# CHAPTER 23

# ANSWERS TO QUESTIONS

**Q23.1** The magnetic flux is $\Phi_B = BA\cos\theta$. Therefore the flux is maximum when **B** is perpendicular to the loop of wire and zero when there is no component of magnetic field perpendicular to the loop.

**Q23.2** The force on positive charges in the bar is $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. If the bar is moving to the left, positive charge will move downward and an electric field will be established upwards.

**Q23.3** No. The magnetic force acts within the bar, but has no influence on the forward motion of the bar.

**Q23.4** Moving magnetic fields create electric fields that, if strong enough, can cause static or communications drop – outs.

**Q23.5** A current could be set up in the bracelet by moving the bracelet through a magnetic field.

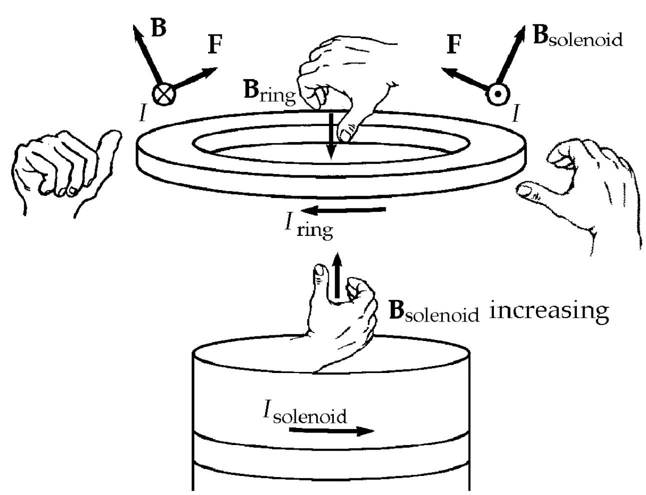
**Q23.6** As water falls, it gains velocity and kinetic energy. It then pushes against turbine blades, transferring its energy to the rotor of a large AC generator. The rotor of the generator has a DC current which powers electromagnets in the rotor. Because the rotor is spinning, the electromagnets create a magnetic flux that changes in time, $\Phi_B = BA\cos\omega t$. Coils of wire that are placed near the rotor experience an induced EMF of $\mathcal{E} = -N\,d\Phi_B/dt$. This induced EMF is the voltage source for the current in our electric power lines.

**Q23.7** Yes. The induced eddy currents on the surface of the aluminum will slow the descent of the aluminum. It will fall very slowly.

**Q23.8** By the magnetic force law $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$: the positive charges in the moving bar will flow downward and therefore clockwise in the circuit. If the bar is moving to the left, the positive charge in the bar will flow upward and therefore counterclockwise in the circuit.

**Q23.9** When the nonmagnetic but conducting sheet moves in the magnetic field, eddy currents are induced in the aluminum. These currents feel forces due to the same magnetic field, contributing to a net force opposite in direction to the motion of the sheet. The force is strong if the sheet is moving fast, efficiently damping its motion, but the force goes to zero as the speed approaches zero. The magnetic damping force cannot affect the equilibrium position of the balance beam.

**Q23.10** The counterclockwise current in the solenoid coil produces a magnetic field that increases rapidly. The increasing upward flux of this field through the ring induces an emf to produce clockwise current in the ring. The magnetic field of the solenoid has a radially outward component at each point on the ring. This field component exerts upward force on the current in the ring there. The whole ring feels a total upward force larger than its weight.



**Q23.11** Oscillating current in the solenoid produces an always — changing magnetic field. Vertical flux through the ring, alternately increasing and decreasing, produces current in it with a direction that is alternately clockwise and counterclockwise. The current through the ring's resistance produces internal energy at the rate $I^2R$.

**Q23.12** The inductance of the series combination of inductor $L_1$ and inductor $L_2$ is $L_1 + L_2 + M_{12}$, where $M_{12}$ is the mutual inductance of the two coils. It can be defined as the emf induced in coil two when the current in coil one changes at one ampere per second, due to the magnetic field of coil one producing flux through coil two.

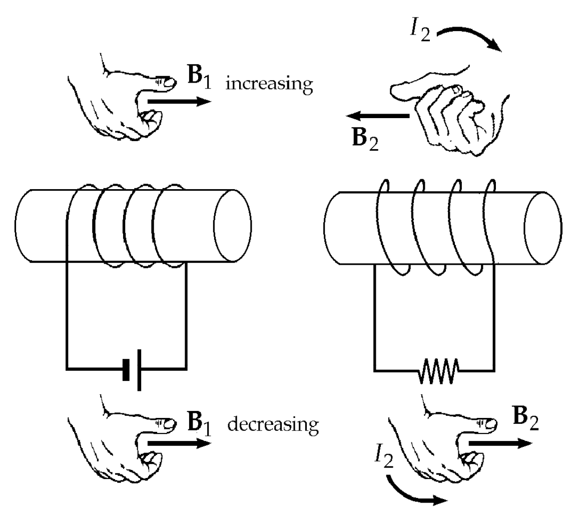
**Q23.13** (a) The south pole of the magnet produces an upward magnetic field that increases as the magnet approaches. The loop opposes change by making its own downward magnetic field; it carries current clockwise, which goes to the left through the resistor.

(b) The north pole of the magnet produces an upward magnetic field. The loop sees decreasing upward flux as the magnet falls away, and tries to make an upward magnetic field of its own by carrying current counterclockwise, to the right in the resistor.

**Q23.14** (a) The battery makes counterclockwise current $I_1$ in the primary coil, so its magnetic field $\mathbf{B}_1$ is to the right and increasing just after the switch is closed. The secondary coil will oppose the change with a leftward field $\mathbf{B}_2$, which comes from an induced clockwise current $I_2$ that goes to the right in the resistor.

(b) At steady state the primary magnetic field is unchanging, so no emf is induced in the secondary.

(c) The primary's field is to the right and decreasing as the switch is opened. The secondary coil opposes this decrease by making its own field to the right, carrying counterclockwise current to the left in the resistor.




**Q23.15** The energy stored in a capacitor is proportional to the square of the electric field, and the energy stored in an induction coil is proportional to the square of the magnetic field. The capacitor's energy is proportional to its capacitance, which depends on its geometry and the dielectric material inside. The coil's energy is proportional to its inductance, which depends on its geometry and the core material. On the other hand, we can think of Henry's discovery of self-inductance as fundamentally new. Before a certain school vacation at the Albany Academy in 1831, one could visualize the universe as consisting of only one thing, matter. All the forms of energy then known (kinetic, gravitational, elastic, internal, electrical) belonged to chunks of matter. But the energy that temporarily maintains a current in a coil after the battery is removed is not energy that belongs to any bit of matter. This energy is vastly larger than the kinetic energy of the drifting electrons in the wires. This energy belongs to the magnetic field around the coil. Beginning in 1831, Nature has forced us to admit that the universe consists of matter and also of fields, massless and invisible, known only by their effects.

**Q23.16** The energy stored in the magnetic field of an inductor is proportional to the square of the current. Doubling $I$ makes $U = \frac{1}{2}LI^2$ get four times larger.

**Q23.17** The current decreases not instantaneously but over some span of time. A spark can appear at the switch as it is opened because the self-induced voltage is maximum at this instant. The voltage can therefore cause breakdown of the air between the contacts.

**Q23.18** The motional emf between the wingtips cannot be used to run a light bulb. To connect the light, an extra insulated wire would have to be run out along each wing, making contact with the wing tip. The wings with the extra wires and the bulb constitute a single-loop circuit. As the plane flies through a uniform magnetic field, the magnetic flux through this loop is constant and zero emf is generated. On the other hand, if the magnetic field is not uniform, a large loop towed through it will generate pulses of positive and negative emf. This phenomenon has been demonstrated with a cable unreeled from the space shuttle.

**Q23.19** An object cannot exert a net force on itself. An object cannot create momentum out of nothing. A coil can induce an emf in itself. When it does so, the actual forces acting on charges in different parts of the loop add as vectors to zero.

**Q23.20** A physicist's list of constituents of the universe in 1829 might include matter, light, heat, the stuff of stars, charge, momentum, and several other entries. Our list today might include quarks, electrons, muons, tauons, and the neutrinos of matter; gravitons of gravitational fields; photons of electric and magnetic fields; W and Z particles; gluons; energy; momentum; angular momentum; charge; baryon number; three different lepton numbers; upness; downness; strangeness; charm; topness; and bottomness. Alternatively, the relativistic interconvertability of mass and energy, and of electric and magnetic fields, can be used to make the list look shorter. Some might think of the conserved quantities energy, momentum, … bottomness as properties of matter, rather than as things with their own existence. The idea of a field is not due to Henry, but rather to Faraday, to whom Henry personally demonstrated self-induction. Still the thesis stated in the question has an important germ of truth. Henry precipitated a basic change if he did not cause it. The biggest difference between the two lists is that the 1829 list does not include fields and today's list does.

# PROBLEM SOLUTIONS

***23.1** $$|\mathcal{E}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{\Delta(\mathbf{B}\cdot\mathbf{A})}{\Delta t} = \frac{(2.50\text{ T} - 0.500\text{ T})(8.00\times10^{-4}\text{ m}^2)}{1.00\text{ s}}\left(\frac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{1\text{ V}\cdot\text{C}}{1\text{ N}\cdot\text{m}}\right)$$

$$|\mathcal{E}| = 1.60\text{ mV} \qquad \text{and} \qquad I_{\text{loop}} = \frac{\mathcal{E}}{R} = \frac{1.60\text{ mV}}{2.00\ \Omega} = \boxed{0.800\text{ mA}}$$

**23.2** $$\mathcal{E} = -N\frac{\Delta BA\cos\theta}{\Delta t} = -NB\pi r^2\left(\frac{\cos\theta_f - \cos\theta_i}{\Delta t}\right) = -25.0(50.0\times10^{-6}\text{ T})\left[\pi(0.500\text{ m})^2\right]\left(\frac{\cos 180° - \cos 0°}{0.200\text{ s}}\right)$$

$$\mathcal{E} = \boxed{+9.82\text{ mV}}$$

**23.3** Noting unit conversions from $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$ and $U = qV$, the induced voltage is

$$\mathcal{E} = -N\frac{d(\mathbf{B}\cdot\mathbf{A})}{dt} = -N\left(\frac{0 - B_iA\cos\theta}{\Delta t}\right) = \frac{+200(1.60\text{ T})(0.200\text{ m}^2)\cos 0°}{20.0\times10^{-3}\text{ s}}\left(\frac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{1\text{ V}\cdot\text{C}}{\text{N}\cdot\text{m}}\right) = 3200\text{ V}$$

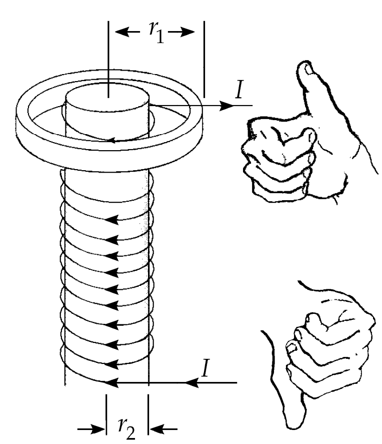
$$I = \frac{\mathcal{E}}{R} = \frac{3200\text{ V}}{20.0\ \Omega} = \boxed{160\text{ A}}$$

**23.4** $$|\mathcal{E}| = \frac{d(BA)}{dt} = 0.500\ \mu_0 nA\frac{dI}{dt} = 0.500\ \mu_0 n\pi r_2^2\frac{\Delta I}{\Delta t}$$

(a) $$I_{\text{ring}} = \frac{\mathcal{E}}{R} = \boxed{\frac{\mu_0 n\pi r_2^2}{2R}\frac{\Delta I}{\Delta t}}$$

(b) $$B = \frac{\mu_0 I}{2r_1} = \boxed{\frac{\mu_0^2 n\pi r_2^2}{4r_1 R}\frac{\Delta I}{\Delta t}}$$

(c) The coil's field points downward, and is increasing, so $\boxed{\text{B}_{\text{ring}}\text{ points upward}}$

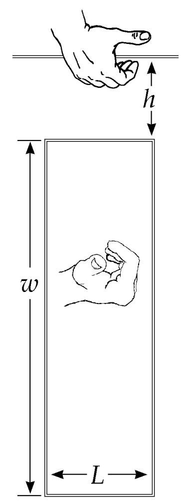


**23.5** (a) $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = \boxed{\frac{AB_{\max}}{\tau}e^{-t/\tau}}$$

(b) $$\mathcal{E} = \frac{(0.160\text{ m})^2(0.350\text{ T})}{2.00\text{ s}}e^{-4.00/2.00} = \boxed{3.79\text{ mV}}$$

(c) At $t = 0$, $\mathcal{E} = \boxed{28.0\text{ mV}}$

***23.6** (a) $d\Phi_B = \mathbf{B}\cdot d\mathbf{A} = \frac{\mu_0 I}{2\pi x}L\,dx$: $\Phi_B = \int_h^{h+w}\frac{\mu_0 IL}{2\pi}\frac{dx}{x} = \boxed{\frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)}$




(b) $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 IL}{2\pi}\ln\left(\frac{h+w}{h}\right)\right] = -\left[\frac{\mu_0 L}{2\pi}\ln\left(\frac{h+w}{h}\right)\right]\frac{dI}{dt}$$

$$\mathcal{E} = -\frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(1.00\text{ m})}{2\pi}\ln\left(\frac{1.00+10.0}{1.00}\right)(10.0\text{ A/s}) = \boxed{-4.80\ \mu\text{V}}$$

The long wire produces magnetic flux into the page through the rectangle, shown by the first hand in the figure to the right.

As the magnetic flux increases, the rectangle produces its own magnetic field out of the page, which it does by carrying $\boxed{\text{counterclockwise}}$ current (second hand in the figure).

***23.7** $$|\mathcal{E}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = N\left(\frac{dB}{dt}\right)A = N(0.0100 + 0.0800t)A$$

At $t = 5.00$ s, $$|\mathcal{E}| = 30.0(0.410\text{ T})\left[\pi(0.0400\text{ m})^2\right] = \boxed{61.8\text{ mV}}$$

**23.8** $$\Phi_B = (\mu_0 nI)A_{\text{solenoid}}$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 n\left(\pi r_{\text{solenoid}}^2\right)\frac{dI}{dt}$$

$$\mathcal{E} = -15.0\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(1.00\times10^3\text{ m}^{-1}\right)\pi(0.0200\text{ m})^2(600\text{ A/s})\cos(120t)$$

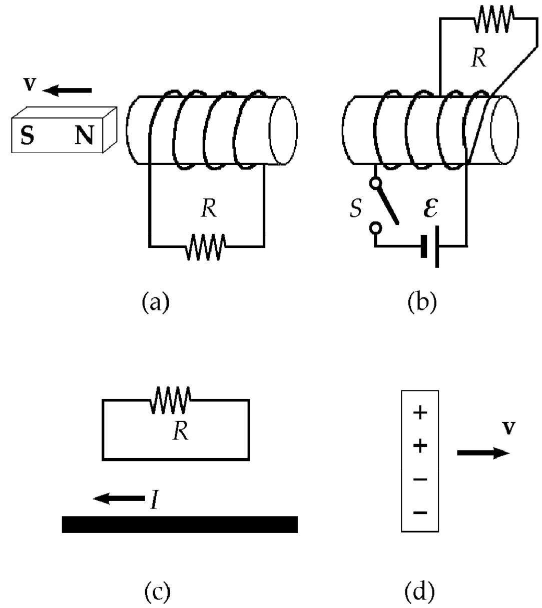
$$\boxed{\mathcal{E} = -14.2\cos(120t)\text{ mV}}$$

**23.9** (a) $\mathbf{B}_{ext} = B_{ext}\mathbf{i}$ and $B_{ext}$ decreases; therefore, the induced field is $\mathbf{B}_0 = B_0\mathbf{i}$ (to the right) and the current in the resistor is directed $\boxed{\text{to the right}}$.

(b) $\mathbf{B}_{ext} = B_{ext}(-\mathbf{i})$ increases; therefore, the induced field $\mathbf{B}_0 = B_0(+\mathbf{i})$ is to the right, and the current in the resistor is directed $\boxed{\text{to the right}}$.

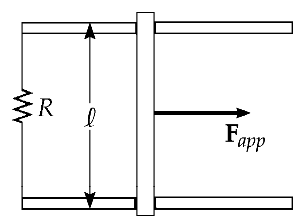
(c) $\mathbf{B}_{ext} = B_{ext}(-\mathbf{k})$ into the paper and $B_{ext}$ decreases; therefore, the induced field is $\mathbf{B}_0 = B_0(-\mathbf{k})$ into the paper, and the current in the resistor is directed $\boxed{\text{to the right}}$

(d) By the magnetic force law, $F_B = q(\mathbf{v} \times \mathbf{B})$. Therefore, a positive charge will move to the top of the bar if $\mathbf{B}$ is $\boxed{\text{into the paper}}$.




**23.10** $$I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$$

$$\boxed{v = 1.00 \text{ m/s}}$$

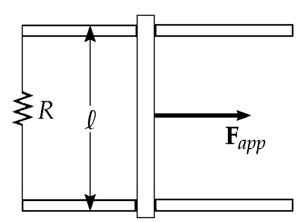



***23.11** (a) $|\mathbf{F}_B| = I|\boldsymbol{\ell} \times \mathbf{B}| = I\ell B$

When $I = \mathcal{E}/R$

and $\mathcal{E} = B\ell v$

we get $$F_B = \frac{B\ell v}{R}(\ell B) = \frac{B^2\ell^2 v}{R} = \frac{(2.50)^2(1.20)^2(2.00)}{6.00} = 3.00 \text{ N}$$




The applied force is $\boxed{3.00 \text{ N to the right}}$

(b) $$\mathcal{P} = I^2 R = \frac{B^2\ell^2 v^2}{R} = 6.00 \text{ W}$$ or $$\mathcal{P} = Fv = \boxed{6.00 \text{ W}}$$

**23.12** $F_B = I\ell B$ and $\mathcal{E} = B\ell v$

$I = \dfrac{\mathcal{E}}{R} = \dfrac{B\ell v}{R}$ so $B = \dfrac{IR}{\ell v}$

(a) $F_B = \dfrac{I^2 \ell R}{\ell v}$ and $I = \sqrt{\dfrac{F_B v}{R}} = \boxed{0.500\text{ A}}$

(b) $I^2 R = \boxed{2.00\text{ W}}$

(c) For constant force, $\mathcal{P} = \mathbf{F}\cdot\mathbf{v} = (1.00\text{ N})(2.00\text{ m/s}) = \boxed{2.00\text{ W}}$

**23.13** (a) For maximum induced emf, with positive charge at the top of the antenna,

$\mathbf{F}_+ = q_+(\mathbf{v}\times\mathbf{B})$, so the auto must move *east*

(b) $\mathcal{E} = B\ell v = \left(5.00\times10^{-5}\text{ T}\right)(1.20\text{ m})\left(\dfrac{65.0\times10^3\text{ m}}{3600\text{ s}}\right)\cos 65.0° = \boxed{4.58\times10^{-4}\text{ V}}$

**23.14** $\omega = (2.00\text{ rev/s})(2\pi\text{ rad/rev}) = 4.00\pi\text{ rad/s}$

$\mathcal{E} = \frac{1}{2}B\omega\ell^2 = \boxed{2.83\text{ mV}}$

***23.15** (a) Suppose, first, that the central wire is long and straight. The enclosed current of unknown amplitude creates a circular magnetic field around it, with the magnitude of the field given by Ampere's Law.

$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I$: $\quad B = \dfrac{\mu_0 I_{\max}\sin\omega t}{2\pi R}$

at the location of the Rogowski coil, which we assume is centered on the wire. This field passes perpendicularly through each turn of the toroid, producing flux

$$\mathbf{B}\cdot\mathbf{A} = \frac{\mu_0 I_{\max} A\sin\omega t}{2\pi R}$$

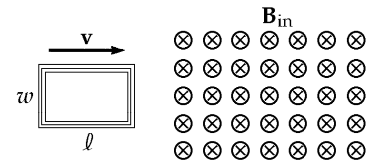
The toroid has $2\pi Rn$ turns. As the magnetic field varies, the emf induced in it is

$$\mathcal{E} = -N\frac{d}{dt}\mathbf{B}\cdot\mathbf{A} = -2\pi Rn\frac{\mu_0 I_{\max}A}{2\pi R}\frac{d}{dt}\sin\omega t = -\mu_0 I_{\max} nA\omega\cos\omega t$$

This is an alternating voltage with amplitude $\mathcal{E}_{\max} = \mu_0 nA\omega I_{\max}$. Measuring the amplitude determines the size $I_{\max}$ of the central current. Our assumptions that the central wire is long and straight and passes perpendicularly through the center of the Rogowski coil are all unnecessary.

(b) If the wire is not centered, the coil will respond to stronger magnetic fields on one side, but to correspondingly weaker fields on the opposite side. The emf induced in the coil is proportional to the line integral of the magnetic field around the circular axis of the toroid. Ampere's Law says that this line integral depends only on the amount of current the coil encloses. It does not depend on the shape or location of the current within the coil, or on any currents outside the coil.

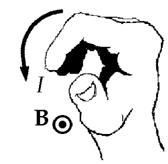
***23.16** (a) The force on the side of the coil entering the field (consisting of $N$ wires) is




$$F = N(ILB) = N(IwB)$$

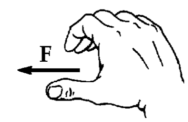
The induced emf in the coil is

$$|\mathcal{E}| = N\frac{d\Phi_B}{dt} = N\frac{d(Bwx)}{dt} = NBwv$$




so the current is $\quad I = \dfrac{|\mathcal{E}|}{R} = \dfrac{NBwv}{R}$ counterclockwise.

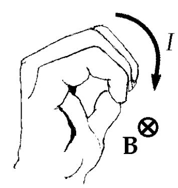
The force on the leading side of the coil is then:

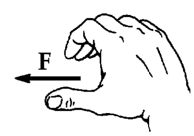



$$F = N\left(\frac{NBwv}{R}\right)wB = \boxed{\frac{N^2B^2w^2v}{R} \text{ to the left}}$$

(b) Once the coil is entirely inside the field, $\Phi_B = NBA = \text{constant}$,

so $\quad \mathcal{E} = 0,\ I = 0, \quad$ and $\quad F = \boxed{0}$

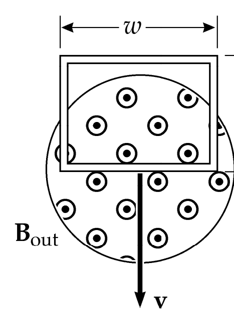



(c) As the coil starts to leave the field, the flux *decreases* at the rate $Bwv$, so the magnitude of the current is the same as in part (a), but now the current is clockwise. Thus, the force exerted on the trailing side of the coil is:

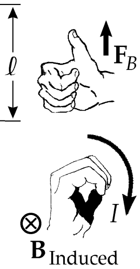
$$F = \boxed{\frac{N^2B^2w^2v}{R} \text{ to the left again}}$$

**23.17** (a) At terminal speed,




$$Mg = F_B = IwB = \left(\frac{\mathcal{E}}{R}\right)wB = \left(\frac{Bwv_T}{R}\right)wB = \frac{B^2w^2v_T}{R}$$

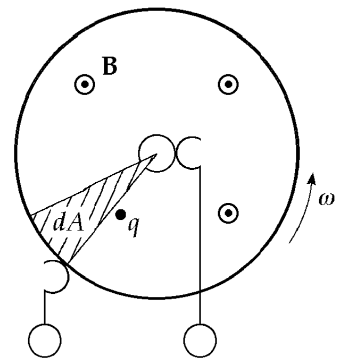
or $\quad \boxed{v_T = \dfrac{MgR}{B^2w^2}}$




(b) The emf is directly proportional to $v_T$, but the current is inversely proportional to $R$. A large $R$ means a small current at a given speed, so the loop must travel faster to get $F_B = mg$.

(c) At a given speed, the current is directly proportional to the magnetic field. But the force is proportional to the product of the current and the field. For a small $B$, the speed must increase to compensate for both the small $B$ and also the current, so $v_T \propto B^2$.

***23.18** Observe that the homopolar generator has no commutator and produces a voltage constant in time: dc with no ripple. In time $dt$, the disk turns by angle $d\theta = \omega\,dt$. The outer brush slides over distance $r\,d\theta$.




The radial line to the outer brush sweeps over area

$$dA = \tfrac{1}{2} r\,r\,d\theta = \tfrac{1}{2} r^2 \omega\,dt$$

The emf generated is $\mathcal{E} = -N \dfrac{d}{dt}\mathbf{B}\cdot\mathbf{A}$

$$\mathcal{E} = -(1)B\cos 0° \frac{dA}{dt} = -B\left(\tfrac{1}{2}r^2\omega\right)$$

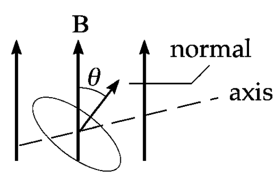
(We could think of this as following from the result of Example 23.3.)

The magnitude of the emf is $|\mathcal{E}| = B\left(\tfrac{1}{2}r^2\omega\right) = (0.9\ \mathrm{N\cdot s/C\cdot m})\left[\tfrac{1}{2}(0.4\ \mathrm{m})^2\left(3200\dfrac{\mathrm{rev}}{\mathrm{min}}\right)\right]\left(\dfrac{2\pi\ \mathrm{rad/rev}}{60\ \mathrm{s/min}}\right)$

$$|\mathcal{E}| = \boxed{24.1\ \mathrm{V}}$$

A free positive charge $q$ shown, turning with the disk, feels a magnetic force $q\mathbf{v}\times\mathbf{B}$  radially outward. Thus the $\boxed{\text{outer contact is positive}}$.

**23.19** (a) $\mathcal{E}_{\max} = NAB\omega = (1000)(0.100)(0.200)(120\pi) = \boxed{7.54\ \mathrm{kV}}$




(b) $\mathcal{E}(t) = NBA\omega\cdot\sin\omega t = NBA\omega\sin\theta$

$|\mathcal{E}|$ is maximal when $|\sin\theta| = 1$

or $\theta = \pm\dfrac{\pi}{2}$

so the $\boxed{\text{plane of coil is parallel to } \mathbf{B}}$

***23.20**

$$B = \mu_0 nI = \left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(200\ \mathrm{m^{-1}}\right)(15.0\ \mathrm{A}) = 3.77\times10^{-3}\ \mathrm{T}$$

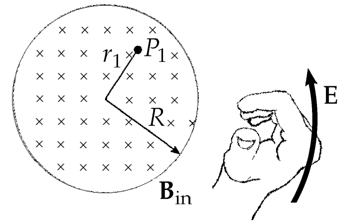
For the small coil, $\Phi_B = N\mathbf{B}\cdot\mathbf{A} = NBA\cos\omega t = NB\left(\pi r^2\right)\cos\omega t$

Thus, $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = NB\pi r^2\omega\sin\omega t$

$$\mathcal{E} = (30.0)\left(3.77\times10^{-3}\ \mathrm{T}\right)\pi(0.0800\ \mathrm{m})^2\left(4.00\pi\ \mathrm{s^{-1}}\right)\sin(4.00\pi t)$$

$$\mathcal{E} = \boxed{(28.6\ \mathrm{mV})\sin(4.00\pi t)}$$

**23.21** $\frac{dB}{dt} = 0.0600t$ $\qquad |\mathcal{E}| = \frac{d\Phi_B}{dt} = \pi r_1^2 \frac{dB}{dt} = 2\pi r_1 E$

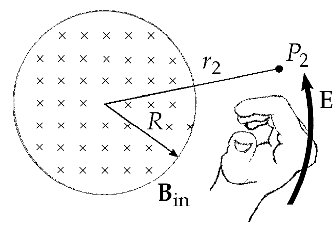



At $t = 3.00$ s,

$$E = \left(\frac{\pi r_1^2}{2\pi r_1}\right)\frac{dB}{dt} = \frac{0.0200 \text{ m}}{2}\left(0.0600 \text{ T/s}^2\right)(3.00 \text{ s})\left(\frac{1 \text{ N}\cdot\text{s}}{1 \text{ T}\cdot\text{C}\cdot\text{m}}\right)$$

$$\mathbf{E} = \boxed{1.80\times10^{-3} \text{ N/C perpendicular to } r_1 \text{ and counterclockwise}}$$

**23.22** (a) $\frac{dB}{dt} = 6.00t^2 - 8.00t$ $\qquad |\mathcal{E}| = \frac{d\Phi_B}{dt}$




At $t = 2.00$ s, $\qquad E = \frac{\pi R^2(dB/dt)}{2\pi r_2} = \frac{8.00\pi(0.0250)^2}{2\pi(0.0500)}$

$$F = qE = \boxed{8.00\times10^{-21} \text{ N}} \text{ clockwise for electron}$$

(b) When $6.00t^2 - 8.00t = 0$, $\qquad t = \boxed{1.33 \text{ s}}$

**23.23** $$|\mathcal{E}| = L\frac{\Delta I}{\Delta t} = \left(3.00\times10^{-3} \text{ H}\right)\left(\frac{1.50 \text{ A} - 0.200 \text{ A}}{0.200 \text{ s}}\right) = 1.95\times10^{-2} \text{ V} = \boxed{19.5 \text{ mV}}$$

***23.24** Treating the telephone cord as a solenoid, we have:

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi\times10^{-7} \text{ T}\cdot\text{ m/A}\right)(70.0)^2\left[\pi\left(6.50\times10^{-3} \text{ m}\right)\right]^2}{0.600 \text{ m}} = \boxed{1.36\ \mu\text{H}}$$

**23.25** $$\mathcal{E}_{back} = -\mathcal{E} = L\frac{dI}{dt} = L\frac{d}{dt}(I_{max}\sin\omega t) = L\omega I_{max}\cos\omega t = \left(10.0\times10^{-3}\right)(120\pi)(5.00)\cos\omega t$$

$$\mathcal{E}_{back} = (6.00\pi)\cos(120\pi t) = \boxed{(18.8 \text{ V})\cos(377t)}$$

***23.26** $$\overline{\mathcal{E}} = -L\frac{\Delta I}{\Delta t} = (-2.00 \text{ H})\left(\frac{0 - 0.500 \text{ A}}{0.0100 \text{ s}}\right)\left(\frac{1 \text{ V}\cdot\text{s}}{1 \text{ H}\cdot\text{A}}\right) = \boxed{100 \text{ V}}$$

**23.27** From $|\mathcal{E}| = L\left(\frac{\Delta I}{\Delta t}\right)$, we have $L = \frac{\mathcal{E}}{(\Delta I / \Delta t)} = \frac{24.0 \times 10^{-3} \text{ V}}{10.0 \text{ A/s}} = 2.40 \times 10^{-3} \text{ H}$

From $L = \frac{N\Phi_B}{I}$, we have $\Phi_B = \frac{LI}{N} = \frac{(2.40 \times 10^{-3} \text{ H})(4.00 \text{ A})}{500} = \boxed{19.2\ \mu\text{T} \cdot \text{m}^2}$

**23.28** $$|\mathcal{E}| = L\frac{dI}{dt} = (90.0 \times 10^{-3})\frac{d}{dt}(t^2 - 6t) \text{ V}$$

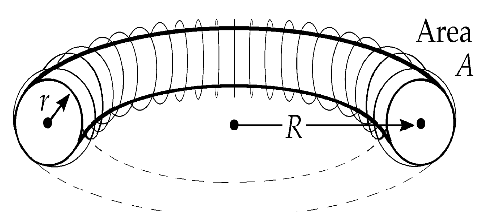
(a) At $t = 1.00$ s, $\mathcal{E} = \boxed{360 \text{ mV}}$

(b) At $t = 4.00$ s, $\mathcal{E} = \boxed{180 \text{ mV}}$

(c) $\mathcal{E} = (90.0 \times 10^{-3})(2t - 6) = 0$

when $\boxed{t = 3.00 \text{ s}}$

**23.29** $$L = \frac{N\Phi_B}{I} = \frac{NBA}{I} \approx \frac{NA}{I} \cdot \frac{\mu_0 NI}{2\pi R} = \boxed{\frac{\mu_0 N^2 A}{2\pi R}}$$

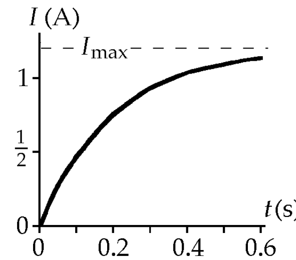



**23.30** $I = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$: $0.900\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left[1 - e^{-R(3.00 \text{ s})/2.50 \text{ H}}\right]$

$$\exp\left(-\frac{R(3.00 \text{ s})}{2.50 \text{ H}}\right) = 0.100$$

$$R = \frac{2.50 \text{ H}}{3.00 \text{ s}} \ln 10.0 = \boxed{1.92\ \Omega}$$

**23.31** At time $t$, $I(t)=\frac{\mathcal{E}\left(1-e^{-t/\tau}\right)}{R}$




where $\tau=L/R=0.200\text{ s}$

After a long time, $I_{max}=\frac{\mathcal{E}\left(1-e^{-\infty}\right)}{R}=\frac{\mathcal{E}}{R}$

At $I(t)=0.500\,I_{max}$ $(0.500)\frac{\mathcal{E}}{R}=\frac{\mathcal{E}\left(1-e^{-t/0.200\text{ s}}\right)}{R}$

so $0.500=1-e^{-t/0.200\text{ s}}$

Isolating the constants on the right, $\ln\left(e^{-t/0.200\text{ s}}\right)=\ln(0.500)$

and solving for $t$, $-\frac{t}{0.200\text{ s}}=-0.693$

or $t=\boxed{0.139\text{ s}}$

(b) Similarly, to reach 90% of $I_{max}$, $0.900=1-e^{-t/\tau}$

and $t=-\tau\ln(1-0.900)$

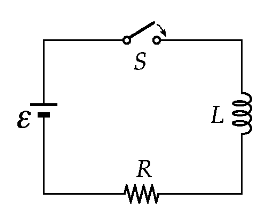
Thus, $t=-(0.200\text{ s})\ln(0.100)=\boxed{0.461\text{ s}}$

**23.32** Taking $\tau=L/R$, $I=I_0e^{-t/\tau}$: $\frac{dI}{dt}=I_0e^{-t/\tau}\left(-\frac{1}{\tau}\right)$

$IR+L\frac{dI}{dt}=0$ will be true if $I_0Re^{-t/\tau}+L\left(I_0e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right)=0$

Because $\tau=L/R$, we have agreement with $0=0$

**23.33** (a) $\tau=L/R=2.00\times10^{-3}\text{ s}=\boxed{2.00\text{ ms}}$

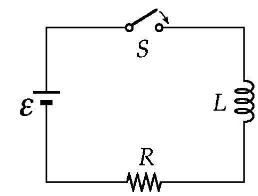



(b) $I=I_{max}\left(1-e^{-t/\tau}\right)=\left(\frac{6.00\text{ V}}{4.00\ \Omega}\right)\left(1-e^{-0.250/2.00}\right)=\boxed{0.176\text{ A}}$

(c) $I_{max}=\frac{\mathcal{E}}{R}=\frac{6.00\text{ V}}{4.00\ \Omega}=\boxed{1.50\text{ A}}$

(d) $0.800=1-e^{-t/2.00\text{ ms}}\rightarrow t=-(2.00\text{ ms})\ln(0.200)=\boxed{3.22\text{ ms}}$

***23.34** $I = I_{\max}\left(1 - e^{-t/\tau}\right)$: $0.980 = 1 - e^{-3.00\times10^{-3}/\tau}$

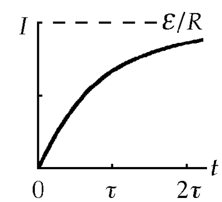



$$0.0200 = e^{-3.00\times10^{-3}/\tau}$$

$$\tau = -\frac{3.00\times10^{-3}}{\ln(0.0200)} = 7.67\times10^{-4}\ \text{s}$$

$\tau = L/R$, so $L = \tau R = \left(7.67\times10^{-4}\right)(10.0) = \boxed{7.67\ \text{mH}}$

**23.35** (a) $\Delta V_R = IR = (8.00\ \Omega)(2.00\ \text{A}) = 16.0\ \text{V}$



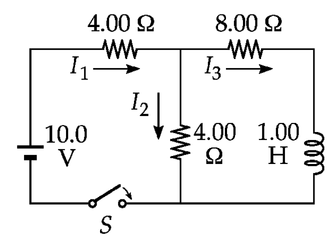

and $\Delta V_L = \mathcal{E} - \Delta V_R = 36.0\ \text{V} - 16.0\ \text{V} = 20.0\ \text{V}$

Therefore, $\dfrac{\Delta V_R}{\Delta V_L} = \dfrac{16.0\ \text{V}}{20.0\ \text{V}} = \boxed{0.800}$

(b) $\Delta V_R = IR = (4.50\ \text{A})(8.00\ \Omega) = 36.0\ \text{V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = \boxed{0}$

***23.36** Name the currents as shown. By Kirchhoff's laws:




$$I_1 = I_2 + I_3 \qquad (1)$$

$$+10.0\ \text{V} - 4.00 I_1 - 4.00 I_2 = 0 \qquad (2)$$

$$+10.0\ \text{V} - 4.00 I_1 - 8.00 I_3 - (1.00)\frac{dI_3}{dt} = 0 \qquad (3)$$

From (1) and (2), $+10.0 - 4.00 I_1 - 4.00 I_1 + 4.00 I_3 = 0$

and $I_1 = 0.500 I_3 + 1.25\ \text{A}$

Then (3) becomes $10.0\ \text{V} - 4.00\left(0.500 I_3 + 1.25\ \text{A}\right) - 8.00 I_3 - (1.00)\dfrac{dI_3}{dt} = 0$

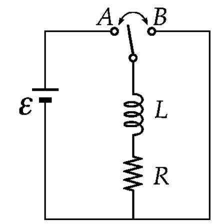
$$(1.00\ \text{H})\left(dI_3/dt\right) + (10.0\ \Omega) I_3 = 5.00\ \text{V}$$

We solve the differential equation using Equations 23.13 and 23.14:

$$I_3(t) = \left(\frac{5.00\ \text{V}}{10.0\ \Omega}\right)\left[1 - e^{-(10.0\ \Omega)t/1.00\ \text{H}}\right] = \boxed{(0.500\ \text{A})\left[1 - e^{-10t/\text{s}}\right]}$$

$$I_1 = 1.25 + 0.500 I_3 = \boxed{1.50\ \text{A} - (0.250\ \text{A})e^{-10t/\text{s}}}$$

***23.37** $\tau = \frac{L}{R} = \frac{0.140}{4.90} = 28.6 \text{ ms}$




$I_{\max} = \frac{\mathcal{E}}{R} = \frac{6.00 \text{ V}}{4.90\ \Omega} = 1.22 \text{ A}$

(a) $I = I_{\max}\left(1 - e^{-t/\tau}\right)$ so $0.220 = 1.22\left(1 - e^{-t/\tau}\right)$

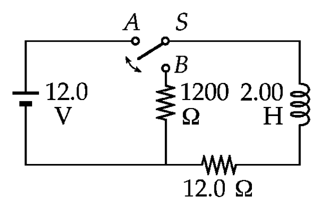
$e^{-t/\tau} = 0.820$: $t = -\tau \ln(0.820) = \boxed{5.66 \text{ ms}}$

(b) $I = I_{\max}\left(1 - e^{-\frac{10.0}{0.0286}}\right) = (1.22 \text{ A})\left(1 - e^{-350}\right) = \boxed{1.22 \text{ A}}$

(c) $I = I_{\max} e^{-t/\tau}$ and $0.160 = 1.22 e^{-t/r}$

so $t = -\tau \ln(0.131) = \boxed{58.1 \text{ ms}}$

**23.38** (a) $I = \frac{\mathcal{E}}{R} = \frac{12.0 \text{ V}}{12.0\ \Omega} = \boxed{1.00 \text{ A}}$




(b) Initial current is 1.00 A: $\Delta V_{12} = (1.00 \text{ A})(12.00\ \Omega) = \boxed{12.0 \text{ V}}$

$\Delta V_{1200} = (1.00 \text{ A})(1200\ \Omega) = \boxed{1.20 \text{ kV}}$

$\Delta V_L = \boxed{1.21 \text{ kV}}$

(c) $I = I_{\max} e^{-Rt/L}$ : $\frac{dI}{dt} = -I_{\max} \frac{R}{L} e^{-Rt/L}$

and $-L\frac{dI}{dt} = \Delta V_L = I_{\max} R e^{-Rt/L}$

Solving $12.0 \text{ V} = (1212 \text{ V}) e^{-1212t/2.00}$

so $9.90 \times 10^{-3} = e^{-606t}$

Thus, $\boxed{t = 7.62 \text{ ms}}$

***23.39** $L = \mu_0 \frac{N^2 A}{\ell} = \mu_0 \frac{(68.0)^2\left[\pi(0.600 \times 10^{-2})^2\right]}{0.0800} = 8.21\ \mu\text{H}$

$U = \frac{1}{2} L I^2 = \frac{1}{2}\left(8.21 \times 10^{-6} \text{ H}\right)(0.770 \text{ A})^2 = \boxed{2.44\ \mu\text{J}}$

**23.40** (a) The magnetic energy density is given by

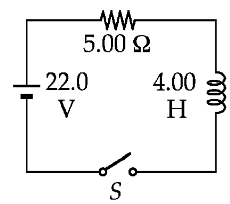
$$u = \frac{B^2}{2\mu_0} = \frac{(4.50\ \text{T})^2}{2\left(1.26\times10^{-6}\ \text{T}\cdot\text{m/A}\right)} = \boxed{8.06\times10^6\ \text{J/m}^3}$$

(b) The magnetic energy stored in the field equals $u$ times the volume of the solenoid (the volume in which $B$ is non-zero).

$$U = uV = \left(8.06\times10^6\ \text{J/m}^3\right)\left[(0.260\ \text{m})\pi(0.0310\ \text{m})^2\right] = \boxed{6.32\ \text{kJ}}$$

**23.41** $u = \epsilon_0 \dfrac{E^2}{2} = \boxed{44.2\ \text{nJ/m}^3}$ $\qquad$ $u = \dfrac{B^2}{2\mu_0} = \boxed{995\ \mu\text{J/m}^3}$

**23.42** (a) $U = \frac{1}{2}LI^2 = \frac{1}{2}(4.00\ \text{H})(0.500\ \text{A})^2$ $\qquad$ $U = \boxed{0.500\ \text{J}}$




(b) When the current is 1.00 A,

Kirchhoff's loop rule reads $\qquad +22.0\ \text{V} - (1.00\ \text{A})(5.00\ \Omega) - \Delta V_L = 0$

Then $\qquad \Delta V_L = 17.0\ \text{V}$

The power being stored in the inductor is $I\Delta V_L = (1.00\ \text{A})(17.0\ \text{V}) = \boxed{17.0\ \text{W}}$

(c) $\mathcal{P} = I\Delta V = (0.500\ \text{A})(22.0\ \text{V})$ $\qquad$ $\mathcal{P} = \boxed{11.0\ \text{W}}$

***23.43** The induced emf in the leading edge of the loop is

$$\mathcal{E} = B\ell v = B(0.2\ \text{m})(400\ \text{km/h})\left(\frac{1000\ \text{m}}{\text{km}}\right)\left(\frac{\text{h}}{3600\ \text{s}}\right) = \left(22.2\ \text{m}^2/\text{s}\right)B$$

The induced current is $\qquad I = \dfrac{\mathcal{E}}{R} = \dfrac{\left(22.2\ \text{m}^2\right)B}{25\ \Omega\cdot\text{s}}$

The magnetic force on the lower section of one loop is $\mathbf{F}_B = I\boldsymbol{\ell}\times\mathbf{B}$:

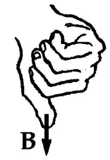
$$\mathbf{F}_B = \left(\frac{22.2\ \text{m}^2\text{A}}{25\ \text{V}\cdot\text{s}}\right)B(0.1\ \text{m})B\sin 90° = \left(0.0889\ \text{m}^3\cdot\text{C}^2/\text{J}\cdot\text{s}^2\right)B^2\ \textbf{up}$$

We require $\Sigma F_y = 0$: $\qquad 100F_B - mg = 0$

$$B^2\left(8.89\ \text{m}^2\cdot\text{C}^2/\text{N}\cdot\text{s}^2\right) = 5\times10^4(9.80\ \text{N})$$

$$B = 235\ \text{N}\cdot\text{s/C}\cdot\text{m} = \boxed{2\times10^2\ \text{T}}$$

***23.44** (a) $B = \dfrac{\mu_0 N I}{\ell} = \dfrac{(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(1400)(2.00\ \mathrm{A})}{1.20\ \mathrm{m}} = \boxed{2.93 \times 10^{-3}\ \mathrm{T}\ \text{(upward)}}$




(b) $u = \dfrac{B^2}{2\mu_0} = \dfrac{(2.93 \times 10^{-3}\ \mathrm{T})^2}{2(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})} = (3.42\ \mathrm{J/m^3})\left(\dfrac{1\ \mathrm{N\cdot m}}{1\ \mathrm{J}}\right) = 3.42\ \mathrm{N/m^2} = \boxed{3.42\ \mathrm{Pa}}$

(c) To produce a downward magnetic field, the surface of the superconductor must carry a $\boxed{\text{clockwise}}$ current.

(d) The vertical component of the field of the solenoid exerts an inward force on the superconductor. The total horizontal force is zero. Over the top end of the solenoid, its field diverges and has a radially outward horizontal component. This component exerts upward force on the clockwise superconductor current. The total force on the core is $\boxed{\text{upward}}$. You can think of it as a force of repulsion between the solenoid with its north end pointing up, and the core, with its north end pointing down.

(e) $F = PA = (3.42\ \mathrm{Pa})\left[\pi(1.10 \times 10^{-2}\ \mathrm{m})^2\right] = \boxed{1.30 \times 10^{-3}\ \mathrm{N}}$

Note that we have not proven that energy density is pressure. In fact, it is not in some cases; Equation 16.13 shows that the pressure is two-thirds of the translational energy density in an ideal gas.

***23.45** $\mathcal{E} = -N\dfrac{d}{dt}(BA\cos\theta) = -N(\pi r^2)\cos 0°\left(\dfrac{dB}{dt}\right)$

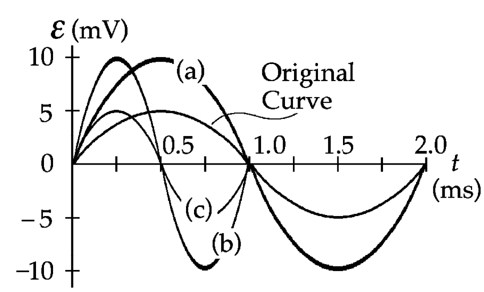
$$\mathcal{E} = -(30.0)\left[\pi(2.70 \times 10^{-3}\ \mathrm{m})^2\right](1)\frac{d}{dt}\left[50.0\ \mathrm{mT} + (3.20\ \mathrm{mT})\sin\left(2\pi\left[523t\ \mathrm{s^{-1}}\right]\right)\right]$$

$$\mathcal{E} = -(30.0)\left[\pi(2.70 \times 10^{-3}\ \mathrm{m})^2\right](3.20 \times 10^{-3}\ \mathrm{T})\left[2\pi(523\ \mathrm{s^{-1}})\cos\left(2\pi\left[523t\ \mathrm{s^{-1}}\right]\right)\right]$$

$$\mathcal{E} = \boxed{-(7.22 \times 10^{-3}\ \mathrm{V})\cos\left[2\pi(523t\ \mathrm{s^{-1}})\right]}$$

**23.46** (a) Doubling the number of turns.

$\boxed{\text{Amplitude doubles: period unchanged}}$




(b) Doubling the angular velocity.

$\boxed{\text{doubles the amplitude: cuts the period in half}}$

(c) Doubling the angular velocity while reducing the number of turns to one half the original value.

$\boxed{\text{Amplitude unchanged: cuts the period in half}}$

**23.47** We are given $\Phi_B = \left(6.00t^3 - 18.0t^2\right)\ \mathrm{T \cdot m^2}$

and $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -18.0t^2 + 36.0t$

Maximum $\mathcal{E}$ occurs when $\dfrac{d\mathcal{E}}{dt} = -36.0t + 36.0 = 0$

which gives $t = 1.00\ \mathrm{s}$

Therefore, the maximum current (at $t = 1.00$ s) is $I = \dfrac{\mathcal{E}}{R} = \dfrac{(-18.0 + 36.0)\ \mathrm{V}}{3.00\ \Omega} = \boxed{6.00\ \mathrm{A}}$

***23.48** The enclosed flux is $\Phi_B = BA = B\pi r^2$

The particle moves according to $\Sigma\mathbf{F} = m\mathbf{a}$: $qvB\sin 90° = \dfrac{mv^2}{r}$

$$r = \frac{mv}{qB}$$

Then $\Phi_B = \dfrac{B\pi m^2 v^2}{q^2 B^2}$

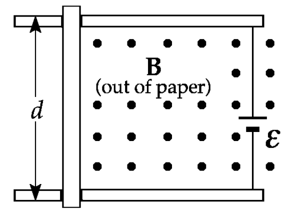
(a) $v = \sqrt{\dfrac{\Phi_B q^2 B}{\pi m^2}} = \sqrt{\dfrac{\left(15 \times 10^{-6}\ \mathrm{T \cdot m^2}\right)\left(30 \times 10^{-9}\ \mathrm{C}\right)^2 (0.6\ \mathrm{T})}{\pi\left(2 \times 10^{-16}\ \mathrm{kg}\right)^2}} = \boxed{2.54 \times 10^5\ \mathrm{m/s}}$

(b) Energy for the particle-electric field system is conserved in the firing process:

$U_i = K_f$: $\quad q\Delta V = \frac{1}{2}mv^2$

$$\Delta V = \frac{mv^2}{2q} = \frac{\left(2 \times 10^{-16}\ \mathrm{kg}\right)\left(2.54 \times 10^5\ \mathrm{m/s}\right)^2}{2\left(30 \times 10^{-9}\ \mathrm{C}\right)} = \boxed{215\ \mathrm{V}}$$

**23.49** $I = \frac{\mathcal{E} + \mathcal{E}_{\text{induced}}}{R}$ and $\mathcal{E}_{\text{induced}} = -\frac{d}{dt}(BA)$




$$F = m\frac{dv}{dt} = IBd$$

$$\frac{dv}{dt} = \frac{IBd}{m} = \frac{Bd}{mR}\left(\mathcal{E} + \mathcal{E}_{\text{induced}}\right)$$

$$\frac{dv}{dt} = \frac{Bd}{mR}\left(\mathcal{E} - Bvd\right)$$

To solve the differential equation, let $u = \mathcal{E} - Bvd$

$$\frac{du}{dt} = -Bd\frac{dv}{dt}$$

$$-\frac{1}{Bd}\frac{du}{dt} = \frac{Bd}{mR}u$$

so $\int_{u_0}^{u} \frac{du}{u} = -\int_0^t \frac{(Bd)^2}{mR}dt$

Integrating from $t = 0$ to $t = t$, $\ln\frac{u}{u_0} = -\frac{(Bd)^2}{mR}t$

or $\frac{u}{u_0} = e^{-B^2d^2t/mR}$

Since $v = 0$ when $t = 0$, $u_0 = \mathcal{E}$

and $u = \mathcal{E} - Bvd$

$$\mathcal{E} - Bvd = \mathcal{E}e^{-B^2d^2t/mR}$$

Therefore, $\boxed{v = \frac{\mathcal{E}}{Bd}\left(1 - e^{-B^2d^2t/mR}\right)}$

***23.50** (a) Consider an annulus of radius $r$, width $dr$, height $b$, and resistivity $\rho$. Around its circumference, a voltage is induced according to

$$\mathcal{E} = -N\frac{d}{dt}\mathbf{B}\cdot\mathbf{A} = -1\frac{d}{dt}B_{\max}(\cos\omega t)\pi r^2 = +B_{\max}\pi r^2\omega\sin\omega t$$

The resistance around the loop is $\dfrac{\rho\ell}{A_x} = \dfrac{\rho(2\pi r)}{b\,dr}$

The eddy current in the ring is $dI = \dfrac{\mathcal{E}}{\text{resistance}} = \dfrac{B_{\max}\pi r^2\omega(\sin\omega t)b\,dr}{\rho(2\pi r)} = \dfrac{B_{\max} r b\omega\,dr\sin\omega t}{2\rho}$

The instantaneous power is $d\mathcal{P}_i = \mathcal{E}\,dI = \dfrac{B_{\max}{}^2\pi r^3 b\omega^2\,dr\sin^2\omega t}{2\rho}$

The time average of the function $\sin^2\omega t = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$ is $\frac{1}{2} - 0 = \frac{1}{2}$

so the time-averaged power delivered to the annulus is

$$d\mathcal{P} = \frac{B_{\max}{}^2\pi r^3 b\omega^2\,dr}{4\rho}$$

The power delivered to the disk is $\mathcal{P} = \int d\mathcal{P} = \int_0^R \dfrac{B_{\max}{}^2\pi b\omega^2}{4\rho} r^3\,dr$

$$\mathcal{P} = \frac{B_{\max}{}^2\pi b\omega^2}{4\rho}\left(\frac{R^4}{4} - 0\right) = \boxed{\frac{\pi B_{\max}{}^2 R^4 b\omega^2}{16\rho}}$$

(b) When $B_{\max}$ gets two times larger, $B_{\max}^2$ and $\mathcal{P}$ get $\boxed{4}$ times larger.

(c) When $f$ and $\omega = 2\pi f$ double, $\omega^2$ and $\mathcal{P}$ get $\boxed{4}$ times larger.

(d) When $R$ doubles, $R^4$ and $\mathcal{P}$ become $2^4 = \boxed{16}$ times larger.

***23.51** $I = \dfrac{\mathcal{E}}{R} = \dfrac{B}{R}\dfrac{|\Delta A|}{\Delta t}$

so $q = I\Delta t = \dfrac{(15.0\ \mu\text{T})(0.200\ \text{m})^2}{0.500\ \Omega} = \boxed{1.20\ \mu\text{C}}$

***23.52** $\mathcal{E} = -N\dfrac{d\Phi_B}{dt} = -N\dfrac{d}{dt}(BA\cos\theta)$

$$\mathcal{E} = -NB\cos\theta\left(\frac{\Delta A}{\Delta t}\right) = -200(50.0\times10^{-6}\ \text{T})(\cos 62.0°)\left(\frac{39.0\times10^{-4}\ \text{m}^2}{1.80\ \text{s}}\right) = \boxed{-10.2\ \mu\text{V}}$$

**23.53** (a) $\mathcal{E} = -N\dfrac{d\Phi_B}{dt} = -NA\dfrac{dB}{dt} = -NA\dfrac{d}{dt}(\mu_0 nI)$ where $A$ = area of coil

$N$ = number of turns in coil

and $n$ = number of turns per unit length in solenoid.

Therefore, $|\mathcal{E}| = N\mu_0 An\dfrac{d}{dt}\left[4\sin(120\pi t)\right] = N\mu_0 An(480\pi)\cos(120\pi t)$

$$|\mathcal{E}| = 40\left(4\pi\times10^{-7}\right)\left[\pi(0.0500\text{ m})^2\right]\left(2.00\times10^3\right)(480\pi)\cos(120\pi t)$$

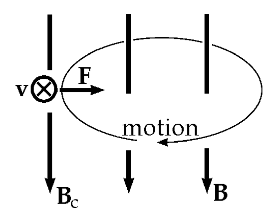
$$|\mathcal{E}| = \boxed{(1.19\text{ V})\cos(120\pi t)}$$

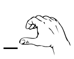
(b) $I = \dfrac{\Delta V}{R}$ and $\mathcal{P} = \Delta VI = \dfrac{(1.19\text{ V})^2\cos^2(120\pi t)}{8.00\ \Omega}$

From $\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$

the average value of $\cos^2\theta$ is $\frac{1}{2}$, so $\overline{\mathcal{P}} = \dfrac{1}{2}\dfrac{(1.19\text{ V})^2}{(8.00\ \Omega)} = \boxed{88.5\text{ mW}}$

***23.54** Suppose the field is vertically down. When an electron is moving away from you the force on it is in the direction given by




$q\mathbf{v}\times\mathbf{B}_c$ as $-(\text{away})\times\text{ down} = -$  $= -\text{left} = \text{right}$

Therefore, the electrons circulate clockwise.

(a) As the downward field increases, an emf is induced to produce some current that in turn produces an upward field. This current is directed  counterclockwise, carried by negative electrons moving clockwise. Therefore the original electron motion speeds up.

(b) At the circumference, we have $\Sigma F_c = ma_c$: $|q|vB_c\sin 90° = \dfrac{mv^2}{r}$

$$mv = |q|rB_c$$

The increasing magnetic field $\mathbf{B}_{av}$ in the area enclosed by the orbit produces a tangential electric field according to

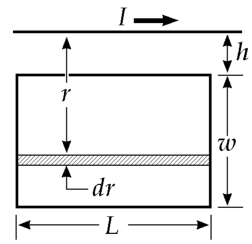
$\left|\oint \mathbf{E}\cdot d\mathbf{s}\right| = \left|-\dfrac{d}{dt}\mathbf{B}_{av}\cdot\mathbf{A}\right|$ $\quad E(2\pi r) = \pi r^2\dfrac{dB_{av}}{dt}$ $\quad E = \dfrac{r}{2}\dfrac{dB_{av}}{dt}$

An electron feels a tangential force according to $\Sigma F_t = ma_t$: $|q|E = m\dfrac{dv}{dt}$

Then $|q|\dfrac{r}{2}\dfrac{dB_{av}}{dt} = m\dfrac{dv}{dt}$ $\quad |q|\dfrac{r}{2}B_{av} = mv = |q|rB_c$

and $B_{av} = 2B_c$

**23.55** The magnetic field produced by the current in the straight wire is perpendicular to the plane of the coil at all points within the coil. The magnitude of the field is $B = \mu_0 I/2\pi r$. Thus, the flux linkage is




$$N\Phi_B = \frac{\mu_0 NIL}{2\pi}\int_h^{h+w}\frac{dr}{r} = \frac{\mu_0 NI_{max}L}{2\pi}\ln\left(\frac{h+w}{h}\right)\sin(\omega t+\phi)$$

Finally, the induced emf is

$$\mathcal{E} = -\frac{\mu_0 N I_{max} L\omega}{2\pi}\ln\left(1+\frac{w}{h}\right)\cos(\omega t+\phi)$$

$$\mathcal{E} = -\frac{\left(4\pi\times10^{-7}\right)(100)(50.0)(0.200\text{ m})\left(200\pi\text{ s}^{-1}\right)}{2\pi}\ln\left(1+\frac{5.00\text{ cm}}{5.00\text{ cm}}\right)\cos(\omega t+\phi)$$

$$\mathcal{E} = \boxed{-(87.1\text{ mV})\cos(200\pi t+\phi)}$$

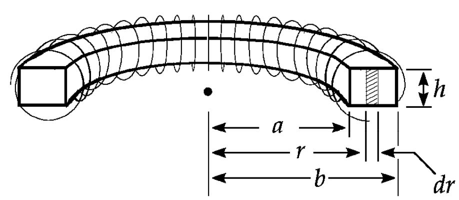
The term $\sin(\omega t+\phi)$ in the expression for the current in the straight wire does not change appreciably when $\omega t$ changes by 0.10 rad or less. Thus, the current does not change appreciably during a time interval

$$\Delta t < \frac{0.10}{\left(200\pi\text{ s}^{-1}\right)} = 1.6\times10^{-4}\text{ s}$$

We define a critical length, $c\Delta t = \left(3.00\times10^{8}\text{ m/s}\right)\left(1.6\times10^{-4}\text{ s}\right) = 4.8\times10^{4}\text{ m}$ equal to the distance to which field changes could be propagated during an interval of $1.6\times10^{-4}$ s. This length is so much larger than any dimension of the coil or its distance from the wire that, although we consider the straight wire to be infinitely long, we can also safely ignore the field propagation effects in the vicinity of the coil. Moreover, the phase angle can be considered to be constant along the wire in the vicinity of the coil.

If the frequency $\omega$ were much larger, say, $200\pi\times10^{5}\text{ s}^{-1}$, the corresponding critical length would be only 48 cm. In this situation propagation effects would be important and the above expression for $\mathcal{E}$ would require modification. As a "rule of thumb" we can consider field propagation effects for circuits of laboratory size to be negligible for frequencies, $f = \omega/2\pi$, that are less than about $10^6$ Hz.

***23.56** Equation 22.31: $B = \dfrac{\mu_0 NI}{2\pi r}$




(a) $\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 NI}{2\pi r}h\,dr = \frac{\mu_0 NIh}{2\pi}\int_a^b\frac{dr}{r} = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$

$$L = \frac{N\Phi_B}{I} = \boxed{\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)}$$

(b) $L = \dfrac{\mu_0(500)^2(0.0100)}{2\pi}\ln\left(\dfrac{12.0}{10.0}\right) = \boxed{91.2\ \mu\text{H}}$

(c) $L_{appx} = \dfrac{\mu_0 N^2}{2\pi}\left(\dfrac{A}{R}\right) = \dfrac{\mu_0(500)^2}{2\pi}\left(\dfrac{2.00\times10^{-4}\text{ m}^2}{0.110}\right) = \boxed{90.9\ \mu\text{H}}$, only 0.3% different.

**23.57** (a) At the center, $B = \frac{N\mu_0 I R^2}{2\left(R^2+0^2\right)^{3/2}} = \frac{N\mu_0 I}{2R}$

So the coil creates flux through itself $\Phi_B \approx BA\cos\theta = \frac{N\mu_0 I}{2R}\pi R^2 \cos 0° = \frac{\pi}{2}N\mu_0 IR$

When the current it carries changes, $\mathcal{E}_L = -N\frac{d\Phi_B}{dt} \approx -N\left(\frac{\pi}{2}\right)N\mu_0 R\frac{dI}{dt} = -L\frac{dI}{dt}$

so $\boxed{L \approx \frac{\pi}{2}N^2\mu_0 R}$

(b) $2\pi r \approx 3(0.3\text{ m})$ so $r \approx 0.14\text{ m}$

$$L \approx \frac{\pi}{2}\left(1^2\right)\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(0.14\text{ m}) = 2.8\times10^{-7}\text{ H}$$

$\boxed{L \sim 100\text{ nH}}$

(c) $\frac{L}{R} = \frac{2.8\times10^{-7}\text{ V}\cdot\text{s/A}}{270\text{ V/A}} = 1.0\times10^{-9}\text{ s}$ $\boxed{\frac{L}{R} \sim 1\text{ ns}}$

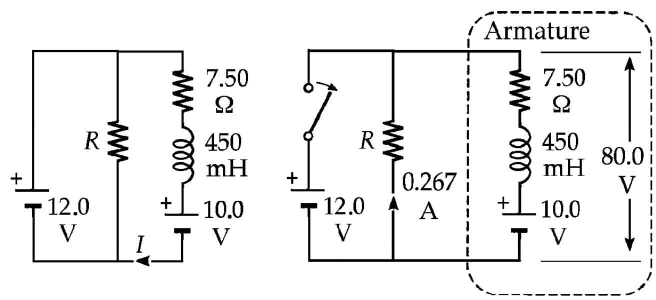
***23.58** When the switch is closed, as shown in figure (a), the current in the inductor is $I$:

$12.0 - 7.50I - 10.0 = 0 \rightarrow I = 0.267\text{ A}$

When the switch is opened, the initial current in the inductor remains at 0.267 A.

$IR = \Delta V$: $(0.267\text{ A})R \le 80.0\text{ V}$

$\boxed{R \le 300\ \Omega}$




(a) (b)

***23.59** For an $RL$ circuit,

$I(t) = I_{\max}e^{-\frac{R}{L}t}$: $\frac{I(t)}{I_{\max}} = 1 - 10^{-9} = e^{-\frac{R}{L}t} \cong 1 - \frac{R}{L}t$

$\frac{R}{L}t = 10^{-9}$ so $R_{\max} = \frac{\left(3.14\times10^{-8}\right)\left(10^{-9}\right)}{(2.50\text{ yr})\left(3.16\times10^{7}\text{ s/yr}\right)} = \boxed{3.97\times10^{-25}\ \Omega}$

(If the ring were of purest copper, of diameter 1 cm, and cross-sectional area 1 mm$^2$, its resistance would be at least $10^{-6}\ \Omega$).

**23.60** (a) $U_B = \frac{1}{2}LI^2 = \frac{1}{2}(50.0\text{ H})\left(50.0\times10^3\text{ A}\right)^2 = \boxed{6.25\times10^{10}\text{ J}}$

(b) Two adjacent turns are parallel wires carrying current in the same direction. Since the loops have such large radius, a one-meter section can be regarded as straight.

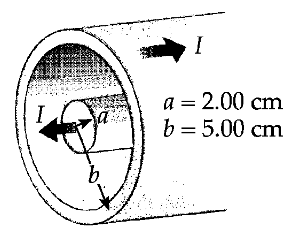
Then one wire creates a field of $B = \dfrac{\mu_0 I}{2\pi r}$

This causes a force on the next wire of $F = I\ell B\sin\theta$

giving $F = I\ell\dfrac{\mu_0 I}{2\pi r}\sin 90° = \dfrac{\mu_0 \ell I^2}{2\pi r}$

Solving for the force $F = \left(4\pi\times10^{-7}\text{ N/A}^2\right)\dfrac{(1.00\text{ m})\left(50.0\times10^3\text{ A}\right)^2}{2\pi(0.250\text{ m})} = \boxed{2000\text{ N}}$

***23.61** $\mathcal{P} = I\Delta V$ $\quad I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{1.00\times10^9\text{ W}}{200\times10^3\text{ V}} = 5.00\times10^3\text{ A}$




From Ampere's law, $B(2\pi r) = \mu_0 I_{\text{enclosed}}$ or $B = \dfrac{\mu_0 I_{\text{enclosed}}}{2\pi r}$

(a) At $r = a = 0.0200$ m, $I_{\text{enclosed}} = 5.00\times10^3$ A

and $B = \dfrac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(5.00\times10^3\text{ A}\right)}{2\pi(0.0200\text{ m})} = 0.0500\text{ T} = \boxed{50.0\text{ mT}}$

(b) At $r = b = 0.0500$ m, $I_{\text{enclosed}} = I = 5.00\times10^3$ A

and $B = \dfrac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(5.00\times10^3\text{ A}\right)}{2\pi(0.0500\text{ m})} = 0.0200\text{ T} = \boxed{20.0\text{ mT}}$

(c) $U = \int u\,dV = \int_a^b \dfrac{[B(r)]^2(2\pi r\ell\,dr)}{2\mu_0} = \dfrac{\mu_0 I^2\ell}{4\pi}\int_a^b\dfrac{dr}{r} = \dfrac{\mu_0 I^2\ell}{4\pi}\ln\left(\dfrac{b}{a}\right)$

$$U = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(5.00\times10^3\text{ A}\right)^2\left(1000\times10^3\text{ m}\right)}{4\pi}\ln\left(\frac{5.00\text{ cm}}{2.00\text{ cm}}\right) = 2.29\times10^6\text{ J} = \boxed{2.29\text{ MJ}}$$

(d) The magnetic field created by the inner conductor exerts a force of repulsion on the current in the outer sheath. The strength of this field, from part (b), is 20.0 mT. Consider a small rectangular section of the outer cylinder of length $\ell$ and width $w$.

It carries a current of $\left(5.00\times10^3\text{ A}\right)\left(\dfrac{w}{2\pi(0.0500\text{ m})}\right)$

and experiences an outward force $F = I\ell B\sin\theta = \dfrac{\left(5.00\times10^3\text{ A}\right)w}{2\pi(0.0500\text{ m})}\ell\left(20.0\times10^{-3}\text{ T}\right)\sin 90.0°$

The pressure on it is $P = \dfrac{F}{A} = \dfrac{F}{w\ell} = \dfrac{\left(5.00\times10^3\text{ A}\right)\left(20.0\times10^{-3}\text{ T}\right)}{2\pi(0.0500\text{ m})} = \boxed{318\text{ Pa}}$

# ANSWERS TO EVEN NUMBERED PROBLEMS

**2.** 9.82 mV

**4.** (a) $\dfrac{\mu_0 n \pi r_2^{\,2}}{2R}\left(\dfrac{\Delta I}{\Delta t}\right)$ (b) $\dfrac{\mu_0^{\,2} n \pi r_2^{\,2}}{4 r_1 R}\left(\dfrac{\Delta I}{\Delta t}\right)$ (c) upward

**6.** (a) $(\mu_0 IL/2\pi)\ln(1+w/h)$ (b) – 4.80 $\mu$V; current is counterclockwise

**8.** –(14.2 mV) cos (120 $t$)

**10.** 1.00 m/s

**12.** (a) 0.500 A (b) 2.00 W (c) 2.00 W

**14.** 2.83 mV

**16.** (a) $F = N^2B^2w^2v/R$ to the left (b) 0
(c) $F = N^2B^2w^2v/R$ to the left

**18.** 24.1 V with the outer contact positive

**20.** (28.6 mV) sin (4.00$\pi$ $t$)

**22.** (a) $8.00\times10^{-21}$ N, clockwise (b) 1.33 s

**24.** 1.36 $\mu$H

**26.** 100 V

**28.** (a) 360 mV (b) 180 mV (c) 3.00 s

**30.** 1.92 Ω

**32.** See the solution

**34.** 7.67 mH

**36.** $(500\text{ mA})\left(1-e^{-10t/\text{s}}\right)$, $1.50\text{A}-(0.250\text{ A})e^{-10t/\text{s}}$

**38.** (a) 1.00 A (b) 12.0 V, 1.20 kV, 1.21 kV (c) 7.62 ms

**40.** (a) $8.06\times10^{6}\text{ J/m}^{3}$ (b) 6.32 kJ

**42.** (a) 0.500 J (b) 17.0 W (c) 11.0 W

**44.** (a) 2.93 mT up (b) 3.42 Pa (c) clockwise
(d) up (e) 1.30 mN

**46.** See the solution. (a) Doubling $N$ doubles amplitude.
(b) Doubling $\omega$ doubles amplitude and halves period.
(c) Doubling $\omega$ and halving $N$ leaves the amplitude the same and cuts the period in half.

**48.** (a) 254 km/s (b) 215 V

**50.** (a) $\dfrac{\pi B_{\max}{}^{2}R^{4}\omega^{2}b}{16\rho}$ (b) 4
(c) 4 (d) 16

**52.** –10.2 $\mu$V

**54.** See the solution

**56.** (a) See the solution (b) 91.2 $\mu$H (c) 90.9 $\mu$H

**58.** 300 Ω

**60.** (a) 62.5 GJ (b) 2000 N