CHAPTER 22

ANSWERS TO QUESTIONS

Q22.1 The charges are of opposite sign.

Q22.2 If the current is in a direction *parallel* or *antiparallel* to the magnetic field, then there is no force.

Q22.3 Yes, if the magnetic field is perpendicular to the plane of the loop, then there is no torque.

Q22.4 The geographic North Pole is a magnetic South Pole.

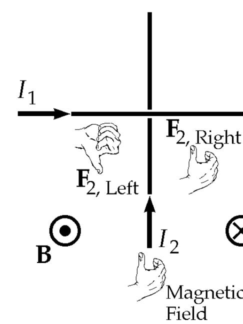
Q22.5 Straight down.

Q22.6 Domain alignment creates a stronger magnetic field, which in turn can align domains in other iron samples.

Q22.7 Yes, either pole. Domains inside the iron nail are aligned along the magnetic field lines of the magnet.

Q22.8 The shock misaligns the domains. Heating will also decrease magnetism.

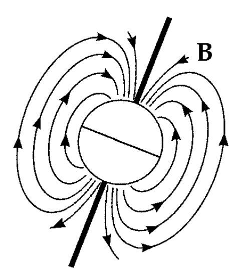
Q22.9 No total force, but a torque. Let wire one carry current in the y–direction, toward the top of the page. Let wire two be a millimeter above the plane of the paper and carry current to the right (in the x–direction). On the left–hand side of wire one, it creates a magnetic field in the z–direction, which exerts force in the $\mathbf{i}\times\mathbf{k}=-\mathbf{j}$ direction on wire two. On the right–hand side, wire one produces magnetic field in the $(-\mathbf{k})$ direction and makes a $\mathbf{i}\times(-\mathbf{k})=+\mathbf{j}$ force of equal magnitude act on wire two. If wire two is free to move, its center section will twist counterclockwise and then be attracted to wire one.



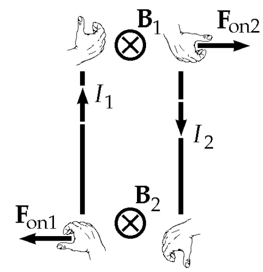
Q22.10 If you can hook a spring balance to the particle and measure the force on it in a known electric field, then $q=F/E$ will tell you its charge. You cannot hook a spring balance to an electron. Measuring the acceleration of small particles by observing their deflection in known electric and magnetic fields can tell you the charge–to–mass ratio, but not separately the charge or mass.

Q22.11 If the current loop feels a torque, it must be caused by a magnetic field. If the current loop feels no torque, try a different orientation – the torque is zero if the field is along the axis of the loop.

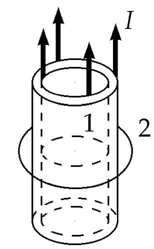
Q22.12 The Earth's magnetic field exerts force on a charged incoming cosmic ray, tending to make it spiral around a magnetic field line. If the particle energy is low enough the spiral will be tight enough that the particle will first hit some matter as it follows a field line down into the atmosphere or to the surface at a high geographic latitude.



Q22.13 The magnetic field created by wire 1 at the position of wire 2 is into the paper. Hence, the magnetic force on wire 2 is in direction down × into the paper = to the right, away from wire 1. Now wire 2 creates a magnetic field into the page at the location of wire 1, so wire 1 feels force up ×into the paper = left, away from wire 2.



Q22.14 Apply Ampere's law to the circular path labeled 1 in the picture. Since there is no current inside this path, the magnetic field inside the tube must be zero. On the other hand, the current through path 2 is the current carried by the conductor. Therefore the magnetic field outside the tube is nonzero.



Q22.15 The magnetic field inside a long solenoid is given by $B = \mu_0 NI / \ell$. (a) If the length ℓ is doubled, the field is cut in half. (b) If N is doubled, the magnetic field is doubled.

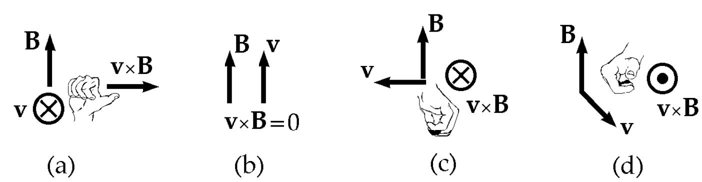
Q22.16 The magnetic field near the Earth's equator is horizontally north.

(a) If the velocity is down, $\mathbf{v} \times \mathbf{B}$ is east and $q\mathbf{v} \times \mathbf{B}$ for the negative electron is west.

(b) If $\mathbf{v}$ is north, $\mathbf{v} \times \mathbf{B}$ is zero and the electron is not deflected.

(c) If $\mathbf{v}$ is west, $\mathbf{v} \times \mathbf{B}$ is west × north in direction, namely down, and the electron is deflected up by the $q\mathbf{v} \times \mathbf{B}$ force.

(d) If $\mathbf{v}$ is southeast, $\mathbf{v} \times \mathbf{B}$ is in direction southeast × north = up, and $q\mathbf{v} \times \mathbf{B}$ deflects the electron down.



Q22.17 If one of the bars has its magnetic moment along its length, its magnetic field will likely be strongest at its ends. Its end may attract the center of the other bar, while one end of the unmagnetized bar does not attract the center of the bar magnet.

Q22.18 Magnetic levitation is illustrated in Figure Q22.20. The Earth's magnetic field is so weak that the floor of his tomb should be magnetized as well as his coffin. Alternatively, the floor of his tomb could be made of superconducting material, which exerts a force of repulsion on any magnet.

Q22.19 The medium for any magnetic recording should be a hard ferromagnetic substance, so that thermal vibrations will not rapidly erase the information.

Q22.20 (a) The magnets repel each other with a force equal to the weight of one of them.

(b) The pencil prevents motion to the side and prevents the magnets from rotating under their mutual torques. Its constraint changes unstable equilibrium into stable.

(c) Most likely, the disks are magnetized perpendicular to their flat faces, making one face a north pole and the other a south pole.

(d) Then if either were inverted they would attract each other and stick firmly together.

Q22.21 See Figure 31.7 in the textbook. A spiral of decreasing radius is the path of a charged particle that is losing kinetic energy, as by collisions with atoms in the medium. Particles with positive and negative charges make tracks curving in opposite directions. A straight line might be produced by an uncharged particle, or by a particle with such high momentum that its deflection cannot be observed.

Q22.22 The $q\mathbf{v}\times\mathbf{B}$ force on each electron is down. Since electrons are negative, $\mathbf{v}\times\mathbf{B}$ must be up. With $\mathbf{v}$ to the right, $\mathbf{B}$ must be (a) into the page, away from you. Reversing the current in the coils would reverse the direction of $\mathbf{B}$, making it toward you. Then $\mathbf{v}\times\mathbf{B}$ is in the direction **right** $\times$ **toward you** = **down**, and $q\mathbf{v}\times\mathbf{B}$ will make the electron beam curve up.

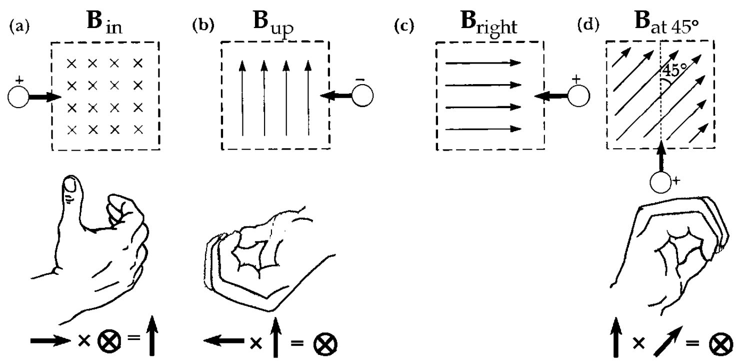
PROBLEM SOLUTIONS

22.1 (a) up

(b) out of the page, since the charge is negative.

(c) no deflection

(d) into the page



22.2 (a) $F_B = qvB\sin\theta = \left(1.60\times10^{-19}\ \text{C}\right)\left(3.00\times10^{6}\ \text{m/s}\right)\left(3.00\times10^{-1}\ \text{T}\right)\sin 37.0°$

$F_B = \boxed{8.67\times10^{-14}\ \text{N}}$

(b) $a = \dfrac{F}{m} = \dfrac{8.67\times10^{-14}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = \boxed{5.19\times10^{13}\ \text{m/s}^2}$

22.3 First find the speed of the electron.

$\Delta K = \frac{1}{2}mv^2 = e\Delta V = \Delta U:$ $\quad v = \sqrt{\dfrac{2e\Delta V}{m}} = \sqrt{\dfrac{2\left(1.60\times10^{-19}\ \text{C}\right)(2400\ \text{J/C})}{\left(9.11\times10^{-31}\ \text{kg}\right)}} = 2.90\times10^{7}\ \text{m/s}$

(a) $F_{B,\max} = qvB = \left(1.60\times10^{-19}\ \text{C}\right)\left(2.90\times10^{7}\ \text{m/s}\right)(1.70\ \text{T}) = \boxed{7.90\times10^{-12}\ \text{N}}$

(b) $F_{B,\min} = \boxed{0}$ occurs when **v** is either parallel to or anti-parallel to **B**.

22.4 Gravitational force: $F_g = mg = \left(9.11\times10^{-31}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = \boxed{8.93\times10^{-30}\ \text{N down}}$

Electric force: $F_e = qE = \left(-1.60\times10^{-19}\ \text{C}\right)(100\ \text{N/C down}) = \boxed{1.60\times10^{-17}\ \text{N up}}$

Magnetic force: $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = \left(-1.60\times10^{-19}\ \text{C}\right)\left(6.00\times10^{6}\ \text{m/s}\ \mathbf{E}\right)\times\left(50.0\times10^{-6}\ \dfrac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}\ \mathbf{N}\right)$

$\mathbf{F}_B = -4.80\times10^{-17}\ \text{N up} = \boxed{4.80\times10^{-17}\ \text{N down}}$

***22.5** $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B}$

$$\mathbf{v}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ +2 & -4 & +1 \\ +1 & +2 & -3 \end{vmatrix} = (12-2)\mathbf{i} + (1+6)\mathbf{j} + (4+4)\mathbf{k} = 10\mathbf{i} + 7\mathbf{j} + 8\mathbf{k}$$

$$\left|\mathbf{v}\times\mathbf{B}\right| = \sqrt{10^2+7^2+8^2} = 14.6\ \mathrm{T\cdot m/s}$$

$$\left|\mathbf{F}_B\right| = q\left|\mathbf{v}\times\mathbf{B}\right| = \left(1.60\times10^{-19}\ \mathrm{C}\right)(14.6\ \mathrm{T\cdot m/s}) = \boxed{2.34\times10^{-18}\ \mathrm{N}}$$

***22.6** (a) We begin with $qvB = \dfrac{mv^2}{R}$

or $qRB = mv$

But $L = mvR = qR^2B$

Therefore, $$R = \sqrt{\frac{L}{qB}} = \sqrt{\frac{4.00\times10^{-25}\ \mathrm{J\cdot s}}{\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(1.00\times10^{-3}\ \mathrm{T}\right)}} = 0.0500\ \mathrm{m} = \boxed{5.00\ \mathrm{cm}}$$

(b) Thus, $$v = \frac{L}{mR} = \frac{4.00\times10^{-25}\ \mathrm{J\cdot s}}{\left(9.11\times10^{-31}\ \mathrm{kg}\right)(0.0500\ \mathrm{m})} = \boxed{8.78\times10^{6}\ \mathrm{m/s}}$$

***22.7** $E = \frac{1}{2}mv^2 = e\Delta V$

and $evB\sin 90° = \dfrac{mv^2}{R}$

$$B = \frac{mv}{eR} = \frac{m}{eR}\sqrt{\frac{2e\Delta V}{m}} = \frac{1}{R}\sqrt{\frac{2m\Delta V}{e}}$$

$$B = \frac{1}{5.80\times10^{10}\ \mathrm{m}}\sqrt{\frac{2(1.67\times10^{-27}\ \mathrm{kg})(10.0\times10^{6}\ \mathrm{V})}{1.60\times10^{-19}\ \mathrm{C}}} = \boxed{7.88\times10^{-12}\ \mathrm{T}}$$

***22.8** $F_B = F_e$

so $qvB = qE$

where $v = \sqrt{2K/m}$ and K is kinetic energy of the electron.

$$E = vB = \sqrt{\frac{2K}{m}}B = \sqrt{\frac{2(750)\left(1.60\times10^{-19}\right)}{9.11\times10^{-31}}}(0.0150) = \boxed{244\ \mathrm{kV/m}}$$

***22.9** In the velocity selector: $v = \dfrac{E}{B} = \dfrac{2500 \text{ V/m}}{0.0350 \text{ T}} = 7.14 \times 10^4 \text{ m/s}$

In the deflection chamber: $r = \dfrac{mv}{qB} = \dfrac{(2.18 \times 10^{-26} \text{ kg})(7.14 \times 10^4 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.0350 \text{ T})} = \boxed{0.278 \text{ m}}$

***22.10** Note that the "cyclotron frequency" is an angular speed. The motion of the proton is described by $\Sigma F = ma$:

$$|q|vB\sin 90° = \frac{mv^2}{r}$$

$$|q|B = m\frac{v}{r} = m\omega$$

(a) $\omega = \dfrac{|q|B}{m} = \dfrac{(1.60 \times 10^{-19} \text{ C})(0.8 \text{ N}\cdot\text{s/C}\cdot\text{m})}{(1.67 \times 10^{-27} \text{ kg})}\left(\dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right) = \boxed{7.66 \times 10^7 \text{ rad/s}}$

(b) $v = \omega r = (7.66 \times 10^7 \text{ rad/s})(0.350 \text{ m})\left(\dfrac{1}{1 \text{ rad}}\right) = \boxed{2.68 \times 10^7 \text{ m/s}}$

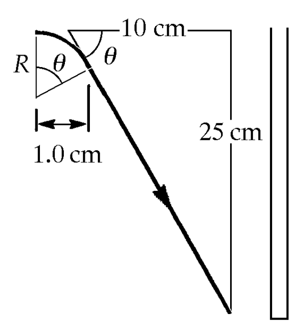
(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(2.68 \times 10^7 \text{ m/s})^2\left(\dfrac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}}\right) = \boxed{3.76 \times 10^6 \text{ eV}}$

(d) The proton gains 600 eV twice during each revolution, so the number of revolutions is

$$\frac{3.76 \times 10^6 \text{ eV}}{2(600 \text{ eV})} = \boxed{3.13 \times 10^3 \text{ revolutions}}$$

(e) $\theta = \omega t$ $\qquad t = \dfrac{\theta}{\omega} = \dfrac{3.13 \times 10^3 \text{ rev}}{7.66 \times 10^7 \text{ rad/s}}\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right) = \boxed{2.57 \times 10^{-4} \text{ s}}$

***22.11** $\theta = \tan^{-1}\left(\dfrac{25.0}{10.0}\right) = 68.2°$ and $R = \dfrac{1.00 \text{ cm}}{\sin 68.2°} = 1.08 \text{ cm}$



Ignoring relativistic correction, the kinetic energy of the electrons is

$\frac{1}{2}mv^2 = q\Delta V$ so $v = \sqrt{\dfrac{2q\Delta V}{m}} = 1.33 \times 10^8 \text{ m/s}$

From Newton's second law $\dfrac{mv^2}{R} = qvB$, we find the magnetic field

$$B = \frac{mv}{|q|R} = \frac{(9.11 \times 10^{-31} \text{ kg})(1.33 \times 10^8 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(1.08 \times 10^{-2} \text{ m})} = \boxed{70.1 \text{ mT}}$$

***22.12** (a) If the charge carriers are negative, to carry current in the x direction they move with drift velocity v_d in the $-x$ direction. The magnetic force $q\mathbf{v}\times\mathbf{B}$ is in the $-(-\mathbf{i})\times\mathbf{j}=\mathbf{k}$ direction, so the negative charges are deflected to the top of the ribbon, point c. Some accumulate there to make V_c negative with respect to V_a, until ...

(b) ... an upward electric field $\mathbf{E}=\dfrac{|V_c-V_a|}{d}\mathbf{k}$ exerts a downward force on the other charge carries to let them drift in equilibrium according to

$\Sigma F_z = 0$:

$$|q|\frac{|V_c-V_a|}{d}(-\mathbf{k})+|q||\mathbf{v}_d||\mathbf{B}|\mathbf{k}=0$$

$$v_d=\frac{|\Delta V_H|}{dB}$$

Since the measured current is $\quad I=n|q|v_d t\,d$

we have $\quad I=n|q|\dfrac{|\Delta V_H|}{dB}t\,d$

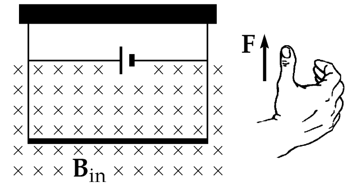
$$n=\frac{IB}{|q||\Delta V_H|t}$$

Since we have shown that ΔV_H is negative if q is negative, this expression simplifies to

$$\boxed{n=\frac{IB}{q\Delta V_H t}}$$

22.13 $\mathbf{F}_B=I\boldsymbol{\ell}\times\mathbf{B}=(2.40\text{ A})(0.750\text{ m})\mathbf{i}\times(1.60\text{ T})\mathbf{k}=\boxed{(-2.88\,\mathbf{j})\text{ N}}$

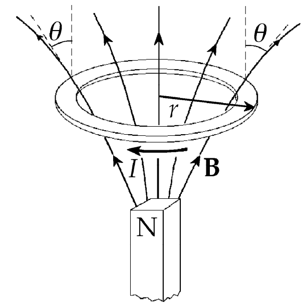
***22.14** $\dfrac{|\mathbf{F}_B|}{\ell}=\dfrac{mg}{\ell}=\dfrac{I|\boldsymbol{\ell}\times\mathbf{B}|}{\ell}$

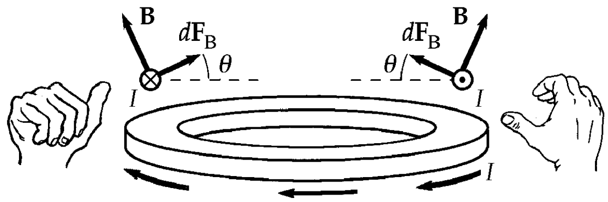


$$I=\frac{mg}{B\ell}=\frac{(0.0400\text{ kg/m})(9.80\text{ m/s}^2)}{3.60\text{ T}}=\boxed{0.109\text{ A}}$$

The direction of I in the bar is $\boxed{\text{to the right}}$.

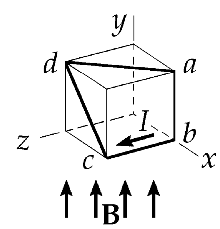
22.15 The magnetic force on each bit of ring is $I\,d\mathbf{s}\times\mathbf{B} = I\,ds\,B$ radially inward and upward, at angle θ above the radial line. The radially inward components tend to squeeze the ring but all cancel out as forces. The upward components $I\,ds\,B\sin\theta$ all add to $\boxed{I\,2\pi rB\sin\theta \text{ up}}$.





22.16 For each segment, $I = 5.00$ A and $\mathbf{B} = 0.0200\ \mathrm{N/A\cdot m}\ \mathbf{j}$

Segment	$\boldsymbol{\ell}$	$\mathbf{F}_B = I(\boldsymbol{\ell}\times\mathbf{B})$
ab	$-0.400\ \mathrm{m}\ \mathbf{j}$	$\boxed{0}$
bc	$0.400\ \mathrm{m}\ \mathbf{k}$	$\boxed{(40.0\ \mathrm{mN})(-\mathbf{i})}$
cd	$-0.400\ \mathrm{m}\ \mathbf{i} + 0.400\ \mathrm{m}\ \mathbf{j}$	$\boxed{(40.0\ \mathrm{mN})(-\mathbf{k})}$
da	$0.400\ \mathrm{m}\ \mathbf{i} - 0.400\ \mathrm{m}\ \mathbf{k}$	$\boxed{(40.0\ \mathrm{mN})(\mathbf{k}+\mathbf{i})}$

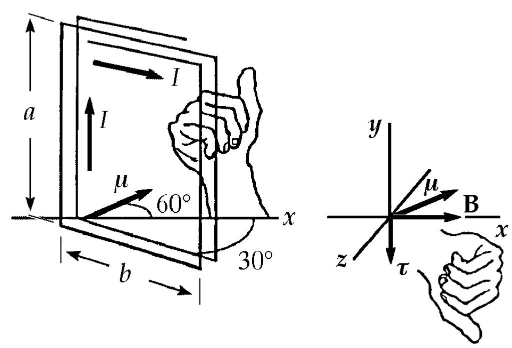


22.17 $\tau = NBAI\sin\theta$

$$\tau = 100(0.800\ \mathrm{T})\left(0.400\ \times 0.300\ \mathrm{m}^2\right)(1.20\ \mathrm{A})\sin 60°$$

$$\tau = \boxed{9.98\ \mathrm{N\cdot m}}$$

Note that θ is the angle between the magnetic moment and the **B** field. The loop will rotate so as to align the magnetic moment with the **B** field. Looking down along the y-axis, the loop will rotate in a $\boxed{\text{clockwise}}$ direction.



***22.18** (a) $2\pi r = 2.00$ m

so $r = 0.318$ m

$$\mu = IA = \left(17.0\times10^{-3}\ \mathrm{A}\right)\left[\pi(0.318)^2\ \mathrm{m}^2\right] = \boxed{5.41\ \mathrm{mA\cdot m^2}}$$

(b) $\boldsymbol{\tau} = \boldsymbol{\mu}\times\mathbf{B}$

so $\tau = \left(5.41\times10^{-3}\ \mathrm{A\cdot m^2}\right)(0.800\ \mathrm{T}) = \boxed{4.33\ \mathrm{mN\cdot m}}$

22.19 (a) Let θ represent the unknown angle; L, the total length of the wire; and d, the length of one side of the square coil. Then, using the definition of magnetic moment and the right-hand rule in Figure 22.21, we find

$\mu = NAI$: $\quad \boldsymbol{\mu} = \left(\frac{L}{4d}\right) d^2 I$ at angle θ with the horizontal.

At equilibrium, $\quad \Sigma \boldsymbol{\tau} = (\boldsymbol{\mu} \times \mathbf{B}) - (\mathbf{r} \times m\mathbf{g}) = 0$

$$\left(\frac{ILBd}{4}\right)\sin(90.0° - \theta) - \left(\frac{mgd}{2}\right)\sin\theta = 0$$

and $\quad \left(\frac{mgd}{2}\right)\sin\theta = \left(\frac{ILBd}{4}\right)\cos\theta$

$$\theta = \tan^{-1}\left(\frac{ILB}{2mg}\right) = \tan^{-1}\left(\frac{(3.40\text{ A})(4.00\text{ m})(0.0100\text{ T})}{2(0.100\text{ kg})(9.80\text{ m/s}^2)}\right) = \boxed{3.97°}$$

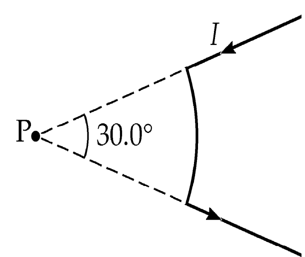
(b) $\tau_m = \left(\frac{ILBd}{4}\right)\cos\theta = \frac{1}{4}(3.40\text{ A})(4.00\text{ m})(0.0100\text{ T})(0.100\text{ m})\cos 3.97° = \boxed{3.39\text{ mN}\cdot\text{m}}$

***22.20** Choose $U = 0$ when the dipole moment is at $\theta = 90.0°$ to the field. The field exerts torque of magnitude $\mu B\sin\theta$ on the dipole, tending to turn the dipole moment in the direction of decreasing θ. According to equations 7.13 and 10.28, the potential energy of the dipole-field system is given by

$$U - 0 = \int_{90.0°}^{\theta} \mu B \sin\theta\, d\theta = \mu B(-\cos\theta)\Big|_{90.0°}^{\theta} = -\mu B\cos\theta + 0 \qquad \text{or} \qquad \boxed{U = -\boldsymbol{\mu}\cdot\mathbf{B}}$$

22.21 $B = \frac{\mu_0 I}{2R} = \frac{\mu_0 q(v/2\pi R)}{2R} = \boxed{12.5\text{ T}}$

22.22 We use the Biot-Savart law. For bits of wire along the straight-line sections, $d\mathbf{s}$ is at 0° or 180° to $\hat{\mathbf{r}}$, so $d\mathbf{s} \times \hat{\mathbf{r}} = 0$. Thus, only the curved section of wire contributes to $\mathbf{B}$ at P. Hence, $d\mathbf{s}$ is tangent to the arc and $\hat{\mathbf{r}}$ is radially inward; so $d\mathbf{s} \times \hat{\mathbf{r}} = |ds|\ell\sin 90° = |ds|\otimes$. All points along the curve are the same distance $r = 0.600$ m from the field point, so



$$B = \int_{\text{all current}} |d\mathbf{B}| = \int \frac{\mu_0}{4\pi}\frac{I|d\mathbf{s}\times\hat{\mathbf{r}}|}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int|ds| = \frac{\mu_0}{4\pi}\frac{I}{r^2}s$$

where s is the arc length of the curved wire,

$$s = r\theta = (0.600\text{ m})(30.0°)\left(\frac{2\pi}{360°}\right) = 0.314\text{ m}$$

Then, $B = \left(10^{-7}\text{ T}\cdot\text{m/A}\right)\frac{(3.00\text{ A})}{(0.600\text{ m})^2}(0.314\text{ m}) \qquad B = \boxed{261\text{ nT into the page}}$

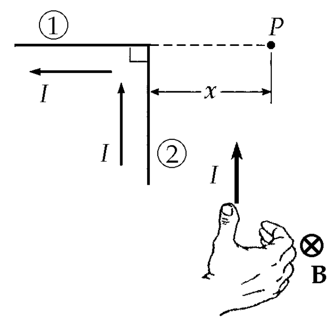
***22.23** $$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})(1.00\text{ A})}{2\pi(1.00\text{ m})} = \boxed{2.00 \times 10^{-7}\text{ T}}$$

***22.24** We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\mu_0 I/2\pi R$ and directed into the page) and the field due to the circular loop (having magnitude $\mu_0 I/2R$ and directed into the page). The resultant magnetic field is:

$$\boxed{\mathbf{B} = \left(1 + \frac{1}{\pi}\right)\frac{\mu_0 I}{2R} \quad \text{(directed into the page)}}$$

22.25 For leg 1, $d\mathbf{s} \times \hat{\mathbf{r}} = 0$, so there is no contribution to the field from this segment. For leg 2, the wire is only semi-infinite; thus,

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x}\right) = \boxed{\frac{\mu_0 I}{4\pi x} \quad \text{into the paper}}$$

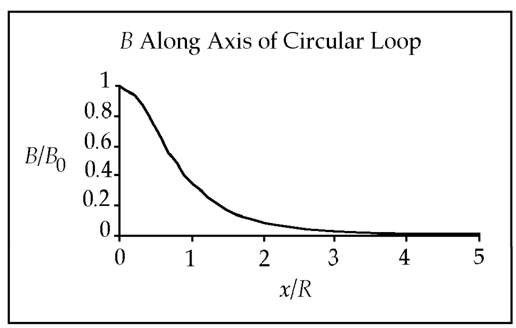


***22.26** Along the axis of a circular loop of radius R,

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

or $$\frac{B}{B_0} = \left[\frac{1}{(x/R)^2 + 1}\right]^{3/2}$$

where $B_0 \equiv \mu_0 I/2R$



x/R	B/B_0
0.00	1.00
1.00	0.354
2.00	0.0894
3.00	0.0316
4.00	0.0143
5.00	0.00754

***22.27** Label the wires 1, 2, and 3 as shown in Figure (a) and let the magnetic field created by the currents in these wires be $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{B}_3$ respectively.

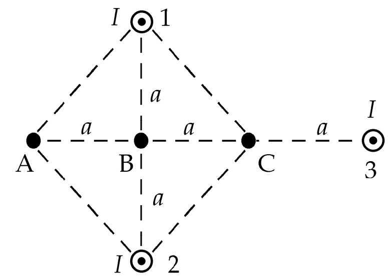


Figure (a)

(a) At Point A:

$$B_1 = B_2 = \frac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$$

and $$B_3 = \frac{\mu_0 I}{2\pi(3a)}$$

The directions of these fields are shown in Figure (b). Observe that the horizontal components of $\mathbf{B}_1$ and $\mathbf{B}_2$ cancel while their vertical components both add to $\mathbf{B}_3$.

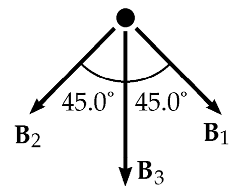


Figure (b)

Therefore, the net field at point A is:

$$B_A = B_1\cos 45.0° + B_2\cos 45.0° + B_3 = \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° + \frac{1}{3}\right]$$

$$B_A = \frac{\left(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}\right)(2.00\ \mathrm{A})}{2\pi\left(1.00\times 10^{-2}\ \mathrm{m}\right)}\left[\frac{2}{\sqrt{2}}\cos 45° + \frac{1}{3}\right]$$

$$B_A = \boxed{53.3\ \mu\mathrm{T}\ \text{toward the bottom of the page}}$$

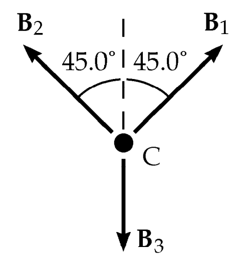


Figure (c)

(b) At point B: $\mathbf{B}_1$ and $\mathbf{B}_2$ cancel,

leaving $$B_B = B_3 = \frac{\mu_0 I}{2\pi(2a)}$$

$$B_B = \frac{\left(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}\right)(2.00\ \mathrm{A})}{2\pi(2)\left(1.00\times 10^{-2}\ \mathrm{m}\right)} = \boxed{20.0\ \mu\mathrm{T}\ \text{toward the bottom of the page}}$$

(c) At point C: $$B_1 = B_2 = \frac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$$

and $B_3 = \dfrac{\mu_0 I}{2\pi a}$ with the directions shown in Figure (c). Again, the horizontal components of $\mathbf{B}_1$ and $\mathbf{B}_2$ cancel. The vertical components both oppose $\mathbf{B}_3$ giving

$$B_C = 2\left[\frac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}\cos 45.0°\right] - \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° - 1\right] = \boxed{0}$$

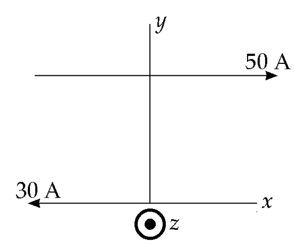
***22.28** Every element of current creates magnetic field in the same direction, into the page, at the center of the arc. The upper straight portion creates one-half of the field that an infinitely long straight wire would create. The curved portion creates one quarter of the field that a circular loop produces at its center. The lower straight segment also creates field $\frac{1}{2}\frac{\mu_0 I}{2\pi r}$.

The total field is

$$\mathbf{B} = \left(\frac{1}{2}\frac{\mu_0 I}{2\pi r} + \frac{1}{4}\frac{\mu_0 I}{2r} + \frac{1}{2}\frac{\mu_0 I}{2\pi r}\right)\text{ into the page} = \boxed{\frac{\mu_0 I}{2r}\left(\frac{1}{\pi} + \frac{1}{4}\right)\text{ into the plane of the paper}}$$

$$= \left(0.28415\mu_0 I / r\right)\text{ into the page}$$

***22.29** (a) Above the pair of wires, the field out of the page of the 50 A current will be stronger than the (−**k**) field of the 30 A current, so they cannot add to zero. Between the wires, both produce fields into the page. They can only add to zero below the wires, at coordinate $y = -|y|$. Here the total field is



$$\mathbf{B} = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2\pi r}:$$

$$0 = \frac{\mu_0}{2\pi}\frac{50\text{ A}}{\left(|y| + 0.28\text{ m}\right)}(-\mathbf{k}) + \frac{30\text{ A}}{|y|}(\mathbf{k})$$

$$50|y| = 30\left(|y| + 0.28\text{ m}\right)$$

$$50(-y) = 30(0.28\text{ m} - y)$$

$$-20y = 30(0.28\text{ m}) \quad \boxed{\text{at } y = -0.420\text{ m}}$$

(b) At $y = 0.1$ m the total field is $\mathbf{B} = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I}{2\pi r}:$

$$\mathbf{B} = \frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{2\pi}\left(\frac{50\text{ A}}{(0.28 - 0.10)\text{ m}}(-\mathbf{k}) + \frac{30\text{ A}}{0.10\text{ m}}(-\mathbf{k})\right) = 1.16\times10^{-4}\text{ T}(-\mathbf{k})$$

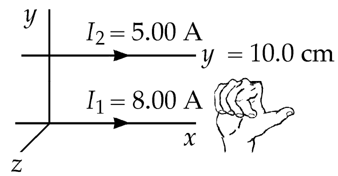
The force on the particle is

$$\mathbf{F} = q\mathbf{v}\times\mathbf{B} = \left(-2\times10^{-6}\text{ C}\right)\left(150\times10^{6}\text{ m/s}\right)(\mathbf{i})\times\left(1.16\times10^{-4}\text{ N}\cdot\text{s/C}\cdot\text{m}\right)(-\mathbf{k}) = \boxed{3.47\times10^{-2}\text{ N}(-\mathbf{j})}$$

(c) We require $\mathbf{F}_e = 3.47\times10^{-2}\text{ N}(+\mathbf{j}) = q\mathbf{E} = \left(-2\times10^{-6}\text{ C}\right)\mathbf{E}$

So $\mathbf{E} = \boxed{-1.73\times10^{4}\ \mathbf{j}\text{ N/C}}$

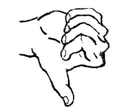
22.30 Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.



(a) $$\mathbf{B} = \frac{\mu_0 I}{2\pi r}\mathbf{k} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(5.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}\mathbf{k}$$

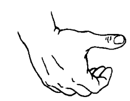
$$\mathbf{B} = \boxed{1.00\times10^{-5}\ \mathrm{T}\ \text{out of the page}}$$

(b) $$\mathbf{F}_B = I_2\boldsymbol{\ell}\times\mathbf{B} = (8.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\mathbf{i}\times\left(1.00\times10^{-5}\ \mathrm{T}\right)\mathbf{k}\right] = \left(8.00\times10^{-5}\ \mathrm{N}\right)(-\mathbf{j})$$



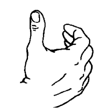
$$\boxed{\mathbf{F}_B = 8.00\times10^{-5}\ \mathrm{N}\ \text{toward the first wire}}$$

(c) $$\mathbf{B} = \frac{\mu_0 I}{2\pi r}(-\mathbf{k}) = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(8.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}(-\mathbf{k}) = \left(1.60\times10^{-5}\ \mathrm{T}\right)(-\mathbf{k})$$



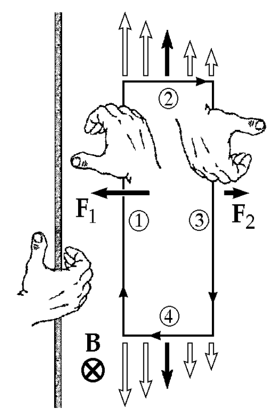
$$\mathbf{B} = \boxed{1.60\times10^{-5}\ \mathrm{T}\ \text{into the page}}$$

(d) $$\mathbf{F}_B = I_1\boldsymbol{\ell}\times\mathbf{B} = (5.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\mathbf{i}\times\left(1.60\times10^{-5}\ \mathrm{T}\right)(-\mathbf{k})\right] = \left(8.00\times10^{-5}\ \mathrm{N}\right)(+\mathbf{j})$$



$$\mathbf{F}_B = \boxed{8.00\times10^{-5}\ \mathrm{N}\ \text{towards the second wire}}$$

22.31 By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (using Equation 22.27)



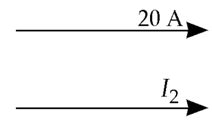
$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\mathbf{i} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{-a}{c(c+a)}\right)\mathbf{i}$$

$$\mathbf{F} = \frac{\left(4\pi\times10^{-7}\ \mathrm{N/A^2}\right)(5.00\ \mathrm{A})(10.0\ \mathrm{A})(0.450\ \mathrm{m})}{2\pi}\left(\frac{-0.150\ \mathrm{m}}{(0.100\ \mathrm{m})(0.250\ \mathrm{m})}\right)\mathbf{i}$$

$$\mathbf{F} = \left(-2.70\times10^{-5}\,\mathbf{i}\right)\ \mathrm{N}$$

or $$\mathbf{F} = \boxed{2.70\times10^{-5}\ \mathrm{N}\ \text{toward the left}}$$

***22.32** To attract, both currents must be to the right. The attraction is described by



$$F = I_2 \ell B \sin 90° = I_2 \ell \frac{\mu_0 I}{2\pi r}$$

So $$I_2 = \frac{F}{\ell}\frac{2\pi r}{\mu_0 I_1} = \left(320 \times 10^{-6}\ \mathrm{N/m}\right)\left(\frac{2\pi(0.5\ \mathrm{m})}{\left(4\pi \times 10^{-7}\ \mathrm{N \cdot s/C \cdot m}\right)(20\ \mathrm{A})}\right) = 40.0\ \mathrm{A}$$

Let y represent the distance of the zero-field point below the upper wire.

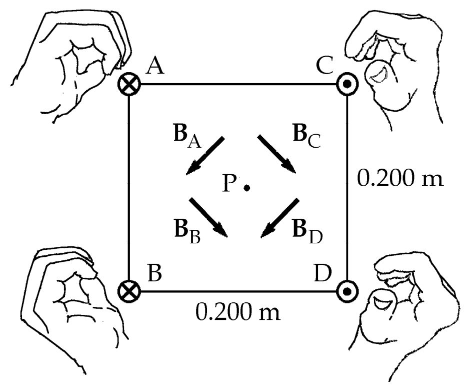
Then $\mathbf{B} = \frac{\mu_0 I}{2\pi r}$  $+ \frac{\mu_0 I}{2\pi r}$ 

$$0 = \frac{\mu_0}{2\pi}\left(\frac{20\ \mathrm{A}}{y}\ (\text{away}) + \frac{40\ \mathrm{A}}{(0.5\ \mathrm{m} - y)}\ (\text{toward})\right)$$

$$20(0.5\ \mathrm{m} - y) = 40y \qquad 20(0.5\ \mathrm{m}) = 60y$$

$$y = \boxed{0.167\ \text{m below the upper wire}}$$

22.33 Each wire is distant from P by



$$(0.200\ \mathrm{m})\cos 45.0° = 0.141\ \mathrm{m}$$

Each wire produces a field at P of equal magnitude:

$$B_A = \frac{\mu_0 I}{2\pi a} = \frac{\left(2.00 \times 10^{-7}\ \mathrm{T \cdot m/A}\right)(5.00\ \mathrm{A})}{(0.141\ \mathrm{m})} = 7.07\ \mu\mathrm{T}$$

Carrying currents into the page, A produces at P a field of 7.07 μT to the left and down at –135°, while B creates a field to the right and down at – 45°. Carrying currents toward you, C produces a field downward and to the right at – 45°, while D's contribution is downward and to the left. The total field is then

$4(7.07\ \mu\mathrm{T})\sin 45.0° = \boxed{20.0\ \mu\mathrm{T}}$ toward the bottom of the page

22.34 Let the current I be to the right. It creates a field $B = \mu_0 I / 2\pi d$ at the proton's location. And we have a balance between the weight of the proton and the magnetic force

$$mg(-\mathbf{j}) + qv(-\mathbf{i}) \times \frac{\mu_0 I}{2\pi d}(\mathbf{k}) = 0 \text{ at a distance } d \text{ from the wire}$$

$$d = \frac{qv\mu_0 I}{2\pi mg} = \frac{\left(1.60 \times 10^{-19}\ \mathrm{C}\right)\left(2.30 \times 10^{4}\ \mathrm{m/s}\right)\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)\left(1.20 \times 10^{-6}\ \mathrm{A}\right)}{2\pi\left(1.67 \times 10^{-27}\ \mathrm{kg}\right)\left(9.80\ \mathrm{m/s^2}\right)} = \boxed{5.40\ \mathrm{cm}}$$

*22.35 From Ampere's law, the magnetic field at point a is given by $B_a = \mu_0 I_a / 2\pi r_a$, where I_a is the net current through the area of the circle of radius r_a. In this case, $I_a = 1.00$ A out of the page (the current in the inner conductor), so

$$B_a = \frac{\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)(1.00 \text{ A})}{2\pi\left(1.00 \times 10^{-3} \text{ m}\right)} = \boxed{200\ \mu\text{T toward top of page}}$$

Similarly at point b: $B_b = \frac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current through the area of the circle having radius r_b.

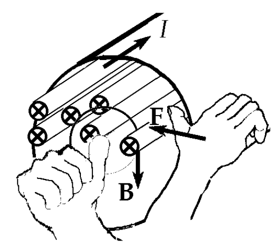
Taking out of the page as positive, $I_b = 1.00 \text{ A} - 3.00 \text{ A} = -2.00 \text{ A}$, or $I_b = 2.00$ A into the page. Therefore,

$$B_b = \frac{\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)(2.00 \text{ A})}{2\pi\left(3.00 \times 10^{-3} \text{ m}\right)} = \boxed{133\ \mu\text{T toward bottom of page}}$$

22.36 (a) $$B_{\text{inner}} = \frac{\mu_0 N I}{2\pi r} = \frac{\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)(900)\left(14.0 \times 10^3 \text{ A}\right)}{2\pi(0.700 \text{ m})} = \boxed{3.60 \text{ T}}$$

(b) $$B_{\text{outer}} = \frac{\mu_0 N I}{2\pi r} = \frac{\left(2 \times 10^{-7} \text{ T}\cdot\text{m/A}\right)(900)\left(14.0 \times 10^3 \text{ A}\right)}{1.30 \text{ m}} = \boxed{1.94 \text{ T}}$$

22.37 (a) One wire feels force due to the field of the other ninety-nine.

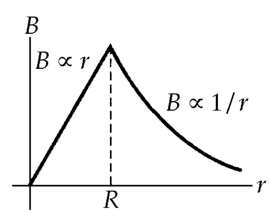


$$B = \frac{\mu_0 I_0 r}{2\pi R^2} = \frac{\left(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}\right)(99)(2.00 \text{ A})\left(0.200 \times 10^{-2} \text{ m}\right)}{2\pi\left(0.500 \times 10^{-2} \text{ m}\right)^2} = 3.17 \times 10^{-3} \text{ T}$$

This field points tangent to a circle of radius 0.200 cm and exerts force $\mathbf{F} = I\boldsymbol{\ell} \times \mathbf{B}$ toward the center of the bundle, on the single hundredth wire:

$$F/\ell = IB\sin\theta = (2.00 \text{ A})\left(3.17 \times 10^{-3} \text{ T}\right)\sin 90° = 6.34 \text{ mN/m}$$

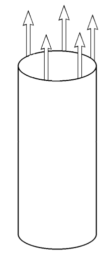
$$\frac{F_B}{\ell} = \boxed{6.34 \times 10^{-3} \text{ N/m inward}}$$



(b) $B \propto r$, so B is greatest at the outside of the bundle. Since each wire carries the same current, F is $\boxed{\text{greatest at the outer surface}}$.

22.38 From $\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 I$, $$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(1.00 \times 10^{-3}\right)(0.100)}{4\pi \times 10^{-7}} = \boxed{500 \text{ A}}$$

***22.39** We assume the current is vertically upward.



(a) Consider a circle of radius r slightly less than R. It encloses no current so from

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{\text{inside}} \qquad\qquad B(2\pi r) = 0$$

we conclude that the magnetic field is zero.

(b) Now let the r be barely larger than R. Ampere's law becomes $B(2\pi R) = \mu_0 I$,

so $\boxed{B = \dfrac{\mu_0 I}{2\pi R}}$

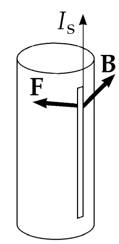
The field's direction is  $\boxed{\text{tangent to the wall of the cylinder in a counterclockwise sense}}$

(c) Consider a strip of the wall of width dx and length ℓ. Its width is so small compared to $2\pi R$ that the field at its location would be essentially unchanged if the current in the strip were turned off.

The current it carries is $I_s = \dfrac{I\,dx}{2\pi R}$ up

The force on it is $\mathbf{F} = I_s \boldsymbol{\ell}\times\mathbf{B} = \dfrac{I\,dx}{2\pi R}\left(\ell\,\dfrac{\mu_0 I}{2\pi R}\right)$ **up** $\times$ **into page** $= \dfrac{\mu_0 I^2 \ell\,dx}{4\pi^2 R^2}$ **radially inward**

The pressure on the strip and everywhere on the cylinder is



$$P = \frac{F}{A} = \frac{\mu_0 I^2 \ell\,dx}{4\pi^2 R^2 \ell\,dx} = \frac{\mu_0 I^2}{(2\pi R)^2} \text{ inward}$$

The pinch effect makes an effective demonstration when an aluminum can crushes itself as it carries a large current along its length.

22.40 The resistance of the wire is $R_e = \dfrac{\rho\ell}{\pi r^2}$, so it carries current $I = \dfrac{\mathcal{E}}{R_e} = \dfrac{\mathcal{E}\pi r^2}{\rho\ell}$

If there is a single layer of windings, the number of turns per length is the reciprocal of the wire diameter: $n = 1/2r$.

So, $$B = n\mu_0 I = \frac{\mu_0\mathcal{E}\pi r^2}{\rho\ell(2r)} = \frac{\mu_0\mathcal{E}\pi r}{2\rho\ell} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(20.0\ \text{V})\pi(2.00\times10^{-3}\ \text{m})}{2(1.70\times10^{-8}\ \Omega\cdot\text{m})(10.0\ \text{m})} = \boxed{464\ \text{mT}}$$

22.41 $$B = \mu_0\frac{N}{\ell}I \quad\text{so}\quad I = \frac{B}{\mu_0 n} = \frac{(1.00\times10^{-4}\ \text{T})0.400\ \text{m}}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})1000} = \boxed{31.8\ \text{mA}}$$

22.42 Let the axis of the solenoid lie along the y–axis from $y = 0$ to $y = \ell$. We will determine the field at $y = a$. This point will be inside the solenoid if $0 < a < \ell$ and outside if $a < 0$ or $a > \ell$. We think of solenoid as formed of rings, each of thickness dy. Now I is the symbol for the current in each turn of wire and the number of turns per length is (N/ℓ). So the number of turns in the ring is $(N/\ell)dy$ and the current in the ring is $I_{\text{ring}} = I(N/\ell)dy$. Now we use the result of Example 22.6 for the field created by one ring:

$$B_{\text{ring}} = \frac{\mu_0 I_{\text{ring}} R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

where x is the name of the distance from the center of the ring, at location y, to the field point $x = a - y$. Each ring creates field in the same direction, along our y–axis, so the whole field of the solenoid is

$$B = \sum_{\text{all rings}} B_{\text{ring}} = \sum \frac{\mu_0 I_{\text{ring}} R^2}{2\left(x^2 + R^2\right)^{3/2}} = \int_0^\ell \frac{\mu_0 I \frac{N}{\ell}\, dy\, R^2}{2\left((a-y)^2 + R^2\right)^{3/2}} = \frac{\mu_0 I N R^2}{2\ell}\int_0^\ell \frac{dy}{2\left((a-y)^2 + R^2\right)^{3/2}}$$

To perform the integral we change variables to $u = a - y$.

$$B = \frac{\mu_0 I N R^2}{2\ell}\int_a^{a-\ell} \frac{-du}{\left(u^2 + R^2\right)^{3/2}}$$

and then use the table of integrals in the appendix:

(a) $$B = \frac{\mu_0 I N R^2}{2\ell}\,\frac{-u}{R^2\sqrt{u^2 + R^2}}\Bigg|_a^{a-\ell} = \boxed{\frac{\mu_0 I N}{2\ell}\left[\frac{a}{\sqrt{a^2 + R^2}} - \frac{a-\ell}{\sqrt{(a-\ell)^2 + R^2}}\right]}$$

(b) If ℓ is much larger than R and $a = 0$,

we have $$B \cong \frac{\mu_0 I N}{2\ell}\left[0 - \frac{-\ell}{\sqrt{\ell^2}}\right] = \frac{\mu_0 I N}{2\ell}$$

This is just half the magnitude of the field deep within the solenoid. We would get the same result by substituting $a = \ell$ to describe the other end.

22.43 (a) $I = \dfrac{ev}{2\pi r}$ $\qquad$ $\mu = IA = \left(\dfrac{ev}{2\pi r}\right)\pi r^2 = \boxed{9.27 \times 10^{-24}\ \text{A} \cdot \text{m}^2}$

The Bohr model predicts the correct magnetic moment. However, the "planetary model" is seriously deficient in other regards.

(b) Because the electron is (–), its [conventional] current is clockwise, as seen from above, and μ points $\boxed{\text{downward}}$.

***22.44** We model a sample of material magnetized to saturation as a collection of equal parallel magnetic moments. The field B they together produce must be proportional to the value of each magnetic moment μ. The field must be proportional to the magnetic permeability of space μ_0, as in equation 22.26. The uniform average field is finally proportional to the number density of the magnetic moments. Although we do not prove it here, the proportionality constant is exactly 1. We have $B = \mu_0 \mu x n$ where n is the number of atoms per volume and x is the number of electrons per atom contributing.

Then $$x = \frac{B}{\mu_0 \mu n} = \frac{2.00 \text{ T}}{\left(8.50 \times 10^{28} \text{ m}^{-3}\right)\left(9.27 \times 10^{-24} \text{ N} \cdot \text{m/T}\right)\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)} = \boxed{2.02}$$

***22.45** Let N be the number of charges. For the vehicle we want

$\Sigma F_y = 0$: $$-mg + NqvB \sin 90° = 0$$

$$N = \frac{mg}{qvB} = \frac{5 \times 10^4 \text{ kg}\left(9.8 \text{ m/s}^2\right)}{10^{-6} \text{ C}(400 \text{ km/h})(1000/\text{k})(1 \text{ h}/3600 \text{ s})(0.1 \text{ N} \cdot \text{s/C} \cdot \text{m})} = \boxed{4 \times 10^{10}}$$

***22.46** The energy per distance is the effective force required to propel the vehicle:

$$\frac{W}{\Delta x} = F = \frac{W/t}{\Delta x/t} = \frac{\mathcal{P}}{v}$$

(a) $$\frac{\mathcal{P}}{v} = \frac{10^2\left(10^3 \text{ J/s}\right)(3600 \text{ s}/1 \text{ h})}{400 \times 10^3 \text{ m/h}} = 900 \text{ N} = (900 \text{ J/m})\left(\frac{1609 \text{ m}}{1 \text{ mi}}\right) = \boxed{1.4 \times 10^6 \text{ J/mi}}$$

(b) We call 20 mi/gal the fuel economy. Then 1 gal/20 mi is the measure of energy use in which we are interested:

$$\frac{W}{\Delta x} = \frac{1 \text{ gal}}{20 \text{ mi}}\left(\frac{3.786 \text{ L}}{1 \text{ gal}}\right)\left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}}\right)\left(\frac{754 \text{ kg}}{\text{m}^3}\right)\left(\frac{40 \times 10^6 \text{ J}}{\text{kg}}\right) = \boxed{5.7 \times 10^6 \text{ J/mi}}$$

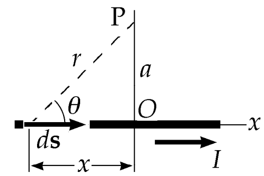
(c) One automobile passenger uses chemical energy at the rate 5.7×10^6 J/mi. One Transrapid passenger uses electric energy at the rate

$$\frac{1.4 \times 10^6 \text{ J/mi}}{100} = 1.4 \times 10^4 \text{ J/mi}, \boxed{\text{smaller by 400 times}}.$$

If we suppose that electric energy must be generated by burning a fossil fuel with limited efficiency, then a fair comparison is between the output energies propelling the vehicles, namely $0.20\left(5.7 \times 10^6 \text{ J/mi}\right) = 1.1 \times 10^6$ J/mi for the car and 1.4×10^4 J/mi for the maglev vehicle. The latter is 80 times smaller.

The dependence of American society on gasoline is a dangerous and destructive addiction which we cannot continue in the long run.

22.47 (a) The element $d\mathbf{s}$ is a distance r from P. The direction of the field at P due to this element is out of the page, since $d\mathbf{s} \times \hat{\mathbf{r}}$ is out of the page. In fact, *all* elements give contributions directly out of the page at P. Therefore we have only to determine the magnitude of the field.



Since $d\mathbf{s} = \mathbf{i}\,dx$ in this case,

we see that $\left|d\mathbf{s}\times\hat{\mathbf{r}}\right| = dx\sin\theta$

using this in Equation 22.17,

we get $$dB = \frac{\mu_0 I}{4\pi}\frac{dx\sin\theta}{r^2} \qquad (1)$$

In order to integrate this expression, we must relate the variables θ, x, and r. One approach is to express x and r in terms of θ. From the geometry in figure 22.47a and some simple differentiation, we obtain the following relationships:

$$r = \frac{a}{\sin\theta} = a\csc\theta \qquad (2)$$

Since $\tan\theta = -\frac{a}{x}$ from the right triangle in Figure 22.47a, we have $x = -a\cot\theta$, so

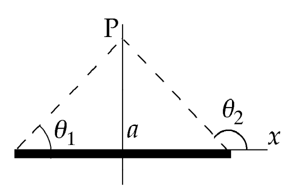
$$dx = a\csc^2\theta\,d\theta \qquad (3)$$

Substitution of (2) and (3) into (1) gives

$$dB = \frac{\mu_0 I}{4\pi a}\sin\theta\,d\theta \qquad (4)$$

Thus, we have reduced the expression to one involving only the variable θ. We can now obtain the total field at P by integrating (4) over all elements that subtend angles ranging from θ_1 to θ_2, as defined in Figure 22.47. This gives

$$B = \frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta = \frac{\mu_0 I}{4\pi a}\left(\cos\theta_1 - \cos\theta_2\right)$$



We can apply this result to find the magnetic field of any straight wire — if we know the geometry and the hence the angles θ_1 and θ_2.

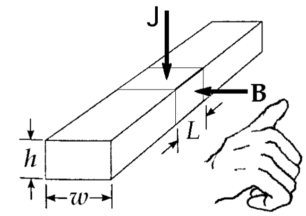
(b) Consider the point P a distance a from the wire. Then for an infinite wire,

$$\theta_1 \to 0$$

and $$\theta_2 \to -\pi \text{ radians}$$

and $$B = \frac{\mu_0 I}{2\pi a}$$

22.48 (a) Define vector $\mathbf{h}$ to have the downward direction of the current, and vector $\mathbf{L}$ to be along the pipe into the page as shown. The $\boxed{\text{electric current experiences a magnetic force}}$



$I(\mathbf{h} \times \mathbf{B})$ in the direction of $\mathbf{L}$.

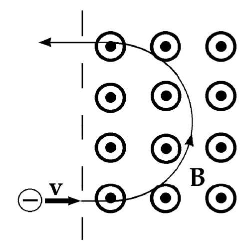
(b) The sodium, consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length L, electrons drift upward to constitute downward electric current $\mathbf{J} \times (\text{area}) = \mathbf{J}Lw$.

The current then feels a magnetic force $I|\mathbf{h} \times \mathbf{B}| = JLwhB \sin 90°$

This force along the pipe axis will make the fluid move, exerting pressure

$$\frac{F}{\text{area}} = \frac{JLwhB}{hw} = \boxed{JLB}$$

22.49 The boundary between a region of strong magnetic field and a region of zero field cannot be perfectly sharp, but we ignore the thickness of the transition zone. In the field the electron moves on an arc of a circle:



$\Sigma F = ma$:

$$|q|vB \sin 90° = \frac{mv^2}{r}$$

$$\frac{v}{r} = \omega = \frac{|q|B}{m} = \frac{(1.60 \times 10^{-19}\ \text{C})(10^{-3}\ \text{N} \cdot \text{s/C} \cdot \text{m})}{(9.11 \times 10^{-31}\ \text{kg})}$$

$$= 1.76 \times 10^{8}\ \text{rad/s}$$

The time for one half revolution is,

from $\Delta\theta = \omega\Delta t$

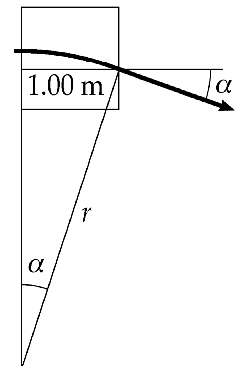
$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{\pi\ \text{rad}}{1.76 \times 10^{8}\ \text{rad/s}} = \boxed{1.79 \times 10^{-8}\ \text{s}}$$

(b) The maximum depth of penetration is the radius of the path.

Then $v = \omega r = (1.76 \times 10^{8}\ \text{s}^{-1})(0.02\ \text{m}) = 3.51 \times 10^{6}\ \text{m/s}$

and $K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(3.51 \times 10^{6}\ \text{m/s})^2 = 5.62 \times 10^{-18}\ \text{J} = \frac{5.62 \times 10^{-18}\ \text{J} \cdot \text{e}}{1.60 \times 10^{-19}\ \text{C}} = \boxed{35.1\ \text{eV}}$

22.50 The magnetic force on each proton, $\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = qvB\sin 90°$ downward perpendicular to velocity, causes centripetal acceleration, guiding it into a circular path of radius r, with



$$qvB = \frac{mv^2}{r}$$

and $$r = \frac{mv}{qB}$$

We compute this radius by first finding the proton's speed:

$$K = \tfrac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(5.00\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{1.67\times10^{-27}\text{ kg}}} = 3.10\times10^{7}\text{ m/s}$$

Now, $$r = \frac{mv}{qB} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.10\times10^{7}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.0500\text{ N}\cdot\text{s/C}\cdot\text{m}\right)} = 6.46\text{ m}$$

(b) From the figure, observe that

$$\sin\alpha = \frac{1.00\text{ m}}{r} = \frac{1\text{ m}}{6.46\text{ m}}$$

$$\boxed{\alpha = 8.90°}$$

(a) The magnitude of the proton momentum stays constant, and its final y component is

$$-\left(1.67\times10^{-27}\text{ kg}\right)\left(3.10\times10^{7}\text{ m/s}\right)\sin 8.90° = \boxed{-8.00\times10^{-21}\text{ kg}\cdot\text{m/s}}$$

***22.51** (a) The net force is the Lorentz force given by

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

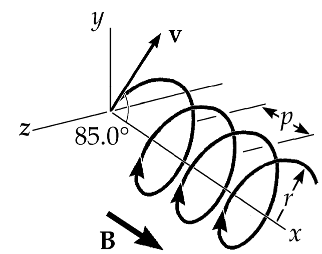
$$\mathbf{F} = \left(3.20\times10^{-19}\right)\left[(4\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}) + (2\mathbf{i} + 3\mathbf{j} - 1\mathbf{k})\times(2\mathbf{i} + 4\mathbf{j} + 1\mathbf{k})\right]\text{ N}$$

Carrying out the indicated operations, we find:

$$\mathbf{F} = \boxed{(3.52\mathbf{i} - 1.60\mathbf{j})\times10^{-18}\text{ N}}$$

(b) $$\theta = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{3.52}{\sqrt{(3.52)^2 + (1.60)^2}}\right) = \boxed{24.4°}$$

***22.52** Let v_x and $v_\perp$ be the components of the velocity of the positron parallel to and perpendicular to the direction of the magnetic field.



(a) The pitch of trajectory is the distance moved along x by the positron during each period, T (see Equation 22.5)

$$p = v_x T = (v \cos 85.0°)\left(\frac{2\pi m}{Bq}\right)$$

$$p = \frac{(5.00\times10^{6})(\cos 85.0°)(2\pi)(9.11\times10^{-31})}{0.150(1.60\times10^{-19})} = \boxed{1.04\times10^{-4}\ \text{m}}$$

(b) From Equation 22.3, $r = \frac{mv_\perp}{Bq} = \frac{mv\sin 85.0°}{Bq}$

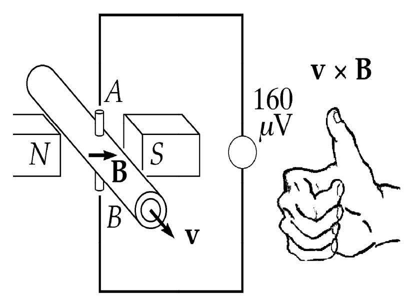
$$r = \frac{(9.11\times10^{-31})(5.00\times10^{6})(\sin 85.0°)}{(0.150)(1.60\times10^{-19})} = \boxed{1.89\times10^{-4}\ \text{m}}$$

22.53 $\Sigma F_y = 0$: $+n - mg = 0$

$\Sigma F_x = 0$: $-\mu_k n + IBd\sin 90.0° = 0$

$$B = \frac{\mu_k mg}{Id} = \frac{0.100(0.200\ \text{kg})(9.80\ \text{m/s}^2)}{(10.0\ \text{A})(0.500\ \text{m})} = \boxed{39.2\ \text{mT}}$$

***22.54** (a) The magnetic force acting on ions in the blood stream will deflect positive charges toward point A and negative charges toward point B. This separation of charges produces an electric field directed from A toward B. At equilibrium, the electric force caused by this field must balance the magnetic force, so



$$qvB = qE = q\left(\frac{\Delta V}{d}\right)$$

or $$v = \frac{\Delta V}{Bd} = \frac{(160\times10^{-6}\ \text{V})}{(0.0400\ \text{T})(3.00\times10^{-3}\ \text{m})} = \boxed{1.33\ \text{m/s}}$$

(b) $\boxed{\text{No}}$. Negative ions moving in the direction of v would be deflected toward point B, giving A a higher potential than B. Positive ions moving in the direction of v would be deflected toward A, again giving A a higher potential than B. Therefore, the sign of the potential difference does not depend on whether the ions in the blood are positively or negatively charged.

***22.55** Suppose the input power is

$120\text{ W} = (120\text{ V})I$: $\boxed{I \sim 1\text{A} = 10^0\text{ A}}$

Suppose $\omega = 2000\text{ rev/min}\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) \sim 200\text{ rad/s}$

and the output power is $20\text{ W} = \tau\omega = \tau(200\text{ rad/s})$ $\boxed{\tau \sim 10^{-1}\text{ N}\cdot\text{m}}$

Suppose the area is about $(3\text{ cm})\times(4\text{ cm})$, or $\boxed{A \sim 10^{-3}\text{ m}^2}$

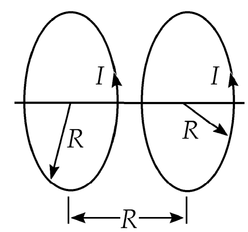
Suppose that the field is $\boxed{B \sim 10^{-1}\text{ T}}$

Then, the number of turns in the coil may be found from $\tau \cong NIAB$:

$$0.1\text{ N}\cdot\text{m} \sim N(1\text{ C/s})\left(10^{-3}\text{ m}^2\right)\left(10^{-1}\text{ N}\cdot\text{s/C}\cdot\text{m}\right)$$

giving $\boxed{N \sim 10^3}$

22.56 (a) Use equation 22.23 twice: $B_x = \dfrac{\mu_0 IR^2}{2\left(x^2+R^2\right)^{3/2}}$



$$B = B_{x1} + B_{x2} = \frac{\mu_0 IR^2}{2}\left[\frac{1}{\left(x^2+R^2\right)^{3/2}} + \frac{1}{\left((R-x)^2+R^2\right)^{3/2}}\right]$$

$$\boxed{B = \frac{\mu_0 IR^2}{2}\left[\frac{1}{\left(x^2+R^2\right)^{3/2}} + \frac{1}{\left(2R^2+x^2-2xR\right)^{3/2}}\right]}$$

If each coil has N turns, the field is just N times larger.

(b) $$\frac{dB}{dx} = \frac{\mu_0 IR^2}{2}\left[-\tfrac{3}{2}(2x)\left(x^2+R^2\right)^{-5/2} - \tfrac{3}{2}\left(2R^2+x^2-2xR\right)^{-5/2}(2x-2R)\right]$$

Substituting $x = \dfrac{R}{2}$ and canceling terms, $\boxed{\dfrac{dB}{dx} = 0}$

$$\frac{d^2B}{dx^2} = \frac{-3\mu_0 IR^2}{2}\left[\left(x^2+R^2\right)^{-5/2} - 5x^2\left(x^2+R^2\right)^{-7/2} + \left(2R^2+x^2-2xR\right)^{-5/2} - 5(x-R)^2\left(2R^2+x^2-2xR\right)^{-7/2}\right]$$

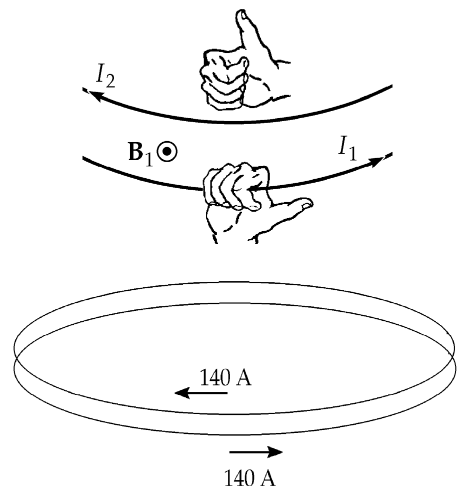
Again substituting $x = \dfrac{R}{2}$ and canceling terms, $\boxed{\dfrac{d^2B}{dx^2} = 0}$

22.57 (a) Number of unpaired electrons $= \dfrac{8.00\times 10^{22}\ \text{A}\cdot\text{m}^2}{9.27\times 10^{-24}\ \text{A}\cdot\text{m}^2} = \boxed{8.63\times 10^{45}}$

Each iron atom has two unpaired electrons, so the number of iron atoms required is $\frac{1}{2}\left(8.63\times 10^{45}\right)$.

(b) $\text{Mass} = \dfrac{\left(4.31\times 10^{45}\ \text{atoms}\right)\left(7900\ \text{kg/m}^3\right)}{\left(8.50\times 10^{28}\ \text{atoms/m}^3\right)} = \boxed{4.01\times 10^{20}\ \text{kg}}$

22.58 Model the two wires as straight parallel wires (!)



(a) $F_B = \dfrac{\mu_0 I^2 \ell}{2\pi a}$ (Equation 22.27)

$F_B = \dfrac{\left(4\pi\times 10^{-7}\right)(140)^2(2\pi)(0.100)}{2\pi\left(1.00\times 10^{-3}\right)} = \boxed{2.46\ \text{N}}$ upward

(b) $a_{\text{loop}} = \dfrac{2.46\ \text{N} - m_{\text{loop}}g}{m_{\text{loop}}} = \boxed{107\ \text{m/s}^2}$ upward

22.59 $B = \dfrac{\mu_0 I}{2\pi R} = \dfrac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.00\times 10^4\ \text{A}\right)}{2\pi(100\ \text{m})} = 2.00\times 10^{-5}\ \text{T} = \boxed{20.0\ \mu\text{T}}$

***22.60** (a) $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(24.0\ \text{A})}{2\pi(0.0175\ \text{m})} = \boxed{2.74\times 10^{-4}\ \text{T}}$

(b) At point C, conductor AB produces a field $\frac{1}{2}\left(2.74\times 10^{-4}\ \text{T}\right)(-\mathbf{j})$,  conductor DE produces a field of $\frac{1}{2}\left(2.74\times 10^{-4}\ \text{T}\right)(-\mathbf{j})$,  BD produces no field, and AE produces negligible field. The total field at C is $\boxed{2.74\times 10^{-4}\ \text{T}(-\mathbf{j})}$

(c) $\mathbf{F}_B = I\boldsymbol{\ell}\times\mathbf{B} = (24.0\ \text{A})(0.0350\ \text{m}\ \mathbf{k})\times\left[5\left(2.74\times 10^{-4}\ \text{T}\right)(-\mathbf{j})\right] = \boxed{\left(1.15\times 10^{-3}\ \text{N}\right)\mathbf{i}}$ 

(d) $\mathbf{a} = \dfrac{\Sigma\mathbf{F}}{m} = \dfrac{\left(1.15\times 10^{-3}\ \text{N}\right)\mathbf{i}}{3.00\times 10^{-3}\ \text{kg}} = \boxed{\left(0.384\ \text{m/s}^2\right)\mathbf{i}}$

(e) The bar is already so far from AE that it moves through nearly constant magnetic field. The force acting on the bar is constant, and therefore the bar's $\boxed{\text{acceleration is constant}}$

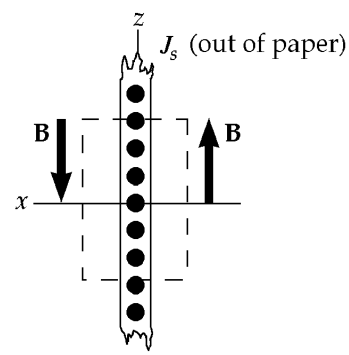
(f) $v_f^{\,2} = v_i^{\,2} + 2ax = 0 + 2\left(0.384\ \text{m/s}^2\right)(1.30\ \text{m})$, so $\mathbf{v}_f = \boxed{(0.999\ \text{m/s})\mathbf{i}}$

22.61 (a) On the axis of a current loop, the magnetic field is given by $B = \dfrac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$

where in this case $I = \dfrac{q}{(2\pi/\omega)}$. Therefore, $B = \dfrac{\mu_0 \omega R^2 q}{4\pi\left(x^2 + R^2\right)^{3/2}}$

when $x = \dfrac{R}{2}$, then $B = \dfrac{\mu_0 \omega R^2 q}{4\pi\left(\frac{5}{4}R^2\right)^{3/2}} = \boxed{\dfrac{\mu_0 q \omega}{2.5\sqrt{5}\pi R}}$

22.62 $J_s = \dfrac{I}{\ell}$



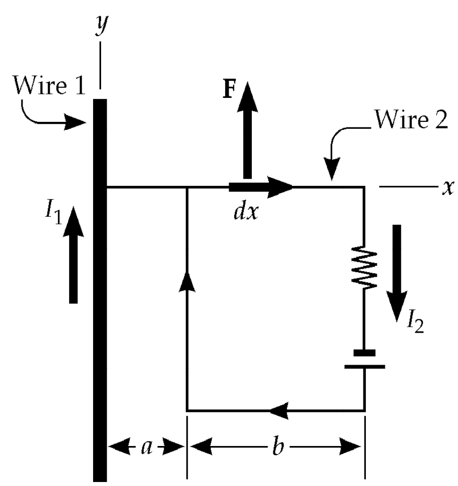
From Ampere's Law, $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$

$B \cdot 2\ell = \mu_0 J_s \ell$ $\therefore$ $\boxed{B = \dfrac{\mu_0 J_s}{2}}$

22.63 $B = \dfrac{\mu_0 I R^2}{2\left(R^2 + R^2\right)^{3/2}} = \dfrac{\mu_0 I}{2^{5/2} R}$ $I = \dfrac{2^{5/2} BR}{\mu_0} = \dfrac{2^{5/2}\left(7.00 \times 10^{-5}\ \text{T}\right)\left(6.37 \times 10^{6}\ \text{m}\right)}{\left(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}\right)}$

so $\boxed{I = 2.01 \times 10^{9}\ \text{A}}$ toward the west

22.64 Start with the force on a small segment of wire 2, given by $d\mathbf{F} = I\,d\mathbf{s} \times \mathbf{B}$ (Equation 22.11), where, in this case, $I = I_2$ and $\mathbf{B}$ is the magnetic field due to wire 1 at the position of the segment of wire 2 of length dx. From Ampere's law, the field at the distance x from wire 1 is



$$\mathbf{B} = \frac{\mu_0 I_1}{2\pi x}(-\mathbf{k})$$

where the field points into the page, as indicated by the unit vector notation ($-\mathbf{k}$). Taking the length of our segment as $d\mathbf{s} = dx\,\mathbf{i}$ we find

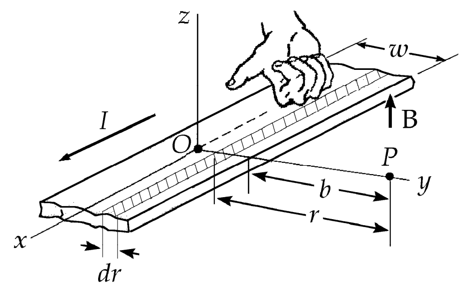
$$d\mathbf{F} = \frac{\mu_0 I_1 I_2}{2\pi x}\left[\mathbf{i} \times (-\mathbf{k})\right]dx = \frac{\mu_0 I_1 I_2}{2\pi}\frac{dx}{x}\mathbf{j}$$

Integrating this equation between the limits $x = a$ to $x = a + b$ gives

$$\mathbf{F} = \frac{\mu_0 I_1 I_2}{2\pi}\left[\ln x\right]_a^{a+b}\mathbf{j} = \frac{\mu_0 I_1 I_2}{2\pi}\ln\left(1 + \frac{b}{a}\right)\mathbf{j}\ .$$

The force points upward, as shown in Figure 22.64.

22.65 Consider a longitudinal filament of the strip of width dr as shown in the sketch. The contribution to the field at point P due to the current dI in the element dr is



$$dB = \frac{\mu_0 dI}{2\pi r}$$

where $dI = I(dr/w)$

$$\mathbf{B} = \int d\mathbf{B} = \int_b^{b+w} \frac{\mu_0 I\, dr}{2\pi w r}\mathbf{k} = \boxed{\frac{\mu_0 I}{2\pi w}\ln\left(1 + \frac{w}{b}\right)\mathbf{k}}$$

ANSWERS TO EVEN NUMBERED PROBLEMS

2. (a) 8.67×10^{-14} N (b) 5.19×10^{13} m/s^2

4. 8.93×10^{-30} N down, 1.60×10^{-17} N up, 4.80×10^{-17} N down

6. (a) 5.00 cm (b) 8.78×10^{6} m/s

8. 244 kV/m

10. (a) 7.66×10^{7} rad/s (b) 26.8 Mm/s (c) 3.76 MeV
(d) 3.13×10^{3} rev (e) 257 μs

12. (a) See the solution (b) $n = IB / qt\Delta V_H$

14. 0.109 A to the right

16. *ab*: 0, *bc*: –40.0 mN **i**, *cd*: –40.0 mN **k**, *da*: (40.0 **i** + 40.0 **k**) mN

18. (a) 5.41 mA $\cdot$ m^2 (b) 4.33 mN·m

20. See the solution

22. 261 nT into the page

24. $\left(1+\frac{1}{\pi}\right)\frac{\mu_0 I}{2R}$ directed into the page

26. See the solution

28. $(\mu_0 I / 2r)(1/\pi + 1/4)$ into the plane of the paper

30. (a) 10.0 μT out of the page (b) 80.0 μN toward the first wire
(c) 16.0 μT into the page (d) 80.0 μN toward the second wire

32. 0.167 m below the upper wire, 0.333 m above the lower wire

34. 5.40 cm

36. (a) 3.60 T (b) 1.94 T

38. 500 A

40. 464 mT

42. (a) $\frac{\mu_0 IN}{2\ell}\left(\frac{a}{\sqrt{a^2+R^2}}-\frac{a-\ell}{\sqrt{(a-\ell)^2+R^2}}\right)$ (b) See the solution

44. 2.02

46. (a) 1.4 MJ/mi (b) 5.7 MJ/mi (c) 1/400

48. (a) The electric current feels a magnetic force. (b) See the solution

50. (a) -8.00×10^{-21} kg·m/s (b) 8.90°

52. (a) 1.04×10^{-4} m (b) 1.89×10^{-4} m

54. (a) 1.33 m/s
(b) No. Positive ions moving toward you in magnetic field to the right feel upward magnetic force, and migrate upward in the blood vessel. Negative ions moving toward you feel downward magnetic force and accumulate at the bottom of this section of vessel. Thus both species can participate in the generation of the same emf.

56. See the solution

58. (a) 2.46 N up (b) 107 m/s^2 up

60. (a) 274 μT (b) -274 **j** μT (c) 1.15 **i** mN
(d) 0.384 **i** m/s^2 (e) acceleration is constant (f) 0.999 **i** m/s

62. $\mathbf{B}=\frac{\mu_0 J_s}{2}\mathbf{k}$ for $x>0$ and $\mathbf{B}=-\frac{\mu_0 J_s}{2}\mathbf{k}$ for $x<0$

64. See the solution