CHAPTER 21

ANSWERS TO QUESTIONS

Q21.1 The number of cars would correspond to charge Q. The rate of flow of cars past a point would correspond to current I.

Q21.2 The temperature, dimensions, the resistivity, and any impurities affect the resistance of a conductor.

Q21.3 A conductor is not in electrostatic equilibrium when it is carrying a current, duh! If charges are placed on an isolated conductor, the electric fields established in the conductor by the charges will cause the charges to move until they are in positions such that there is zero electric field throughout the conductor. A conductor carrying a steady current is not an isolated conductor—its ends must be connected to a source of emf, such as a battery. The battery maintains a potential difference across the conductor and, therefore, an electric field in the conductor. The steady current is due to the response of the electrons in the conductor due to this constant electric field.

Q21.4 The radius of wire B is $\sqrt{3}$ times the radius of wire A, to make its cross–sectional area 3 times larger.

Q21.5 The amplitude of atomic vibrations increases with temperature thereby scattering electrons more efficiently.

Q21.6 A current will continue to exist in a superconductor without voltage because there is no resistance loss.

Q21.7 Suppose in a normal metal, we could proceed to a limit of zero resistance by lengthening the average time between collisions. The classical model of conduction then suggests that a constant applied voltage would cause constant acceleration of the free electrons, and a current steadily increasing in time.

On the other hand, we can actually switch to zero resistance by substituting a superconducting wire for the normal metal. In this case, the drift velocity of electrons is established by vibrations of atoms in the crystal lattice; the maximum current is limited; and it becomes impossible to establish a potential difference across the superconductor.

Q21.8 Because there are so many electrons in a conductor (approximately 10^{28} electrons/m^3) the average velocity of charges is very slow. When you connect a wire to a potential difference, you establish an electric field everywhere in the wire nearly instantaneously, to make electrons start drifting all at once.

Q21.9 Choose the voltage of the power supply you will use to drive the heater. Next calculate the required resistance R as $\Delta V^2/\mathcal{P}$. Knowing the resistivity ρ of the material, choose a combination of wire length and cross–sectional area to make $(\ell/A)=(R/\rho)$. You will have to pay for less material if you make both ℓ and A smaller, but if you go too far the wire will have too little surface area to radiate away the energy; the resistor will melt.

Q21.10 One ampere–hour is 3600 coulombs. The ampere–hour rating is the quantity of charge that the battery can lift though its nominal potential difference.

Q21.11 Resistors of 5.0 kΩ, 7.5 kΩ and 2.2 kΩ connected in series present equivalent resistance 14.7 kΩ.

Q21.12 Resistors of 5.0 kΩ, 7.5 kΩ and 2.2 kΩ connected in parallel present equivalent resistance 1.3 kΩ.

Q21.13 The whole wire is very nearly at one uniform potential. There is essentially zero *difference* is potential between the bird's feet. Then negligible current goes through the bird. The resistance through the bird's body between its feet is much larger than resistance through the wire between the same two points.

Q21.14 A short circuit can develop when the last bit of insulation frays away between the two conductors in a lamp cord. Then the two conductors touch each other, opening a low–resistance branch in parallel with the lamp. The lamp will immediately go out, carrying no current and presenting no danger. A very large current exists in the power supply, the house wiring, and the rest of the lamp cord up to the contact point. Before it blows the fuse or pops the circuit breaker it can quickly raise the temperature in the short–circuit path.

Q21.15 Suppose $\mathcal{E} = 12\text{ V}$ and each lamp has $R = 2\ \Omega$. Before the switch is closed the current is $(12\text{ V})/(6\ \Omega) = 2\text{ A}$. The potential difference across each lamp is $(2\text{ A})(2\ \Omega) = 4\text{ V}$. The power of each lamp is $(2\text{ A})(4\text{ V}) = 8\text{ W}$, totaling 24 W for the circuit. Closing the switch makes the switch and the wires connected to it a zero–resistance branch. All of the current through A and B will go through the switch and (b) lamp C goes out, with zero voltage across it. With less total resistance, the (c) current in the battery $(12\text{ V})/(4\ \Omega) = 3\text{ A}$ becomes larger than before and (a) lamps A and B get brighter. (d) The voltage across each of A and B is $(3\text{ A})(2\ \Omega) = 6\text{ V}$, larger than before. Each converts power $(3\text{ A})(6\text{ V}) = 18\text{ W}$, totaling 36 W, which is (e) an increase.

Q21.16 Since $\mathcal{P} = \Delta V^2/R$, and ΔV is the same for both bulbs, the 25 W bulb must have the higher resistance. From $\mathcal{P} = \Delta V I$, the 100 W bulb carries the greater current.

Q21.17 A wire or cable in a transmission line is thick and made of material with very low resistivity. Only when its length is very large does its resistance become significant. To transmit power over a long distance it is most efficient to use low current at high voltage, minimizing the I^2R power loss in the transmission line. Alternating current, as opposed to the direct current we study, can be stepped up in voltage and then down again, with high–efficiency transformers at both ends of the power line.

Q21.18 The bulbs of set A are wired in parallel. The bulbs of set B are wired in series, so removing one bulb produces an open circuit with infinite resistance and zero current.

Q21.19 Car headlights are in parallel. If they were in series, both would go out when the filament of one failed. An important safety factor would be lost.

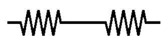
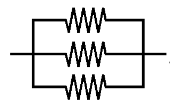
Q21.20 Kirchhoff's junction rule expresses conservation of electric charge. If the total current into a point were different from the total current out, then charge would be continuously created or annihilated at that point.

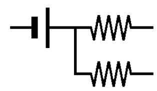
Kirchhoff's loop rule expresses conservation of energy. For a single-loop circle with two resistors, the loop rule reads $+\mathcal{E} - IR_1 - IR_2 = 0$. This is algebraically equivalent to $q\mathcal{E} = qIR_1 + qIR_2$, where $q = I\Delta t$ is the charge passing a point in the loop in the time interval Δt. The equivalent equation states that the power supply injects energy into the circuit equal in amount to that which the resistors degrade into internal energy.

Q21.21 The bulb will light up for a while immediately after the switch is closed. As the capacitor charges, the bulb gets progressively dimmer. When the capacitor is fully charged the current in the circuit is zero and the bulb does not glow at all.

Q21.22 At their normal operating temperatures, from $\mathcal{P} = \Delta V^2/R$, the bulbs present resistances $R = \Delta V^2/\mathcal{P} = (120\text{ V})^2/60\text{ W} = 240\ \Omega$, and $(120\text{ V})^2/75\text{W} = 190\ \Omega$, and $(120\text{ V})^2/200\text{ W} = 72\ \Omega$. The nominal 60 W lamp has greatest resistance. When they are connected in series, they all carry the same small current. Here the highest–resistance bulb glows most brightly and the one with lowest resistance is faintest. This is just the reverse of their order of intensity if they were connected in parallel, as they are designed to be.

Q21.23 Speak no word. If the student is just looking at a diagram, provide the materials to build the circuit. If you are looking at a circuit where the second bulb really is fainter, get the student to unscrew them both and interchange them. But check that the student's understanding of potential has not been impaired: if you patch past the first bulb to short it out, the second gets brighter. And if with a nine–volt battery the potential difference across the first bulb is four volts, then the voltage across the second is… .

Q21.24 Two runs in series:  . Three runs in parallel: .

Junction of one lift and two runs: 

Gustav Robert Kirchhoff, Professor of Physics at Berlin, was master of the obvious. A junction rule: The number of skiers coming into any junction must be equal to the number of skiers leaving. A loop rule: the total change in altitude must be zero for any skiers completing a closed path.

PROBLEM SOLUTIONS

21.1 $I = \dfrac{\Delta Q}{\Delta t}$ $\quad \Delta Q = I\Delta t = \left(30.0\times10^{-6}\ \text{A}\right)(40.0\ \text{s}) = 1.20\times10^{-3}\ \text{C}$

$$N = \frac{Q}{e} = \frac{1.20\times10^{-3}\ \text{C}}{1.60\times10^{-19}\ \text{C/electron}} = \boxed{7.50\times10^{15}\ \text{electrons}}$$

21.2 The molar mass of silver = 107.9 g/mole and the volume V is

$$V = (\text{area})(\text{thickness}) = \left(700\times10^{-4}\ \text{m}^2\right)\left(0.133\times10^{-3}\ \text{m}\right) = 9.31\times10^{-6}\ \text{m}^3$$

The mass of silver deposited is $m_{\text{Ag}} = \rho V = \left(10.5\times10^{3}\ \text{kg/m}^3\right)\left(9.31\times10^{-6}\ \text{m}^3\right) = 9.78\times10^{-2}\ \text{kg}$

And the number of silver atoms deposited is

$$N = \left(9.78\times10^{-2}\ \text{kg}\right)\left(\frac{6.02\times10^{23}\ \text{atoms}}{107.9\ \text{g}}\right)\left(\frac{1000\ \text{g}}{1\ \text{kg}}\right) = 5.45\times10^{23}\ \text{atoms}$$

$$I = \frac{\Delta V}{R} = \frac{12.0\ \text{V}}{1.80\ \Omega} = 6.67\ \text{A} = 6.67\ \text{C/s}$$

$$\Delta t = \frac{\Delta Q}{I} = \frac{Ne}{I} = \frac{\left(5.45\times10^{23}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{6.67\ \text{C/s}} = 1.31\times10^{4}\ \text{s} = \boxed{3.64\ \text{h}}$$

***21.3** The period of revolution for the sphere is $T = \dfrac{2\pi}{\omega}$, and the average current represented by this revolving charge is $I = \dfrac{q}{T} = \boxed{\dfrac{q\omega}{2\pi}}$.

***21.4** $q = 4t^3 + 5t + 6$

$$A = \left(2.00\ \text{cm}^2\right)\left(\frac{1.00\ \text{m}}{100\ \text{cm}}\right)^2 = 2.00\times10^{-4}\ \text{m}^2$$

(a) $I(1.00\ \text{s}) = \left.\dfrac{dq}{dt}\right|_{t=1.00\ \text{s}} = \left.\left(12t^2 + 5\right)\right|_{t=1.00\ \text{s}} = \boxed{17.0\ \text{A}}$

(b) $J = \dfrac{I}{A} = \dfrac{17.0\ \text{A}}{2.00\times10^{-4}\ \text{m}^2} = \boxed{85.0\ \text{kA/m}^2}$

21.5 $Q(t)=\int_0^t I\,dt = I_0\tau\left(1-e^{-t/\tau}\right)$

(a) $Q(\tau)=I_0\tau\left(1-e^{-1}\right)=\boxed{(0.632)I_0\tau}$

(b) $Q(10\tau)=I_0\tau\left(1-e^{-10}\right)=\boxed{(0.99995)I_0\tau}$

(c) $Q(\infty)=I_0\tau\left(1-e^{-\infty}\right)=\boxed{I_0\tau}$

21.6 We use $I=nqAv_d$ where n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molar mass of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27.0 g. Thus, the mass per atom is

$$\frac{27.0\text{ g}}{N_A}=\frac{27.0\text{ g}}{6.02\times10^{23}}=4.49\times10^{-23}\text{ g/atom}$$

Thus, $$n=\frac{\text{density of aluminum}}{\text{mass per atom}}=\frac{2.70\text{ g/cm}^3}{4.49\times10^{-23}\text{ g/atom}}$$

$$n=6.02\times10^{22}\text{ atoms/cm}^3=6.02\times10^{28}\text{ atoms/m}^3$$

Therefore, $$v_d=\frac{I}{nqA}=\frac{5.00\text{ A}}{\left(6.02\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(4.00\times10^{-6}\text{ m}^2\right)}=1.30\times10^{-4}\text{ m/s}$$

or, $$v_d=\boxed{0.130\text{ mm/s}}$$

21.7 $I=\dfrac{\Delta V}{R}=\dfrac{120\text{ V}}{240\ \Omega}=0.500\text{ A}=\boxed{500\text{ mA}}$

21.8 (a) Given $M=\rho_d V=\rho_d A\ell$ where $\rho_d\equiv$ mass density,

we obtain: $A=\dfrac{M}{\rho_d\ell}$ Taking $\rho_r\equiv$ resistivity, $R=\dfrac{\rho_r\ell}{A}=\dfrac{\rho_r\ell}{M/\rho_d\ell}=\dfrac{\rho_r\rho_d\ell^2}{M}$

Thus, $\ell=\sqrt{\dfrac{MR}{\rho_r\rho_d}}=\sqrt{\dfrac{\left(1.00\times10^{-3}\right)(0.500)}{(1.70\times10^{-8})(8.92\times10^3)}}$ $\ell=\boxed{1.82\text{ m}}$

(b) $V=\dfrac{M}{\rho_d}$, or $\pi r^2\ell=\dfrac{M}{\rho_d}$

Thus, $r=\sqrt{\dfrac{M}{\pi\rho_d\ell}}=\sqrt{\dfrac{1.00\times10^{-3}}{\pi(8.92\times10^3)(1.82)}}$ $r=1.40\times10^{-4}\text{ m}$

The diameter is twice this distance: diameter $=\boxed{280\ \mu\text{m}}$

21.9 $\Delta V = IR$

and $R = \dfrac{\rho \ell}{A}$: $A = (0.600\text{ mm})^2\left(\dfrac{1.00\text{ m}}{1000\text{ mm}}\right)^2 = 6.00 \times 10^{-7}\text{ m}^2$

$\Delta V = \dfrac{I\rho\ell}{A}$: $I = \dfrac{\Delta V A}{\rho \ell} = \dfrac{(0.900\text{ V})\left(6.00 \times 10^{-7}\text{ m}^2\right)}{\left(5.60 \times 10^{-8}\ \Omega \cdot \text{m}\right)(1.50\text{ m})}$

$$I = \boxed{6.43\text{ A}}$$

21.10 At the low temperature T_C we write $R_C = \dfrac{\Delta V}{I_C} = R_0\left[1 + \alpha(T_C - T_0)\right]$ where $T_0 = 20.0°\text{C}$

At the high temperature T_h, $R_h = \dfrac{\Delta V}{I_h} = \dfrac{\Delta V}{1\text{ A}} = R_0\left[1 + \alpha(T_h - T_0)\right]$

Then $$\frac{(\Delta V)/(1.00\text{ A})}{(\Delta V)/I_C} = \frac{1 + \left(3.90 \times 10^{-3}\right)(38.0)}{1 + \left(3.90 \times 10^{-3}\right)(-108)}$$

and $$I_C = (1.00\text{ A})\left(\frac{1.15}{0.579}\right) = \boxed{1.98\text{ A}}$$

***21.11** (a) $\rho = \rho_0\left[1 + \alpha(T - T_0)\right] = \left(2.82 \times 10^{-8}\ \Omega \cdot \text{m}\right)\left[1 + 3.90 \times 10^{-3}(30.0°)\right] = \boxed{3.15 \times 10^{-8}\ \Omega \cdot \text{m}}$

(b) $J = \dfrac{E}{\rho} = \dfrac{0.200\text{ V/m}}{3.15 \times 10^{-8}\ \Omega \cdot \text{m}} = \boxed{6.35 \times 10^{6}\text{ A/m}^2}$

(c) $I = JA = J\left(\dfrac{\pi d^2}{4}\right) = \left(6.35 \times 10^{6}\text{ A/m}^2\right)\left[\dfrac{\pi\left(1.00 \times 10^{-4}\text{ m}\right)^2}{4}\right] = \boxed{49.9\text{ mA}}$

(d) $n = \dfrac{6.02 \times 10^{23}\text{ electrons}}{\left(\dfrac{26.98\text{ g}}{2.70 \times 10^{6}\text{ g/m}^3}\right)} = 6.02 \times 10^{28}\text{ electrons/m}^3$

$$v_d = \frac{J}{ne} = \frac{(6.35 \times 10^{6}\text{ A/m}^2)}{(6.02 \times 10^{28}\text{ electrons/m}^3)(1.60 \times 10^{-19}\text{ C})} = \boxed{659\ \mu\text{m/s}}$$

(e) $\Delta V = E\ell = (0.200\text{ V/m})(2.00\text{ m}) = \boxed{0.400\text{ V}}$

21.12 For aluminum,

$$\alpha_E = 3.90\times10^{-3}\ ^\circ\text{C}^{-1} \qquad \text{(Table 21.1)}$$

$$\alpha = 24.0\times10^{-6}\ ^\circ\text{C}^{-1} \qquad \text{(Table 16.1)}$$

$$R = \frac{\rho\ell}{A} = \frac{\rho_0(1+\alpha_E\Delta T)\ell(1+\alpha\Delta T)}{A(1+\alpha\Delta T)^2} = R_0\frac{(1+\alpha_E\Delta T)}{(1+\alpha\Delta T)} = (1.234\ \Omega)\left(\frac{1.39}{1.0024}\right) = \boxed{1.71\ \Omega}$$

21.13 $$\rho = \frac{m}{nq^2\tau}$$

so $$\tau = \frac{m}{\rho n q^2} = \frac{(9.11\times10^{-31})}{(1.70\times10^{-8})(8.49\times10^{28})(1.60\times10^{-19})^2} = 2.47\times10^{-14}\ \text{s}$$

$$v_d = \frac{qE}{m}\tau$$

so $$7.84\times10^{-4} = \frac{(1.60\times10^{-19})E(2.47\times10^{-14})}{9.11\times10^{-31}}$$

Therefore, $E = \boxed{0.181\ \text{V/m}}$

21.14 (a) n is $\boxed{\text{unaffected}}$

(b) $|J| = \dfrac{I}{A} \propto I$

so it $\boxed{\text{doubles}}$

(c) $J = nev_d$

so v_d $\boxed{\text{doubles}}$

(d) $\tau = \dfrac{m\sigma}{nq^2}$ is $\boxed{\text{unchanged}}$ as long as σ does not change due to a temperature change in the conductor.

21.15 $$I = \frac{\mathcal{P}}{\Delta V} = \frac{600\ \text{W}}{120\ \text{V}} = \boxed{5.00\ \text{A}}$$

and $$R = \frac{\Delta V}{I} = \frac{120\ \text{V}}{5.00\ \text{A}} = \boxed{24.0\ \Omega}$$

21.16 $\mathcal{P} = 0.800(1500\text{ hp})(746\text{ W/hp}) = 8.95\times10^5\text{ W}$

$\mathcal{P} = I\Delta V \qquad 8.95\times10^5 = I(2000) \qquad I = \boxed{448\text{ A}}$

21.17 $$\frac{\mathcal{P}}{\mathcal{P}_0} = \frac{(\Delta V)^2/R}{(\Delta V_0)^2/R} = \left(\frac{\Delta V}{\Delta V_0}\right)^2 = \left(\frac{140}{120}\right)^2 = 1.361$$

$$\Delta\% = \left(\frac{\mathcal{P}-\mathcal{P}_0}{\mathcal{P}_0}\right)(100\%) = \left(\frac{\mathcal{P}}{\mathcal{P}_0}-1\right)(100\%) = (1.361-1)100\% = \boxed{36.1\%}$$

***21.18** $\mathcal{P} = I\Delta V = (2.00\text{ A})(120\text{ V}) = 240\text{ W}$

$\Delta E_{\text{int}} = (0.500\text{ kg})(4186\text{ J/kg}\cdot{}^\circ\text{C})(77.0\text{ }^\circ\text{C}) = 161\text{ kJ}$

$$\Delta t = \frac{\Delta E_{\text{int}}}{\mathcal{P}} = \frac{1.61\times10^5\text{ J}}{240\text{ W}} = \boxed{672\text{ s}}$$

***21.19** At operating temperature,

(a) $\mathcal{P} = I\Delta V = (1.53\text{ A})(120\text{ V}) = \boxed{184\text{ W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$R = R_0(1+\alpha\Delta T) \qquad \dfrac{120}{1.53} = \dfrac{120}{1.80}\left[1+\left(0.400\times10^{-3}\right)\Delta T\right]$

$\Delta T = 441^\circ\text{C} \qquad T = 20.0^\circ\text{C} + 441^\circ\text{C} = \boxed{461^\circ\text{C}}$

***21.20** The total clock power is

$$\left(270\times10^6\text{ clocks}\right)\left(2.50\ \frac{\text{J/s}}{\text{clock}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 2.43\times10^{12}\text{ J/h}$$

From $e = \dfrac{W_{out}}{Q_{in}}$, the power input to the generating plants must be:

$$\frac{Q_{in}}{t} = \frac{W_{out}/t}{e} = \frac{2.43\times10^{12}\text{ J/h}}{0.250} = 9.72\times10^{12}\text{ J/h}$$

and the rate of coal consumption is

$$\text{Rate} = \left(9.72\times10^{12}\text{ J/h}\right)\left(\frac{1.00\text{ kg coal}}{33.0\times10^6\text{ J}}\right) = 2.95\times10^5\text{ kg coal/h} = \boxed{295\text{ metric ton/h}}$$

***21.21** Consider a 400-W blow dryer used for ten minutes daily for a year. The energy transferred to the dryer is

$$\mathcal{P}\Delta t = (400\ \text{J/s})(600\ \text{s/d})(365\ \text{d}) \cong 9\times 10^{7}\ \text{J}\left(\frac{1\ \text{kWh}}{3.6\times 10^{6}\ \text{J}}\right) \cong 20\ \text{kWh}$$

We suppose that electrical energy costs on the order of ten cents per kilowatt-hour. Then the cost of using the dryer for a year is on the order of

$$\text{Cost} \cong (20\ \text{kWh})(\$\ 0.10/\text{kWh}) = \$\ 2\ \boxed{\sim \$\ 1}$$

***21.22** You pay the electric company for energy transferred in the amount $E = \mathcal{P}\Delta t$.

(a) $$\mathcal{P}\Delta t = 40\ \text{W}(2\ \text{weeks})\left(\frac{7\ \text{d}}{1\ \text{week}}\right)\left(\frac{86\,400\ \text{s}}{1\ \text{d}}\right)\left(\frac{1\ \text{J}}{1\ \text{W}\cdot\text{s}}\right) = 48.4\ \text{MJ}$$

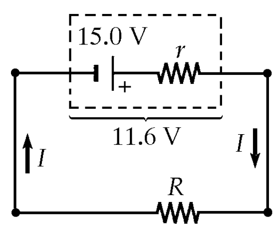
$$\mathcal{P}\Delta t = 40\ \text{W}(2\ \text{weeks})\left(\frac{7\ \text{d}}{1\ \text{week}}\right)\left(\frac{24\ \text{h}}{1\ \text{d}}\right)\left(\frac{\text{k}}{1000}\right) = 13.4\ \text{kWh}$$

$$\mathcal{P}\Delta t = 40\ \text{W}(2\ \text{weeks})\left(\frac{7\ \text{d}}{1\ \text{week}}\right)\left(\frac{24\ \text{h}}{1\ \text{d}}\right)\left(\frac{\text{k}}{1000}\right)\left(\frac{0.12\ \$}{\text{kWh}}\right) = \boxed{\$\ 1.61}$$

(b) $$\mathcal{P}\Delta t = 970\ \text{W}(3\ \text{min})\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{\text{k}}{1000}\right)\left(\frac{0.12\ \$}{\text{kWh}}\right) = \boxed{\$\ 0.00582} = 0.582\ ¢$$

(c) $$\mathcal{P}\Delta t = 5200\ \text{W}(40\ \text{min})\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{\text{k}}{1000}\right)\left(\frac{0.12\ \$}{\text{kWh}}\right) = \boxed{\$\ 0.416}$$

21.23 (a) $$\mathcal{P} = \frac{(\Delta V)^2}{R}$$



becomes $$20.0\ \text{W} = \frac{(11.6\ \text{V})^2}{R}$$

so $$R = \boxed{6.73\ \Omega}$$

(b) $$\Delta V = IR$$

so $$11.6\ \text{V} = I(6.73\ \Omega)$$

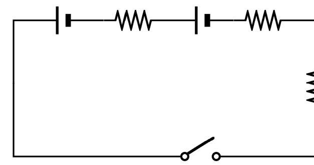
and $$I = 1.72\ \text{A}$$

$$\mathcal{E} = IR + Ir$$

so $$15.0\ \text{V} = 11.6\ \text{V} + (1.72\ \text{A})r$$

$$r = \boxed{1.97\ \Omega}$$

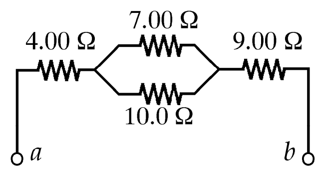
21.24 The total resistance is $R = \dfrac{3.00\text{ V}}{0.600\text{ A}} = 5.00\ \Omega$



(a) $R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\ \Omega - 0.408\ \Omega = \boxed{4.59\ \Omega}$

(b) $\dfrac{\mathcal{P}_{\text{batteries}}}{\mathcal{P}_{\text{total}}} = \dfrac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2} = 0.0816 = \boxed{8.16\%}$

21.25 (a) $R_p = \dfrac{1}{(1/7.00\ \Omega)+(1/10.0\ \Omega)} = 4.12\ \Omega$



$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\ \Omega}$

(b) $\Delta V = IR$

$34.0\text{ V} = I(17.1\ \Omega)$

$I = \boxed{1.99\text{ A}}$ for $4.00\ \Omega$, $9.00\ \Omega$ resistors

Applying $\Delta V = IR$, $(1.99\text{ A})(4.12\ \Omega) = 8.18\text{ V}$

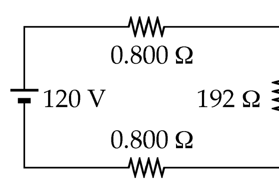
$8.18\text{ V} = I(7.00\ \Omega)$

so $I = \boxed{1.17\text{ A}}$ for $7.00\ \Omega$ resistor

$8.18\text{ V} = I(10.0\ \Omega)$

so $I = \boxed{0.818\text{ A}}$ for $10.0\ \Omega$ resistor

21.26 For the bulb in use as intended,



$$I = \frac{\mathcal{P}}{\Delta V} = \frac{75.0\text{ W}}{120\text{ V}} = 0.625\text{ A}$$

and $$R = \frac{\Delta V}{I} = \frac{120\text{ V}}{0.625\text{ A}} = 192\ \Omega$$

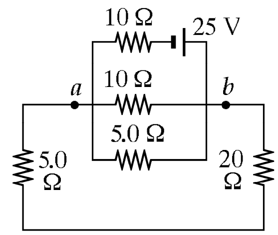
Now, presuming the bulb resistance is unchanged,

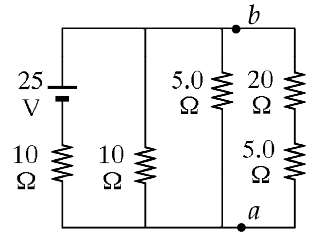
$$I = \frac{120\text{ V}}{193.6\ \Omega} = 0.620\text{ A}$$

Across the bulb is $\Delta V = IR = 192\ \Omega(0.620\text{ A}) = 119\text{ V}$

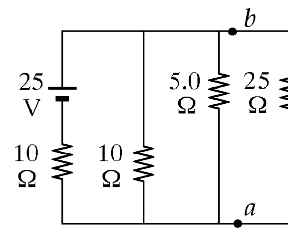
so its power is $\mathcal{P} = I\Delta V = 0.620\text{ A}(119\text{ V}) = \boxed{73.8\text{ W}}$

21.27 If we turn the given diagram on its side, we find that it is the same as figure (a). The 20.0 Ω and 5.00 Ω resistors are in series, so the first reduction is shown in (b). In addition, since the 10.0 Ω, 5.00 Ω, and 25.0 Ω resistors are then in parallel, we can solve for their equivalent resistance as:

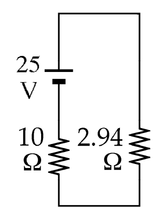




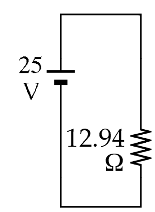
(a)



(b)



(c)



(d)

$$R_{eq} = \frac{1}{\left(\frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} + \frac{1}{25.0\ \Omega}\right)} = 2.94\ \Omega$$

This is shown in figure (c), which in turn reduces to the circuit shown in figure (d).

Next, we work backwards through the diagrams applying $I = \Delta V/R$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The 12.94 Ω resistor is connected across 25.0 V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}$$

In figure (c), this 1.93 A goes through the 2.94 Ω equivalent resistor to give a potential difference of:

$$\Delta V = IR = (1.93\ \text{A})(2.94\ \Omega) = 5.68\ \text{V}$$

From figure (b), we see that this potential difference is the same across ΔV_{ab}, the 10 Ω resistor, and the 5.00 Ω resistor.

(b) Therefore, $\Delta V_{ab} = \boxed{5.68\ \text{V}}$

(a) Since the current through the 20.0 Ω resistor is also the current through the 25.0 Ω line ab,

$$I = \frac{\Delta V_{ab}}{R_{ab}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = 0.227\ \text{A} = \boxed{227\ \text{mA}}$$

***21.28** We assume that the metal wand makes low-resistance contact with the person's hand and that the resistance through the person's body is negligible compared to the resistance R_{shoes} of the shoe soles. The equivalent resistance seen by the power supply is $1.00\ \text{M}\Omega + R_{\text{shoes}}$. The current through both resistors is $50.0\ \text{V}/(1.00\ \text{M}\Omega + R_{\text{shoes}})$. The voltmeter displays

$$\Delta V = I(1.00\ \text{M}\Omega) = \frac{50.0\ \text{V}(1.00\ \text{M}\Omega)}{1.00\ \text{M}\Omega + R_{\text{shoes}}} = \Delta V$$

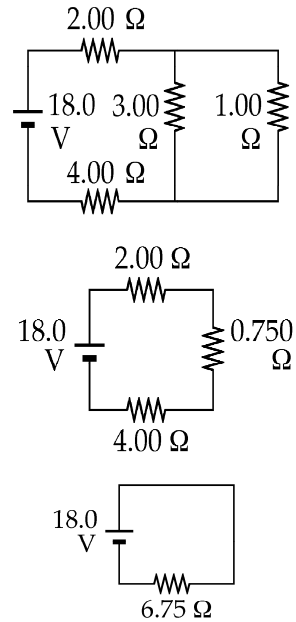
(a) We solve to obtain

$$50.0\ \text{V}(1.00\ \text{M}\Omega) = \Delta V(1.00\ \text{M}\Omega) + \Delta V(R_{\text{shoes}})$$

$$R_{\text{shoes}} = \frac{1.00\ \text{M}\Omega(50.0 - \Delta V)}{\Delta V}$$

(c) With $R_{\text{shoes}} \to 0$, the current through the person's body is

$50.0\ \text{V}/1.00\ \text{M}\Omega = 50.0\ \mu\text{A}$. $\boxed{\text{The current will never exceed } 50\ \mu\text{A}}$.



***21.29** $R_p = \left(\dfrac{1}{3.00} + \dfrac{1}{1.00}\right)^{-1} = 0.750\ \Omega$

$R_s = (2.00 + 0.750 + 4.00)\ \Omega = 6.75\ \Omega$

$I_{\text{battery}} = \dfrac{\Delta V}{R_s} = \dfrac{18.0\ \text{V}}{6.75\ \Omega} = 2.67\ \text{A}$

$\mathcal{P} = I^2R$: $\quad \mathcal{P}_2 = (2.67\ \text{A})^2(2.00\ \Omega)$

$\mathcal{P}_2 = \boxed{14.2\ \text{W}}$ in $2.00\ \Omega$

$\mathcal{P}_4 = (2.67\ \text{A})^2(4.00\ \text{A}) = \boxed{28.4\ \text{W}}$ in $4.00\ \Omega$

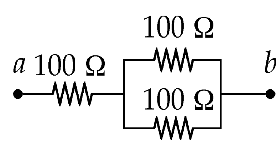
$\Delta V_2 = (2.67\ \text{A})(2.00\ \Omega) = 5.33\ \text{V},$

$\Delta V_4 = (2.67\ \text{A})(4.00\ \Omega) = 10.67\ \text{V}$

$\Delta V_p = 18.0\ \text{V} - \Delta V_2 - \Delta V_4 = 2.00\ \text{V}\ (= \Delta V_3 = \Delta V_1)$

$\mathcal{P}_3 = \dfrac{(\Delta V_3)^2}{R_3} = \dfrac{(2.00\ \text{V})^2}{3.00\ \Omega} = \boxed{1.33\ \text{W}}$ in $3.00\ \Omega$

$\mathcal{P}_1 = \dfrac{(\Delta V_1)}{R_1} = \dfrac{(2.00\ \text{V})^2}{1.00\ \Omega} = \boxed{4.00\ \text{W}}$ in $1.00\ \Omega$



21.30 (a) Since all the current in the circuit must pass through the series $100\ \Omega$ resistor, $\mathcal{P} = RI^2$

$\mathcal{P}_{\max} = RI_{\max}^{\ 2}$

so $\quad I_{\max} = \sqrt{\dfrac{\mathcal{P}}{R}} = \sqrt{\dfrac{25.0\ \text{W}}{100\ \Omega}} = 0.500\ \text{A}$

$R_{eq} = 100\ \Omega + \left(\dfrac{1}{100} + \dfrac{1}{100}\right)^{-1}\ \Omega = 150\ \Omega$

$\Delta V_{\max} = R_{eq}I_{\max} = \boxed{75.0\ \text{V}}$

(b) $\mathcal{P} = I\Delta V = (0.500\ \text{A})(75.0\ \text{V}) = \boxed{37.5\ \text{W}}$ total power

$\mathcal{P}_1 = \boxed{25.0\ \text{W}}$

$\mathcal{P}_2 = \mathcal{P}_3 = RI^2 = (100\ \Omega)(0.250\ \text{A})^2 = \boxed{6.25\ \text{W}}$

21.31 (a) $\mathcal{P} = I\Delta V$: So for the Heater, $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{1500\text{ W}}{120\text{ V}} = \boxed{12.5\text{ A}}$

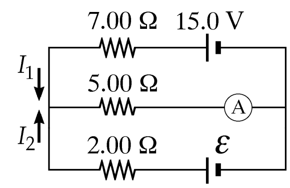
For the Toaster, $I = \dfrac{750\text{ W}}{120\text{ V}} = \boxed{6.25\text{ A}}$

And for the Grill, $I = \dfrac{1000\text{ W}}{120\text{ V}} = \boxed{8.33\text{ A}}$

(b) $12.5 + 6.25 + 8.33 = \boxed{27.1\text{ A}}$.

The current draw is greater than 25.0 amps, so this circuit breaker would not be sufficient.

21.32 $+15.0 - (7.00)I_1 - (2.00)(5.00) = 0$



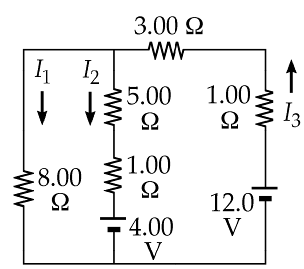
$5.00 = 7.00\,I_1$ so $\boxed{I_1 = 0.714\text{ A}}$

$I_3 = I_1 + I_2 = 2.00\text{ A}$

$0.714 + I_2 = 2.00$ so $\boxed{I_2 = 1.29\text{ A}}$

$+\mathcal{E} - 2.00(1.29) - 5.00(2.00) = 0$ $\boxed{\mathcal{E} = 12.6\text{ V}}$

21.33 We name currents I_1, I_2, and I_3 as shown.



From Kirchhoff's current rule, $I_3 = I_1 + I_2$

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$12.0\text{ V} - (4.00)I_3 - (6.00)I_2 - 4.00\text{ V} = 0$

$8.00 = (4.00)I_3 + (6.00)I_2$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

$-(6.00)I_2 - 4.00\text{ V} + (8.00)I_1 = 0$ $(8.00)I_1 = 4.00 + (6.00)I_2$

Solving the above linear system, we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = 1.33I_1 - 0.667 \end{cases}$$

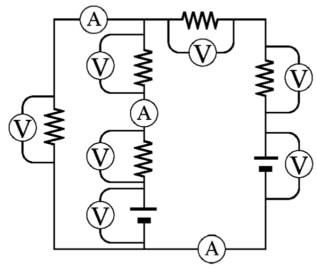
and to the single equation $8 = 4I_1 + 13.3I_1 - 6.67$

$I_1 = \dfrac{14.7\text{ V}}{17.3\ \Omega} = 0.846\text{ A}$ Then $I_2 = 1.33(0.846\text{ A}) - 0.667$

and $I_3 = I_1 + I_2$ give $\boxed{I_1 = 846\text{ mA},\ I_2 = 462\text{ mA},\ I_3 = 1.31\text{ A}}$

All currents are in the directions indicated by the arrows in the circuit diagram.

21.34 The solution figure is shown to the right.



21.35 We use the results of Problem 33

(a) By the 4.00-V battery:

$$\Delta U = (\Delta V) I \Delta t = (4.00 \text{ V})(-0.462 \text{ A})120 \text{ s} = \boxed{-222 \text{ J}}$$

By the 12.0-V battery:

$$(12.0 \text{ V})(1.31 \text{ A})120 \text{ s} = \boxed{1.88 \text{ kJ}}$$

(b) By the 8.00-Ω resistor:

$$I^2 R \Delta t = (0.846 \text{ A})^2 (8.00 \ \Omega)120 \text{ s} = \boxed{687 \text{ J}}$$

By the 5.00-Ω resistor:

$$(0.462 \text{ A})^2 (5.00 \ \Omega)120 \text{ s} = \boxed{128 \text{ J}}$$

By the 1.00-Ω resistor:

$$(0.462 \text{ A})^2 (1.00 \ \Omega)120 \text{ s} = \boxed{25.6 \text{ J}}$$

By the 3.00-Ω resistor:

$$(1.31 \text{ A})^2 (3.00 \ \Omega)120 \text{ s} = \boxed{616 \text{ J}}$$

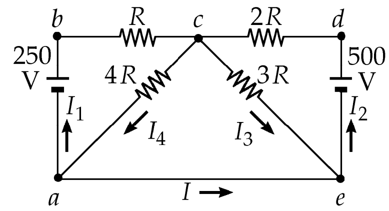
By the 1.00-Ω resistor:

$$(1.31 \text{ A})^2 (1.00 \ \Omega)120 \text{ s} = \boxed{205 \text{ J}}$$

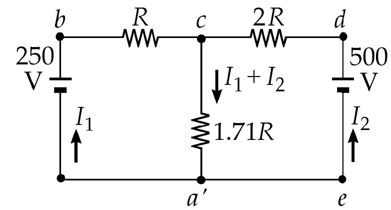
(c) $-222 \text{ J} + 1.88 \text{ kJ} = \boxed{1.66 \text{ kJ}}$ from chemical to electrical.

$687 \text{ J} + 128 \text{ J} + 25.6 \text{ J} + 616 \text{ J} + 205 \text{ J} = 1.66 \text{ kJ}$ from electrical to internal.

***21.36** Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure.



(a)



(b)

Apply Kirchhoff's loop rule to both loops in Figure (b) to obtain:

$$(2.71R)I_1 + (1.71R)I_2 = 250$$

and $$(1.71R)I_1 + (3.71R)I_2 = 500$$

With $R = 1000\ \Omega$, simultaneous solution of these equations yields:

$$I_1 = 10.0\text{ mA}$$

and $$I_2 = 130.0\text{ mA}$$

From Figure (b), $$V_c - V_a = (I_1 + I_2)(1.71R) = 240\text{ V}$$

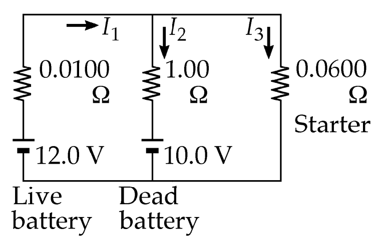
Thus, from Figure (a), $$I_4 = \frac{V_c - V_a}{4R} = \frac{240\text{ V}}{4000\ \Omega} = 60.0\text{ mA}$$

Finally, applying Kirchhoff's point rule at point a in Figure (a) gives:

$$I = I_4 - I_1 = 60.0\text{ mA} - 10.0\text{ mA} = +50.0\text{ mA},$$

or $$I = \boxed{50.0\text{ mA from point } a \text{ to point } e}.$$

***21.37** Using Kirchhoff's rules,



$$12.0 - (0.0100)I_1 - (0.0600)I_3 = 0$$

$$10.0 + (1.00)I_2 - (0.0600)I_3 = 0$$

and $$I_1 = I_2 + I_3$$

$$12.0 - (0.0100)I_2 - (0.0700)I_3 = 0$$

$$10.0 + (1.00)I_2 - (0.0600)I_3 = 0$$

Solving simultaneously,

$$I_2 = \boxed{0.283\text{ A downward}}\text{ in the dead battery}$$

and $$I_3 = \boxed{171\text{ A downward}}\text{ in the starter.}$$

The currents are forward in the live battery and in the starter, relative to normal starting operation. The current is backward in the dead battery, tending to charge it up.

21.38 (a) $I(t) = -I_0 e^{-t/RC}$

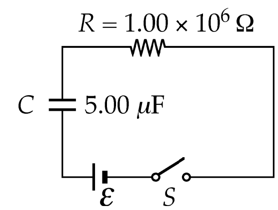
$$I_0 = \frac{Q}{RC} = \frac{5.10\times10^{-6}\text{ C}}{(1300\ \Omega)\left(2.00\times10^{-9}\text{ F}\right)} = 1.96\text{ A}$$

$$I(t) = -(1.96\text{ A})\exp\left[\frac{-9.00\times10^{-6}\text{ s}}{(1300\ \Omega)\left(2.00\times10^{-9}\text{ F}\right)}\right] = \boxed{-61.6\text{ mA}}$$

(b) $$q(t) = Qe^{-t/RC} = (5.10\ \mu\text{C})\exp\left[\frac{-8.00\times10^{-6}\text{ s}}{(1300\ \Omega)\left(2.00\times10^{-9}\text{ F}\right)}\right] = \boxed{0.235\ \mu\text{C}}$$

(c) The magnitude of the maximum current is $I_0 = \boxed{1.96\text{ A}}$

21.39 (a) $RC = \left(1.00\times10^{6}\ \Omega\right)\left(5.00\times10^{-6}\text{ F}\right) = \boxed{5.00\text{ s}}$



(b) $Q = C\mathcal{E} = \left(5.00\times10^{-6}\text{ C}\right)(30.0\text{ V}) = \boxed{150\ \mu\text{C}}$

(c) $$I(t) = \frac{\mathcal{E}}{R}e^{-t/RC} = \left(\frac{30.0}{1.00\times10^{6}}\right)\exp\left[\frac{-10.0}{\left(1.00\times10^{6}\right)\left(5.00\times10^{-6}\right)}\right] = \boxed{4.06\ \mu\text{A}}$$

21.40 (a) $\tau = RC = \left(1.50\times10^{5}\ \Omega\right)\left(10.0\times10^{-6}\text{ F}\right) = \boxed{1.50\text{ s}}$

(b) $\tau = \left(1.00\times10^{5}\ \Omega\right)\left(10.0\times10^{-6}\text{ F}\right) = \boxed{1.00\text{ s}}$

(c) The battery carries current

$$\frac{10.0\text{ V}}{50.0\times10^{3}\ \Omega} = 200\ \mu\text{A}$$

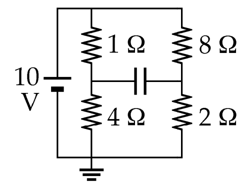
The 100 kΩ carries current of magnitude

$$I = I_0e^{-t/RC} = \left(\frac{10.0\text{ V}}{100\times10^{3}\ \Omega}\right)e^{-t/1.00\text{ s}}$$

So the switch carries downward current

$$\boxed{200\ \mu\text{A} + (100\ \mu\text{A})e^{-t/1.00\text{ s}}}$$

21.41 (a) Call the potential at the left junction V_L and at the right V_R. After a "long" time, the capacitor is fully charged.



$V_L = 8.00$ V because of voltage divider:

$$I_L = \frac{10.0\text{ V}}{5.00\ \Omega} = 2.00\text{ A}$$

$$V_L = 10.0\text{ V} - (2.00\text{ A})(1.00\ \Omega) = 8.00\text{ V}$$

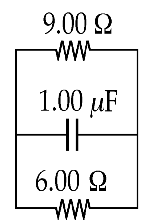
Likewise, $$V_R = \left(\frac{2.00\ \Omega}{2.00\ \Omega + 8.00\ \Omega}\right)(10.0\text{ V}) = 2.00\text{ V}$$

or $$I_R = \frac{10.0\text{ V}}{10.0\ \Omega} = 1.00\text{ A}$$

$$V_R = (10.0\text{ V}) - (8.00\ \Omega)(1.00\text{ A}) = 2.00\text{ V}$$

Therefore, $$\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00\text{ V}}$$

(b) Redraw the circuit $$R = \frac{1}{(1/9.00\ \Omega) + (1/6.00\ \Omega)} = 3.60\ \Omega$$



$$RC = 3.60 \times 10^{-6}\text{ s}$$

and $$e^{-t/RC} = \frac{1}{10}$$

so $$t = RC\ \ln 10 = \boxed{8.29\ \mu\text{s}}$$

21.42 The potential difference across the capacitor $$\Delta V(t) = \Delta V_{\max}\left(1 - e^{-t/RC}\right)$$

Using 1 Farad = 1 s/Ω, $$4.00\text{ V} = (10.0\text{ V})\left[1 - e^{-(3.00\text{ s})/(R(10.0\times10^{-6}\text{ s}/\Omega))}\right]$$

Therefore, $$0.400 = 1.00 - e^{-\left(3.00\times10^{5}\ \Omega\right)/R}$$

Or $$e^{-\left(3.00\times10^{5}\ \Omega\right)/R} = 0.600$$

Taking the natural logarithm of both sides, $$-\frac{3.00\times10^{5}\ \Omega}{R} = \ln(0.600)$$

and $$R = -\frac{3.00\times10^{5}\ \Omega}{\ln(0.600)} = +5.87\times10^{5}\ \Omega = \boxed{587\text{ k}\Omega}$$

***21.43** $J = \sigma E$ so $\sigma = \frac{J}{E} = \frac{6.00 \times 10^{-13} \text{ A / m}^2}{100 \text{ V / m}} = \boxed{6.00 \times 10^{-15} \ (\Omega \cdot \text{m})^{-1}}$

***21.44** (a) $\mathcal{P} = \Delta VI = \left(300 \times 10^3 \text{ J/C}\right)\left(1.00 \times 10^3 \text{ C/s}\right) = \boxed{3.00 \times 10^8 \text{ W}}$

A large electric generating station, fed by a trainload of coal each day, converts energy faster.

(b) $I = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}}{\pi r^2}$ $\mathcal{P} = I\left(\pi r^2\right) = \left(1340 \text{ W/m}^2\right)\left[\pi(6.37 \times 10^6 \text{ m})^2\right] = \boxed{1.71 \times 10^{17} \text{ W}}$

Terrestrial solar power is immense compared to lightning and compared to all human energy conversions.

21.45 (a) $I = \frac{\Delta V}{R}$ so $\mathcal{P} = I\Delta V = \frac{(\Delta V)^2}{R}$

$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120 \text{ V})^2}{25.0 \text{ W}} = \boxed{576 \ \Omega}$ and $R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120 \text{ V})^2}{100 \text{ W}} = \boxed{144 \ \Omega}$

(b) $I = \frac{\mathcal{P}}{\Delta V} = \frac{25.0 \text{ W}}{120 \text{ V}} = 0.208 \text{ A} = \frac{Q}{\Delta t} = \frac{1.00 \text{ C}}{\Delta t}$

$\Delta t = \frac{1.00 \text{ C}}{0.208 \text{ A}} = \boxed{4.80 \text{ s}}$

The bulb takes in charge at high potential and puts out the same amount of charge at low potential.

(c) $\mathcal{P} = 25.0 \text{ W} = \frac{\Delta U}{\Delta t} = \frac{1.00 \text{ J}}{\Delta t}$ $\Delta t = \frac{1.00 \text{ J}}{25.0 \text{ W}} = \boxed{0.0400 \text{ s}}$

The bulb takes in energy by electrical transmission and puts out the same amount of energy by heat and light.

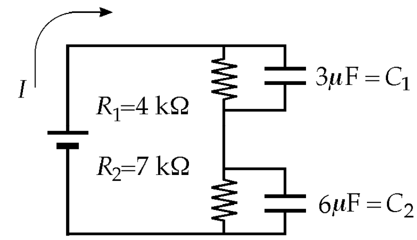
(d) $\Delta U = \mathcal{P}\Delta t = (25.0 \text{ J/s})(86400 \text{ s/d})(30.0 \text{ d}) = 64.8 \times 10^6 \text{ J}$

The electric company sells $\boxed{\text{energy}}$

$\text{Cost} = 64.8 \times 10^6 \text{ J}\left(\frac{\$0.0700}{\text{kWh}}\right)\left(\frac{\text{k}}{1000}\right)\left(\frac{\text{W} \cdot \text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3600 \text{ s}}\right) = \boxed{\$1.26}$

$\text{Cost per joule} = \frac{\$0.0700}{\text{kWh}}\left(\frac{\text{kWh}}{3.60 \times 10^6 \text{ J}}\right) = \boxed{\$1.94 \times 10^{-8} \ / \text{J}}$

***21.46** (a) With the switch closed, current exists in a simple series circuit as shown. The capacitors carry no current. For R_2 we have



$$\mathcal{P} = I^2 R_2 \qquad I = \sqrt{\frac{\mathcal{P}}{R_2}} = \sqrt{\frac{2.40\text{ V}\cdot\text{A}}{7000\text{ V/A}}} = 18.5\text{ mA}$$

The potential difference across R_1 and C_1 is

$$\Delta V = IR_1 = \left(1.85\times10^{-2}\text{ A}\right)\left(4000\text{ V/A}\right) = 74.1\text{ V}$$

The charge on C_1 is

$$Q = C_1\Delta V = \left(3.00\times10^{-6}\text{ C/V}\right)\left(74.1\text{ V}\right) = \boxed{222\ \mu\text{C}}$$

The potential difference across R_2 and C_2 is

$$\Delta V = IR_2 = \left(1.85\times10^{-2}\text{ A}\right)\left(7000\ \Omega\right) = 130\text{ V}$$

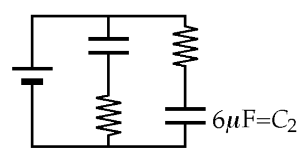
The charge on C_2 is

$$Q = C_2\Delta V = \left(6.00\times10^{-6}\text{ C/V}\right)\left(130\text{ V}\right) = 778\ \mu\text{C}$$

The battery emf is

$$IR_{eq} = I\left(R_1 + R_2\right) = 1.85\times10^{-2}\text{ A}\left(4000+7000\right)\text{ V/A} = 204\text{ V}$$

(b) In equilibrium after the switch has been opened, no current exists. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge C_2 is



$$Q = C_2\Delta V = \left(6.00\times10^{-6}\text{ C/V}\right)\left(204\text{ V}\right) = 1222\ \mu\text{C}$$

for a change of $\quad 1222\ \mu\text{C} - 778\ \mu\text{C} = \boxed{444\ \mu\text{C}}$

***21.47** (a) $\mathbf{E} = -\dfrac{dV}{dx}\mathbf{i} = -\dfrac{(0-4.00)\text{V}}{(0.500-0)\text{ m}} = \boxed{8.00\,\mathbf{i}\text{ V/m}}$

(b) $R = \dfrac{\rho\ell}{A} = \dfrac{\left(4.00\times10^{-8}\ \Omega\cdot\text{m}\right)\left(0.500\text{ m}\right)}{\pi\left(1.00\times10^{-4}\text{ m}\right)^2} = \boxed{0.637\ \Omega}$

(c) $I = \dfrac{\Delta V}{R} = \dfrac{4.00\text{ V}}{0.637\ \Omega} = \boxed{6.28\text{ A}}$

(d) $\boldsymbol{J} = \dfrac{I}{A}\mathbf{i} = \dfrac{6.28\text{ A}}{\pi\left(1.00\times10^{-4}\text{ m}\right)^2} = 2.00\times10^{8}\,\mathbf{i}\text{ A/m}^2 = \boxed{200\,\mathbf{i}\text{ MA/m}^2}$

(e) $\rho\boldsymbol{J} = \left(4.00\times10^{-8}\ \Omega\cdot\text{m}\right)\left(2.00\times10^{8}\,\mathbf{i}\text{ A/m}^2\right) = 8.00\,\mathbf{i}\text{ V/m} = \mathbf{E}$

21.48 (a) $\mathbf{E} = -\frac{dV(x)}{dx}\mathbf{i} = \boxed{\frac{V}{L}\mathbf{i}}$

(b) $R = \frac{\rho\ell}{A} = \boxed{\frac{4\rho L}{\pi d^2}}$

(c) $I = \frac{\Delta V}{R} = \boxed{\frac{V\pi d^2}{4\rho L}}$

(d) $\boldsymbol{J} = \frac{I}{A}\mathbf{i} = \boxed{\frac{V}{\rho L}\mathbf{i}}$

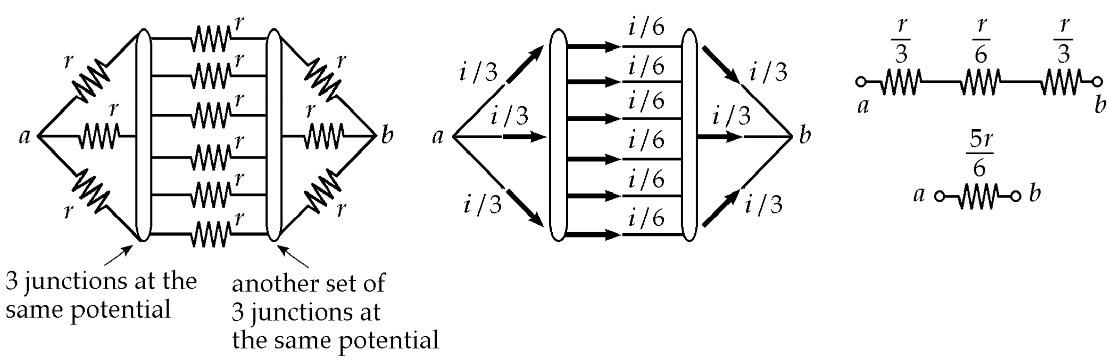
(e) $\rho\boldsymbol{J} = \frac{V}{L}\mathbf{i} = \boxed{\mathbf{E}}$

21.49 $\rho = \frac{RA}{\ell} = \frac{(\Delta V)}{I}\frac{A}{\ell}$

ℓ (m)	R (Ω)	ρ ($\Omega \cdot$ m)
0.540	10.4	1.41×10^{-6}
1.028	21.1	1.50×10^{-6}
1.543	31.8	1.50×10^{-6}

$\bar{\rho} = \boxed{1.47 \times 10^{-6}\ \Omega \cdot \text{m}}$ (in agreement with tabulated value of $1.50 \times 10^{-6}\ \Omega \cdot$ m in Table 21.1)

***21.50**



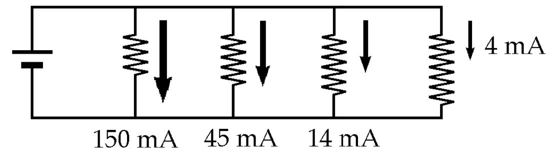
***21.51** (a) $\mathcal{P} = I\Delta V$

so $I = \frac{\mathcal{P}}{\Delta V} = \frac{8.00 \times 10^3\ \text{W}}{12.0\ \text{V}} = \boxed{667\ \text{A}}$

(b) $\Delta t = \frac{\Delta U}{\mathcal{P}} = \frac{2.00 \times 10^7\ \text{J}}{8.00 \times 10^3\ \text{W}} = 2.50 \times 10^3\ \text{s}$

and $\Delta x = v\Delta t = (20.0\ \text{m/s})(2.50 \times 10^3\ \text{s}) = \boxed{50.0\ \text{km}}$

***21.52** The battery current is



$$(150 + 45 + 14 + 4) \text{ mA} = 213 \text{ mA}$$

(a) The resistor with highest resistance is that carrying 4 mA. Doubling its resistance will reduce the current it carries to 2 mA. Then the total current is

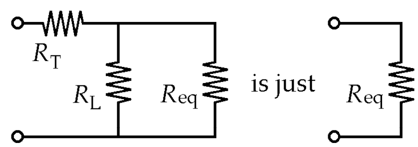
$(150 + 45 + 14 + 2) \text{ mA} = 211 \text{ mA}$, nearly the same as before. The ratio is $211/213 = \boxed{0.991}$

(b) The resistor with least resistance carries 150 mA. Doubling its resistance changes this current to 75 mA and changes the total to

$(75 + 45 + 14 + 4) \text{ mA} = 138 \text{ mA}$. The ratio is $138/213 = \boxed{0.648}$, representing a much larger reduction (35.2% instead of 0.9%).

(c) This problem is precisely analogous. As a battery maintained a potential difference in parts (a) and (b), a furnace maintains a temperature difference here. Energy flow by heat is analogous to current and takes place through thermal resistances in parallel. Each resistance can have its "*R*-value" increased by adding insulation. Doubling the thermal resistance of the attic door will produce only a negligible (0.9%) saving in fuel. Doubling the thermal resistance of $\boxed{\text{the ceiling}}$ will produce a much larger saving. The ceiling originally has the smallest thermal resistance.

***21.53** From the hint, the equivalent resistance of



That is, $$R_T + \frac{1}{1/R_L + 1/R_{eq}} = R_{eq}$$

$$R_T + \frac{R_L R_{eq}}{R_L + R_{eq}} = R_{eq}$$

$$R_T R_L + R_T R_{eq} + R_L R_{eq} = R_L R_{eq} + R_{eq}{}^2$$

$$R_{eq}{}^2 - R_T R_{eq} - R_T R_L = 0$$

$$R_{eq} = \frac{R_T \pm \sqrt{R_T{}^2 - 4(1)(-R_T R_L)}}{2(1)}$$

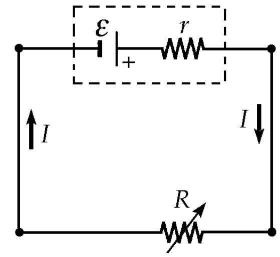
Only the + sign is physical:

$$R_{eq} = \tfrac{1}{2}\left(\sqrt{4R_T R_L + R_T{}^2} + R_T\right)$$

For example, if $R_T = 1\ \Omega$

And $R_L = 20\ \Omega$, $R_{eq} = 5\ \Omega$.

***21.54** The current in the simple loop circuit will be $I = \dfrac{\mathcal{E}}{R+r}$



(a) $\Delta V_{ter} = \mathcal{E} - Ir = \dfrac{\mathcal{E}R}{R+r}$ and $\Delta V_{ter} \to \mathcal{E}$ as $\boxed{R \to \infty}$

(b) $I = \dfrac{\mathcal{E}}{R+r}$ and $I \to \dfrac{\mathcal{E}}{r}$ as $\boxed{R \to 0}$

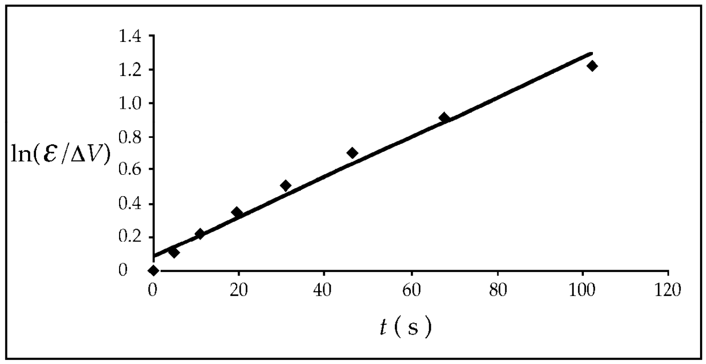
(c) $\mathcal{P} = I^2R = \mathcal{E}^2 \dfrac{R}{(R+r)^2}$ $\quad \dfrac{d\mathcal{P}}{dR} = \dfrac{-2\mathcal{E}^2R}{(R+r)^3} + \dfrac{\mathcal{E}^2}{(R+r)^2} = 0$

Then $2R = R + r$ and $\boxed{R = r}$

***21.55** $\Delta V = \mathcal{E}e^{-t/RC}$

so $\ln\left(\dfrac{\mathcal{E}}{\Delta V}\right) = \left(\dfrac{1}{RC}\right)t$

A plot of $\ln(\mathcal{E}/\Delta V)$ versus t should be a straight line with slope equal to $1/RC$



Using the given data values:

t (s)	ΔV (V)	$\ln(\mathcal{E}/\Delta V)$
0	6.19	0
4.87	5.55	0.109
11.1	4.93	0.228
19.4	4.34	0.355
30.8	3.72	0.509
46.6	3.09	0.695
67.3	2.47	0.919
102.2	1.83	1.219

(a) A least-square fit to this data yields the graph above.

$\Sigma x_i = 282,$ $\quad \Sigma x_i^2 = 1.86 \times 10^4,$

$\Sigma x_i y_i = 244,$ $\quad \Sigma y_i = 4.03,$ $\quad N = 8$

$$\text{Slope} = \frac{N(\Sigma x_i y_i) - (\Sigma x_i)(\Sigma y_i)}{N(\Sigma x_i^2) - (\Sigma x_i)^2} = 0.0118$$

$$\text{Intercept} = \frac{(\Sigma x_i^2)(\Sigma y_i) - (\Sigma x_i)(\Sigma x_i y_i)}{N(\Sigma x_i^2) - (\Sigma x_i)^2} = 0.0882$$

The equation of the best fit line is: $\boxed{\ln\left(\dfrac{\mathcal{E}}{\Delta V}\right) = (0.0118)t + 0.0882}$

(b) Thus, the time constant is $\tau = RC = \dfrac{1}{\text{slope}} = \dfrac{1}{0.0118} = \boxed{84.7 \text{ s}}$

and the capacitance is $C = \dfrac{\tau}{R} = \dfrac{84.7 \text{ s}}{10.0 \times 10^6\ \Omega} = \boxed{8.47\ \mu\text{F}}$

***21.56** (a) $q = C\Delta V\left(1 - e^{-t/RC}\right)$

$$q = \left(1.00\times10^{-6}\text{ F}\right)(10.0\text{ V})\left[1 - e^{\frac{-10.0}{\left(2.00\times10^{6}\right)\left(1.00\times10^{-6}\right)}}\right] = \boxed{9.93\ \mu\text{C}}$$

(b) $I = \dfrac{dq}{dt} = \left(\dfrac{\Delta V}{R}\right)e^{-t/RC}$

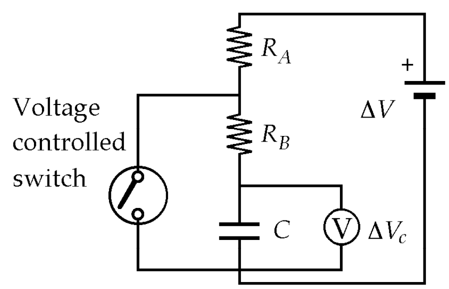
$$I = \left(\frac{10.0\text{ V}}{2.00\times10^{6}\ \Omega}\right)e^{-5.00} = 3.37\times10^{-8}\text{ A} = \boxed{33.7\text{ nA}}$$

(c) $\dfrac{dU}{dt} = \dfrac{d}{dt}\left(\dfrac{1}{2}\dfrac{q^2}{C}\right) = \left(\dfrac{q}{C}\right)\dfrac{dq}{dt} = \left(\dfrac{q}{C}\right)I$

$$\frac{dU}{dt} = \left(\frac{9.93\times10^{-6}\text{ C}}{1.00\times10^{-6}\text{ C/V}}\right)\left(3.37\times10^{-8}\text{ A}\right) = 3.34\times10^{-7}\text{ W} = \boxed{334\text{ nW}}$$

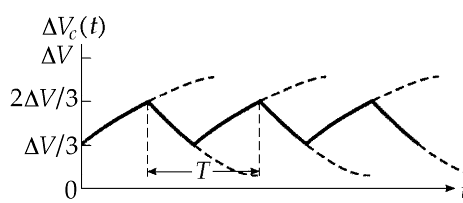
(d) $\mathcal{P}_{\text{battery}} = I\mathcal{E} = \left(3.37\times10^{-8}\text{ A}\right)(10.0\text{ V}) = 3.37\times10^{-7}\text{ W} = \boxed{337\text{ nW}}$

***21.57** Start at the point when the voltage has just reached $\frac{2}{3}\Delta V$ and the switch has just closed. The voltage is $\frac{2}{3}\Delta V$ and is decaying towards 0 V with a time constant R_BC



$$\Delta V_C(t) = \left[\tfrac{2}{3}\Delta V\right]e^{-t/R_BC}$$

We want to know when $\Delta V_C(t)$ will reach $\frac{1}{3}\Delta V$.



Therefore, $\quad \frac{1}{3}\Delta V = \left[\frac{2}{3}\Delta V\right]e^{-t/R_BC}$

or $\quad e^{-t/R_BC} = \frac{1}{2}$

or $\quad t_1 = R_BC\ln 2$

After the switch opens, the voltage is $\frac{1}{3}\Delta V$, increasing toward ΔV with time constant $(R_A + R_B)C$:

$$\Delta V_C(t) = \Delta V - \left[\tfrac{2}{3}\Delta V\right]e^{-t/(R_A+R_B)C}$$

When $\quad \Delta V_C(t) = \frac{2}{3}\Delta V$

$\frac{2}{3}\Delta V = \Delta V - \frac{2}{3}\Delta V e^{-t/(R_A+R_B)C} \quad$ or $\quad e^{-t/(R_A+R_B)C} = \frac{1}{2}$

So $\quad t_2 = (R_A + R_B)C\ln 2 \quad$ and $\quad T = t_1 + t_2 = \boxed{(R_A + 2R_B)C\ln 2}$

21.58 The battery supplies energy at a changing rate $\frac{dE}{dt} = \mathcal{P} = \mathcal{E} I = \mathcal{E}\left(\frac{\mathcal{E}}{R} e^{-1/RC}\right)$

Then the total energy put out by the battery is $\int dE = \int_{t=0}^{\infty} \frac{\mathcal{E}^2}{R} \exp\left(-\frac{t}{RC}\right) dt$

$$\int dE = \frac{\mathcal{E}^2}{R}(-RC)\int_0^{\infty} \exp\left(-\frac{t}{RC}\right)\left(-\frac{dt}{RC}\right) = -\mathcal{E}^2 C \exp\left(-\frac{t}{RC}\right)\Big|_0^{\infty} = -\mathcal{E}^2 C[0-1] = \mathcal{E}^2 C$$

The power delivered to the resistor is $\frac{dE}{dt} = \mathcal{P} = \Delta V_R I = I^2 R = R\frac{\mathcal{E}^2}{R^2}\exp\left(-\frac{2t}{RC}\right)$

So the total internal energy appearing in the resistor is $\int dE = \int_0^{\infty} \frac{\mathcal{E}^2}{R}\exp\left(-\frac{2t}{RC}\right) dt$

$$\int dE = \frac{\mathcal{E}^2}{R}\left(-\frac{RC}{2}\right)\int_0^{\infty} \exp\left(-\frac{2t}{RC}\right)\left(-\frac{2dt}{RC}\right) = -\frac{\mathcal{E}^2 C}{2}\exp\left(-\frac{2t}{RC}\right)\Big|_0^{\infty} = -\frac{\mathcal{E}^2 C}{2}[0-1] = \frac{\mathcal{E}^2 C}{2}$$

The energy finally stored in the capacitor is $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C\mathcal{E}^2$. Thus, energy of the circuit is conserved $\mathcal{E}^2 C = \frac{1}{2}\mathcal{E}^2 C + \frac{1}{2}\mathcal{E}^2 C$ and resistor and capacitor share equally in the energy from the battery.

21.59 $I = \frac{\mathcal{E}}{R+r}$, so $\mathcal{P} = I^2 R = \frac{\mathcal{E}^2 R}{(R+r)^2}$ or $(R+r)^2 = \left(\frac{\mathcal{E}^2}{\mathcal{P}}\right)R$

Let $x \equiv \frac{\mathcal{E}^2}{\mathcal{P}}$, then $(R+r)^2 = xR$ or $R^2 + (2r - x)R - r^2 = 0$

With $r = 1.20\ \Omega$, this becomes $R^2 + (2.40 - x)R - 1.44 = 0$,

which has solutions of $R = \frac{-(2.40-x) \pm \sqrt{(2.40-x)^2 - 5.76}}{2}$

(a) With $\mathcal{E} = 9.20$ V and $\mathcal{P} = 12.8$ W, $x = 6.61$:

$$R = \frac{+4.21 \pm \sqrt{(4.21)^2 - 5.76}}{2} = \boxed{3.84\ \Omega} \quad \text{or} \quad \boxed{0.375\ \Omega}$$

(b) For $\mathcal{E} = 9.20$ V and $\mathcal{P} = 21.2$ W, $x \equiv \frac{\mathcal{E}^2}{\mathcal{P}} = 3.99$

$$R = \frac{+1.59 \pm \sqrt{(1.59)^2 - 5.76}}{2} = \frac{1.59 \pm \sqrt{-3.22}}{2}$$

The equation for the load resistance yields a complex number, so $\boxed{\text{there is no resistance}}$ that will extract 21.2 W from this battery. The maximum power output occurs when $R = r = 1.20\ \Omega$, and that maximum is: $\mathcal{P}_{max} = \mathcal{E}^2 / 4r = 17.6$ W

***21.60** (a) A thin cylindrical shell of radius r, thickness dr, and length L contributes resistance

$$dR = \frac{\rho\, d\ell}{A} = \frac{\rho\, dr}{(2\pi r)L} = \left(\frac{\rho}{2\pi L}\right)\frac{dr}{r}$$

The resistance of the whole annulus is the series summation of the contributions of the thin shells:

$$R = \frac{\rho}{2\pi L}\int_{r_a}^{r_b}\frac{dr}{r} = \boxed{\frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)}$$

(b) In this equation $\dfrac{\Delta V}{I} = \dfrac{\rho}{2\pi L}\ln\left(\dfrac{r_b}{r_a}\right)$

we solve for $\boxed{\rho = \dfrac{2\pi L \Delta V}{I \ln(r_b / r_a)}}$

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 3.64 h

4. (a) 17.0 A (b) 85.0 kA/m^2

6. 0.130 mm/s

8. (a) 1.82 m (b) 280 μm

10. 1.98 A

12. 1.71 Ω

14. (a) unaffected (b) doubles
(c) doubles (d) unchanged

16. 448 A

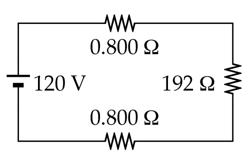
18. 672 s

20. 295 metric ton/h

22. (a) \$1.61 (b) \$0.005 82 (c) \$0.416

24. (a) 4.59 Ω (b) 8.16%

26. 73.8 W



28. (a) See the solution (b) no

30. (a) 75.0 V (b) 25.0 W, 6.25 W, and 6.25 W; 37.5 W

32. 0.714 A, 1.29 A, 12.6 V

34. See the solution

36. 50.0 mA from a to e

38. (a) –61.6 mA (b) 0.235 μC (c) 1.96 A

40. (a) 1.50 s (b) 1.00 s (c) $\left(200+100e^{-t/1.00\text{ s}}\right)\mu$A

42. 587 kΩ

44. (a) 300 MW (b) 171 PW

46. (a) 222 μC (b) increases by 444 μC

48. (a) $\frac{V}{L}\,\mathbf{i}$ (b) $\frac{4\rho L}{\pi d^2}$ (c) $\frac{V\pi d^2}{4\rho L}$
(d) $\frac{V}{\rho L}\,\mathbf{i}$ (e) See the solution

50. See the solution

52. (a) 0.991 (b) 0.648
(c) Insulation should be added to the ceiling.

54. (a) $R \to \infty$ (b) $R \to 0$ (c) $R = r$

56. (a) 9.93 μC (b) 33.7 nA
(c) 334 nW (d) 337 nW

58. See the solution

60. (a) $(\rho/2\pi L)\ln(r_b/r_a)$ (b) $2\pi L\Delta V/\left[I\ln(r_b/r_a)\right]$