property.
Differences: The electrical force can either attract or repel, while the gravitational force can only attract. The electrical force between elementary particles is vastly stronger than the gravitational force.

Q19.8 So the electric field from the test charge does not distort the electric field you are trying to measure, by moving the charges that create it.

Q19.9 At a point exactly midway between the two changes.

Q19.10 The negative charge will be drawn to the center of the positively charged ring. Since it will then have velocity, it will continue on, to an equidistant point on the opposite side of the ring. It will then start moving back and arrive again at point P. This periodic motion will continue. If x is much less than a, the motion can be shown from the solution for the electric field to be simple harmonic.

Q19.11 Four times as many electric field lines start at the surface of the larger charge as end at the smaller charge. The extra lines extend away from the pair of charges. They may never end, or they may terminate on more distant negative charges. (Figure 19.20 shows the situation for charges $+2q$ and $-q$).

Q19.12 The fields are equal. The Equation 19.25 $E = \sigma_{\text{conductor}} / \epsilon_0$ for the field outside the aluminum looks different from Equation 19.24 $E = \sigma_{\text{insulator}} / 2\epsilon_0$ for the field around glass. But its charge will spread out to cover both sides of the aluminum plate, so the density is $\sigma_{\text{conductor}} = Q/2A$. The glass carries charge only on area A, with $\sigma_{\text{insulator}} = Q/A$. The two fields are $Q/2A\epsilon_0$ the same in magnitude, and both are perpendicular to the plates.

Q19.13 The surface must enclose a positive total charge.

Q19.14 The net flux through any gaussian surface is zero. We can argue it two ways. Any surface contains zero charge so Gauss's law says the total flux is zero. The field is uniform, so the field lines entering one side of the closed surface come out the other side and the net flux is zero.

Q19.15 Gauss's law cannot tell the different values of the electric field at different points on the surface. When E is an unknown number, then we can say $\int E\cos\theta\, dA = E\int \cos\theta\, dA$. When $E(x, y, z)$ is an unknown function, then there is no such simplification.

Q19.16 The electric flux through a sphere around a point charge is independent of the size of the sphere. A sphere of larger radius has a larger area, but a smaller field at its surface, so that the product of field strength and area is independent of radius. If the surface is not spherical, some parts are closer to the charge than others. In this case as well, smaller projected areas go with stronger fields, so that the net flux is unaffected.

Q19.17 There is zero force. The huge charged sheet creates a uniform field. The field can polarize the neutral sheet, creating in effect a film of opposite charge on the near face and a film with an equal amount of like charge on the far face of the neutral sheet. Since the field is uniform, the films of charge feel equal-magnitude forces of attraction and repulsion to the charged sheet. The forces add to zero.

Q19.18 Inject some charge at arbitrary places within a conducting object. Every bit of the charge repels every other bit, so each bit runs away as far as it can, stopping only when it reaches the outer surface of the conductor.

Q19.19 If a charge distribution is small compared to the distance of a field point from it, the charge distribution can be modeled as a single particle with charge equal to the net charge of the distribution. Further, if a charge distribution is spherically symmetric, it will create a field at exterior points just as if all of its charge were a point charge at its center.

Q19.20 If the person is uncharged, the electric field inside the sphere is zero. The interior wall of the shell carries no charge. The person is not harmed by touching this wall. If the person carries a (small) charge q, the electric field inside the sphere is no longer zero. Charge $-q$ is induced on the inner wall of the sphere. The person will get a (small) shock when touching the sphere, as all the charge on his body jumps to the metal.

Q19.21 In special orientations the force between two dipoles can be zero or a force of repulsion. In general each dipole will exert a torque on the other, tending to align its axis with the field created by the first dipole. After this alignment, each dipole exerts a force of attraction on the other.

Q19.22 The electric fields outside are identical. The electric fields inside are very different. We have $\mathbf{E} = 0$ everywhere inside the conducting sphere while E decreases gradually as you go below the surface of the sphere with uniform volume charge density.

PROBLEM SOLUTIONS

***19.1** (a) $N = \left(\dfrac{10.0 \text{ grams}}{107.87 \text{ grams/mol}}\right)\left(6.02\times10^{23}\ \dfrac{\text{atoms}}{\text{mol}}\right)\left(47\ \dfrac{\text{electrons}}{\text{atom}}\right) = \boxed{2.62\times10^{24}}$

(b) $\#\text{ electrons added} = \dfrac{Q}{e} = \dfrac{1.00\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C/electron}} = 6.25\times10^{15}$

or $\boxed{2.38 \text{ electrons for every } 10^9 \text{ already present}}$

***19.2** $F = k_e\dfrac{q_1q_2}{r^2} = \dfrac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2\left(6.02\times10^{23}\right)^2}{\left[2(6.37\times10^6\text{ m})\right]^2} = \boxed{514 \text{ kN}}$

19.3 If each person has a mass of ≈ 70 kg and is (almost) composed of water, then each person contains

$$N \cong \left(\frac{70\,000\text{ grams}}{18\text{ grams/mol}}\right)\left(6.02\times10^{23}\ \frac{\text{molecules}}{\text{mol}}\right)\left(10\ \frac{\text{protons}}{\text{molecule}}\right) \cong 2.3\times10^{28}\text{ protons}$$

With an excess of 1% electrons over protons, each person has a charge

$$q = 0.01\left(1.6\times10^{-19}\text{ C}\right)\left(2.3\times10^{28}\right) = 3.7\times10^{7}\text{ C}$$

So $F = k_e\dfrac{q_1q_2}{r^2} = (9\times10^9)\dfrac{(3.7\times10^7)^2}{0.6^2}\text{ N} = 4\times10^{25}\text{ N}\ \boxed{\sim 10^{26}\text{ N}}$

This force is almost enough to lift a weight equal to that of the Earth:

$$Mg = 6\times10^{24}\text{ kg}\left(9.8\text{ m/s}^2\right) = 6\times10^{25}\text{ N} \sim 10^{26}\text{ N}$$

19.4 (a) The force is one of $\boxed{\text{attraction}}$. The distance r in Coulomb's law is the distance between centers. The magnitude of the force is

$$F = \frac{k_eq_1q_2}{r^2} = \left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(12.0\times10^{-9}\text{ C}\right)\left(18.0\times10^{-9}\text{ C}\right)}{(0.300\text{ m})^2} = \boxed{2.16\times10^{-5}\text{ N}}$$

(b) The net charge of -6.00×10^{-9} C will be equally split between the two spheres, or -3.00×10^{-9} C on each. The force is one of $\boxed{\text{repulsion}}$, and its magnitude is

$$F = \frac{k_eq_1q_2}{r^2} = \left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(3.00\times10^{-9}\text{ C}\right)\left(3.00\times10^{-9}\text{ C}\right)}{(0.300\text{ m})^2} = \boxed{8.99\times10^{-7}\text{ N}}$$

***19.5** The force on one proton is $\mathbf{F} = \dfrac{k_e q_1 q_2}{r^2}$ away from the other proton. Its magnitude is

$$\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{1.6\times10^{-19}\ \mathrm{C}}{2\times10^{-15}\ \mathrm{m}}\right)^2 = \boxed{57.5\ \mathrm{N}}$$

19.6 (a) $F = \dfrac{k_e e^2}{r^2} = \left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\dfrac{\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{\left(0.529\times10^{-10}\ \mathrm{m}\right)^2} = \boxed{8.22\times10^{-8}\ \mathrm{N}}$

(b) We have $F = \dfrac{mv^2}{r}$ from which

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{8.22\times10^{-8}\ \mathrm{N}\left(0.529\times10^{-10}\ \mathrm{m}\right)}{9.11\times10^{-31}\ \mathrm{kg}}} = \boxed{2.19\times10^{6}\ \mathrm{m/s}}$$

***19.7**

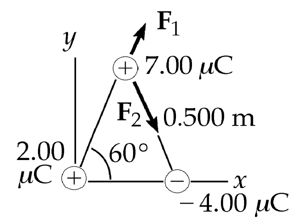
$$F_1 = k_e\frac{q_1q_2}{r^2} = \frac{(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})(7.00\times10^{-6}\ \mathrm{C})(2.00\times10^{-6}\ \mathrm{C})}{(0.500\ \mathrm{m})^2} = 0.503\ \mathrm{N}$$

$$F_2 = k_e\frac{q_1q_2}{r^2} = \frac{(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})(7.00\times10^{-6}\ \mathrm{C})(4.00\times10^{-6}\ \mathrm{C})}{(0.500\ \mathrm{m})^2} = 1.01\ \mathrm{N}$$

$$F_x = 0.503\cos 60.0^\circ + 1.01\cos 60.0^\circ = 0.755\ \mathrm{N}$$

$$F_y = 0.503\sin 60.0^\circ - 1.01\sin 60.0^\circ = -0.436\ \mathrm{N}$$

$$\mathbf{F} = (0.755\ \mathrm{N})\mathbf{i} - (0.436\ \mathrm{N})\mathbf{j} = \boxed{0.872\ \mathrm{N\ at\ an\ angle\ of\ 330^\circ}}$$



***19.8** For equilibrium, $\mathbf{F}_e = -\mathbf{F}_g$

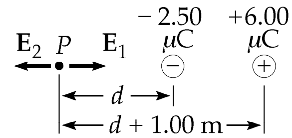
or $q\mathbf{E} = -mg(-\mathbf{j})$

Thus, $\mathbf{E} = \dfrac{mg}{q}\mathbf{j}$

(a) $\mathbf{E} = \dfrac{mg}{q}\mathbf{j} = \dfrac{(9.11\times10^{-31}\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{\left(-1.60\times10^{-19}\ \mathrm{C}\right)}\mathbf{j} = \boxed{-\left(5.58\times10^{-11}\ \mathrm{N/C}\right)\mathbf{j}}$

(b) $\mathbf{E} = \dfrac{mg}{q}\mathbf{j} = \dfrac{\left(1.67\times10^{-27}\ \mathrm{kg}\right)\left(9.80\ \mathrm{m/s^2}\right)}{\left(1.60\times10^{-19}\ \mathrm{C}\right)}\mathbf{j} = \boxed{\left(1.02\times10^{-7}\ \mathrm{N/C}\right)\mathbf{j}}$

19.9 The point is designated in the sketch. The magnitudes of the electric fields, E_1, (due to the -2.50×10^{-6} C charge) and E_2 (due to the 6.00×10^{-6} C charge) are



$$E_1 = \frac{k_e q}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.50\times10^{-6}\ \text{C})}{d^2} \tag{1}$$

$$E_2 = \frac{k_e q}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.00\times10^{-6}\ \text{C})}{(d+1.00\ \text{m})^2} \tag{2}$$

Equate the right sides of (1) and (2)

to get $\quad (d+1.00\ \text{m})^2 = 2.40\,d^2$

or $\quad d + 1.00\ \text{m} = \pm 1.55d$

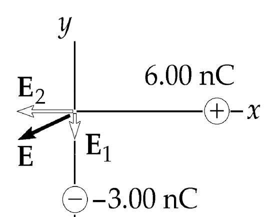
which yields $\quad d = 1.82$ m

or $\quad d = -0.392$ m

The negative value for d is unsatisfactory because that locates a point between the charges where both fields are in the same direction.

Thus, $\quad d = \boxed{1.82\ \text{m to the left of the } -2.50\ \mu\text{C charge}}$.

***19.10** (a) $\mathbf{E}_1 = \dfrac{k_e|q_1|}{r_1^2}(-\mathbf{j}) = \dfrac{(8.99\times10^9)(3.00\times10^{-9})}{(0.100)^2}(-\mathbf{j}) = -(2.70\times10^3\ \text{N/C})\mathbf{j}$



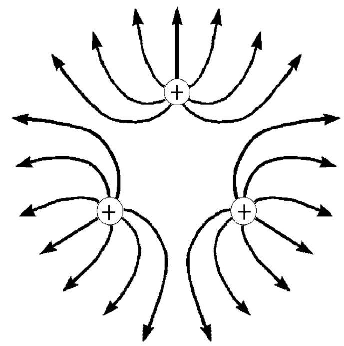
$$\mathbf{E}_2 = \frac{k_e|q_2|}{r_2^2}(-\mathbf{i}) = \frac{(8.99\times10^9)(6.00\times10^{-9})}{(0.300)^2}(-\mathbf{i}) = -(5.99\times10^2\ \text{N/C})\mathbf{i}$$

$$\mathbf{E} = \mathbf{E}_2 + \mathbf{E}_1 = \boxed{-(5.99\times10^2\ \text{N/C})\mathbf{i} - (2.70\times10^3\ \text{N/C})\mathbf{j}}$$

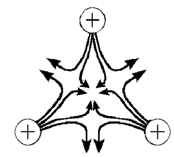
(b) $\mathbf{F} = q\mathbf{E} = (5.00\times10^{-9}\ \text{C})(-599\,\mathbf{i} - 2700\,\mathbf{j})\ \text{N/C}$

$$\mathbf{F} = (-3.00\times10^{-6}\,\mathbf{i} - 13.5\times10^{-6}\,\mathbf{j})\text{N} = \boxed{(-3.00\,\mathbf{i} - 13.5\,\mathbf{j})\ \mu\text{N}}$$

19.11 (a) The electric field has the general appearance shown. It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.



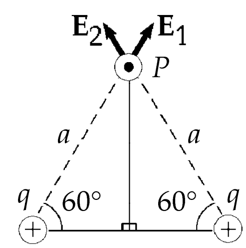
In addition to the center of the triangle, the electric field lines in the second figure to the right indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.



(b) You may need to review vector addition in Chapter One. The electric field at point P can be found by adding the electric field vectors due to each of the two lower point charges: $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

The electric field from a point charge is $\mathbf{E} = k_e \dfrac{q}{r^2}\hat{\mathbf{r}}$

As shown in the solution figure at right,

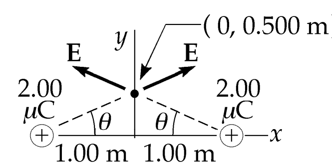


$\mathbf{E}_1 = k_e \dfrac{q}{a^2}$ to the right and upward at 60°

$\mathbf{E}_2 = k_e \dfrac{q}{a^2}$ to the left and upward at 60°

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = k_e \frac{q}{a^2}\left[(\cos 60°\mathbf{i} + \sin 60°\mathbf{j}) + (-\cos 60°\mathbf{i} + \sin 60°\mathbf{j})\right] = k_e \frac{q}{a^2}\left[2(\sin 60°\mathbf{j})\right] = \boxed{1.73 k_e \frac{q}{a^2}\mathbf{j}}$$

***19.12** (a) $E = \dfrac{k_e q}{r^2} = \dfrac{(8.99\times10^9)(2.00\times10^{-6})}{(1.12)^2} = 14\,400 \text{ N/C}$



$E_x = 0$ and $E_y = 2(14\,400)\sin 26.6° = 1.29\times10^4 \text{ N/C}$

so $\boxed{\mathbf{E} = 1.29\times10^4\ \mathbf{j}\text{ N/C}}$

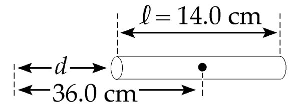
(b) $\mathbf{F} = q\mathbf{E} = \left(-3.00\times10^{-6}\right)\left(1.29\times10^4\,\mathbf{j}\right) = \boxed{-3.86\times10^{-2}\,\mathbf{j}\text{ N}}$

***19.13** (a) $\mathbf{E} = \dfrac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \dfrac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \dfrac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3 = \dfrac{k_e(2q)}{a^2}\mathbf{i} + \dfrac{k_e(3q)}{2a^2}(\mathbf{i}\cos 45.0° + \mathbf{j}\sin 45.0°) + \dfrac{k_e(4q)}{a^2}\mathbf{j}$

$\mathbf{E} = 3.06\dfrac{k_e q}{a^2}\mathbf{i} + 5.06\dfrac{k_e q}{a^2}\mathbf{j} = \boxed{5.91\dfrac{k_e q}{a^2} \text{ at } 58.8°}$

(b) $\mathbf{F} = q\mathbf{E} = \boxed{5.91\dfrac{k_e q^2}{a^2} \text{ at } 58.8°}$

19.14 $$E = \frac{k_e \lambda \ell}{d(\ell + d)} = \frac{k_e (Q/\ell)\ell}{d(\ell + d)} = \frac{k_e Q}{d(\ell + d)} = \frac{(8.99\times 10^{9})(22.0\times 10^{-6})}{(0.290)(0.140 + 0.290)}$$



$\mathbf{E} = \boxed{1.59\times 10^{6}\ \text{N/C, directed toward the rod.}}$

19.15 The electric field at any point x is

$$E = \frac{k_e q}{(x-a)^2} - \frac{k_e q}{(x-(-a))^2} = \frac{k_e q(4ax)}{(x^2-a^2)^2}$$

When x is much, much greater than a, we find $E \cong \boxed{\dfrac{4a(k_e q)}{x^3}}$

19.16 $$E = \frac{k_e x Q}{\left(x^2+a^2\right)^{3/2}} = \frac{\left(8.99\times 10^{9}\right)\left(75.0\times 10^{-6}\right)x}{\left(x^2+0.100^2\right)^{3/2}} = \frac{6.74\times 10^{5}\,x}{\left(x^2+0.0100\right)^{3/2}}$$

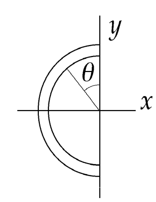
(a) At $x = 0.0100$ m, $\mathbf{E} = 6.64\times 10^{6}\,\mathbf{i}$ N/C $= \boxed{6.64\,\mathbf{i}\ \text{MN/C}}$

(b) At $x = 0.0500$ m, $\mathbf{E} = 2.41\times 10^{7}\,\mathbf{i}$ N/C $= \boxed{24.1\,\mathbf{i}\ \text{MN/C}}$

(c) At $x = 0.300$ m, $\mathbf{E} = 6.40\times 10^{6}\,\mathbf{i}$ N/C $= \boxed{6.40\,\mathbf{i}\ \text{MN/C}}$

(d) At $x = 1.00$ m, $\mathbf{E} = 6.64\times 10^{5}\,\mathbf{i}$ N/C $= \boxed{0.664\,\mathbf{i}\ \text{MN/C}}$

19.17 Due to symmetry $E_y = \int dE_y = 0$, and $E_x = \int dE\sin\theta = k_e \int \frac{dq\sin\theta}{r^2}$



where $dq = \lambda ds = \lambda r\,d\theta$,

so that, $$E_x = \frac{k_e\lambda}{r}\int_0^{\pi}\sin\theta\,d\theta = \frac{k_e\lambda}{r}(-\cos\theta)\Big|_0^{\pi} = \frac{2k_e\lambda}{r}$$

where $\lambda = \dfrac{q}{L}$ and $r = \dfrac{L}{\pi}$.

Thus, $$E_x = \frac{2k_e q\pi}{L^2} = \frac{2\left(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.50\times 10^{-6}\ \text{C}\right)\pi}{(0.140\ \text{m})^2}$$

Solving, $E_x = 2.16\times 10^{7}$ N/C

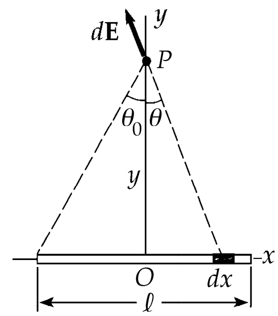
Since the rod has a negative charge, $\mathbf{E} = (-2.16\times 10^{7}\,\mathbf{i})$ N/C $= \boxed{-21.6\,\mathbf{i}\ \text{MN/C}}$

19.18 $E = \int \frac{k_e dq}{x^2}$, where $dq = \lambda_0 dx$

$$E = k_e \lambda_0 \int_{x_0}^{\infty} \frac{dx}{x^2} = k_e \left(-\frac{1}{x} \right)\Bigg|_{x_0}^{\infty} = \boxed{\frac{k_e \lambda_0}{x_0}}$$

$\boxed{\text{The direction is } -\mathbf{i} \text{ or left for } \lambda_0 > 0}$

***19.19** (a) The electric field at point P due to each element of length dx, is $dE = \frac{k_e dq}{x^2 + y^2}$ and is directed along the line joining the element to point P. By symmetry,



$E_x = \int dE_x = 0$ and since $dq = \lambda dx$,

$E = E_y = \int dE_y = \int dE \cos\theta$ where $\cos\theta = \frac{y}{\sqrt{x^2 + y^2}}$

Therefore, $E = 2k_e \lambda y \int_0^{\ell/2} \frac{dx}{(x^2 + y^2)^{3/2}} = \boxed{\frac{2k_e \lambda \sin\theta_0}{y}}$

(b) For a bar of infinite length, $\theta_0 \to 90°$ and $E_y = \boxed{\frac{2k_e \lambda}{y}}$

***19.20** (a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r + L)$.

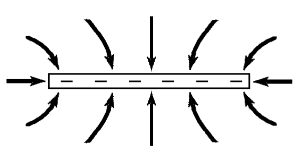
$$Q = \sigma A = \left(15.0 \times 10^{-9} \text{ C/m}^2\right) 2\pi (0.0250 \text{ m})[0.0250 \text{ m} + 0.0600 \text{ m}] = \boxed{2.00 \times 10^{-10} \text{ C}}$$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$$Q = \sigma A = \left(15.0 \times 10^{-9} \text{ C/m}^2\right)\left[2\pi (0.0250 \text{ m})(0.0600 \text{ m})\right] = \boxed{1.41 \times 10^{-10} \text{ C}}$$

(c) $Q = \rho V = \rho \pi r^2 L = \left(500 \times 10^{-9} \text{ C/m}^3\right)\left[\pi (0.0250 \text{ m})^2 (0.0600 \text{ m})\right] = \boxed{5.89 \times 10^{-11} \text{ C}}$

***19.21**



***19.22** (a) $\frac{q_1}{q_2} = -\frac{6}{18} = \boxed{-\frac{1}{3}}$

(b) $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$

***19.23** (a) $a = \dfrac{qE}{m} = \dfrac{1.602 \times 10^{-19}(640)}{1.67 \times 10^{-27}} = \boxed{6.14 \times 10^{10}\ \text{m/s}^2}$

(b) $v_f = v_i + at$ $\qquad 1.20 \times 10^6 = \left(6.14 \times 10^{10}\right)t$ $\qquad t = \boxed{1.95 \times 10^{-5}\ \text{s}}$

(c) $x_f - x_i = \frac{1}{2}\left(v_i + v_f\right)t$ $\qquad x_f = \frac{1}{2}\left(1.20 \times 10^6\right)\left(1.95 \times 10^{-5}\right) = \boxed{11.7\ \text{m}}$

(d) $K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(1.20 \times 10^6\ \text{m/s}\right)^2 = \boxed{1.20 \times 10^{-15}\ \text{J}}$

19.24 The required electric field will be $\boxed{\text{in the direction of motion}}$.

Work done = ΔK

so, $\qquad -Fd = -\frac{1}{2}mv_i^2$ (since the final velocity = 0)

which becomes $\qquad eEd = K$

and $\qquad E = \boxed{\dfrac{K}{ed}}$

19.25 The acceleration is given by

$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$ $\qquad$ or $\qquad v_f^2 = 0 + 2a(-h)$

Solving $\qquad a = -\dfrac{v_f^2}{2h}$

Now $\Sigma \mathbf{F} = m\mathbf{a}$: $\qquad -mg\mathbf{j} + q\mathbf{E} = -\dfrac{mv_f^2\mathbf{j}}{2h}$

Therefore $\qquad q\mathbf{E} = \left(-\dfrac{mv_f^2}{2h} + mg\right)\mathbf{j}$

(a) Gravity alone would give the bead downward impact velocity

$$\sqrt{2\left(9.80\ \text{m/s}^2\right)(5.00\ \text{m})} = 9.90\ \text{m/s}$$

To change this to 21.0 m/s down, a $\boxed{\text{downward}}$ electric field must exert a downward electric force.

(b) $q = \dfrac{m}{E}\left(\dfrac{v_f^2}{2h} - g\right) = \dfrac{1.00 \times 10^{-3}\ \text{kg}}{1.00 \times 10^4\ \text{N/C}}\left(\dfrac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}\right)\left[\dfrac{(21.0\ \text{m/s})^2}{2(5.00\ \text{m})} - 9.80\ \text{m/s}^2\right] = \boxed{3.43\ \mu\text{C}}$

***19.26** (a) $t = \frac{x}{v_x} = \frac{0.0500}{4.50 \times 10^5} = 1.11 \times 10^{-7} \text{ s} = \boxed{111 \text{ ns}}$

(b) $a_y = \frac{qE}{m} = \frac{\left(1.602 \times 10^{-19}\right)\left(9.60 \times 10^3\right)}{\left(1.67 \times 10^{-27}\right)} = 9.21 \times 10^{11} \text{ m/s}^2$

$y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2$: $\quad y_f = \frac{1}{2}\left(9.21 \times 10^{11}\right)\left(1.11 \times 10^{-7}\right)^2 = 5.67 \times 10^{-3} \text{ m} = \boxed{5.67 \text{ mm}}$

(c) $v_x = \boxed{4.50 \times 10^5 \text{ m/s}}$ $\quad v_{yf} = v_{yi} + a_y t = \left(9.21 \times 10^{11}\right)\left(1.11 \times 10^{-7}\right) = \boxed{1.02 \times 10^5 \text{ m/s}}$

***19.27** The particle feels a constant force: $\mathbf{F} = q\mathbf{E} = (1 \times 10^{-6} \text{ C})(2000 \text{ N/C})(-\mathbf{j}) = 2 \times 10^{-3} \text{ N } (-\mathbf{j})$

and moves with acceleration : $\quad \mathbf{a} = \frac{\Sigma \mathbf{F}}{m} = \frac{(2 \times 10^{-3} \text{ kg} \cdot \text{m/s}^2)(-\mathbf{j})}{2 \times 10^{-16} \text{ kg}} = \left(1 \times 10^{13} \text{m/s}^2\right)(-\mathbf{j})$

Its x-component of velocity is constant at $\left(1.00 \times 10^5 \text{ m/s}\right)\cos 37° = 7.99 \times 10^4 \text{ m/s}$. Thus it moves in a parabola opening downward. The maximum height it attains above the bottom plate is described by

$v_{yf}{}^2 = v_{yi}{}^2 + 2a_y\left(y_f - y_i\right)$: $\quad 0 = \left(6.02 \times 10^4 \text{ m/s}\right)^2 - \left(2 \times 10^{13} \text{ m/s}^2\right)\left(y_f - 0\right)$

$y_f = 1.81 \times 10^{-4} \text{ m}$

Since this is less than 10 mm, the particle does not strike the top plate, but moves in a symmetric parabola and strikes the bottom plate after a time given by

$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$ $\quad 0 = 0 + \left(6.02 \times 10^4 \text{ m/s}\right)t + \frac{1}{2}\left(-1 \times 10^{13} \text{ m/s}^2\right)t^2$

since $t > 0$, $\quad t = 1.20 \times 10^{-8} \text{ s}$

The particle's range is $\quad x_f = x_i + v_x t = 0 + \left(7.99 \times 10^4 \text{ m/s}\right)\left(1.20 \times 10^{-8} \text{ s}\right) = 9.61 \times 10^{-4} \text{ m}$

In sum,

The particle strikes the negative plate after moving in a parabola with a height of 0.181 mm and a width of 0.961 mm

19.28 $\Phi_E = EA\cos\theta = (2.00 \times 10^4 \text{ N/C})(18.0 \text{ m}^2)\cos 10.0° = \boxed{355 \text{ kN} \cdot \text{m}^2/\text{C}}$

19.29 $\Phi_E = EA\cos\theta$ $\quad A = \pi r^2 = \pi(0.200)^2 = 0.126 \text{ m}^2$

$5.20 \times 10^5 = E(0.126) \cos 0°$ $\quad E = 4.14 \times 10^6 \text{ N/C} = \boxed{4.14 \text{ MN/C}}$

19.30 (a) $E = \frac{k_e Q}{r^2}$: $\quad 8.90 \times 10^2 = \frac{(8.99\times 10^9)Q}{(0.750)^2}$

But Q is negative since **E** points inward. $\quad Q = -5.56 \times 10^{-8}\ \text{C} = \boxed{-55.6\ \text{nC}}$

(b) The $\boxed{\text{negative}}$ charge has a $\boxed{\text{spherically symmetric}}$ charge distribution.

19.31 (a) $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{(+5.00\ \mu\text{C} - 9.00\ \mu\text{C} + 27.0\ \mu\text{C} - 84.0\ \mu\text{C})}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -6.89 \times 10^6\ \text{N}\cdot\text{m}^2/\text{C}^2$

$\Phi_E = \boxed{-6.89\ \text{MN}\cdot\text{m}^2/\text{C}}$

(b) Since the net electric flux is negative, more lines enter than leave the surface.

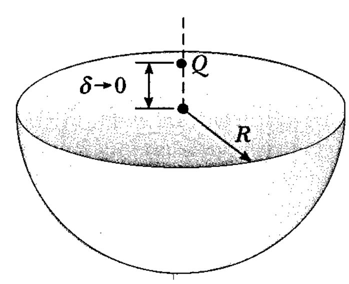
***19.32** $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{170\times 10^{-6}\ \text{C}}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.92\times 10^7\ \text{N}\cdot\text{m}^2/\text{C}$

(a) $(\Phi_E)_{\text{one face}} = \frac{1}{6}\Phi_E = \frac{1.92\times 10^7\ \text{N}\cdot\text{m}^2/\text{C}}{6}$ $\quad (\Phi_E)_{\text{one face}} = \boxed{3.20\ \text{MN}\cdot\text{m}^2/\text{C}}$

(b) $\Phi_E = \boxed{19.2\ \text{MN}\cdot\text{m}^2/\text{C}}$

(c) The answer to (a) would change because the flux through each face of the cube would not be equal with an asymmetric charge distribution. The sides of the cube nearer the charge would have more flux and the ones farther away would have less. The answer to (b) would remain the same, since the overall flux would remain the same.

19.33 (a) With δ very small, all points on the hemisphere are nearly at a distance R from the charge, so the field everywhere on the curved surface is $k_e Q/R^2$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:



$$\Phi_{\text{curved}} = \int \mathbf{E}\cdot d\mathbf{A} = E_{\text{local}} A_{\text{hemisphere}}$$

$$\Phi_{\text{curved}} = \left(k_e \frac{Q}{R^2}\right)\left(\frac{1}{2}4\pi R^2\right) = \frac{1}{4\pi\epsilon_0}Q(2\pi) = \boxed{\frac{+Q}{2\epsilon_0}}$$

(b) The closed surface encloses zero charge so Gauss's law gives

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0 \quad \text{or} \quad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = \boxed{\frac{-Q}{2\epsilon_0}}$$

19.34 (a) $E = \frac{k_e Q r}{a^3} = \boxed{0}$

(b) $E = \frac{k_e Q r}{a^3} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})(0.100)}{(0.400)^3} = \boxed{365 \text{ kN/C}}$

(c) $E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.400)^2} = \boxed{1.46 \text{ MN/C}}$

(d) $E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.600)^2} = \boxed{649 \text{ kN/C}}$

The direction for each electric field is $\boxed{\text{radially outward}}$.

***19.35** (a) $\mathbf{E} = \boxed{0}$

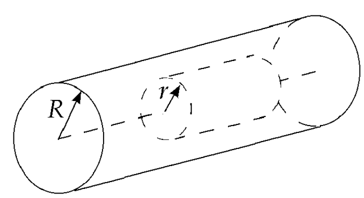
(b) $E = \frac{k_e Q}{r^2} = \frac{(8.99 \times 10^9)(32.0 \times 10^{-6})}{(0.200)^2} = 7.19 \text{ MN/C}$ $\mathbf{E} = \boxed{7.19 \text{ MN/C radially outward}}$

19.36 (a) $E = \frac{2k_e \lambda}{r}$ $3.60 \times 10^4 = \frac{2(8.99 \times 10^9)(Q/2.40)}{0.190}$

$Q = +9.13 \times 10^{-7} \text{ C} = \boxed{+913 \text{ nC}}$

(b) $\mathbf{E} = \boxed{0}$

19.37 If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho \pi r^2 L$. Because the charge distribution is long, no electric flux passes through the circular end caps; $\mathbf{E} \cdot d\mathbf{A} = E\,dA \cos 90.0° = 0$. The curved surface has $\mathbf{E} \cdot d\mathbf{A} = E\,dA \cos 0°$, and E must be the same strength everywhere over the curved surface.



Gauss's law, $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\epsilon_0}$, becomes $E \int_{\text{Curved Surface}} dA = \frac{\rho \pi r^2 L}{\epsilon_0}$

Now the lateral surface area of the cylinder is $2\pi r L$:

$E(2\pi r)L = \frac{\rho \pi r^2 L}{\epsilon_0}$ Thus, $\mathbf{E} = \boxed{\frac{\rho r}{2\epsilon_0} \text{ radially away from the cylinder axis}}$

19.38 $mg = qE = q\left(\frac{\sigma}{2\epsilon_0}\right) = q\left(\frac{Q/A}{2\epsilon_0}\right)$ $\frac{Q}{A} = \frac{2\epsilon_0 mg}{q} = \frac{2\left(8.85\times10^{-12}\right)(0.01)(9.8)}{-0.7\times10^{-6}} = \boxed{-2.48\ \mu\text{C/m}^2}$

19.39 The distance between centers is $2 \times 5.90 \times 10^{-15}$ m. Each produces a field as if it were a point charge at its center, and each feels a force as if all its charge were a point at its center.

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(46)^2(1.60\times10^{-19}\ \text{C})^2}{(2\times5.90\times10^{-15}\ \text{m})^2} = 3.50\times10^3\ \text{N} = \boxed{3.50\ \text{kN}}$$

19.40 $\oint E\,dA = E(2\pi rl) = \frac{q_{in}}{\epsilon_0}$ $E = \frac{q_{in}/l}{2\pi\epsilon_0 r} = \frac{\lambda}{2\pi\epsilon_0 r}$

(a) $r = 3.00$ cm $\mathbf{E} = \boxed{0}$ inside the conductor

(b) $r = 10.0$ cm $\mathbf{E} = \frac{30.0\times10^{-9}}{2\pi\left(8.85\times10^{-12}\right)(0.100)} = \boxed{5400\ \text{N/C, outward}}$

(c) $r = 100$ cm $\mathbf{E} = \frac{30.0\times10^{-9}}{2\pi\left(8.85\times10^{-12}\right)(1.00)} = \boxed{540\ \text{N/C, outward}}$

19.41 (a) $E = \sigma/\epsilon_0$ $\sigma = \left(8.00\times10^4\right)\left(8.85\times10^{-12}\right) = 7.08\times10^{-7}\ \text{C/m}^2$

$\sigma = \boxed{708\ \text{nC/m}^2}$, positive on one face and negative on the other.

(b) $\sigma = \frac{Q}{A}$ $Q = \sigma A = (7.08\times10^{-7})(0.500)^2\ \text{C}$

$Q = 1.77 \times 10^{-7}\ \text{C} = \boxed{177\ \text{nC}}$, positive on one face and negative on the other.

19.42 The electric field on the surface of a conductor varies inversely with the radius of curvature of the surface. Thus, the field is most intense where the radius of curvature is smallest and vice-versa. The local charge density and the electric field intensity are related by

$E = \frac{\sigma}{\epsilon_0}$ or $\sigma = \epsilon_0 E$

(a) Where the radius of curvature is the greatest,

$$\sigma = \epsilon_0 E_{\text{min}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.80\times10^4\ \text{N/C}\right) = \boxed{248\ \text{nC/m}^2}$$

(b) Where the radius of curvature is the smallest,

$$\sigma = \epsilon_0 E_{\text{max}} = \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(5.60\times10^4\ \text{N/C}\right) = \boxed{496\ \text{nC/m}^2}$$

***19.43** (a) The charge density on each of the surfaces (upper and lower) of the plate is:

$$\sigma = \frac{1}{2}\left(\frac{q}{A}\right) = \frac{1}{2}\frac{(4.00 \times 10^{-8}\ \text{C})}{(0.500\ \text{m})^2} = 8.00 \times 10^{-8}\ \text{C/m}^2 = \boxed{80.0\ \text{nC/m}^2}$$

(b) $$\mathbf{E} = \left(\frac{\sigma}{\epsilon_0}\right)\mathbf{k} = \left(\frac{8.00 \times 10^{-8}\ \text{C/m}^2}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right)\mathbf{k} = \boxed{(9.04\ \text{kN/C})\mathbf{k}}$$

(c) $$\mathbf{E} = \boxed{(-9.04\ \text{kN/C})\mathbf{k}}$$

19.44 (a) Inside surface: consider a cylindrical surface within the metal. Since E inside the conducting shell is zero, the total charge inside the gaussian surface must be zero, so the inside charge/length $= -\lambda$.

$0 = \lambda\ell + q_{\text{in}}$ so $\dfrac{q_{\text{in}}}{\ell} = \boxed{-\lambda}$

Outside surface: The total charge on the metal cylinder is $2\lambda\ell = q_{\text{in}} + q_{\text{out}}$

$q_{\text{out}} = 2\lambda\ell + \lambda\ell$ so the outside charge/length is $\boxed{3\lambda}$

(b) $$E = \frac{2k_e(3\lambda)}{r} = \frac{6k_e\lambda}{r} = \boxed{\frac{3\lambda}{2\pi\epsilon_0 r}\ \text{radially outward}}$$

***19.45** (a) $\mathbf{E} = \boxed{0}$

(b) $$E = \frac{k_eQ}{r^2} = \frac{(8.99\times10^9)(8.00\times10^{-6})}{(0.0300)^2} = 7.99\times10^7\ \text{N/C} \qquad \mathbf{E} = \boxed{79.9\ \text{MN/C radially outward}}$$

(c) $\mathbf{E} = \boxed{0}$

(d) $$E = \frac{k_eQ}{r^2} = \frac{(8.99\times10^9)(4.00\times10^{-6})}{(0.0700)^2} = 7.34\times10^6\ \text{N/C} \qquad \mathbf{E} = \boxed{7.34\ \text{MN/C radially outward}}$$

***19.46** $\mathbf{E} = \frac{\sigma}{\epsilon_0}$ toward a $\boxed{\text{negative}}$ charge.

$$\sigma = E\epsilon_0 = (120\ \text{N/C})\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right) = \boxed{1.06\times10^{-9}\ \text{C/m}^2}$$

(b) $$Q = \sigma A = \sigma\left(4\pi R^2\right) = \left(-1.06\times10^{-9}\ \text{C/m}^2\right)\left[4\pi\left(6.37\times10^6\ \text{m}\right)^2\right] = \boxed{-5.42\times10^5\ \text{C}}$$

$$-5.42\times10^5\ \text{C}\left(\frac{1\ \text{electron}}{-1.6\times10^{-19}\ \text{C}}\right) = \boxed{3.38\times10^{24}\ \text{excess electrons}}$$

***19.47** Consider as a gaussian surface a box with horizontal area A, lying between 500 and 600 m elevation.

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q}{\epsilon_0}: \qquad (+120\ \text{N/C})A + (-100\ \text{N/C})A = \frac{\rho A(100\ \text{m})}{\epsilon_0}$$

$$\rho = \frac{(20\ \text{N/C})\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{100\ \text{m}} = \boxed{1.77\times10^{-12}\ \text{C/m}^3}$$

The charge is $\boxed{\text{positive}}$, to produce the net outward flux of electric field.

***19.48** (a) The Moon would feel a force $\boxed{\text{away from Earth}}$ of magnitude

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{\left(-5\times10^5\ \text{C}\right)\left(-1.37\times10^5\ \text{C}\right)}{\left(3.84\times10^8\ \text{m}\right)^2}\right) = \boxed{4.18\times10^3\ \text{N}}$$

(b) The gravitational force is

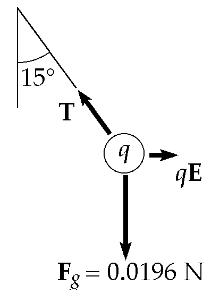
$$F = \frac{Gm_1 m_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(7.36\times10^{22}\ \text{kg}\right)}{\left(3.84\times10^8\ \text{m}\right)^2}$$

$F = 1.99\times10^{20}\ \text{N}$ toward Earth.

Thus, $\boxed{\text{the electric force is weaker by}}$

$$\frac{1.99\times10^{20}\ \text{N}}{4.18\times10^3\ \text{N}} = \boxed{4.77\times10^{16}\ \text{times and in the opposite direction}}$$

19.49 From the free-body diagram shown,



$\Sigma F_y = 0$: $\quad T\cos 15.0° = 1.96 \times 10^{-2}$ N

So $\quad T = 2.03 \times 10^{-2}$ N

From $\Sigma F_x = 0$, we have $\quad qE = T\sin 15.0°$

or $\quad q = \dfrac{T\sin 15.0°}{E} = \dfrac{(2.03\times10^{-2}\ \text{N})\sin 15.0°}{1.00\times10^{3}\ \text{N/C}} = 5.25\times10^{-6}\ \text{C} = \boxed{5.25\ \mu\text{C}}$

19.50 From Figure A: $\quad d\cos 30.0° = 15.0$ cm,

or $\quad d = \dfrac{15.0\ \text{cm}}{\cos 30.0°}$

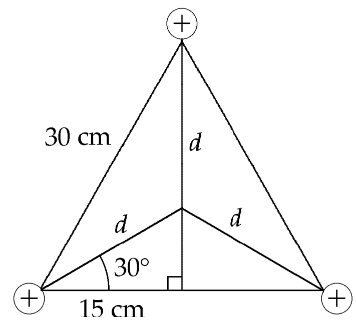


Figure A

From Figure B: $\quad \theta = \sin^{-1}\left(\dfrac{d}{50.0\ \text{cm}}\right)$

$$\theta = \sin^{-1}\left(\frac{15.0\ \text{cm}}{50.0\ \text{cm}(\cos 30.0°)}\right) = 20.3°$$

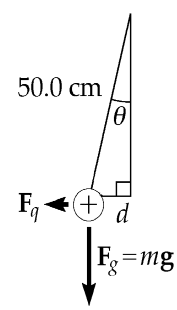


Figure B

$$\frac{F_q}{mg} = \tan\theta$$

or $\quad F_q = mg\tan 20.3° \qquad (1)$

From Figure C: $\quad F_q = 2F\cos 30.0°$

$$F_q = 2\left[\frac{k_e q^2}{(0.300\ \text{m})^2}\right]\cos 30.0° \qquad (2)$$

Combining equations (1) and (2),

$$2\left[\frac{k_e q^2}{(0.300\ \text{m})^2}\right]\cos 30.0° = mg\tan 20.3°$$

$$q^2 = \frac{mg(0.300\ \text{m})^2\tan 20.3°}{2k_e\cos 30.0°}$$

$$q^2 = \frac{(2.00\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)(0.300\ \text{m})^2\tan 20.3°}{2(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\cos 30.0°}$$

$$q = \sqrt{4.20\times10^{-14}\ \text{C}^2} = 2.05\times10^{-7}\ \text{C} = \boxed{0.205\ \mu\text{C}}$$

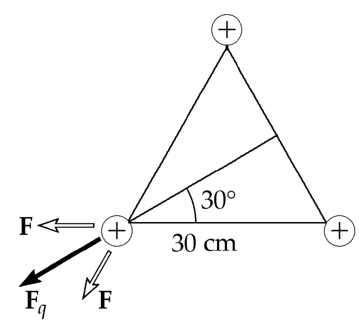


Figure C

***19.51** Charge $Q/2$ resides on each block, which repel as point charges:

$$F = \frac{k_e(Q/2)(Q/2)}{L^2} = k(L - L_i)$$

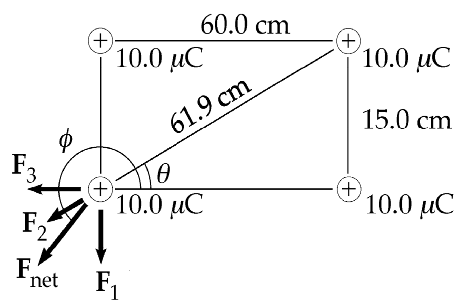
$$Q = 2L\sqrt{\frac{k(L-L_i)}{k_e}} = 2(0.400\text{ m})\sqrt{\frac{(100\text{ N/m})(0.100\text{ m})}{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)}} = \boxed{26.7\ \mu\text{C}}$$

***19.52** Charge $Q/2$ resides on each block, which repel as point charges:

$$F = \frac{k_e(Q/2)(Q/2)}{L^2} = k(L - L_i)$$

Solving for Q, $\quad Q = \boxed{2L\sqrt{\dfrac{k(L-L_i)}{k_e}}}$

19.53 $F = \dfrac{k_e q_1 q_2}{r^2}$: $\quad \tan\theta = \dfrac{15.0}{60.0}$



$$\theta = 14.0°$$

$$F_1 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.150)^2} = 40.0\text{ N}$$

$$F_3 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.600)^2} = 2.50\text{ N}$$

$$F_2 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.619)^2} = 2.35\text{ N}$$

$$F_x = -F_3 - F_2\cos 14.0° = -2.50 - 2.35\cos 14.0° = -4.78\text{ N}$$

$$F_y = -F_1 - F_2\sin 14.0° = -40.0 - 2.35\sin 14.0° = -40.6\text{ N}$$

$$F_{\text{net}} = \sqrt{F_x^{\,2} + F_y^{\,2}} = \sqrt{(4.78)^2 + (40.6)^2} = \boxed{40.9\text{ N}}$$

$$\tan\phi = \frac{F_y}{F_x} = \frac{-40.6}{-4.78}$$

$$\phi = \boxed{263°}$$

***19.54** At an equilibrium position, the net force on the charge Q is zero. The equilibrium position can be located by determining the angle θ corresponding to equilibrium.

In terms of lengths s, $\frac{1}{2}a\sqrt{3}$, and r, shown in Figure P19.54, the charge at the origin exerts an attractive force

$$k_e Qq / \left(s + \tfrac{1}{2}a\sqrt{3}\right)^2$$

The other two charges exert equal repulsive forces of magnitude $k_e Qq/r^2$. The horizontal components of the two repulsive forces add, balancing the attractive force,

$$F_{\text{net}} = k_e Qq\left[\frac{2\cos\theta}{r^2} - \frac{1}{\left(s + \frac{1}{2}a\sqrt{3}\right)^2}\right] = 0$$

From Figure P19.54 $\quad r = \dfrac{\frac{1}{2}a}{\sin\theta} \qquad s = \frac{1}{2}a\cot\theta$

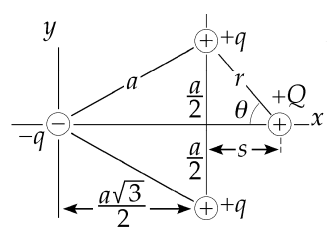
The equilibrium condition, in terms of θ, is $\quad F_{\text{net}} = \left(\dfrac{4}{a^2}\right)k_e Qq\left(2\cos\theta\sin^2\theta - \dfrac{1}{\left(\sqrt{3} + \cot\theta\right)^2}\right) = 0$

Thus the equilibrium value of θ satisfies $\quad 2\cos\theta\sin^2\theta\left(\sqrt{3} + \cot\theta\right)^2 = 1$

One method for solving for θ is to tabulate the left side. To three significant figures a value of θ corresponding to equilibrium is $81.7°$.

The distance from the vertical side of the triangle to the equilibrium position is

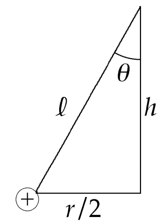
$$s = \tfrac{1}{2}a\cot 81.7° = \boxed{0.0729\,a}$$



θ	$2\cos\theta\sin^2\theta\left(\sqrt{3} + \cot\theta\right)^2$
60°	4
70°	2.654
80°	1.226
90°	0
81°	1.091
81.5°	1.024
81.7°	0.997

A second zero-field point is on the negative side of the x-axis, where $\theta = -9.16°$ and $s = -3.10\,a$.

***19.55** (a) From the $2Q$ charge we have $F_e - T_2 \sin\theta_2 = 0$ and $mg - T_2 \cos\theta_2 = 0$



Combining these we find $\dfrac{F_e}{mg} = \dfrac{T_2 \sin\theta_2}{T_2 \cos\theta_2} = \tan\theta_2$

From the Q charge we have $F_e - T_1 \sin\theta_1 = 0$ and $mg - T_1 \cos\theta_1 = 0$

Combining these we find $\dfrac{F_e}{mg} = \dfrac{T_1 \sin\theta_1}{T_1 \cos\theta_1} = \tan\theta_1$ or $\boxed{\theta_2 = \theta_1}$

(b) $F_e = \dfrac{k_e 2QQ}{r^2} = \dfrac{2k_e Q^2}{r^2}$

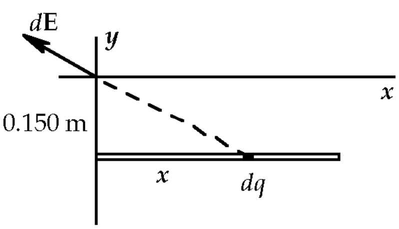
If we assume θ is small then $\tan\theta \cong \dfrac{r/2}{\ell}$

Substitute expressions for F_e and $\tan\theta$ into either equation found in part (a) and solve for r.

$\dfrac{F_e}{mg} = \tan\theta$ then $\dfrac{2k_e Q^2}{r^2}\left(\dfrac{1}{mg}\right) \cong \dfrac{r}{2\ell}$ and solving for r we find $r \cong \left(\dfrac{4k_e Q^2 \ell}{mg}\right)^{1/3}$

***19.56**

$$d\mathbf{E} = \frac{k_e\,dq}{x^2 + (0.150\text{ m})^2}\left(\frac{-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right) = \frac{k_e\lambda(-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j})dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$



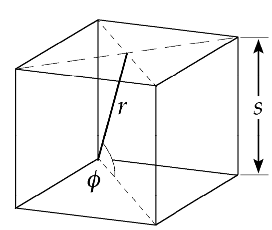
$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = k_e\lambda \int_{x=0}^{0.400\text{ m}} \frac{(-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j})dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$

$$\mathbf{E} = k_e\lambda\left[\left.\frac{+\mathbf{i}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}} + \left.\frac{(0.150\text{ m})\,\mathbf{j}\,x}{(0.150\text{ m})^2\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}}\right]$$

$$\mathbf{E} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(35.0\times10^{-9}\ \text{C/m}\right)\left[\mathbf{i}(2.34 - 6.67)\ \text{m}^{-1} + \mathbf{j}(6.24 - 0)\ \text{m}^{-1}\right]$$

$$\mathbf{E} = (-1.36\,\mathbf{i} + 1.96\,\mathbf{j})\times10^3\ \text{N/C} = \boxed{(-1.36\,\mathbf{i} + 1.96\,\mathbf{j})\ \text{kN/C}}$$

***19.57** (a) Zero contribution from the same face due to symmetry, opposite face contributes



$4\left(\dfrac{k_e q}{r^2}\sin\phi\right)$ where $r = \sqrt{\left(\dfrac{s}{2}\right)^2 + \left(\dfrac{s}{2}\right)^2 + s^2} = \sqrt{1.5}\ s = 1.22\ s$

$\sin\phi = s/r$ $\quad E = 4\dfrac{k_e q s}{r^3} = \dfrac{4}{(1.22)^3}\dfrac{k_e q}{s^2} = \boxed{2.18\dfrac{k_e q}{s^2}}$

(b) The direction is the **k** direction.

***19.58** The field on the axis of the ring is calculated in Example 19.5, $E = E_x = \dfrac{k_e x Q}{\left(x^2 + a^2\right)^{3/2}}$

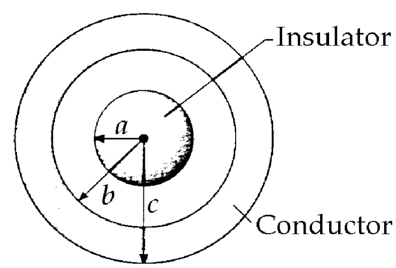
The force experienced by a charge $-q$ placed along the axis of the ring is $F = -k_e Q q\left[\dfrac{x}{\left(x^2 + a^2\right)^{3/2}}\right]$

and when $x << a$, this becomes $F = \left(\dfrac{k_e Q q}{a^3}\right)x$

This expression for the force is in the form of Hooke's law,
with an effective spring constant of $k = k_e Q q / a^3$

Since $\omega = 2\pi f = \sqrt{k/m}$, we have $f = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e Q q}{m a^3}}}$

19.59 (a) $\oint \mathbf{E} \cdot d\mathbf{A} = E\left(4\pi r^2\right) = q_{in} / \epsilon_0$



For $r < a$, $q_{in} = \rho\left(\frac{4}{3}\pi r^3\right)$

so $E = \boxed{\dfrac{\rho r}{3\epsilon_0}}$

For $a < r < b$ and $c < r$, $q_{in} = Q$

So $E = \boxed{\dfrac{Q}{4\pi r^2 \epsilon_0}}$

For $b \le r \le c$, $E = 0$, since $\boxed{E = 0}$ inside a conductor.

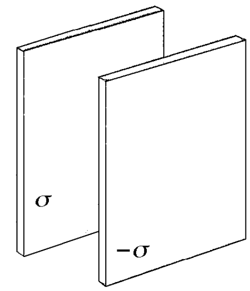
(b) Let q_1 = induced charge on the inner surface of the hollow sphere. Since $E = 0$ inside the conductor, the total charge enclosed by a spherical surface of radius $b \le r \le c$ must be zero.

Therefore, $q_1 + Q = 0$ and $\sigma_1 = \dfrac{q_1}{4\pi b^2} = \boxed{\dfrac{-Q}{4\pi b^2}}$

Let q_2 = induced charge on the outside surface of the hollow sphere. Since the hollow sphere is uncharged, we require

$q_1 + q_2 = 0$ and $\sigma_2 = \dfrac{q_1}{4\pi c^2} = \boxed{\dfrac{Q}{4\pi c^2}}$

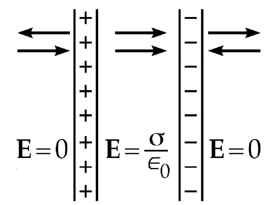
19.60 Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by Equation 19.24:



$$|E_+| = |E_-| = \frac{\sigma}{2\epsilon_0}$$

(a) To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\mathbf{E} = \boxed{0}$.

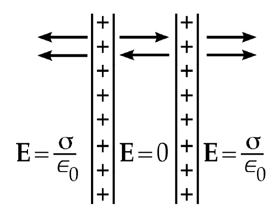
(b) In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is



$$\mathbf{E} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

(c) To the right of the negative sheet, E_+ and E_- are again oppositely directed and $\mathbf{E} = \boxed{0}$.

19.61 The magnitude of the field due to the each sheet given by Equation 19.24 is



$$E = \frac{\sigma}{2\epsilon_0} \text{ directed perpendicular to the sheet.}$$

(a) In the region to the left of the pair of sheets, both fields are directed toward the left and the net field is

$$\mathbf{E} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the left}}$$

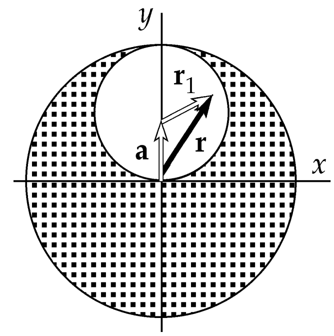
(b) In the region between the sheets, the fields due to the individual sheets are oppositely directed and the net field is

$$\mathbf{E} = \boxed{0}$$

(c) In the region to the right of the pair of sheets, both are fields are directed toward the right and the net field is

$$\mathbf{E} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

19.62 The resultant field within the cavity is the superposition of two fields, one $\mathbf{E}_+$ due to a uniform sphere of positive charge of radius $2a$, and the other $\mathbf{E}_-$ due to a sphere of negative charge of radius a centered within the cavity.



$$\frac{4}{3}\left(\frac{\pi r^3 \rho}{\epsilon_0}\right) = 4\pi r^2 E_+ \qquad \text{so} \qquad \mathbf{E}_+ = \frac{\rho r}{3\epsilon_0}\hat{\mathbf{r}} = \frac{\rho \mathbf{r}}{3\epsilon_0}$$

$$-\frac{4}{3}\left(\frac{\pi r_1^3 \rho}{\epsilon_0}\right) = 4\pi r_1^2 E_- \qquad \text{so} \qquad \mathbf{E}_- = \frac{\rho r_1}{3\epsilon_0}\left(-\hat{\mathbf{r}}_1\right) = \frac{-\rho}{3\epsilon_0}\mathbf{r}_1$$

Since $\mathbf{r} = \mathbf{a} + \mathbf{r}_1$, $\qquad \mathbf{E}_- = \dfrac{-\rho(\mathbf{r} - \mathbf{a})}{3\epsilon_0}$

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \frac{\rho \mathbf{r}}{3\epsilon_0} - \frac{\rho \mathbf{r}}{3\epsilon_0} + \frac{\rho \mathbf{a}}{3\epsilon_0} = \frac{\rho \mathbf{a}}{3\epsilon_0} = 0\,\mathbf{i} + \frac{\rho a}{3\epsilon_0}\mathbf{j}$$

Thus, $\boxed{E_x = 0}$

and $\boxed{E_y = \dfrac{\rho a}{3\epsilon_0}}$ at all points within the cavity.

***19.63** $\oint \mathbf{E} \cdot d\mathbf{A} = E(4\pi r^2) = \dfrac{q_{\text{in}}}{\epsilon_0}$

(a) For $r > R$, $\qquad q_{\text{in}} = \displaystyle\int_0^R Ar^2(4\pi r^2)\,dr = 4\pi \frac{AR^5}{5}$

and $\qquad E = \boxed{\dfrac{AR^5}{5\epsilon_0 r^2}}$

(b) For $r < R$, $\qquad q_{\text{in}} = \displaystyle\int_0^r Ar^2(4\pi r^2)\,dr = \frac{4\pi A r^5}{5}$

and $\qquad E = \boxed{\dfrac{Ar^3}{5\epsilon_0}}$

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 514 kN

4. (a) 2.16×10^{-5} N toward the other (b) 8.99×10^{-7} N away from the other

6. (a) 82.2 nN (b) 2.19 Mm/s

8. (a) 55.8 pN/C down (b) 102 nN/C up

10. (a) (–599**i** – 2 700**j**) N/C (b) (–3.00**i** – 13.5**j**) μN

12. (a) $1.29 \times 10^{4}\ \mathbf{j}$ N/C (b) $-3.86 \times 10^{-2}\mathbf{j}$ N

14. 1.59×10^{6} N/C toward the rod

16. (a) 6.64 **i** MN/C (b) 24.1 **i** MN/C
(c) 6.40 **i** MN/C (d) 0.664 **i** MN/C, taking the axis of the ring as the x-axis

18. $-k_e \lambda_0 \mathbf{i} / x_0$

20. (a) 2.00×10^{-10} C (b) 1.41×10^{-10} C (c) 5.89×10^{-11} C

22. (a) –1/3 (b) $q_1 < 0,\ q_2 > 0$

24. K/ed in the direction of motion

26. (a) 111 ns (b) 5.67 mm (c) (450 **i** + 102 **j**) km/s

28. $355\ \text{kN} \cdot \text{m}^2/\text{C}$

30. (a) –55.6 nC (b) negative, with a spherically symmetric distribution

32. (a) $3.20\ \text{MN} \cdot \text{m}^2/\text{C}$ (b) $19.2\ \text{MN} \cdot \text{m}^2/\text{C}$
(c) The answer to (a) could change, but the answer to (b) would stay the same.

34. (a) 0 (b) 365 kN/C radially outward
(c) 1.46 MN/C outward (d) 649 kN/C radially outward

36. (a) 913 nC (b) 0

38. $-2.48\ \mu C/m^2$

40. (a) 0 (b) 5 400 N/C outward (c) 540 N/C outward

42. (a) $248\ nC/m^2$ (b) $496\ nC/m^2$

44. (a) $-\lambda,\ +3\lambda$ (b) $3\lambda/2\pi\epsilon_0 r$ radially outward

46. (a) negative $1.06\ nC/m^2$ (b) -542 kC, 3.38×10^{24} excess electrons

48. (a) 4.18 kN away from Earth
(b) The gravitational force is 4.77×10^{16} times stronger and in the opposite direction

50. $0.205\ \mu C$

52. $Q = 2L\sqrt{\dfrac{k(L-L_i)}{k_e}}$

54. $0.0729\ a$

56. $(-1.36\ \mathbf{i} + 1.96\ \mathbf{j})$ kN/C

58. See the solution

60. (a) 0 (b) σ/ϵ_0 to the right (c) 0

62. See the solution