CHAPTER 20

ANSWERS TO QUESTIONS

- **Q20.1** When one object B with electric charge is immersed in the electric field of another charge or charges A, the system possesses electric potential energy. The energy can be measured by seeing how much work the field does on the charge B as it moves to a reference location. We choose not to visualize A's effect on B as an action-at-a-distance, but as the result of a two-step process: Charge A creates electric potential throughout the surrounding space. Then the potential acts on B to inject the system with energy.
- **Q20.2** To move like charges together from an infinite separation, at which the potential energy of the system of two charges is zero, requires *work* to be done on the system, hence energy is stored, and potential energy is positive. As unlike charges move together from an infinite separation, energy is released, and the potential energy of the set of charges becomes negative.
- **Q20.3** If there were a potential difference between two points on the conductor, the free electrons in the conductor would move until the potential difference disappears.
- **Q20.4** A sharp point in a charged conductor would imply a large electric field in that region. An electric discharge could most easily take place at that sharp point.
- **Q20.5** Cold, snowy, windy weather would provide the most favorable environment for charge separation and possible car battery discharge.
- **Q20.6** Use a conductive box, to shield the equipment. Any stray electric field will cause charges on the outer surface of the conductor to rearrange and cancel the stray field inside the volume it encloses.
- **Q20.7** Seventeen combinations:

Individual

\n
$$
C_{1}, C_{2}, C_{3}
$$
\nParallel

\n
$$
C_{1} + C_{2} + C_{3}, C_{1} + C_{2}, C_{1} + C_{3}, C_{2} + C_{3}
$$
\nSeries – Parallel

\n
$$
\left(\frac{1}{C_{1}} + \frac{1}{C_{2}}\right)^{-1} + C_{3}, \left(\frac{1}{C_{1}} + \frac{1}{C_{3}}\right)^{-1} + C_{2}, \left(\frac{1}{C_{2}} + \frac{1}{C_{3}}\right)^{-1} + C_{1}
$$
\n
$$
\left(\frac{1}{C_{1} + C_{2}} + \frac{1}{C_{3}}\right)^{-1}, \left(\frac{1}{C_{1} + C_{3}} + \frac{1}{C_{2}}\right)^{-1}, \left(\frac{1}{C_{2} + C_{3}} + \frac{1}{C_{1}}\right)^{-1}
$$
\nSeries

\n
$$
\left(\frac{1}{C_{1}} + \frac{1}{C_{2}} + \frac{1}{C_{3}}\right)^{-1}, \left(\frac{1}{C_{1}} + \frac{1}{C_{2}}\right)^{-1}, \left(\frac{1}{C_{2}} + \frac{1}{C_{3}}\right)^{-1}, \left(\frac{1}{C_{1}} + \frac{1}{C_{3}}\right)^{-1}
$$

- **Q20.8** Nothing happens to the charge if the wires are disconnected. If the wires are connected to each other, charges in the single conductor which now exists move between the wires and the plates until the entire conductor is at a single potential and the capacitor is discharged.
- **Q20.9** Put a material with higher dielectric strength between the plates, or evacuate the space between the plates. At very high voltages, you may want to cool off the plates or choose to make them of a different chemically stable material, because atoms in the plates themselves can ionize, showing *thermionic emission* under high electric fields.
- **Q20.10** Energy is proportional to voltage squared. It gets four times larger.
- **Q20.11** The work you do to pull the plates apart becomes additional electric energy stored in the capacitor. The charge is constant and the capacitance decreases but the potential difference increases to drive up the potential energy $\frac{1}{2}Q\Delta V$. The electric field between the plates is constant in strength but fills more volume as you pull the plates apart.
- **Q20.12** Make the plate separation very small with a thin sheet of material with high dielectric constant.
- **Q20.13** The material of the dielectric may be able to support a larger electric field than air, without breaking down to pass a spark between the capacitor plates.
- **Q20.14** The parallel-connected capacitors store more energy, since they have higher equivalent capacitance.
- **Q20.15** The charge *Q* and voltage ΔV both double to make $U = \frac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C}$ $2\sqrt{Q^2}$ $(\Delta V)^2 = \frac{Q}{2C}$ four times larger.

PROBLEM SOLUTIONS

20.1 (a) Energy of the proton-field system is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V.

$$
K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f \qquad 0 + qV + 0 = \frac{1}{2} m v_p^2 + 0
$$

$$
(1.60 \times 10^{-19} \text{ C})(120 \text{ V}) \left(\frac{1 \text{ J}}{1 \text{ V} \cdot \text{C}}\right) = \frac{1}{2} (1.67 \times 10^{-27} \text{ kg}) v_p^2
$$

$$
v_p = 1.52 \times 10^5 \text{ m/s}
$$

(b) The electron will gain speed in moving the other way,

from
$$
V_i = 0
$$
 to $V_f = 120$ V:
\n
$$
K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f
$$
\n
$$
0 + 0 + 0 = \frac{1}{2} m v_e^2 + qV
$$
\n
$$
0 = \frac{1}{2} (9.11 \times 10^{-31} \text{ kg}) v_e^2 + (-1.60 \times 10^{-19} \text{ C})(120 \text{ J/C})
$$
\n
$$
v_e = 6.49 \times 10^6 \text{ m/s}
$$

20.2
$$
\Delta V = -14.0 \text{ V}
$$
 and $Q = -N_A e = -(6.02 \times 10^{23})(1.60 \times 10^{-19}) = -9.63 \times 10^4 \text{ C}$
\n $\Delta V = \frac{W}{Q'}$ so $W = Q\Delta V = (-9.63 \times 10^4 \text{ C})(-14.0 \text{ J/C}) = \boxed{1.35 \text{ MJ}}$

20.3 For speeds larger than one-tenth the speed of light, $\frac{1}{2}$ $\frac{1}{2}mv^2$ gives noticeably wrong answers for kinetic energy, so we use

$$
K = mc^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right) = \left(9.11 \times 10^{-31} \text{ kg}\right) \left(3.00 \times 10^8 \text{ m/s}\right)^2 \left(\frac{1}{\sqrt{1 - 0.400^2}} - 1\right) = 7.47 \times 10^{-15} \text{ J}
$$

Energy of the electron-field system is conserved during acceleration

 $K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

$$
0 + qV_i + 0 = 7.47 \times 10^{-15} \text{ J} + qV_f
$$

The change in potential is

$$
V_f - V_i
$$
:
$$
V_f - V_i = \frac{-7.47 \times 10^{-15} \text{ J}}{q} = \frac{-7.47 \times 10^{-15} \text{ J}}{-1.60 \times 10^{-19} \text{ C}} = \boxed{+46.7 \text{ kV}}
$$

The positive answer means that the electron speeds up in moving toward higher potential.

20.4 (a)
$$
(K + U_e)_i = (K + U_e)_j
$$
:
\n
$$
0 + 0 = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)mc^2 - qV
$$
\n
$$
\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right) 0.511 \text{ MeV} = 20 \text{ keV}
$$
\n
$$
\frac{1}{\sqrt{1 - v^2/c^2}} = 1.0391 \text{ so } 1 - \frac{v^2}{c^2} = 0.926
$$
\n
$$
v = 0.272c = \boxed{81.6 \text{ Mm/s}}
$$
\n(b) With $\frac{1}{2}mv^2 = |q|V$,
\n
$$
\frac{1}{2}(0.511 \text{ MeV})\left(\frac{v^2}{c^2}\right) = 20 \text{ keV}
$$
\nWe find $v = 0.280c = \boxed{83.9 \text{ Mm/s, too large by } 2.91\%$

20.5 (a) We follow the path from (0, 0) to (20.0 cm, 0) to (20.0 cm, 50.0 cm). $ΔU = - (work done)$ ∆*U* = – (work from origin to (20.0 cm, 0)) – (work from (20.0 cm, 0) to (20.0 cm, 50.0 cm)) Note that the last term is equal to 0 because the force is perpendicular to the displacement. $\Delta U = -(qE_x)\Delta x = -(12.0 \times 10^{-6} \text{ C})(250 \text{ V/m})(0.200 \text{ m}) = -6.00 \times 10^{-4} \text{ J}$ (b) ∆*V* = $\frac{\Delta U}{q}$ = – – 6.00×10 12.0×10 4 6 . . × × − − $\frac{U}{C}$ = -50.0 J/C = $\boxed{-50.0 \text{ V}}$

20.6
$$
E = \frac{|\Delta V|}{d} = \frac{25.0 \times 10^3 \text{ J/C}}{1.50 \times 10^{-2} \text{ m}} = 1.67 \times 10^6 \text{ N/C} = 1.67 \text{ MN/C}
$$

20.7
$$
\Delta U = -\frac{1}{2}m(v_f^2 - v_i^2) = -\frac{1}{2}(9.11 \times 10^{-31} \text{ kg}) \left[(1.40 \times 10^5 \text{ m/s})^2 - (3.70 \times 10^6 \text{ m/s})^2 \right] = 6.23 \times 10^{-18} \text{ J}
$$

$$
\Delta U = q \Delta V: \qquad +6.23 \times 10^{-18} = (-1.60 \times 10^{-19}) \Delta V
$$

$$
\Delta V = -38.9 \text{ V} \text{ The origin is at higher potential.}
$$

20.8 (a)
$$
|\Delta V| = Ed = (5.90 \times 10^3 \text{ V/m})(0.0100 \text{ m}) = 59.0 \text{ V}
$$

\n(b) $\frac{1}{2} m v_f^2 = |q\Delta V|$: $\frac{1}{2} (9.11 \times 10^{-31}) v_f^2 = (1.60 \times 10^{-19})(59.0)$
\n $v_f = 4.55 \times 10^6 \text{ m/s}$

***20.9** (a) Arbitrarily choose $V = 0$ at 0. Then at other points $V = -Ex$ and $U_e = QV = -QEx$. Between the endpoints of the motion, $(K + U_s + U_e)_i = (K + U_s + U_e)_f$ $0 + 0 + 0 = 0 + \frac{1}{2}kx_{\text{max}}^2 - QEx_{\text{max}}$ so : x_{max} \overline{a} 2*QE k* (b) At equilibrium, $\Sigma F_x = -F_s + F_e = 0$ or $kx = QE.$ So the equilibrium position is at *x* = *QE k*

(c) The block's equation of motion is
$$
\Sigma F_x = -kx + QE = m\frac{d^2x}{dt^2}
$$
.

Let
$$
x' = x - \frac{QE}{k}
$$
, or $x = x' + \frac{QE}{k}$,

so the equation of motion becomes:

$$
-k\left(x' + \frac{QE}{k}\right) + QE = m\frac{d^2(x' + QE/k)}{dt^2}, \quad \text{or} \quad \frac{d^2x'}{dt^2} = -\left(\frac{k}{m}\right)x'
$$

This is the equation for simple harmonic motion $a_{x'} = -\omega^2 x'$,

with

The period of the motion is then

(d) $(K + U_s + U_e)_i + \Delta E_{\text{mech}} = (K + U_s + U_e)_f$ $0 + 0 + 0 - \mu_k mgx_{\text{max}} = 0 + \frac{1}{2}kx_{\text{max}}^2 - QEx_{\text{max}}$ $x_{\text{max}} =$ L 2(QE – μ_k mg *k* $(QE - \mu_k mg)$

$$
x = x' + \frac{QE}{k},
$$

 $\omega = \sqrt{k/m}$.

$$
T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{k}}}
$$

20.10 (a) The potential at 1.00 cm is
$$
V_1 = k_e \frac{q}{r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})}{1.00 \times 10^{-2} \text{ m}} = 1.44 \times 10^{-7} \text{ V}
$$

(b) The potential at 2.00 cm is
$$
V_2 = k_e \frac{q}{r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})}{2.00 \times 10^{-2} \text{ m}} = 0.719 \times 10^{-7} \text{ V}
$$

Thus, the difference in potential between the two points is $\Delta V = V_2 - V_1 = \frac{9.719 \times 10^{-8} \text{ V}}{2 \times 10^{-8} \text{ V}}$

(c) The approach is the same as above except the charge is -1.60×10^{-19} C. This changes the sign of all the answers, with the magnitudes remaining the same.

That is, the potential at 1.00 cm is $\vert -1.44 \times 10^{-7} \mathrm{V} \vert$.

The potential at 2.00 cm is
$$
-0.719 \times 10^{-7}
$$
 V, so $\Delta V = V_2 - V_1 = \boxed{7.19 \times 10^{-8} \text{ V}}$.

2.00 μ C
 \rightarrow 2.00 μ C **20.11** (a) Since the charges are equal and placed symmetrically, *F* = 0 (b) Since $F = qE = 0, |E = 0$ $2k_e \frac{q}{r} = 2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{2.00 \times 10^{-19}}{0.800 \text{ N}}\right)$.99 × 10⁹ N·m² / C² $\left(\frac{2.00 \times 10^{-6} \text{ C}}{0.800 \text{ m}}\right)$ $V = 2k_e \frac{q}{r} = 2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{2.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{0.80}\right)$ ľ 2 / C^{2} (c) $\overline{}$ $V = 4.50 \times 10^4$ V = 45.0 kV

20.12 (a)
$$
E_x = \frac{k_e q_1}{x^2} + \frac{k_e q_2}{(x - 2.00)^2} = 0
$$
 becomes
$$
E_x = k_e \left(\frac{+q}{x^2} + \frac{-2q}{(x - 2.00)^2}\right) = 0
$$

Dividing by k_e ,
$$
2qx^2 = q(x - 2.00)^2 \quad x^2 + 4.00x - 4.00 = 0
$$

Therefore $E = 0$ when
$$
x = \frac{-4.00 \pm \sqrt{16.0 + 16.0}}{2} = \boxed{-4.83 \text{ m}}
$$

(Note that the positive root does not correspond to a physically valid situation.)

(b) $V = \frac{k_e q}{x}$ *k q* $=\frac{\kappa_e q_1}{x} + \frac{\kappa_e q_2}{2.00 - x} = 0$ or $V = k_e \left(\frac{+q}{x} \right)$ $k_e \left(\frac{+q}{x} - \frac{2q}{2.00 - x} \right)$ l $\frac{2q}{2.00-x}$ = 0

 0.667 m

Again solving for *x*, $2qx = q(2.00 - x)$

For
$$
0 \le x \le 2.00
$$
 $V = 0$ when $x =$

and $\overline{}$ *q x* $=\frac{-2q}{|2-x|}$ 2 2 For $x < 0$ $x = \frac{1}{200}$ m

20.13
$$
V = \sum_{i} k \frac{q_i}{r_i}
$$

\n
$$
V = (8.99 \times 10^{9})(7.00 \times 10^{-6}) \left[\frac{-1}{0.0100} - \frac{1}{0.0100} + \frac{1}{0.0387} \right]
$$

\n
$$
V = \left[\frac{-1.10 \times 10^{7} \text{ V} = -11.0 \text{ MV}}{1000} \right]
$$

\n4.00
\n
$$
V = \left[\frac{-1.10 \times 10^{7} \text{ V} = -11.0 \text{ MV}}{1000} \right]
$$

20.14 (a)
$$
U = \frac{k_e q_1 q_2}{r} = \frac{-\left(8.99 \times 10^9\right) \left(1.60 \times 10^{-19}\right)^2}{0.0529 \times 10^{-9}} = -4.35 \times 10^{-18} \text{ J} = \boxed{-27.2 \text{ eV}}
$$

\n(b) $U = \frac{k_e q_1 q_2}{r} = \frac{-\left(8.99 \times 10^9\right) \left(1.60 \times 10^{-19}\right)^2}{2^2 \left(0.0529 \times 10^{-9}\right)} = \boxed{-6.80 \text{ eV}}$
\n(c) $U = \frac{k_e q_1 q_2}{r} = \frac{-k_e e^2}{\infty} = \boxed{0}$

20.15
$$
U_e = q_4 V_1 + q_4 V_2 + q_4 V_3 = q_4 \left(\frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)\right)
$$

$$
U_e = \left(10.0 \times 10^{-6} \text{ C}\right)^2 \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{ C}^2 \left(\frac{1}{0.600 \text{ m}} + \frac{1}{0.150 \text{ m}} + \frac{1}{\sqrt{(0.600 \text{ m})^2 + (0.150 \text{ m})^2}}\right)
$$

$$
U_e = \boxed{8.95 \text{ J}}
$$

***20.16** (a)
$$
V = \frac{k_e q_1}{r_1} + \frac{k_e q_2}{r_2} = 2\left(\frac{k_e q}{r}\right)
$$

\n
$$
V = 2\left(\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.00 \times 10^{-6} \text{ C})}{\sqrt{(1.00 \text{ m})^2 + (0.500 \text{ m})^2}}\right)
$$
\n
$$
V = 3.22 \times 10^4 \text{ V} = \boxed{32.2 \text{ kV}}
$$
\n(b) $U = qV = (-3.00 \times 10^{-6} \text{ C})(3.22 \times 10^4 \text{ C}) = \boxed{-9.65 \times 10^{-2} \text{ J}}$ (1.00 m, 0)

20.17
$$
U = U_1 + U_2 + U_3 + U_4
$$
\n
$$
U = 0 + U_{12} + (U_{13} + U_{23}) + (U_{14} + U_{24} + U_{34})
$$
\n
$$
U = 0 + \frac{k_e Q^2}{s} + \frac{k_e Q^2}{s} \left(\frac{1}{\sqrt{2}} + 1\right) + \frac{k_e Q^2}{s} \left(1 + \frac{1}{\sqrt{2}} + 1\right)
$$
\n
$$
U = \frac{k_e Q^2}{s} \left(4 + \frac{2}{\sqrt{2}}\right) = \left[5.41 \frac{k_e Q^2}{s}\right]
$$
\n(1)

An alternate way to get the term $\left(4+2/\sqrt{2}\right)$ is to recognize that there are 4 side pairs and 2 face diagonal pairs.

- ***20.18** (a) Each charge separately creates positive potential everywhere. The total potential produced by the three charges together is then the sum of three positive terms. There is \mid no point \mid located at a finite distance from the charges, at which this total potential is zero.
	- (b) $V = \frac{k_e q}{a} + \frac{k_e q}{a} =$ 2*k q a e*
- **20.19** Consider the two spheres as a system.
	- (a) Conservation of momentum: $0 = m_1 v_1 \mathbf{i} + m_2 v_2(-\mathbf{i})$ or $v_2 = \frac{m_1 v}{m_2}$ $n_2 = \frac{m_1 v_1}{m_2}$ 2 = By conservation of energy, $0 = \frac{k_e(-q_1)q_2}{d} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{k_e(-q_1)q_2}{r_1 + r_2}$ $1 + 72$ $k_e(-q_1)q$ $\frac{q_1}{q_2} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{k_e(-q_1)q_1}{r_1 + r_2}$ $e^{-q_1/q_2} = 1_{m,7} 2 + 1_{m,7} 2 + \frac{\kappa_e}{\kappa}$ and *kqq* $r_1 + r_2$ *kqq* $\frac{d^{2}u}{dt^{2}} = \frac{1}{2}m_{1}v_{1}^{2} + \frac{1}{2}\frac{m_{1}^{2}v}{m_{2}}$ *m* <u>e 4142</u> _ <u>^e</u> $1 - r_2$ $\frac{192}{1} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2}$ 2 $1^2v_1^2$ $+\frac{r_1r_2}{r_2}-\frac{r_2r_1r_2}{d}=\frac{1}{2}m_1v_1^2+\frac{1}{2}\frac{m_1v_2}{m_2}$ $v_1 = \sqrt{\frac{2m_2k_eq_1q_2}{m_1(m_1 + m_2)}}$ 1 $r_1 + r_2$ $-\frac{1}{d}$ ſ $\left(\frac{1}{r_1+r_2}-\frac{1}{d}\right)$ *v*1 9 N \cdot m²/ C^{2} | 2×10^{-6} C | 3×10^{-6} 3 $2(0.700 \text{ kg}) \le 8.99 \times 10^{7} \text{ N} \cdot \text{m}^{2}/\text{C}^{2} \le 2 \times 10^{-6} \text{ C} \le 3 \times 10^{10} \text{ C}$ 0.100 kg)(0.800 1 8×10 $= \sqrt{\frac{2(0.700 \text{ kg})(8.99 \times 10^{9} \text{ N} \cdot \text{m}^{2}/\text{C}^{2})(2 \times 10^{-6} \text{ C})(3 \times 10^{-6} \text{ C})}{(0.100 \text{ kg})(0.800 \text{ kg})}} \left(\frac{1}{8 \times 10^{-3} \text{ m}} - \frac{1}{1.00}\right)$ $\Big) =$ − − − .700 kg) $(8.99 \times 10^5 \text{ N} \cdot \text{m}^2)$.100 kg)(0.800 kg) $(8 \times 10^{-3} \text{ m} \quad 1.$ kg $(8.99 \times 10^{7} \text{ N} \cdot \text{m}^{2}/\text{C}^{2})$ $(2 \times 10^{-6} \text{ C})$ $(3 \times 10^{-6} \text{ C})$ kg (0.800 kg) $(8 \times 10^{-3} \text{ m}$ 1.00 m 2 $\frac{10.8 \text{ m/s}}{2}$ $v_2 = \frac{m_1 v}{m_2}$ $n_2 = \frac{m_1 v_1}{m_2}$ 2 $=\frac{m_1c_1}{m_1}$ = 0.100 0.700 . . $kg(10.8 \text{ m/s})$ $\frac{(10.8 \text{ m/s})}{20 \text{ kg}} = \boxed{1.55 \text{ m/s}}$
	- (b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving l faster than calculated in (a) .
- ***20.20** Consider the two spheres as a system.
	- (a) Conservation of momentum: $0 = m_1v_1$ **i** + m_2v_2 (-**i**) or $v_2 = \frac{m_1 v}{m}$ $n_2 = \frac{m_1 v_1}{m_2}$ 2 = By conservation of energy, $0 = \frac{\kappa_e(-q_1)q_2}{d} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{\kappa_e(-q_1)q_2}{r_1+r_2}$ $1'$ $'$ 2 $=\frac{k_e(-q_1)q_2}{d}=\frac{1}{2}m_1v_1^2+\frac{1}{2}m_2v_2^2+\frac{k_e(-q_1)}{r_1+r_2}$ $k_e(-q_1)q$ $\frac{q_1}{q_2} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{k_e(-q_1)q_1}{r_1 + r_2}$ $e^{-q_1/q_2} - 1$ $m_{1,2}$, $2 + 1$ $m_{1,2}$, $2 + \frac{R_e}{r}$ and *kqq* $r_1 + r$ *kqq* $\frac{d^2M_2}{dt} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}\frac{m_1^2v_1^2}{m_2^2}$ *m* <u>e 4142</u> _ <u>^e</u> $1 + r_2$ $\frac{142}{1} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2}$ 2 $\frac{1^2v_1^2}{1}$ $+\frac{r_1r_2}{r_2}-\frac{r_2r_1r_2}{d}=\frac{1}{2}m_1v_1^2+\frac{1}{2}\frac{m_1}{m_2}$ $\frac{2m_2k_eq_1q_2}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ m₂k_eq₁q $\frac{m_2 k_e q_1 q_2}{(m_1 + m_2)} \left(\frac{1}{r_1 + r_2} - \frac{1}{d} \right)$

 $v_1 =$ \overline{a}

 $v_2 = \frac{m}{2}$ $v_2 = \left(\frac{m_1}{m_2}\right)v_1$ $\left(\frac{m_1}{m_2}\right)v_1 =$

- (b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving l faster than calculated in (a) \vert
- ***20.21** Using conservation of energy for the alpha particle-nucleus system,

we have **been** and the contract of the contrac

But

$$
U_i = \frac{k_e q_\alpha q_{\text{gold}}}{r_i}
$$

 $U_f = K_i$

 $K_f + U_f = K_i + U_i$

 $1^{n_1 + n_2}$ $\binom{n_1 + n_2}{n_1 + n_2}$

l,

 $m_1(m_1 + m_2) (r_1 + r_2) d$

 $\frac{2m_1k_eq_1q_2}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $2(n_1 + n_2) \setminus (1 + n_2)$

 $m_2(m_1 + m_2) (r_1 + r_2) d$ $\frac{m_1 k_e q_1 q_2}{(m_1 + m_2)} \left(\frac{1}{r_1 + r_2} - \frac{1}{d} \right)$

mkqq

and *and* $r_i \cong \infty$

Thus, $U_i = 0$

Also $K_f = 0$ ($v_f = 0$ at turning point),

so

or

$$
\frac{k_e q_\alpha q_{\text{gold}}}{r_{\text{min}}} = \frac{1}{2} m_\alpha v_\alpha^2
$$

$$
r_{\min} = \frac{2k_e q_\alpha q_{\text{gold}}}{m_\alpha v_\alpha^2} = \frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2)(79)(1.60 \times 10^{-19} \text{ C})^2}{(6.64 \times 10^{-27} \text{ kg})(2.00 \times 10^7 \text{ m/s})^2} = 2.74 \times 10^{-14} \text{ m} = \boxed{27.4 \text{ fm}}
$$

***20.22** In an empty universe, the 20-nC charge can be placed at its location with no energy investment. At a distance of 4 cm, it creates a potential

$$
V_1 = \frac{k_e q_1}{r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(20 \times 10^{-9} \text{ C})}{0.04 \text{ m}} = 4.50 \text{ kV}
$$

To place the 10-nC charge there we must put in energy

$$
U_{12} = q_2 V_1 = (10 \times 10^{-9} \text{ C})(4.5 \times 10^3 \text{ V}) = 4.50 \times 10^{-5} \text{ J}
$$

Next, to bring up the –20-nC charge requires energy

$$
U_{23} + U_{13} = q_3 V_2 + q_3 V_1 = q_3 (V_2 + V_1) = -20 \times 10^{-9} \text{ C} \left(8.89 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \left(\frac{10 \times 10^{-9} \text{ C}}{0.04 \text{ m}} - \frac{20 \times 10^{-9} \text{ C}}{0.08 \text{ m}} \right)
$$

= -4.50 × 10⁻⁵ J – 4.50 × 10⁻⁵ J

The total energy of the three charges is

$$
U_{12} + U_{23} + U_{13} = \boxed{-4.50 \times 10^{-5} \text{ J}}
$$

(b) The three fixed charges create this potential at the location where the fourth is released:

$$
V = V_1 + V_2 + V_3 = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right) \left(\frac{20 \times 10^{-9}}{\sqrt{0.04^2 + 0.03^2}} + \frac{10 \times 10^{-9}}{0.03} - \frac{20 \times 10^{-9}}{0.05}\right) \text{C/m}
$$

 $V = 3.00 \times 10^3$ V

Energy of the system of four charged objects is conserved as the fourth charge flies away:

$$
\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f
$$

$$
0 + \left(40 \times 10^{-9} \text{ C}\right)\left(3.00 \times 10^3 \text{ V}\right) = \frac{1}{2}\left(2.00 \times 10^{-13} \text{ kg}\right)v^2 + 0
$$

$$
v = \sqrt{\frac{2\left(1.20 \times 10^{-4} \text{ J}\right)}{2 \times 10^{-13} \text{ kg}}} = \boxed{3.46 \times 10^4 \text{ m/s}}
$$

20.23

$$
V = a + bx = 10.0 \text{ V} + (-7.00 \text{ V/m})x
$$

(a) At
$$
x = 0
$$
, $V = \boxed{10.0 \text{ V}}$
\nAt $x = 3.00 \text{ m}$, $V = \boxed{-11.0 \text{ V}}$
\nAt $x = 6.00 \text{ m}$, $V = \boxed{-32.0 \text{ V}}$
\n(b) $E = -\frac{dV}{dx} = -b = -(-7.00 \text{ V/m}) = \boxed{7.00 \text{ N/C in the } +x \text{ direction}}$

*20.24 (a) For
$$
r < R
$$
 $V = \frac{k_e Q}{R}$
\n
$$
E_r = -\frac{dV}{dr} = \boxed{0}
$$
\n(b) For $r \ge R$ $V = \frac{k_e Q}{r}$
\n
$$
E_r = -\frac{dV}{dr} = -\left(-\frac{k_e Q}{r^2}\right) = \boxed{\frac{k_e Q}{r^2}}
$$

20.25
$$
V = 5x - 3x^{2}y + 2yz^{2}
$$

Evaluate *E* at (1, 0 – 2)

$$
E_{x} = -\frac{\partial V}{\partial x} = \boxed{-5 + 6xy} = -5 + 6(1)(0) = -5
$$

$$
E_{y} = -\frac{\partial V}{\partial y} = \boxed{+3x^{2} - 2z^{2}} = 3(1)^{2} - 2(-2)^{2} = -5
$$

$$
E_{z} = -\frac{\partial V}{\partial z} = \boxed{-4yz} = -4(0)(-2) = 0
$$

$$
E = \sqrt{E_{x}^{2} + E_{y}^{2} + E_{z}^{2}} = \sqrt{(-5)^{2} + (-5)^{2} + 0^{2}} = \boxed{7.07 \text{ N/C}}
$$

20.26
$$
\Delta V = V_{2R} - V_0 = \frac{k_e Q}{\sqrt{R^2 + (2R)^2}} - \frac{k_e Q}{R} = \frac{k_e Q}{R} \left(\frac{1}{\sqrt{5}} - 1\right) = \left[-0.533 \frac{k_e Q}{R}\right]
$$

20.27 (a)
$$
\left[\alpha\right] = \left[\frac{\lambda}{x}\right] = \frac{C}{m} \cdot \left(\frac{1}{m}\right) = \frac{C}{m^2}
$$

\n(b) $V = k_e \int \frac{dq}{r} = k_e \int \frac{\lambda dx}{r} = k_e \alpha \int_0^L \frac{x dx}{d+x} = \left[k_e \alpha \left[L - d \ln\left(1 + \frac{L}{d}\right)\right]\right]$

$$
20.28
$$

$$
V = \int \frac{k_e \, dq}{r} = k_e \int \frac{\alpha \, x \, dx}{\sqrt{b^2 + (L/2 - x)^2}}
$$

Let $z=\frac{L}{2}-x$.

Then $x = \frac{L}{2} - z$, and $dx = -dz$

$$
V = k_e \alpha \int \frac{(L/2 - z)(-dz)}{\sqrt{b^2 + z^2}} = -\frac{k_e \alpha L}{2} \int \frac{dz}{\sqrt{b^2 + z^2}} + k_e \alpha \int \frac{z \, dz}{\sqrt{b^2 + z^2}} = -\frac{k_e \alpha L}{2} \ln(z + \sqrt{z^2 + b^2}) + k_e \alpha \sqrt{z^2 + b^2}
$$

\n
$$
V = -\frac{k_e \alpha L}{2} \ln\left[(L/2 - x) + \sqrt{(L/2 - x)^2 + b^2}\right]_0^L + k_e \alpha \sqrt{(L/2 - x)^2 + b^2}\Big|_0^L
$$

\n
$$
V = -\frac{k_e \alpha L}{2} \ln\left[\frac{L/2 - L + \sqrt{(L/2)^2 + b^2}}{L/2 + \sqrt{(L/2)^2 + b^2}}\right] + k_e \alpha \left[\sqrt{(L/2 - L)^2 + b^2} - \sqrt{(L/2)^2 + b^2}\right]
$$

\n
$$
V = \left[-\frac{k_e \alpha L}{2} \ln\left[\frac{\sqrt{b^2 + (L^2/4)} - L/2}{\sqrt{b^2 + (L^2/4)} + L/2}\right]\right]
$$

20.29

$$
V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}
$$

All bits of charge are at the same distance from *O*,

So
$$
V = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R}\right) = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \left(\frac{-7.50 \times 10^{-6} \text{ C}}{0.140 \text{ m}/\pi}\right) = \boxed{-1.51 \text{ MV}}
$$

***20.30** Substituting given values into *V* = *k q r ^e* ,

$$
7.50 \times 10^3 \text{ V} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)q}{0.300 \text{ m}}
$$

Substituting $q = 2.50 \times 10^{-7}$ C,

$$
N = \frac{2.50 \times 10^{-7} \text{ C}}{1.60 \times 10^{-19} \text{ C/e}^{-}} = \boxed{1.56 \times 10^{12} \text{ electrons}}
$$

20.31 (a)
$$
E = \frac{0}{0}
$$

\n
$$
V = \frac{k_e q}{R} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{0.140} = \frac{1.67 \text{ MV}}{1.67 \text{ MV}}
$$
\n(b) $E = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.200)^2} = \frac{5.84 \text{ MN/C}}{5.84 \text{ MN/C}} \text{ away}$ \n
$$
V = \frac{k_e q}{R} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{0.200} = \frac{1.17 \text{ MV}}{1.17 \text{ MV}}
$$
\n(c) $E = \frac{k_e q}{R^2} = \frac{(8.99 \times 10^9)(26.0 \times 10^{-6})}{(0.140)^2} = \frac{11.9 \text{ MN/C}}{11.9 \text{ MN/C}} \text{ away}$

***20.32** (a) Both spheres must be at the same potential according to *k q r k q r e e* 1 1 2 2 =

where also

$$
q_1 + q_2 = 1.20 \times 10^{-6} \text{ C}
$$

Then

$$
q_1 = q_2 r_1 / r_2
$$

\n
$$
q_2 r_1 / r_2 + q_2 = 1.20 \times 10^{-6} \text{ C}
$$

\n
$$
q_2 = \frac{1.20 \times 10^{-6} \text{ C}}{1 + 6 \text{ cm} / 2 \text{ cm}} = 0.300 \times 10^{-6} \text{ C on the smaller sphere}
$$

\n
$$
q_1 = 1.20 \times 10^{-6} \text{ C} - 0.300 \times 10^{-6} \text{ C} = 0.900 \times 10^{-6} \text{ C}
$$

\n
$$
V = \frac{k_e q_1}{r_1} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(0.900 \times 10^{-6} \text{ C}\right)}{6 \times 10^{-2} \text{ m}} = \boxed{1.35 \times 10^5 \text{ V}}
$$

(b) Outside the larger sphere,

$$
\mathbf{E}_1 = \frac{k_e q_1}{r_1^2} \hat{\mathbf{r}} = \frac{V_1}{r_1} \hat{\mathbf{r}} = \frac{1.35 \times 10^5 \text{ V}}{0.06 \text{ m}} \hat{\mathbf{r}} = 2.25 \times 10^6 \text{ V/m away}
$$

Outside the smaller sphere,

$$
\mathbf{E}_2 = \frac{1.35 \times 10^5 \text{ V}}{0.02 \text{ m}} \hat{\mathbf{r}} = \boxed{6.74 \times 10^6 \text{ V/m away}}
$$

The smaller sphere carries less charge but creates a much stronger electric field than the larger sphere.

20.33 (a)
$$
Q = C\Delta V = (4.00 \times 10^{-6} \text{ F})(12.0 \text{ V}) = 4.80 \times 10^{-5} \text{ C} = 48.0 \mu\text{C}
$$

\n(b) $Q = C\Delta V = (4.00 \times 10^{-6} \text{ F})(1.50 \text{ V}) = 6.00 \times 10^{-6} \text{ C} = 6.00 \mu\text{C}$

20.34 (a)
$$
C = \frac{Q}{\Delta V} = \frac{10.0 \times 10^{-6} \text{ C}}{10.0 \text{ V}} = 1.00 \times 10^{-6} \text{ F} = 1.00 \mu\text{F}
$$

\n(b) $\Delta V = \frac{Q}{C} = \frac{100 \times 10^{-6} \text{ C}}{1.00 \times 10^{-6} \text{ F}} = 100 \text{ V}$

$$
20.35 \qquad (a) \quad \Delta V = Ed
$$

$$
E = \frac{20.0 \text{ V}}{1.80 \times 10^{-3} \text{ m}} = 11.1 \text{ kV/m}
$$

(b) $E = \frac{\sigma}{\sqrt{2}}$ *e*0

$$
\sigma = (1.11 \times 10^4 \text{ N/C})(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) = 98.3 \text{ nC/m}^2
$$

(c)
$$
C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(7.60 \text{ cm}^2)(1.00 \text{ m}/100 \text{ cm})^2}{1.80 \times 10^{-3} \text{ m}} = 3.74 \text{ pF}
$$

(d)
$$
\Delta V = \frac{Q}{C}
$$

$$
Q = (20.0 \text{ V})(3.74 \times 10^{-12} \text{ F}) = 74.7 \text{ pC}
$$

*20.36

\n
$$
E = \frac{k_e q}{r^2}:
$$
\n
$$
q = \frac{(4.80 \times 10^4 \text{ N/C})(0.210 \text{ m})^2}{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)} = 0.240 \ \mu\text{C}
$$
\n(a)

\n
$$
\sigma = \frac{q}{A} = \frac{0.240 \times 10^{-6}}{4\pi (0.120)^2} = 1.33 \ \mu\text{C/m}^2
$$
\n(b)

\n
$$
C = 4\pi \epsilon_0 r = 4\pi \left(8.85 \times 10^{-12}\right)(0.120) = 13.3 \ \text{pF}
$$

***20.37** With $\theta = \pi$, the plates are out of mesh and the overlap area is zero. With θ = 0, the overlap area is that of a semicircle, $\pi R^2/2$. By proportion, the effective area of a single sheet of charge is $(\pi - \theta) R^2 / 2$.

When there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are *N* plates on each comb, the number of parallel capacitors is $2N - 1$ and the total capacitance is

$$
C = (2N-1)\frac{\epsilon_0 A_{\text{effective}}}{\text{distance}} = \frac{(2N-1)\epsilon_0(\pi-\theta)R^2/2}{d/2} = \frac{(2N-1)\epsilon_0(\pi-\theta)R^2}{d}
$$

20.38 (a)
$$
C = \frac{l}{2k_e \ln(\frac{b}{a})} = \frac{50.0}{2(8.99 \times 10^9) \ln(\frac{7.27}{2.58})} = 2.68 \text{ nF}
$$

(b) Method 1: $\Delta V = 2k \lambda \ln(\frac{b}{a})$

(b) Method 1:
$$
\Delta V = 2k_e \lambda \ln \left(\frac{b}{a} \right)
$$

$$
\lambda = q/\ell = \frac{8.10 \times 10^{-6} \text{ C}}{50.0 \text{ m}} = 1.62 \times 10^{-7} \text{ C/m}
$$

 8.10×10 2.68×10

× ×

. .

 $\frac{Q}{C}$ =

$$
\Delta V = 2(8.99 \times 10^{9})(1.62 \times 10^{-7}) \ln \left(\frac{7.27}{2.58}\right) = 3.02 \text{ kV}
$$

 $\frac{1}{-9}$ = 3.02 kV

6 9

−

Method 2:

zF_y = 0:
$$
T \cos \theta - mg = 0
$$

\n $\Sigma F_x = 0$: $T \sin \theta - Eq = 0$
\nDividing, $\tan \theta = \frac{Eq}{mg}$
\nso $E = \frac{mg}{q} \tan \theta$
\nand $\Delta V = Ed = \frac{mgd \tan \theta}{q}$

20.40 The electric field created by the charges points radially outward in the space between radius *a* and radius *b*. To find it, use a spherical Gaussian surface of radius *r*. The magnitude of the field must be uniform over this surface, so

$$
\oint \mathbf{E} \, d\mathbf{A} = \frac{q_{\text{in}}}{\epsilon_0}
$$

gives

$$
E = \frac{Q}{4\pi\epsilon_0 r^2} = k_e \frac{Q}{r^2}
$$

 $E(4\pi r^2) = \frac{Q}{r}$ $\left(4\pi r^2\right) = \frac{8}{\epsilon_0}$

Now to find the difference in electric potential between the conducting spheres, we use

$$
V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s}
$$

along a radius line

$$
V_b - V_a = -\int_{r=a}^{b} \frac{k_e Q}{r^2} (\cos 0) \, dr = -k_e Q \frac{r^{-1}}{-1} \bigg|_a^b = k_e Q \bigg(\frac{1}{b} - \frac{1}{a} \bigg) = -k_e Q \bigg(\frac{1}{a} - \frac{1}{b} \bigg) = -k_e Q \bigg(\frac{b - a}{ab} \bigg)
$$

$$
V_a - V_b = +k_e Q \bigg(\frac{b - a}{ab} \bigg)
$$

Then the capacitance is

(b) As $b \rightarrow \infty$, $b - a \rightarrow b$, to give

 $k_e = \frac{1}{4\pi\epsilon_0}$

with

$$
C = \frac{Q}{V_a - V_b} = \frac{ab}{k_e(b - a)}
$$

 $C \rightarrow \frac{ab}{1}$ $\rightarrow \frac{hc}{k_e b}$ *a ke*

 $C = 4\pi\epsilon_0 a$, in agreement with Equation 20.20.

20.41 (a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

, this is

$$
C_{eq} = C_1 + C_2 = 5.00 \ \mu\text{F} + 12.0 \ \mu\text{F} = \boxed{17.0 \ \mu\text{F}}
$$

(b) The potential difference across each branch is the same and equal to the voltage of the battery.

 $\Delta V = | 9.00 \text{ V}$

(c)
$$
Q_5 = C\Delta V = (5.00 \,\mu\text{F})(9.00 \text{ V}) = 45.0 \,\mu\text{C}
$$

and $Q_{12} = C\Delta V = (12.0 \,\mu\text{F})(9.00 \text{ V}) = 108 \,\mu\text{C}$

 $Q_{12} = C\Delta V = (12.0 \,\mu\text{F})(9.00 \text{ V}) = | 108 \,\mu\text{C}$

20.42 (a) In series capacitors add as

 $1 \t1 \t1 \t1$ 5 00 1 $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00 \ \mu F} + \frac{1}{12.0 \ \mu F}$ $C_{eq} = 3.53 \, \mu \text{F}$

$$
Q_{eq} = C_{eq} \Delta V = (3.53 \ \mu\text{F})(9.00 \ \text{V}) = 31.8 \ \mu\text{C}
$$

Each of the series capacitors has this same charge on it.

So and the state of the sta

and

and

(b) The potential difference across each is

(c) The charge on the equivalent capacitor is

$$
\Delta V_1 = \frac{Q_1}{C_1} = \frac{31.8 \, \mu\text{C}}{5.00 \, \mu\text{F}} = \boxed{6.35 \, \text{V}}
$$
\n
$$
\Delta V_2 = \frac{Q_2}{C_2} = \frac{31.8 \, \mu\text{C}}{12.0 \, \mu\text{F}} = \boxed{2.65 \, \text{V}}
$$

 $Q_1 = Q_2 = 31.8 \mu C$

20.43 (a)
$$
\frac{1}{C_s} = \frac{1}{15.0} + \frac{1}{3.00}
$$

\n $C_s = 2.50 \mu\text{F}$
\n $C_p = 2.50 + 6.00 = 8.50 \mu\text{F}$
\n $C_{eq} = \left(\frac{1}{8.50 \mu\text{F}} + \frac{1}{20.0 \mu\text{F}}\right)^{-1} = \left[\frac{5.96 \mu\text{F}}{3.96 \mu\text{F}}\right]$
\n(b) $Q = C\Delta V = (5.96 \mu\text{F})(15.0 \text{ V}) = \left[\frac{89.5 \mu\text{C}}{89.5 \mu\text{C}}\right] \text{ on } 20.0 \mu\text{F}$
\n $\Delta V = \frac{Q}{C} = \frac{89.5 \mu\text{C}}{20.0 \mu\text{F}} = 4.47 \text{ V}$
\n $15.0 - 4.47 = 10.53 \text{ V}$
\n $Q = C\Delta V = (6.00 \mu\text{F})(10.53 \text{ V}) = \left[\frac{63.2 \mu\text{C}}{63.2 \mu\text{C}}\right] \text{ on } 6.00 \mu\text{F}$
\n $89.5 - 63.2 = \left[\frac{26.3 \mu\text{C}}{26.3 \mu\text{C}}\right] \text{ on } 15.0 \mu\text{F}$
\nand $3.00 \mu\text{F}$

121

20.44
$$
C_p = C_1 + C_2
$$

and $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$

 $C_2 = C_p - C_1$:

Substitute

and

$$
\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_p - C_1} = \frac{C_p - C_1 + C_1}{C_1 (C_p - C_1)}
$$

Simplifying,

$$
C_1^2 - C_1 C_p + C_p C_s = 0
$$

and the contract of the contra

$$
C_1 = \frac{C_p \pm \sqrt{C_p^2 - 4C_p C_s}}{2} = \boxed{\frac{1}{2}C_p + \sqrt{\frac{1}{4}C_p^2 - C_p C_s}}
$$

 \boldsymbol{b}

 $\chi^{6.00}_{\mu \rm F}$

where the positive sign was arbitrarily chosen (choosing the negative sign gives the same values for the capacitances, with the names reversed).

Then, from

$$
C_2 = C_p - C_1
$$

$$
C_2 = \frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_pC_s}
$$

20.45
\n
$$
C = \frac{Q}{\Delta V} \qquad \text{so} \qquad \qquad 6.00 \times 10^{-6} = \frac{Q}{20.0}
$$
\nand\n
$$
Q = \boxed{120 \ \mu C}
$$
\n
$$
Q_1 = 120 \ \mu C - Q_2
$$
\nand\n
$$
\Delta V = \frac{Q}{C}:
$$
\n
$$
\Delta V = \frac{Q}{C}:
$$
\n
$$
\Delta V = \frac{Q_2}{Q_1} = \frac{Q_2}{C_2}
$$
\nor\n
$$
\frac{120 - Q_2}{6.00} = \frac{Q_2}{3.00}
$$
\n
$$
(3.00)(120 - Q_2) = (6.00)Q_2
$$
\n
$$
Q_2 = \frac{360}{9.00} = \boxed{40.0 \ \mu C}
$$
\n
$$
Q_1 = 120 \ \mu C - 40.0 \ \mu C = \boxed{80.0 \ \mu C}
$$
\n
$$
Q_2 = \frac{360}{5.00} + \frac{1}{7.00} \text{ m}^{-1} = 2.92 \ \mu \text{F}
$$
\n
$$
C_p = 2.92 + 4.00 + 6.00 = \boxed{12.9 \ \mu \text{F}}
$$
\n
$$
Q_p = \frac{5.00}{4.00} = \frac{4.00}{4.00} \text{ m}^{-1} = 2.92 \ \mu \text{F}
$$
\n
$$
Q_p = \frac{4.00}{4.00} \text{ m}^{-1} = 2.92 \ \mu \text{F}
$$
\n
$$
Q_p = 2.92 + 4.00 + 6.00 = \boxed{12.9 \ \mu \text{F}}
$$

 $C_p = 2.92 + 4.00 + 6.00 = 12.9 \,\mu\text{F}$

***20.47**

According to the hint, this combination of capacitors is equivalent to $\mathcal{A} \longrightarrow \mathcal{A}$.

Then $1 \t1 \t1 \t1$ *C C*₀ *C* + *C*₀ *C* $C + C_0 + C_0 + C + C$ $=\frac{1}{C_0} + \frac{1}{C+C_0} + \frac{1}{C_0} = \frac{C+C_0+C_0+C+C_0}{C_0(C+C_0)}$ C_0 C + C₀ C₀ C₀(C + C₀) $0 - 6 - 6$ $0(- + 6)$ $C_0C + C_0^2 = 2C^2 + 3C_0C$ $2C^2 + 2C_0C - C_0^2 = 0$ $C = \frac{-2C_0 \pm \sqrt{4C_0^2 + 4(2C_0^2)}}{4}$ 4 $\sqrt{4C_0^2+4(2C_0^2)}$

Only the positive root is physical

$$
C = \frac{C_0}{2} \left(\sqrt{3} - 1\right)
$$

20.48 (a)
$$
U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00 \ \mu\text{F})(12.0 \ \text{V})^2 = 216 \ \mu\text{J}
$$

\n(b) $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00 \ \mu\text{F})(6.00 \ \text{V})^2 = 54.0 \ \mu\text{J}$

*20.49
$$
U = \frac{1}{2} C \Delta V^2
$$

 $U = \frac{1}{2} C \Delta V^2$ (300 J)

$$
\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(300 \text{ J})}{30 \times 10^{-6} \text{ C/V}}} = \boxed{4.47 \times 10^3 \text{ V}}
$$

20.50
$$
U = \frac{1}{2}C(\Delta V)^2
$$
The circuit diagram is shown at the right.
\n(a) $C_p = C_1 + C_2 = 25.0 \ \mu\text{F} + 5.00 \ \mu\text{F} = 30.0 \ \mu\text{F}$
\n
$$
U = \frac{1}{2}(30.0 \times 10^{-6})(100)^2 = 0.150 \text{ J}
$$
\n(b) $C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{25.0 \ \mu\text{F}} + \frac{1}{5.00 \ \mu\text{F}}\right)^{-1} = 4.17 \ \mu\text{F}$
\n
$$
U = \frac{1}{2}C(\Delta V)^2
$$
\n
$$
\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(0.150)}{4.17 \times 10^{-6}}} = 268 \text{ V}
$$

$$
\overline{a}
$$

20.51 $W = U = \int F dx$

so
$$
F = \frac{dU}{dx} = \frac{d}{dx} \left(\frac{Q^2}{2C} \right) = \frac{d}{dx} \left(\frac{Q^2 x}{2\epsilon_0 A} \right) = \boxed{\frac{Q^2}{2\epsilon_0 A}}
$$

*20.52 (a)
$$
Q = C\Delta V = (150 \times 10^{-12} \text{ F})(10 \times 10^3 \text{ V}) = 1.50 \times 10^{-6} \text{ C}
$$

\n(b) $U = \frac{1}{2}C(\Delta V)^2$
\n $\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(250 \times 10^{-6} \text{ J})}{150 \times 10^{-12} \text{ F}}} = 1.83 \times 10^3 \text{ V}$

20.53 $u = \frac{U}{V} = \frac{1}{2} \epsilon_0 E^2$ $\frac{1.00 \times 10^{-7}}{11} = \frac{1}{2} (8.85 \times 10^{-12}) (3000$ $\frac{.00 \times 10^{-7}}{V} = \frac{1}{2} (8.85 \times 10^{-12}) (3000)^2$ $|V| = |2.51 \times 10^{-3} \text{ m}^3$ $=$ $(2.51\times10^{-3} \text{ m}^3)$ $\left(\frac{1000 \text{ L}}{\text{m}^3}\right)$ ſ l $= 2.51 L$

20.54 (a)
$$
C = \frac{\kappa \epsilon_0 A}{d} = \frac{2.10(8.85 \times 10^{-12} \text{ F/m})(1.75 \times 10^{-4} \text{ m}^2)}{4.00 \times 10^{-5} \text{ m}} = 8.13 \times 10^{-11} \text{ F} = 81.3 \text{ pF}
$$

\n(b) $\Delta V_{\text{max}} = E_{\text{max}} d = (60.0 \times 10^6 \text{ V/m})(4.00 \times 10^{-5} \text{ m}) = 2.40 \text{ kV}$

20.55

$$
Q_{\text{max}} = C \Delta V_{\text{max}}
$$

 $\Delta V_{\text{max}} = E_{\text{max}} d$

Also, $C = \frac{\kappa \epsilon_0 A}{d}$

Thus,
$$
Q_{\text{max}} = \frac{\kappa \epsilon_0 A}{d} (E_{\text{max}} d) = \kappa \epsilon_0 A E_{\text{max}}
$$

(a) With air between the plates, $\kappa = 1.00$

and
$$
E_{\text{max}} = 3.00 \times 10^6 \text{ V/m}.
$$

Therefore,
$$
Q_{\text{max}} = \kappa \epsilon_0 A E_{\text{max}} = (8.85 \times 10^{-12} \text{ F/m})(5.00 \times 10^{-4} \text{ m}^2)(3.00 \times 10^6 \text{ V/m}) = 13.3 \text{ nC}
$$

(b) With polystyrene between the plates, $\kappa = 2.56$ and $E_{\text{max}} = 24.0 \times 10^6 \text{ V/m}$.

$$
Q_{max} = \kappa \epsilon_0 A E_{max} = 2.56 \left(8.85 \times 10^{-12} \text{ F/m} \right) \left(5.00 \times 10^{-4} \text{ m}^2 \right) \left(24.0 \times 10^6 \text{ V/m} \right) = \boxed{272 \text{ nC}}
$$

***20.56**

$$
C = \frac{\kappa \epsilon_0 A}{d}
$$

or
$$
95.0 \times 10^{-9} = \frac{3.70(8.85 \times 10^{-12})(0.0700) \ell}{0.0250 \times 10^{-3}}
$$

$$
\ell = 1.04 \text{ m}
$$

20.57

$$
\kappa = 3.00
$$
, $E_{\text{max}} = 2.00 \times 10^8$ V/m = $\Delta V_{\text{max}}/d$

For
$$
C = \frac{\kappa \epsilon_0 A}{d} = 0.250 \times 10^{-6} \text{ F}
$$
,
\n
$$
A = \frac{Cd}{\kappa \epsilon_0} = \frac{C\Delta V_{\text{max}}}{\kappa \epsilon_0 E_{\text{max}}} = \frac{(0.250 \times 10^{-6})(4000)}{3.00(8.85 \times 10^{-12})(2.00 \times 10^8)} = 0.188 \text{ m}^2
$$

*20.58 Originally,
$$
C = \frac{\epsilon_0 A}{d} = \frac{Q}{(\Delta V)_i}
$$

(a) The charge is the same before and after immersion, with value $Q = \frac{\epsilon_0 A(\Delta V)}{I}$ $=\frac{\epsilon_0 A(\Delta V)_i}{d}$.

$$
Q = \frac{\left(8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2\right) \left(25.0 \times 10^{-4} \text{ m}^2\right) \left(250 \text{ V}\right)}{\left(1.50 \times 10^{-2} \text{ m}\right)} = 369 \text{ pC}
$$

(b) Finally,

$$
C_f = \frac{\kappa \epsilon_0 A}{d} = \frac{Q}{(\Delta V)_f}
$$

\n
$$
C_f = \frac{80.0(8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2)(25.0 \times 10^{-4} \text{ m}^2)}{(1.50 \times 10^{-2} \text{ m})} = \boxed{118 \text{ pF}}
$$

\n
$$
(\Delta V)_f = \frac{Qd}{\kappa \epsilon_0 A} = \frac{\epsilon_0 A (\Delta V)_i d}{\kappa \epsilon_0 A d} = \frac{(\Delta V)_i}{\kappa} = \frac{250 \text{ V}}{80.0} = \boxed{3.12 \text{ V}}
$$

\n(c) Originally,
\n
$$
U_i = \frac{1}{2} C (\Delta V)_i^2 = \frac{\epsilon_0 A (\Delta V)_i^2}{2d}
$$

Finally,

$$
U_f = \frac{1}{2}C_f(\Delta V)_f^2 = \frac{\kappa \epsilon_0 A (\Delta V)_i^2}{2d\kappa^2} = \frac{\epsilon_0 A (\Delta V)_i^2}{2d\kappa}
$$

So,
$$
\Delta U = U_f - U_i = \frac{-\epsilon_0 A (\Delta V)_i^2 (\kappa - 1)}{2d\kappa}
$$

$$
\Delta U = -\frac{\left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)\left(25.0 \times 10^{-4} \text{ m}^2\right)\left(250 \text{ V}\right)^2 (79.0)}{2(1.50 \times 10^{-2} \text{ m})(80.0)} = \boxed{-45.5 \text{ nJ}}
$$

***20.59** (a)
$$
E_{\text{max}} = 3.00 \times 10^6 \text{ V/m} = \frac{k_e Q}{r^2} = \frac{k_e Q}{r} \left(\frac{1}{r}\right) = V_{\text{max}} \left(\frac{1}{r}\right)
$$

\n $V_{\text{max}} = E_{\text{max}} r = 3.00 \times 10^6 (0.150) = \boxed{450 \text{ kV}}$
\n(b) $\frac{k_e Q_{\text{max}}}{r^2} = E_{\text{max}} \qquad \left\{\text{or } \frac{k_e Q_{\text{max}}}{r} = V_{\text{max}}\right\} \qquad Q_{\text{max}} = \frac{E_{\text{max}} r^2}{k_e} = \frac{3.00 \times 10^6 (0.150)^2}{8.99 \times 10^9} = \boxed{7.51 \,\mu\text{C}}$

*20.60 The energy transferred is
$$
H_{ET} = \frac{1}{2}Q\Delta V = \frac{1}{2}(50.0 \text{ C})(1.00 \times 10^8 \text{ V}) = 2.50 \times 10^9 \text{ J}
$$

and 1% of this (or $\Delta E_{int} = 2.50 \times 10^7$ J) is absorbed by the tree. If *m* is the amount of water boiled away,

then
$$
\Delta E_{int} = m(4186 \text{ J/kg} \cdot {}^{\circ}\text{C})(100 {}^{\circ}\text{C} - 30.0 {}^{\circ}\text{C}) + m(2.26 \times 10^{6} \text{ J/kg}) = 2.50 \times 10^{7} \text{ J}
$$
giving
$$
m = 9.79 \text{ kg}
$$

***20.61** From Example 20.5, the potential at the center of the ring is $V_i = k_e Q / R$ and the potential at an infinite distance from the ring is $V_f = 0$. Thus, the initial and final potential energies of the point charge-ring system are:

 v_f

l

 $U_f = QV_f = 0$

and

From conservation of energy,

or
$$
\frac{1}{2}Mv_f^2 + 0 = 0 + \frac{k_eQ^2}{R}
$$

 $2k_eQ^2$ *MR e*

 $K_f + U_f = K_i + U_i$

giving

$$
\Delta V = Ed = (3 \times 10^6 \text{ V/m})(5 \times 10^{-3} \text{ m}) = 1.5 \times 10^4 \text{ V} \boxed{\sim 10^4 \text{ V}}
$$

(b) The area of your skin is perhaps 1.5 m^2 , so model your body as a sphere with this surface area. Its radius is given by $1.5 \text{ m}^2 = 4\pi r^2$, $r = 0.35 \text{ m}$. We require that you are at the potential found in part (a):

$$
V = \frac{k_e q}{r}
$$

\n
$$
q = \frac{Vr}{k_e} = \frac{1.5 \times 10^4 \text{ V}(0.35 \text{ m})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} \left(\frac{\text{J}}{\text{V} \cdot \text{C}}\right) \left(\frac{\text{N} \cdot \text{m}}{\text{J}}\right)
$$

\n
$$
q = 5.8 \times 10^{-7} \text{ C} \sim 10^{-6} \text{ C}
$$

***20.63**

$$
W = \int_{0}^{Q} V \, dq
$$

where
$$
V = \frac{k_e q}{R}
$$
;
Therefore, $W = \frac{k_e Q^2}{2R}$

***20.64** The original kinetic energy of the particle is

$$
K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2 \times 10^{-16} \text{ kg}\right)\left(2 \times 10^6 \text{ m/s}\right)^2 = 4.00 \times 10^{-4} \text{ J}
$$

The potential difference across the capacitor is $\Delta V = \frac{Q}{C} = \frac{1000 \ \mu C}{10 \ \mu F} = 100 \ V$

For the particle to reach the negative plate, the particle-capacitor system would need energy

$$
U = q\Delta V = \left(-3 \times 10^6 \, \text{C}\right) \left(-100 \, \text{V}\right) = 3.00 \times 10^{-4} \, \text{J}
$$

Since its original kinetic energy is greater than this, \mid the particle will reach the negative plate \mid

As the particle moves, the system keeps constant total energy

$$
(K+U)_{\text{at +plate}} = (K+U)_{\text{at -plate}}:
$$
\n
$$
4.00 \times 10^{-4} \text{ J} + (-3 \times 10^{-6} \text{ C})(+100 \text{ V}) = \frac{1}{2} (2 \times 10^{-16}) v_f^2 + 0
$$
\n
$$
v_f = \sqrt{\frac{2(1.00 \times 10^{-4} \text{ J})}{2 \times 10^{-16} \text{ kg}}} = \boxed{1.00 \times 10^6 \text{ m/s}}
$$

20.65 (a) We use Equation 20.29 to find the potential energy of the capacitor. As we will see, the potential difference ∆*V* changes as the dielectric is withdrawn. The initial and final energies are

$$
U_i = \frac{1}{2} \left(\frac{Q^2}{C_i} \right) \qquad \text{and} \qquad U_f = \frac{1}{2} \left(\frac{Q^2}{C_f} \right)
$$

But the initial capacitance (with the dielectric) is C_i = $\kappa C_f.$ Therefore,

$$
U_f = \frac{1}{2} \kappa \left(\frac{Q^2}{C_i} \right)
$$

Since the work done by the external force in removing the dielectric equals the change in potential energy, we have

$$
W = U_f - U_i = \frac{1}{2}\kappa \left(\frac{Q^2}{C_i}\right) - \frac{1}{2}\left(\frac{Q^2}{C_i}\right) = \frac{1}{2}\left(\frac{Q^2}{C_i}\right)(\kappa - 1)
$$

To express this relation in terms of potential difference ΔV_i , we substitute $Q = C_i(\Delta V_i)$, and evaluate:

$$
W = \frac{1}{2}C_i(\Delta V_i)^2(\kappa - 1) = \frac{1}{2}(2.00 \times 10^{-9} \text{ F})(100 \text{ V})^2(5.00 - 1.00) = 4.00 \times 10^{-5} \text{ J}
$$

The positive result confirms that the final energy of the capacitor is greater than the initial energy. The extra energy comes from the work done *on* the system by the external force that pulled out the dielectric.

(b) The final potential difference across the capacitor is $\Delta V_f = \frac{Q}{C_f}$ $=\frac{\infty}{\cdot}$.

Substituting
$$
C_f = \frac{C_i}{\kappa}
$$
 and $Q = C_i(\Delta V_i)$ gives $\Delta V_f = \kappa \Delta V_i = 5.00(100 \text{ V}) = 500 \text{ V}$

Even though the capacitor is isolated and its charge remains constant, the potential difference across the plates does increase in this case.

***20.66** (a)
$$
C = \frac{\epsilon_0}{d} [(l - x)l + \kappa l x] = \frac{\epsilon_0}{d} [l^2 + l x(\kappa - 1)]
$$

\n(b) $U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} \left[\frac{\epsilon_0 (\Delta V)^2}{d} \right] [l^2 + l x(\kappa - 1)]$
\n(c) $\mathbf{F} = -\left(\frac{dU}{dx}\right) \mathbf{i} = \frac{\epsilon_0 (\Delta V)^2}{2d} l(\kappa - 1)$ to the left (out of the capacitor)
\n(d) $F = \frac{(2000)^2 (8.85 \times 10^{-12})(0.0500)(4.50 - 1)}{2(2.00 \times 10^{-3})} = \boxed{1.55 \times 10^{-3} \text{ N}}$

***20.67** The portion of the capacitor nearly filled by metal has capacitance $\kappa \in (l x) / d \rightarrow \infty$ and stored energy $Q^2 / 2C \rightarrow 0$.

The unfilled portion has

capacitance $\epsilon_0 \ell(\ell - x) / d$

 $Q = (\ell - x)Q_0 / \ell$

The charge on this portion is

(a) The stored energy is

$$
U = \frac{Q^2}{2C} = \frac{\left[(\ell - x)Q_0 / \ell \right]^2}{2\epsilon_0 \ell (\ell - x) / d} = \frac{Q_0^2 (\ell - x) d}{2\epsilon_0 \ell^3}
$$

\n(b)
$$
F = -\frac{dU}{dx} = -\frac{d}{dx} \left(\frac{Q_0^2 (\ell - x) d}{2\epsilon_0 \ell^3} \right) = +\frac{Q_0^2 d}{2\epsilon_0 \ell^3}
$$

$$
\mathbf{F} = \frac{Q_0^2 d}{2\epsilon_0 \ell^3} \text{ to the right} \qquad \text{(into the capacitor)}
$$

\n(c)
$$
\text{Stress} = \frac{F}{\ell d} = \frac{Q_0^2}{2\epsilon_0 \ell^4}
$$

\n(d)
$$
u = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 \left(\frac{\sigma}{\epsilon_0} \right)^2 = \frac{1}{2} \epsilon_0 \left(\frac{Q_0}{\epsilon_0 \ell^2} \right)^2 = \frac{Q_0^2}{2\epsilon_0 \ell^4}
$$

***20.68** By symmetry, the potential difference across 3*C* is zero, so the circuit reduces to

$$
C_{eq} = \left(\frac{1}{2C} + \frac{1}{4C}\right)^{-1} = \frac{8}{6}C = \frac{4}{3}C
$$
\n
$$
\bullet \longrightarrow \left[\begin{array}{c|c}\n\text{L} & \text{L} & \text{L} \\
\text{C} & \text{L} & \text{L} \\
\text{D} & \text{D} & \text{D} \\
\text{E} & \text{D} & \text{D} \\
\text{E} & \text{D} & \text{D} \\
\text{E} & \text{E} & \text{E} \\
\text{
$$

Gasoline has 194 times the specific energy content of the battery and 727000 times that of the capacitor

***20.70** The initial charge on the larger capacitor is

$$
Q = C\Delta V = 10 \mu F(15 V) = 150 \mu C
$$

An additional charge *q* is pushed through the 50-V battery, giving the smaller capacitor charge *q* and the larger charge $150 \ \mu C + q$.

Then
\n
$$
50 \text{ V} = \frac{q}{5 \mu\text{F}} + \frac{150 \mu\text{C} + q}{10 \mu\text{F}}
$$
\n
$$
500 \mu\text{C} = 2q + 150 \mu\text{C} + q
$$
\n
$$
q = 117 \mu\text{C}
$$
\nSo across the 5- μ F capacitor
\n
$$
\Delta V = \frac{q}{C} = \frac{117 \mu\text{C}}{5 \mu\text{F}} = \boxed{23.3 \text{ V}}
$$
\n
$$
\Delta V = \frac{150 \mu\text{C} + 117 \mu\text{C}}{10 \mu\text{F}} = \boxed{26.7 \text{ V}}
$$

***20.69**

l

***20.71** Let *C* = the capacitance of an individual capacitor, and *Cs* represent the equivalent capacitance of the group in series. While being charged in parallel, each capacitor receives charge

$$
Q = C\Delta V_{\text{charge}} = (500 \times 10^{-4} \text{ F})(800 \text{ V}) = 0.400 \text{ C}
$$

While being discharged in series,

$$
\Delta V_{\text{discharge}} = \frac{Q}{C_s} = \frac{Q}{C/10} = \frac{0.400 \text{ C}}{5.00 \times 10^{-5} \text{ F}} = \boxed{8.00 \text{ kV}}
$$
 (or 10 times the original voltage)

***20.72** (a) $V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$ and the field at distance *r* from a uniformly charged rod (where *r* > radius of charged rod) is

$$
E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k_e\lambda}{r}
$$

In this case, the field between the central wire and the coaxial cylinder is directed perpendicular to the line of charge so that

$$
V_B - V_A = -\int_{r_a}^{r_b} \frac{2k_e \lambda}{r} dr = 2k_e \lambda \ln\left(\frac{r_a}{r_b}\right),
$$

$$
\Delta V = 2k_e \lambda \ln\left(\frac{r_a}{r_b}\right)
$$

(b) From part (a), when the outer cylinder is considered to be at zero potential, the potential at a distance *r* from the axis is

$$
V = 2k_e \lambda \ln\left(\frac{r_a}{r}\right)
$$

The field at *r* is given by

or

$$
E = -\frac{\partial V}{\partial r} = -2k_e \lambda \left(\frac{r}{r_a}\right) \left(-\frac{r_a}{r^2}\right) = \frac{2k_e \lambda}{r}
$$

ſ l \overline{a} 1

But, from part (a), $2k_e\lambda = \frac{\Delta V}{\ln(r_a/r_b)}.$

Therefore, \overline{a} $E = \frac{\Delta V}{1 - \frac{V}{V}}$ r_a / r_b) $\left(r_a - r_a\right)$ $=\frac{\Delta}{1}$ $\ln(r_a/r_b)(r)$

131

$$
*20.73
$$

so $E = \frac{\lambda}{2\pi r \epsilon_0}$

2

 $\boldsymbol{0}$

 $\pi r \ell E = \frac{q_{in}}{\epsilon_0}$

$$
\Delta V = -\int_{r_1}^{r_2} \mathbf{E} \cdot d\mathbf{r} = \int_{r_1}^{r_2} \frac{\lambda}{2\pi r \epsilon_0} dr = \frac{\lambda}{2\pi \epsilon_0} \ln \left(\frac{r_1}{r_2} \right)
$$

$$
\frac{\lambda_{\text{max}}}{2\pi\epsilon_0} = E_{\text{max}} r_{\text{inner}}
$$
\n
$$
\Delta V = (1.20 \times 10^6 \text{ V/m})(0.100 \times 10^{-3} \text{ m}) \ln\left(\frac{25.0}{0.200}\right)
$$
\n
$$
\Delta V_{\text{max}} = \boxed{579 \text{ V}}
$$

***20.74** (a) From Problem 72,

or

$$
E = \frac{\Delta V}{\ln(r_a / r_b)} \frac{1}{r}
$$

We require just outside the central wire

$$
5.50 \times 10^{6} \text{ V/m} = \frac{50.0 \times 10^{3} \text{ V}}{\ln(0.850 \text{ m}/r_{b})} \left(\frac{1}{r_{b}}\right)
$$

$$
(110 \text{ m}^{-1}) r_{b} \ln\left(\frac{0.850 \text{ m}}{r_{b}}\right) = 1
$$

We solve by homing in on the required value

Thus, to three significant figures,

$$
r_b = 1.42 \text{ mm}
$$

(b) At *ra*,

$$
E = \frac{50.0 \text{ kV}}{\ln(0.850 \text{ m}/0.00142 \text{ m})} \left(\frac{1}{0.850 \text{ m}}\right) = 9.20 \text{ kV/m}
$$

20.75
$$
U = qV = k_e \frac{q_1 q_2}{r_{12}} = (8.99 \times 10^9) \frac{(38)(54)(1.60 \times 10^{-19})^2}{(5.50 + 6.20) \times 10^{-15}} = 4.04 \times 10^{-11} \text{ J} = \boxed{253 \text{ MeV}}
$$

20.75

***20.76** (a)
$$
V = \frac{k_e q}{r_1} - \frac{k_e q}{r_2} = \frac{k_e q}{r_1 r_2} (r_2 - r_1)
$$

\nFrom the figure, for $r >> a$, $r_2 - r_1 \approx 2a \cos \theta$.
\nThen $V \approx \frac{k_e q}{r_1 r_2} 2a \cos \theta \approx \frac{k_e p \cos \theta}{r^2}$
\n(b) $E_r = -\frac{\partial V}{\partial r} = \frac{2k_e p \cos \theta}{r^3}$
\nIn spherical coordinates, the θ component of the gradient is $\frac{1}{2}(\frac{\partial}{\partial r})$.

In spherical coordinates, the θ component of the gradient is *r* ∂θ $\left(\frac{\partial}{\partial \theta}\right)$.

Therefore,
$$
E_{\theta} = -\frac{1}{r} \left(\frac{\partial V}{\partial \theta} \right) = \frac{k_e p \sin \theta}{r^3}
$$

For $r \gg a$,

$$
E_r(0^\circ) = \frac{2k_e p}{r^3}
$$

and

and
$$
E_r(90^\circ) = 0
$$
,
\n $E_\theta(0^\circ) = 0$
\nand $E_\theta(90^\circ) = \frac{k_e p}{r^3}$

 \overline{a}

These results are \vert reasonable for $r \gg a \vert$. Their directions are as shown in Figure 20.8 (c).

However, for $r \to 0$, $E(0) \to \infty$. This is unreasonable, $|$ since r is not much greater than a if it is 0.

(c)
\n
$$
V = \frac{k_e py}{(x^2 + y^2)^{3/2}}
$$
\nand
\n
$$
E_x = -\frac{\partial V}{\partial x} = \frac{3k_e pxy}{(x^2 + y^2)^{5/2}}
$$

$$
E_y = -\frac{\partial V}{\partial y} = \frac{k_e p (2y^2 - x^2)}{(x^2 + y^2)^{5/2}}
$$

to P

 $r₂$

***20.77** (a)
$$
C_0 = \frac{\epsilon_0 A}{d} = \frac{Q_0}{\Delta V_0}
$$

When the dielectric is inserted at constant voltage,

$$
C = \kappa C_0 = \frac{Q}{\Delta V_0};
$$

\n
$$
U_0 = \frac{C_0 (\Delta V_0)^2}{2}
$$

\n
$$
U = \frac{C(\Delta V_0)^2}{2} = \frac{\kappa C_0 (\Delta V_0^2)}{2}
$$

\nand
\n
$$
\frac{U}{U_0} = \kappa
$$

The extra energy comes from (part of the) electrical work done by the battery in separating the extra charge.

$$
Q_0 = C_0 \Delta V_0
$$

so

and

$$
Q = C \Delta V_0 = \kappa C_0 \Delta V_0
$$

$$
Q/Q_0 = \kappa
$$

134

ANSWERS TO EVEN NUMBERED PROBLEMS

- **2.** 1.35 MJ
- **4.** (a) 81.6 Mm/s (b) 83.9 Mm/s, too large by 2.91%
- **6.** 1.67 MN/C
- **8.** (a) 59.0 V (b) 4.55×10^6 m/s
- **10.** (a) 1.44×10^{-7} V 1.44×10^{-7} V (b) (b) -7.19×10^{-8} V (c) -1.44×10^{-7} V, +7.19 $\times 10^{-8}$ V
- **12.** (a) –4.83 m (b) 0.667 m and –2.00 m **14.** (a) –27.2 eV (b) –6.80 eV (c) 0
- **16.** (a) 32.2 kV (b) -9.65×10^{-2} J
- **18.** (a) no point at a finite distance from the charges (b) $2k_{e}q/a$

20. (a)
$$
v_1 = \sqrt{\frac{2m_2k_eq_1q_2}{m_1(m_1+m_2)} \left(\frac{1}{r_1+r_2} - \frac{1}{d}\right)}
$$
 $v_2 = \sqrt{\frac{2m_1k_eq_1q_2}{m_2(m_1+m_2)} \left(\frac{1}{r_1+r_2} - \frac{1}{d}\right)}$
\n(b) Factor than calculated in (a)
\n22. (a) -45.0 μ J (b) 34.6 km/s

24. (a) 0 (b)
$$
k_e Q / r^2
$$

26. $-0.553 k_eQ/R$

28.
$$
V = -\left(\frac{k_e \alpha L}{2}\right) \ln \left[\frac{\sqrt{(L^2/4) + b^2} - L/2}{\sqrt{(L^2/4) + b^2} + L/2}\right]
$$

30. 1.56×10^{12} electrons removed

- **32**. (a) 135 kV (b) 2.25 MV/m away for the lar*g*e sphere and 6.*7*4MV/m away for the small sphere
- **34.** (a) 1.00μ F (b) $100 V$
- **36.** (a) $1.33 \mu C/m^2$ (b) 13.3 pF

