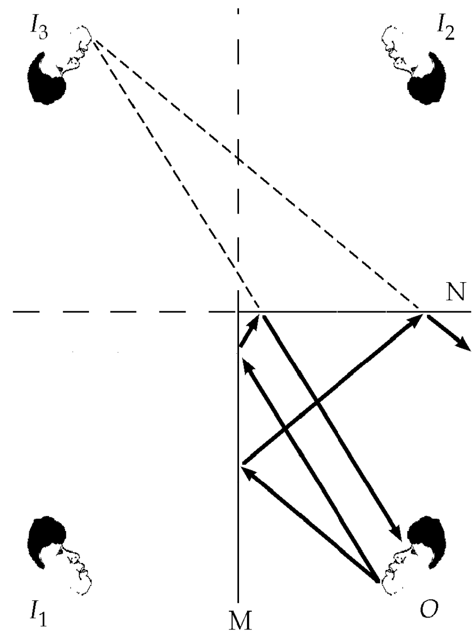
CHAPTER 25

ANSWERS TO QUESTIONS

Q25.1 The three mirrors, two of which are shown as M and N in the figure to the right, reflect any incident ray back parallel to its original direction. When you look into the corner you see image I_3 of yourself.



Q25.2 The beam reflected from the corner reflector goes directly back along the straight-line path from which it came. Therefore, by measuring the length of time Δt from transmission to reception, one can determine the distance to the moon as $d = \frac{1}{2} c \Delta t$.

Q25.3 The right-hand fish image is light from the right side of the actual fish, refracted toward the observer, and the second image is light from the left side of the fish refracted toward the observer.

Q25.4 Yes. The wavelength decreases. The frequency remains constant. The speed diminishes by a factor equal to the index of refraction.

Q25.5 Light coming up from underwater is bent away from the normal. Therefore the part of the oar that is submerged appears bent upward.

Q25.6 A faceted diamond or a stone of cubic zirconia sparkles because the light entering the stone from above is totally internally reflected and the stone is cut so the light can only escape back out the top. If the diamond or the cubic zirconia is immersed in a high index of refraction liquid, then the total internal reflection is thwarted and the diamond loses its "sparkle". For an exact match of index of refraction between cubic zirconia and corn syrup, the cubic zirconia stone would be invisible.

Q25.7 If a laser beam enters a sugar solution with a concentration gradient (density and index of refraction increasing with depth) then the laser beam will be progressively bent downward (toward the normal) as it passes into regions of greater index of refraction.

Q25.8 The ray approximation (sharp shadows) is valid for $\lambda << d$. For $\lambda \sim d$ diffraction effects become important, and the light waves will spread out beyond the slit.

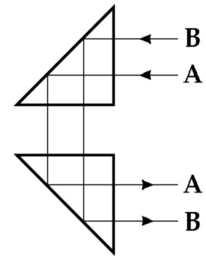
Q25.9 The index of refraction of water is 1.33, quite different from 1.00 for air. Babies learn that the refraction of light going through the water indicates the water is there. On the other hand, the index of refraction of liquid helium is close to that of air, so it gives little visible evidence of its presence.

Q25.10 Take a half-circular disk of plastic. Center it on a piece of polar-coordinate paper on a horizontal corkboard. Slowly move a pin around the curved side while you look for it, gazing at the center of the flat wall. When you can barely see the pin as your line of sight grazes the flat side of the block, the light from the pin is reaching the origin at the critical angle θ_c. You can conclude that the index of refraction of the plastic is $1/\sin\theta_c$.

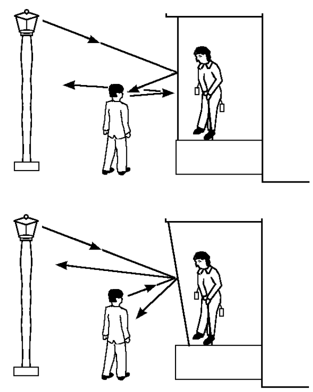
Q25.11 The index of refraction of diamond varies with the frequency of the light. Different color-components of the white light are refracted off in different directions by the jewel. The diamond disperses light to form a spectrum, as any prism does.

Q25.12 Diamond has higher index of refraction than glass and consequently a smaller critical angle for total internal reflection. A brilliant-cut diamond is shaped to admit light from above, reflect it totally at the converging facets on the underside of the jewel, and let the light escape only at the top. Glass will have less light internally reflected.

Q25.13 The image is upside down. As is shown in the figure to the right, ray *B*, from the upper part of the field of view, comes out in the lower part of the image.



Q25.14 With a vertical shop window, streetlights and his own reflection can impede the window shopper's clear view of the display. The tilted shop window can put these reflections out of the way. Windows of airport control towers are also tilted like this, as are automobile windshields.



Q25.15 Refer to Figure 25.16 in the textbook. Suppose the Sun is low in the sky and an observer faces away from the Sun toward a large uniform rain shower. A ray of light passing overhead strikes a drop of water. The light is refracted first at the front surface of the drop, with the violet light deviating the most and the red light the least. At the back of the drop the light is reflected and it returns to the front surface where it again undergoes refraction with additional dispersion as it moves from water into air. The rays leave the drop so that the angle between the incident white light and the most intense returning violet light is 40°, and the angle between the white light and the most intense returning red light is 42°. The observer can see a ring of raindrops shining violet, a ring with angular radius 40° around her shadow. From the locus of directions at 42° away from the antisolar direction the observer receives red light. The other spectral colors make up the rainbow in between.

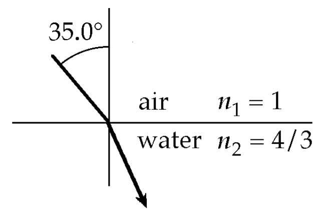
Q25.16 Refer to the answer to question 15. An observer of a rainbow sees violet light at 40° angular separation from the direction opposite the Sun, then the other spectral colors, and then red light on the outside the rainbow, with angular radius 42°.

Q25.17 A mirage occurs when light changes direction as it moves between batches of air having different indices of refraction because they have different densities at different temperatures. When the sun makes a blacktop road hot, an apparent wet spot is bright due to refraction of light from bright sky. The light, originally headed a little below the horizontal, always bends up as it first enters and then leaves sequentially hotter, lower-density, lower-index layers of air closer to the road surface.

Q25.18 The light with the greater change in speed will have the larger deviation. If the glass has a higher index than the surrounding medium, *X* travels slower in the glass.

PROBLEM SOLUTIONS

25.1 Using Snell's law, $\sin\theta_2 = \dfrac{n_1}{n_2}\sin\theta_1$



$$\theta_2 = \boxed{25.5°}$$

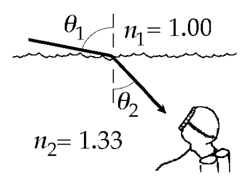
$$\lambda_2 = \frac{\lambda_1}{n_1} = \boxed{442\text{ nm}}$$

25.2 (a) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\text{ m/s}}{6.328\times10^{-7}\text{ m}} = \boxed{4.74\times10^{14}\text{ Hz}}$

(b) $\lambda_{\text{glass}} = \dfrac{\lambda_{\text{air}}}{n} = \dfrac{632.8\text{ nm}}{1.50} = \boxed{422\text{ nm}}$

(c) $v_{\text{glass}} = \dfrac{c_{\text{air}}}{n} = \dfrac{3.00\times10^{8}\text{ m/s}}{1.50} = 2.00\times10^{8}\text{ m/s} = \boxed{200\text{ Mm/s}}$

25.3 $n_1\sin\theta_1 = n_2\sin\theta_2$



$\sin\theta_1 = 1.333\sin 45°$

$\sin\theta_1 = (1.33)(0.707) = 0.943$

$\theta_1 = 70.5° \rightarrow \boxed{19.5°\text{ above the horizon}}$

25.4 (a) $n_1\sin\theta_1 = n_2\sin\theta_2$

$1.00\sin 30.0° = n\sin 19.24°$

$n = \boxed{1.52}$

(c) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\text{ m/s}}{6.328\times10^{-7}\text{ m}} = \boxed{4.74\times10^{14}\text{ Hz}}$ in air and in syrup.

(d) $v = \dfrac{c}{n} = \dfrac{3.00\times10^{8}\text{ m/s}}{1.52} = 1.98\times10^{8}\text{ m/s} = \boxed{198\text{ Mm/s}}$

(b) $\lambda = \dfrac{v}{f} = \dfrac{1.98\times10^{8}\text{ m/s}}{4.74\times10^{14}\text{ /s}} = \boxed{417\text{ nm}}$

***25.5** (a) Flint Glass: $v=\frac{c}{n}=\frac{3.00\times10^8\text{ m/s}}{1.66}=1.81\times10^8\text{ m/s}=\boxed{181\text{ Mm/s}}$

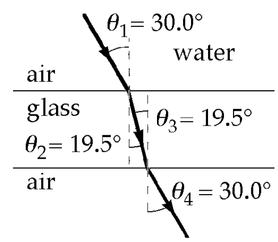
(b) Water: $v=\frac{c}{n}=\frac{3.00\times10^8\text{ m/s}}{1.333}=2.25\times10^8\text{ m/s}=\boxed{225\text{ Mm/s}}$

(c) Cubic Zirconia: $v=\frac{c}{n}=\frac{3.00\times10^8\text{ m/s}}{2.20}=1.36\times10^8\text{ m/s}=\boxed{136\text{ Mm/s}}$

***25.6** $n_1\sin\theta_1=n_2\sin\theta_2$: $1.333\sin 37.0°=n_2\sin 25.0°$

$n_2=1.90=\frac{c}{v}$: $v=\frac{c}{1.90}=1.58\times10^8\text{ m/s}=\boxed{158\text{ Mm/s}}$

25.7 $n_1\sin\theta_1=n_2\sin\theta_2$: $\theta_2=\sin^{-1}\left(\frac{n_1\sin\theta_1}{n_2}\right)$



$$\theta_2=\sin^{-1}\left\{\frac{1.00\sin 30°}{1.50}\right\}=\boxed{19.5°}$$

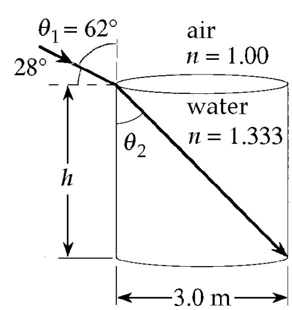
θ_2 and θ_3 are alternate interior angles formed by the ray cutting parallel normals.

So, $\theta_3=\theta_2=\boxed{19.5°}$

$$1.50\sin\theta_3=1.00\sin\theta_4$$

$$\theta_4=\boxed{30.0°}$$

25.8 $\sin\theta_1=n_w\sin\theta_2$



$$\sin\theta_2=\frac{1}{1.333}\sin\theta_1=\frac{1}{1.333}\sin(90.0°-28.0°)=0.662$$

$$\theta_2=\sin^{-1}(0.662)=41.5°$$

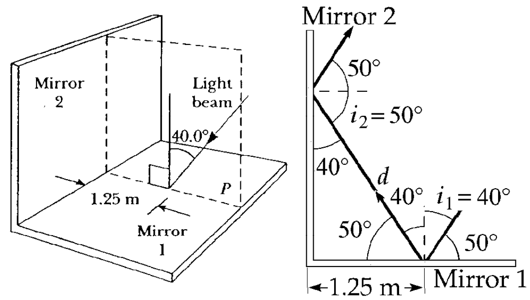
$$h=\frac{d}{\tan\theta_2}=\frac{3.00\text{ m}}{\tan 41.5°}=\boxed{3.39\text{ m}}$$

25.9 (a) From geometry, $1.25 \text{ m} = d \sin 40.0°$

so $d = \boxed{1.94 \text{ m}}$

(b) $\boxed{50.0° \text{ above the horizontal}}$

or parallel to the incident ray

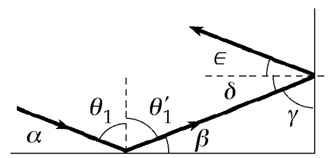


***25.10** (a) **Method One:**

The incident ray makes angle $\alpha = 90° - \theta_1$

with the first mirror. In the picture, the law of reflection implies that

$$\theta_1 = \theta_1'$$



Then $\beta = 90° - \theta_1' = 90 - \theta_1 = \alpha$

In the triangle made by the mirrors and the ray passing between them,

$$\beta + 90° + \gamma = 180°$$

$$\gamma = 90° - \beta$$

Further, $\delta = 90° - \gamma = \beta = \alpha$

and $\epsilon = \delta = \alpha$

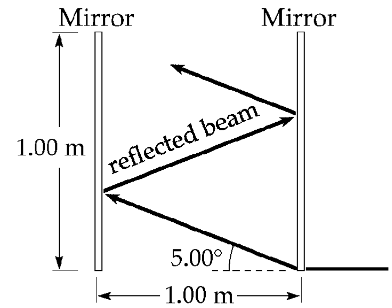
Thus the final ray makes the same angle with the first mirror as did the incident ray. Its direction is opposite to the incident ray.

Method Two:

The vector velocity of the incident light has a component v_y perpendicular to the first mirror and a component v_x perpendicular to the second. The v_y component is reversed upon the first reflection, which leaves v_x unchanged. The second reflection reverses v_x and leaves v_y unchanged. The doubly reflected ray then has velocity opposite to the incident ray.

(b) The incident ray has velocity $v_x \mathbf{i} + v_y \mathbf{j} + v_z \mathbf{k}$. Each reflection reverses one component and leaves the other two unchanged. After all the reflections, the light has velocity $-v_x \mathbf{i} - v_y \mathbf{j} - v_z \mathbf{k}$, opposite to the incident ray.

25.11 The incident light reaches the left-hand mirror at distance



$$(1.00\text{ m})\tan 5.00° = 0.0875\text{ m}$$

above its bottom edge. The reflected light first reaches the right-hand mirror at height

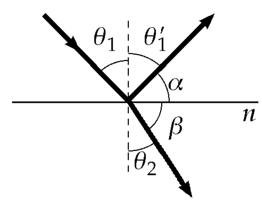
$$2(0.0875\text{ m}) = 0.175\text{ m}$$

It bounces between the mirrors with this distance between points of contact with either.

Since $\dfrac{1.00\text{ m}}{0.175\text{ m}} = 5.72$

the light reflects $\boxed{\text{five times from the right-hand mirror and six times from the left.}}$

***25.12** For $\alpha + \beta = 90°$



with $\theta_1' + \alpha + \beta + \theta_2 = 180°$

we have $\theta_1' + \theta_2 = 90°$

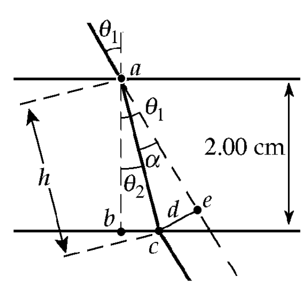
Also, $\theta_1' = \theta_1$

and $1\sin\theta_1 = n\sin\theta_2$

Then, $\sin\theta_1 = n\,\sin(90 - \theta_1) = n\,\cos\theta_1$

$$\frac{\sin\theta_1}{\cos\theta_1} = n = \tan\theta_1 \qquad \boxed{\theta_1 = \tan^{-1} n}$$

25.13 At entry, $n_1\sin\theta_1 = n_2\sin\theta_2$



or $1.00\sin 30.0° = 1.50\sin\theta_2$

$\theta_2 = 19.5°$

The distance h the light travels in the medium is given by

$$\cos\theta_2 = \frac{2.00\text{ cm}}{h}$$

or $h = \dfrac{2.00\text{ cm}}{\cos 19.5°} = 2.12\text{ cm}$

The angle of deviation upon entry is $\alpha = \theta_1 - \theta_2 = 30.0° - 19.5° = 10.5°$

The offset distance comes from $\sin\alpha = \dfrac{d}{h}$: $d = (2.21\text{ cm})\sin 10.5° = \boxed{0.388\text{ cm}}$

25.14 The distance, h, traveled by the light is $h = \dfrac{2.00\text{ cm}}{\cos 19.5°} = 2.12\text{ cm}$

The speed of light in the material is $v = \dfrac{c}{n} = \dfrac{3.00\times10^{8}\text{ m/s}}{1.50} = 2.00\times10^{8}\text{ m/s}$

Therefore, $t = \dfrac{h}{v} = \dfrac{2.12\times10^{-2}\text{ m}}{2.00\times10^{8}\text{ m/s}} = 1.06\times10^{-10}\text{ s} = \boxed{106\text{ ps}}$

25.15 Applying Snell's law at the air-oil interface,

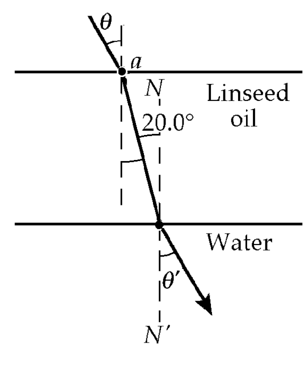
$$n_{\text{air}}\sin\theta = n_{\text{oil}}\sin 20.0°$$

yields $\boxed{\theta = 30.4°}$

Applying Snell's law at the oil-water interface

$$n_w \sin\theta' = n_{\text{oil}}\sin 20.0°$$

yields $\boxed{\theta' = 22.3°}$



25.16 Consider glass with an index of refraction of 1.5, which is 3 mm thick. The speed of light in the glass is

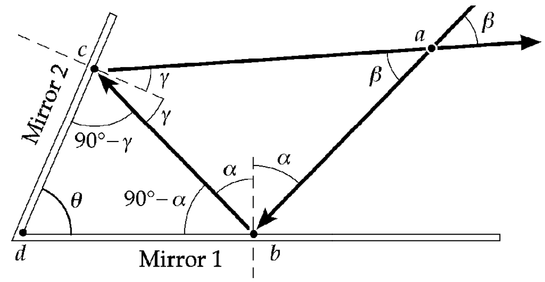
$$\frac{3\times10^{8}\text{ m/s}}{1.5} = 2\times10^{8}\text{ m/s}$$

The extra travel time is $\dfrac{3\times10^{-3}\text{ m}}{2\times10^{8}\text{ m/s}} - \dfrac{3\times10^{-3}\text{ m}}{3\times10^{8}\text{ m/s}}\ \boxed{\sim10^{-11}\text{ s}}$

For light of wavelength 600 nm in vacuum and wavelength $\dfrac{600\text{ nm}}{1.5} = 400\text{ nm}$ in glass,

the extra optical path, in wavelengths, is $\dfrac{3\times10^{-3}\text{ m}}{4\times10^{-7}\text{ m}} - \dfrac{3\times10^{-3}\text{ m}}{6\times10^{-7}\text{ m}}\ \boxed{\sim10^{3}\text{ wavelengths}}$

***25.17** See the sketch showing the path of the light ray. α and γ are angles of incidence at mirrors 1 and 2.



For triangle abca,

$$2\alpha + 2\gamma + \beta = 180°$$

or $$\beta = 180° - 2(\alpha + \gamma) \quad (1)$$

Now for triangle bcdb,

$$(90.0° - \alpha) + (90.0° - \gamma) + \theta = 180°$$

or $$\theta = \alpha + \gamma \quad (2)$$

Substituting Equation (2) into Equation (1) gives $\boxed{\beta = 180° - 2\theta}$

Note: From Equation (2), $\gamma = \theta - \alpha$. Thus, the ray will follow a path like that shown only if $\alpha < \theta$. For $\alpha > \theta$, γ is negative and multiple reflections from each mirror will occur before the incident and reflected rays intersect.

***25.18** Let $n(x)$ be the index of refraction at distance x below the top of the atmosphere and $n(x = h) = n$ be its value at the planet surface.

Then, $$n(x) = 1.000 + \left(\frac{n - 1.000}{h}\right)x$$

(a) The total time interval required to traverse the atmosphere is

$$\Delta t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}dx: \qquad \Delta t = \frac{1}{c}\int_0^h \left[1.000 + \left(\frac{n-1.000}{h}\right)x\right]dx$$

$$\Delta t = \frac{h}{c} + \frac{(n-1.000)}{ch}\left(\frac{h^2}{2}\right) = \boxed{\frac{h}{c}\left(\frac{n+1.000}{2}\right)}$$

(b) The travel time in the absence of an atmosphere would be h / c.

Thus, the time in the presence of an atmosphere is $\boxed{\left(\frac{n+1.000}{2}\right) \text{ times larger}}$

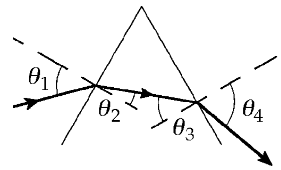
25.19 From Fig 25.13 $\quad n_v = 1.470$ at 400 nm $\quad$ and $\quad n_r = 1.458$ at 700 nm

Then $\quad 1.00 \sin\theta = 1.470 \sin\theta_v \quad$ and $\quad 1.00 \sin\theta = 1.458 \sin\theta_r$

$$\delta_r - \delta_v = \theta_r - \theta_v = \sin^{-1}\left(\frac{\sin\theta}{1.458}\right) - \sin^{-1}\left(\frac{\sin\theta}{1.470}\right)$$

$$\Delta\delta = \sin^{-1}\left(\frac{\sin 30.0°}{1.458}\right) - \sin^{-1}\left(\frac{\sin 30.0°}{1.470}\right) = \boxed{0.171°}$$

25.20 $n_1 \sin\theta_1 = n_2 \sin\theta_2$: $\theta_2 = \sin^{-1}\left(\dfrac{n_1 \sin\theta_1}{n_2}\right)$



$$\theta_2 = \sin^{-1}\left(\frac{1.00 \sin 30.0°}{1.50}\right) = \boxed{19.5°}$$

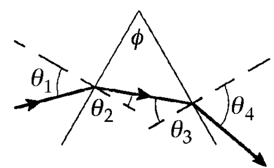
The surface of entry, the surface of exit, and the ray within the prism form a triangle. Inside the triangle the angles must add up according to

$$90.0° - \theta_2 + 60.0° + 90.0° - \theta_3 = 180°$$

$$\theta_3 = \left(\left[(90.0° - 19.5°) + 60.0°\right] - 180°\right) + 90.0° = \boxed{40.5°}$$

$n_3 \sin\theta_3 = n_4 \sin\theta_4$: $\theta_4 = \sin^{-1}\left(\dfrac{n_3 \sin\theta_3}{n_4}\right) = \sin^{-1}\left(\dfrac{1.50 \sin 40.5°}{1.00}\right) = \boxed{77.1°}$

25.21 For the incoming ray, $\sin\theta_2 = \dfrac{\sin\theta_1}{n}$



Using the figure to the right, $(\theta_2)_{violet} = \sin^{-1}\left(\dfrac{\sin 50.0°}{1.66}\right) = 27.48°$

$$(\theta_2)_{red} = \sin^{-1}\left(\frac{\sin 50.0°}{1.62}\right) = 28.22°$$

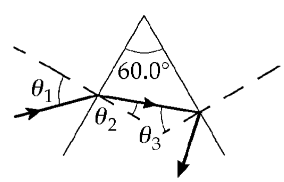
For the outgoing ray, $\theta_3 = 60.0° - \theta_2$

and $\sin\theta_4 = n \sin\theta_3$: $(\theta_4)_{violet} = \sin^{-1}[1.66 \sin 32.52°] = 63.17°$

$$(\theta_4)_{red} = \sin^{-1}[1.62 \sin 31.78°] = 58.56°$$

The angular dispersion is the difference $\Delta\theta_4 = (\theta_4)_{violet} - (\theta_4)_{red} = 63.17° - 58.56° = \boxed{4.61°}$

25.22 At the first refraction, $1.00 \sin\theta_1 = n \sin\theta_2$



The critical angle at the second surface is given by $n \sin\theta_3 = 1.00$:

or $\theta_3 = \sin^{-1}\left(\dfrac{1.00}{1.50}\right) = 41.8°$

But, $\theta_2 = 60.0° - \theta_3$

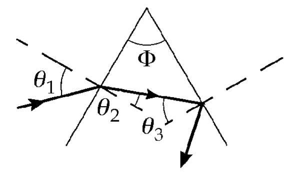
Thus, to avoid total internal reflection at the second surface (i.e., have $\theta_3 < 41.8°$)

it is necessary that $\theta_2 > 18.2°$

Since $\sin\theta_1 = n \sin\theta_2$, this becomes $\sin\theta_1 > 1.50 \sin 18.2° = 0.468$

or $\theta_1 > \boxed{27.9°}$

25.23 At the first refraction, $1.00\sin\theta_1 = n\sin\theta_2$



The critical angle at the second surface is given by

$n\sin\theta_3 = 1.00$, or $\theta_3 = \sin^{-1}\left(\frac{1.00}{n}\right)$

But $\left(90.0° - \theta_2\right) + \left(90.0° - \theta_3\right) + \Phi = 180°$

which gives $\theta_2 = \Phi - \theta_3$

Thus, to have $\theta_3 < \sin^{-1}(1.00/n)$ and avoid total internal reflection at the second surface,

it is necessary that $\theta_2 > \Phi - \sin^{-1}\left(\frac{1.00}{n}\right)$

Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes $\sin\theta_1 > n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right]$

or $\theta_1 > \boxed{\sin^{-1}\left(n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right]\right)}$

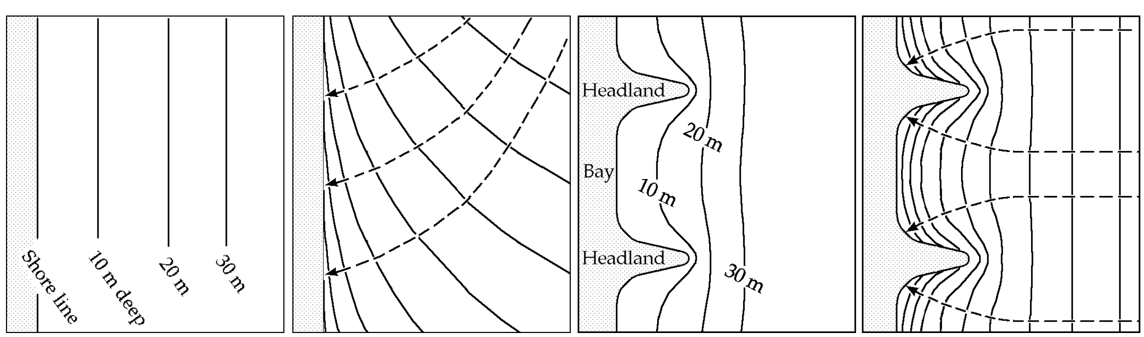
Through the application of trigonometric identities, $\theta_1 > \boxed{\sin^{-1}\left(\sqrt{n^2-1}\sin\Phi - \cos\Phi\right)}$

***25.24** (a) For the diagrams of contour lines and wave fronts and rays, see figures (a) and (b) below.

As the waves move to shallower water, the wave fronts bend to become more nearly parallel to the contour lines.

(b) For the diagrams of contour lines and wave fronts and rays, see figures (c) and (d) below.

We suppose that the headlands are steep underwater, as they are above water. The rays are everywhere perpendicular to the wave fronts of the incoming refracting waves. As shown, the rays bend toward the headlands and deliver more energy per length at the headlands.



(a) Contour lines (b) Wave fronts and rays (c) Contour lines (d) Wave fronts and rays

25.25 $n\sin\theta = 1$. From Table 25.1,

(a) $\theta = \sin^{-1}\left(\frac{1}{2.419}\right) = \boxed{24.4°}$

(b) $\theta = \sin^{-1}\left(\frac{1}{1.66}\right) = \boxed{37.0°}$

(c) $\theta = \sin^{-1}\left(\frac{1}{1.309}\right) = \boxed{49.8°}$

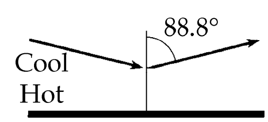
***25.26** $\sin\theta_c = \frac{n_2}{n_1}$: $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

(a) Diamond: $\theta_c = \sin^{-1}\left(\frac{1.333}{2.419}\right) = \boxed{33.4°}$

(b) Flint glass: $\theta_c = \sin^{-1}\left(\frac{1.333}{1.66}\right) = \boxed{53.4°}$

(c) Ice: Since $n_2 > n_1$, $\boxed{\text{there is no critical angle}}$.

25.27 $\sin\theta_c = \frac{n_2}{n_1}$ (Equation 25.8)



$n_2 = n_1 \sin 88.8° = (1.0003)(0.9998) = \boxed{1.00008}$

***25.28** (a) $\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$

and $\theta_2 = 90.0°$ at the critical angle

$$\frac{\sin 90.0°}{\sin\theta_c} = \frac{1850\ \text{m/s}}{343\ \text{m/s}}$$

so $\theta_c = \sin^{-1}(0.185) = \boxed{10.7°}$

(b) Sound can be totally reflected if it is traveling in the medium where it travels slower: $\boxed{\text{air}}$

(c) $\boxed{\text{Sound in air falling on the wall from most directions is 100\% reflected}}$, so the wall is a good mirror.

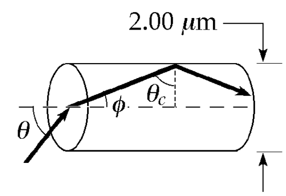
***25.29** For plastic with index of refraction $\boxed{n \geq 1.42}$ surrounded by air, the critical angle for total internal reflection is given by

$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) \leq \sin^{-1}\left(\frac{1}{1.42}\right) = 44.8°$$

In the gasoline gauge, skylight from above travels down the plastic. The rays close to the vertical are totally reflected from the sides of the slab and from both facets at the lower end of the plastic, where it is not immersed in gasoline. This light returns up inside the plastic and makes it look bright. Where the plastic is immersed in gasoline, with index of refraction about 1.50, total internal reflection should not happen. The light passes out of the lower end of the plastic with little reflected, making this part of the gauge look dark. To frustrate total internal reflection in the gasoline, the index of refraction of the plastic should be $\boxed{n < 2.12}$

since $\theta_c = \sin^{-1}\left(\frac{1.50}{2.12}\right) = 45.0°$

25.30 $\sin\theta_c = \frac{n_{\text{air}}}{n_{\text{pipe}}} = \frac{1.00}{1.36} = 0.735$ $\qquad \theta_c = 47.3°$



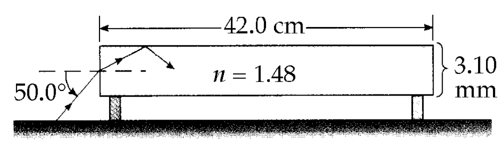
Geometry shows that the angle of refraction at the end is

$\phi = 90.0° - \theta_c = 90.0° - 47.3° = 42.7°$

Then, Snell's law at the end, $\qquad 1.00 \sin\theta = 1.36 \sin 42.7°$

gives $\qquad \boxed{\theta = 67.2°}$

25.31 As the beam enters the slab,



$$1.00 \sin 50.0° = 1.48 \sin\theta_2$$

giving $\qquad \theta_2 = 31.2°$

The beam then strikes the top of the slab at $x_1 = 1.55 \text{ mm}/\tan 31.2°$ from the left end. Thereafter, the beam strikes a face each time it has traveled a distance of $2x_1$ along the length of the slab. Since the slab is 420 mm long, the beam has an additional $420 \text{ mm} - x_1$ to travel after the first reflection. The number of additional reflections is

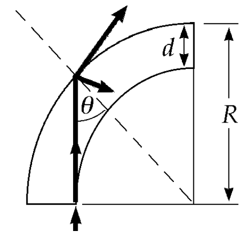
$$\frac{420 \text{ mm} - x_1}{2x_1} = \frac{420 \text{ mm} - 1.55 \text{ mm}/\tan 31.2°}{3.10 \text{ mm}/\tan 31.2°} = 81.5 \qquad \text{or 81 reflections}$$

since the answer must be an integer. The total number of reflections made in the slab is then $\boxed{82}$.

***25.32** For total internal reflection, $n_1 \sin\theta_1 = n_2 \sin 90.0°$

$1.50 \sin\theta_1 = 1.33(1.00)$ or $\theta_1 = \boxed{62.4°}$

***25.33** (a) A ray along the inner edge will escape if any ray escapes. Its angle of incidence is described by $\sin\theta = (R-d)/R$ and by $n\sin\theta > 1\sin 90°$. Then



$$\frac{n(R-d)}{R} > 1 \qquad nR - nd > R \qquad nR - R > nd \qquad R > \boxed{\frac{nd}{n-1}}$$

(b) As $d \to 0$, $R_{\min} \to 0$. This is reasonable.

As n increases, $R_{\min}$ decreases. This is reasonable.

As n decreases toward 1, $R_{\min}$ increases. This is reasonable.

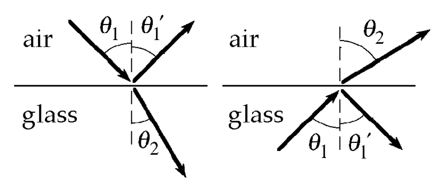
(c) $$R_{\min} = \frac{1.40\left(100\times10^{-6}\text{ m}\right)}{0.40} = \boxed{350\times10^{-6}\text{ m}}$$

25.34 (a) $\theta_1' = \theta_1 = \boxed{30.0°}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.00 \sin 30.0° = 1.55 \sin\theta_2$

$\theta_2 = \boxed{18.8°}$



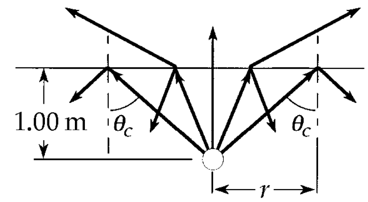
(b) $\theta_1' = \theta_1 = \boxed{30.0°}$ $\qquad \theta_2 = \sin^{-1}\left(\frac{n_1\sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{1.55\sin 30.0°}{1}\right) = \boxed{50.8°}$

(c) and (d) The other entries are computed similarly, and are shown in the table below.

(c) air into glass, angles in degrees			(d) glass into air, angles in degrees		
incidence	reflection	refraction	incidence	reflection	refraction
0	0	0	0	0	0
10.0	10.0	6.43	10.0	10.0	15.6
20.0	20.0	12.7	20.0	20.0	32.0
30.0	30.0	18.8	30.0	30.0	50.8
40.0	40.0	24.5	40.0	40.0	85.1
50.0	50.0	29.6	50.0	50.0	none*
60.0	60.0	34.0	60.0	60.0	none*
70.0	70.0	37.3	70.0	70.0	none*
80.0	80.0	39.4	80.0	80.0	none*
90.0	90.0	40.2	90.0	90.0	none*

*total internal reflection

25.35 For water, $\sin\theta_c = \dfrac{1}{4/3} = \frac{3}{4}$

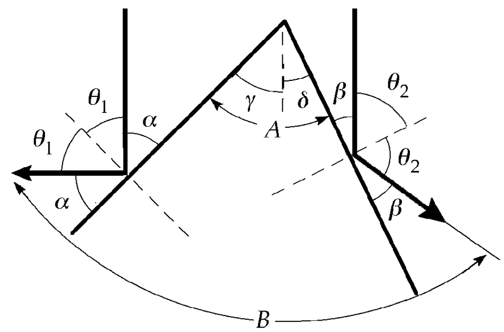


Thus $\theta_c = \sin^{-1}(0.750) = 48.6°$

and $d = 2\left[(1.00\text{ m})\tan\theta_c\right]$

$$d = (2.00\text{ m})\tan 48.6° = \boxed{2.27\text{ m}}$$

25.36 Call θ_1 the angle of incidence and of reflection on the left face and θ_2 those angles on the right face. Let α represent the complement of θ_1 and β be the complement of θ_2. Now $\alpha = \gamma$ and $\beta = \delta$ because they are pairs of alternate interior angles. We have



$$A = \gamma + \delta = \alpha + \beta$$

and $$B = \alpha + A + \beta = \alpha + \beta + A = \boxed{2A}$$

***25.37** (a) We see the Sun moving from east to west across the sky. Its angular speed is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\text{ rad}}{86\,400\text{ s}} = 7.27\times10^{-5}\text{ rad/s}$$

The direction of sunlight crossing the cell from the window changes at this rate, moving on the opposite wall at speed

$$v = r\omega = (2.37\text{ m})\left(7.27\times10^{-5}\text{ rad/s}\right) = 1.72\times10^{-4}\text{ m/s} = \boxed{0.172\text{ mm/s}}$$

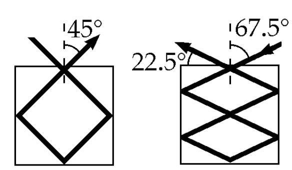
(b) The mirror folds into the cell the motion that would occur in a room twice as wide:

$$v = r\omega = 2(0.174\text{ mm/s}) = \boxed{0.345\text{ mm/s}}$$

(c) and (d)

As the Sun moves southward and upward at 50.0°, we may regard the corner of the window as fixed, and both patches of light move $\boxed{\text{northward and downward at 50.0°}}$.

25.38 (a) $\boxed{45.0°}$ as shown in the first figure to the right.



(b) $\boxed{\text{Yes}}$

If grazing angle is halved, the number of reflections from the side faces is doubled.

25.39 Horizontal light rays from the setting Sun pass above the hiker. The light rays are twice refracted and once reflected, as in Figure (b). The most intense light reaching the hiker, that which represents the visible rainbow, is located between angles of 40° and 42° from the hiker's shadow.

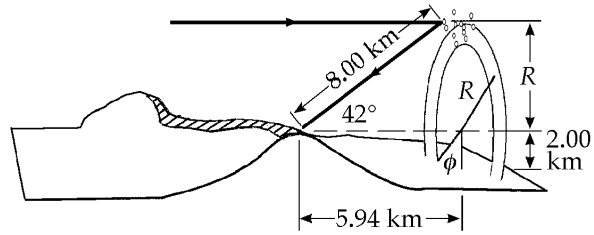


Figure (a)

The hiker sees a greater percentage of the violet inner edge, so we consider the red outer edge. The radius R of the circle of droplets is

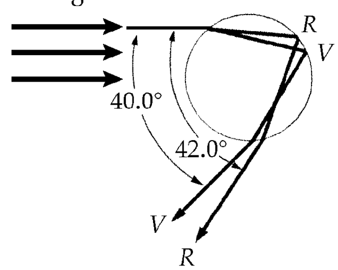


Figure (b)

$$R = (8.00\text{ km})\sin 42.0° = 5.35\text{ km}$$

Then the angle ϕ, between the vertical and the radius where the bow touches the ground, is given by

$$\cos\phi = \frac{2.00\text{ km}}{R} = \frac{2.00\text{ km}}{5.35\text{ km}} = 0.374$$

or $\phi = 68.1°$

The angle filled by the visible bow is $360° - (2 \times 68.1°) = 224°$

so the visible bow is $\frac{224°}{360°} = \boxed{62.2\% \text{ of a circle}}$

25.40 By Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$

With $v = \frac{c}{n}$

$$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$

or $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$

This is also true for sound.

Here, $\frac{\sin 12.0°}{340\text{ m/s}} = \frac{\sin\theta_2}{1510\text{ m/s}}$

$$\theta_2 = \sin^{-1}(4.44\,\sin 12.0°) = \boxed{67.4°}$$

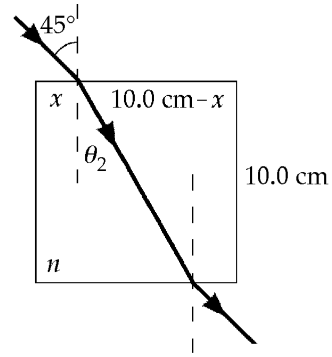
25.41 Entry: $1.00\sin 45° = 1.52\sin\theta_2$

$\theta_2 = 27.7°$

$$\tan\theta_2 = \frac{10\text{ cm} - x}{10\text{ cm}}$$

$10\text{ cm} - x = (10\text{ cm})\tan 27.7°$

$x = \boxed{4.74\text{ cm}}$



***25.42** (a) $$\frac{S_1'}{S_1} = \left[\frac{n_2 - n_1}{n_2 + n_1}\right]^2 = \left[\frac{1.52 - 1.00}{1.52 + 1.00}\right]^2 = \boxed{0.0426}$$

(b) If medium 1 is glass and medium 2 is air, $$\frac{S_1'}{S_1} = \left[\frac{n_2 - n_1}{n_2 + n_1}\right]^2 = \left[\frac{1.00 - 1.52}{1.00 + 1.52}\right]^2 = 0.0426$$

There is $\boxed{\text{no difference}}$

***25.43** (a) With $n_1 = 1$

and $n_2 = n$

the reflected fractional intensity is $$\frac{S_1'}{S_1} = \left(\frac{n-1}{n+1}\right)^2$$

The remaining intensity must be transmitted:

$$\frac{S_2}{S_1} = 1 - \left(\frac{n-1}{n+1}\right)^2 = \frac{(n+1)^2 - (n-1)^2}{(n+1)^2} = \frac{n^2 + 2n + 1 - n^2 + 2n - 1}{(n+1)^2} = \boxed{\frac{4n}{(n+1)^2}}$$

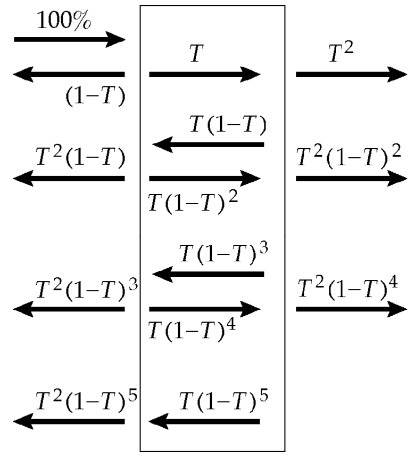
(b) At entry, $$\frac{S_2}{S_1} = 1 - \left(\frac{n-1}{n+1}\right)^2 = \frac{4(2.419)}{(2.419+1)^2} = 0.828$$

At exit, $$\frac{S_3}{S_2} = 0.828$$

Overall, $$\frac{S_3}{S_1} = \left(\frac{S_3}{S_2}\right)\left(\frac{S_2}{S_1}\right) = (0.828)^2 = 0.685$$

or $\boxed{68.5\%}$

***25.44** Define $T = 4n/(n+1)^2$ as the transmission coefficient for one encounter with an interface. For diamond and air, it is 0.828, as in problem 43.



As shown in the figure, the total amount transmitted is

$$T^2 + T^2(1-T)^2 + T^2(1-T)^4 + T^2(1-T)^6 + \dots + T^2(1-T)^{2n} + \dots$$

We have $1 - T = 1 - 0.828 = 0.172$ so the total transmission is

$$(0.828)^2\left[1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \dots\right]$$

To sum this series, define $F = 1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \dots$

Note that $(0.172)^2 F = (0.172)^2 + (0.172)^4 + (0.172)^6 + \dots,$

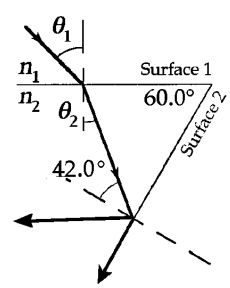
and $1 + (0.172)^2 F = 1 + (0.172)^2 + (0.172)^4 + (0.172)^6 + \dots = F.$

Then, $1 = F - (0.172)^2 F$

or $F = \dfrac{1}{1-(0.172)^2}$

The overall transmission is then $\dfrac{(0.828)^2}{1-(0.172)^2} = 0.706$ or $\boxed{70.6\%}$

25.45 Define n_1 to be the index of refraction of the surrounding medium and n_2 to be that for the prism material. We can use the critical angle of 42.0° to find the ratio n_2/n_1:



$$n_2 \sin 42.0° = n_1 \sin 90.0°$$

So, $$\frac{n_2}{n_1} = \frac{1}{\sin 42.0°} = 1.49$$

Call the angle of refraction θ_2 at the surface 1. The ray inside the prism forms a triangle with surfaces 1 and 2, so the sum of the interior angles of this triangle must be 180°.

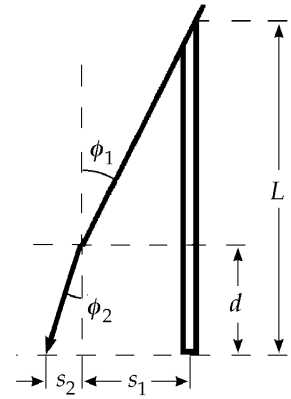
Thus, $(90.0° - \theta_2) + 60.0° + (90.0° - 42.0°) = 180°$

Therefore, $\theta_2 = 18.0°$

Applying Snell's law at surface 1, $n_1 \sin\theta_1 = n_2 \sin 18.0°$

$$\sin\theta_1 = \left(\frac{n_2}{n_1}\right)\sin\theta_2 = 1.49 \sin 18.0° \qquad \boxed{\theta_1 = 27.5°}$$

25.46 Light passing the top of the pole makes an angle of incidence $\phi_1 = 90.0° - \theta$. It falls on the water surface at distance from the pole



$$s_1 = \frac{L-d}{\tan\theta}$$

and has an angle of refraction ϕ_2 from $1.00 \sin\phi_1 = n \sin\phi_2$

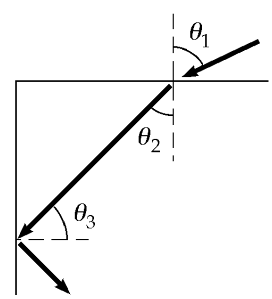
Then $s_2 = d\tan\phi_2$

and the whole shadow length is

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\sin\phi_1}{n}\right)\right)$$

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\cos\theta}{n}\right)\right) = \frac{2.00\text{ m}}{\tan 40.0°} + (2.00\text{ m})\tan\left(\sin^{-1}\left(\frac{\cos 40.0°}{1.33}\right)\right) = \boxed{3.79\text{ m}}$$

25.47 (a) For polystyrene *surrounded by air*, internal reflection requires



$$\theta_3 = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°$$

Then from geometry, $\theta_2 = 90.0° - \theta_3 = 47.8°$

From Snell's law, $\sin\theta_1 = 1.49\sin 47.8° = 1.10$

This has no solution.

Therefore, total internal reflection $\boxed{\text{always happens}}$.

(b) For polystyrene *surrounded by water*, $\theta_3 = \sin^{-1}\left(\frac{1.33}{1.49}\right) = 63.2°$

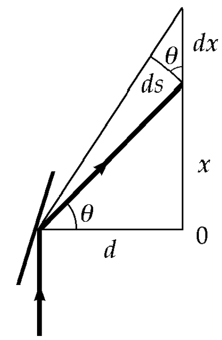
and $\theta_2 = 26.8°$

From Snell's law, $\theta_1 = \boxed{30.3°}$

(c) $\boxed{\text{No internal refraction is possible}}$

since the beam is initially traveling in a medium of lower index of refraction.

***25.48** (a) As the mirror turns through angle θ, the angle of incidence increases by θ and so does the angle of reflection. The incident ray is stationary, so the reflected ray turns through angle 2θ. The angular speed of the reflected ray is $2\omega_m$. The speed of the dot of light on the circular wall is $\boxed{2\omega_m R}$.



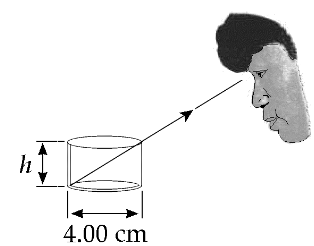
(b) The two angles marked θ in the figure to the right are equal because their sides are perpendicular, right side to right side and left side to left side.

We have $\quad \cos\theta = \dfrac{d}{\sqrt{x^2+d^2}} = \dfrac{ds}{dx}$

and $\quad \dfrac{ds}{dt} = 2\omega_m\sqrt{x^2+d^2}$

So $\quad \dfrac{dx}{dt} = \dfrac{ds}{dt}\dfrac{\sqrt{x^2+d^2}}{d} = \boxed{2\omega_m\dfrac{x^2+d^2}{d}}$

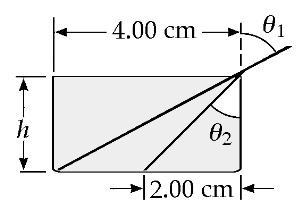
25.49 $\quad \tan\theta_1 = \dfrac{4.00\text{ cm}}{h}$



and $\quad \tan\theta_2 = \dfrac{2.00\text{ cm}}{h}$

$$\tan^2\theta_1 = (2.00\tan\theta_2)^2 = 4.00\tan^2\theta_2$$

$$\frac{\sin^2\theta_1}{1-\sin^2\theta_1} = 4.00\left(\frac{\sin^2\theta_2}{1-\sin^2\theta_2}\right) \qquad (1)$$



Snell's law in this case is: $\quad n_1\sin\theta_1 = n_2\sin\theta_2$

$\sin\theta_1 = 1.333\sin\theta_2$

Squaring both sides, $\quad \sin^2\theta_1 = 1.777\sin^2\theta_2 \qquad (2)$

Substituting (2) into (1), $\quad \dfrac{1.777\sin^2\theta_2}{1-1.777\sin^2\theta_2} = 4.00\left(\dfrac{\sin^2\theta_2}{1-\sin^2\theta_2}\right)$

Defining $x = \sin^2\theta$, $\quad \dfrac{0.444}{1-1.777x} = \dfrac{1}{1-x}$

Solving for x, $\quad 0.444 - 0.444x = 1 - 1.777x \quad$ and $\quad x = 0.417$

From x we can solve for θ_2: $\quad \theta_2 = \sin^{-1}\sqrt{0.417} = 40.2°$

Thus, the height is $\quad h = \dfrac{2.00\text{ cm}}{\tan\theta_2} = \dfrac{2.00\text{ cm}}{\tan 40.2°} = \boxed{2.37\text{ cm}}$

25.50 $\delta = \theta_1 - \theta_2 = 10.0°$ and $n_1 \sin\theta_1 = n_2 \sin\theta_2$

with $n_1 = 1,\ n_2 = \frac{4}{3}$

Thus, $\theta_1 = \sin^{-1}(n_2 \sin\theta_2) = \sin^{-1}[n_2 \sin(\theta_1 - 10.0°)]$

(You can use a calculator to home in on an approximate solution to this equation, testing different values of θ_1 until you find that $\theta_1 = \boxed{36.5°}$. Alternatively, you can solve for θ_1 exactly, as shown below.)

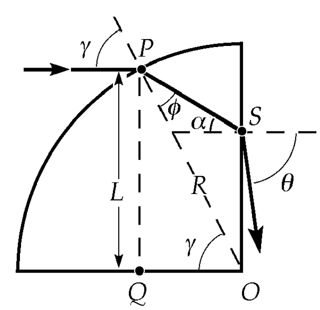
We are given that $\sin\theta_1 = \frac{4}{3}\sin(\theta_1 - 10.0°)$

This is the sine of a difference, so $\frac{3}{4}\sin\theta_1 = \sin\theta_1 \cos 10.0° - \cos\theta_1 \sin 10.0°$

Rearranging, $\sin 10.0° \cos\theta_1 = \left(\cos 10.0° - \frac{3}{4}\right)\sin\theta_1$

$$\frac{\sin 10.0°}{\cos 10.0° - 0.750} = \tan\theta_1 \quad \text{and} \quad \theta_1 = \tan^{-1}(0.740) = \boxed{36.5°}$$

***25.51** Observe in the sketch that the angle of incidence at point P is γ, and using triangle OPQ:



$$\sin\gamma = L/R.$$

Also, $\cos\gamma = \sqrt{1-\sin^2\gamma} = \dfrac{\sqrt{R^2 - L^2}}{R}$

Applying Snell's law at point P, $1.00 \sin\gamma = n \sin\phi$

Thus, $\sin\phi = \dfrac{\sin\gamma}{n} = \dfrac{L}{nR}$

and $\cos\phi = \sqrt{1-\sin^2\phi} = \dfrac{\sqrt{n^2R^2 - L^2}}{nR}$

From triangle OPS, $\phi + (\alpha + 90.0°) + (90.0° - \gamma) = 180°$ or the angle of incidence at point S is $\alpha = \gamma - \phi$. Then, applying Snell's law at point S

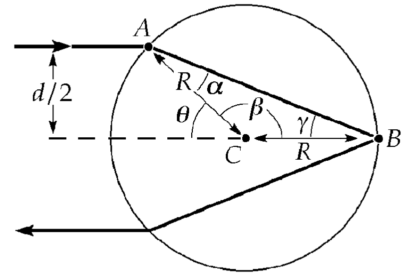
gives $1.00 \sin\theta = n \sin\alpha = n \sin(\gamma - \phi)$

or $$\sin\theta = n[\sin\gamma\cos\phi - \cos\gamma\sin\phi] = n\left[\left(\frac{L}{R}\right)\frac{\sqrt{n^2R^2 - L^2}}{nR} - \frac{\sqrt{R^2 - L^2}}{R}\left(\frac{L}{nR}\right)\right]$$

$$\sin\theta = \frac{L}{R^2}\left(\sqrt{n^2R^2 - L^2} - \sqrt{R^2 - L^2}\right)$$

and $$\theta = \boxed{\sin^{-1}\left[\frac{L}{R^2}\left(\sqrt{n^2R^2 - L^2} - \sqrt{R^2 - L^2}\right)\right]}$$

***25.52** As shown in the sketch, the angle of incidence at point A is:



$$\theta = \sin^{-1}\left(\frac{d/2}{R}\right) = \sin^{-1}\left(\frac{1.00\text{ m}}{2.00\text{ m}}\right) = 30.0°$$

If the emerging ray is to be parallel to the incident ray, the path must be symmetric about the centerline CB of the cylinder. In the isosceles triangle ABC,

$$\gamma = \alpha \qquad \text{and} \qquad \beta = 180° - \theta$$

Therefore, $\alpha + \beta + \gamma = 180°$

becomes $2\alpha + 180° - \theta = 180°$

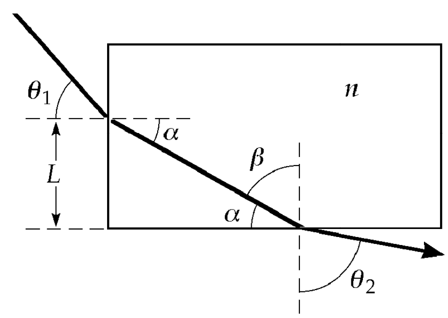
or $\alpha = \dfrac{\theta}{2} = 15.0°$

Then, applying Snell's law at point A,

$$n\sin\alpha = 1.00\sin\theta$$

or $$n = \frac{\sin\theta}{\sin\alpha} = \frac{\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$$

***25.53** (a) Given that $\theta_1 = 45.0°$ and $\theta_2 = 76.0°$



Snell's law at the first surface gives

$$n\sin\alpha = 1.00\sin 45.0° \qquad (1)$$

Observe that the angle of incidence at the second surface is

$$\beta = 90.0° - \alpha$$

Thus, Snell's law at the second surface yields

$$n\sin\beta = n\sin(90.0° - \alpha) = 1.00\sin 76.0°$$

or $$n\cos\alpha = \sin 76.0° \qquad (2)$$

Dividing Equation (1) by Equation (2), $\tan\alpha = \dfrac{\sin 45.0°}{\sin 76.0°} = 0.729$

or $\alpha = 36.1°$

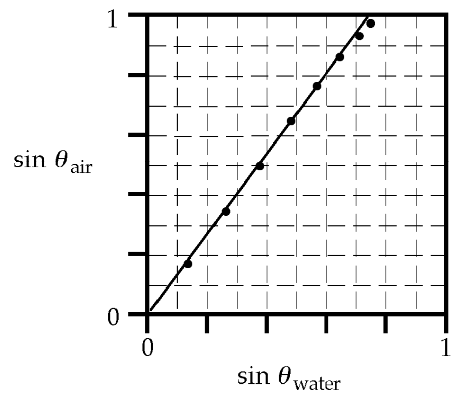
Then, from Equation (1), $n = \dfrac{\sin 45.0°}{\sin\alpha} = \dfrac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$

(b) From the sketch, observe that the distance the light travels in the plastic is $d = \dfrac{L}{\sin\alpha}$. Also, the speed of light in the plastic is $v = c/n$, so the time required to travel through the plastic is

$$\Delta t = \frac{d}{v} = \frac{nL}{c\sin\alpha} = \frac{1.20(0.500\text{ m})}{\left(3.00\times10^{8}\text{ m/s}\right)\sin 36.1°} = 3.40\times10^{-9}\text{ s} = \boxed{3.40\text{ ns}}$$

25.54

$\sin\theta_1$	$\sin\theta_2$	$\sin\theta_1 / \sin\theta_2$
0.174	0.131	1.3304
0.342	0.261	1.3129
0.500	0.379	1.3177
0.643	0.480	1.3385
0.766	0.576	1.3289
0.866	0.647	1.3390
0.940	0.711	1.3220
0.985	0.740	1.3315



The straightness of the graph line demonstrates Snell's proportionality.

The slope of the line is $\bar{n} = 1.3276 \pm 0.01$

and $n = \boxed{1.328 \pm 0.8\%}$

ANSWERS TO EVEN NUMBERED PROBLEMS

2. (a) 474 THz (b) 422 nm (c) 200 Mm/s

4. (a) 1.52 (b) 417 nm
(c) 474 THz (d) 198 Mm/s

6. 158 Mm/s

8. 3.39 m

10. See the solution

12. $\tan^{-1} n$

14. 106 ps

16. $\sim 10^{-11}$ s, $\sim 10^{3}$ wavelengths

18. (a) $\dfrac{h}{c}\left(\dfrac{n+1.00}{2}\right)$ (b) $\left(\dfrac{n+1.00}{2}\right)$ times longer

20. 30.0° and 19.5° at entry, 40.5° and 77.1° at exit

22. 27.9°

24. See the solution

26. (a) 33.4° (b) 53.4° (c) there is no critical angle

28. (a) 10.7° (b) air
(c) Sound in air falling on the wall from most directions is 100% reflected.

30. 67.2°

32. 62.4°

34. See the solution. The angles for the first two parts of the problem are
(a) $\theta_1' = 30.0°$, $\theta_2 = 18.8°$ (b) $\theta_1' = 30.0°$, $\theta_2 = 50.8°$

36. See the solution

38. (a) 45.0° (b) Yes, such as 67.5°; see the solution

40. 67.4°

42. (a) 0.0426 or 4.26% (b) no difference

44. 70.6%

46. 3.79 m

48. (a) $2R\omega_m$ (b) $2\omega_m\left(x^2+d^2\right)/d$

50. 36.5°

52. 1.93

54. See the solution. n = slope = 1.328 ± 0.8%