CHAPTER 26

ANSWERS TO QUESTIONS

Q26.1 Because when you look at the ƎƆИA⅃UᗺMA in your rear view mirror, the apparent left-right inversion clearly displays the name of the AMBULANCE behind you. Do not jam on your brakes when a MIAMI city bus is right behind you.

Q26.2 This is a convex mirror. The mirror gives the driver a wide field of view and an upright image with the possible disadvantage of having objects appear farther away than they really are.

Q26.3 With a concave spherical mirror, for objects beyond the focal length the image will be real and inverted. For objects inside the focal length, the image will be virtual, upright, and magnified. Try a shaving mirror as an example.

Q26.4 The mirror equation and the magnification equation apply to plane mirrors. A curved mirror is made flat by increasing its radius of curvature without bound, so that its focal length goes to infinity. From $1/p+1/q=1/f=0$ we have $1/p=-1/q$; therefore, $p=-q$. The virtual image is as far behind the mirror as the object is in front. The magnification is $M=-q/p=p/p=1$. The image is right side up and actual size.

Q26.5 Stones at the bottom of a clear stream always appears closer to the surface because light is refracted away from the normal at the surface.

Q26.6 Because of the refraction of light, the fish is actually deeper than its appears. The spear fisherman should aim *below* the observed position of the fish.

Q26.7 The spherical goldfish bowl acts as a refracting surface. The magnification M of the fish is then

$$M=1+\left(\frac{n_W-1}{R}\right)q\approx 1+\frac{q}{3R} \qquad \text{(Equations 26.8 and 26.9)}$$

where R is the radius of curvature of the goldfish bowl (and is negative) and the image distance q (a negative number) is the distance the fish appears behind the glass. Therefore, $M>1$.

Q26.8 Both words are inverted. However OXIDE has up-down symmetry whereas LEAD does not.

Q26.9 For air that gets *warmer* with height, the light rays will bend downward. Therefore objects will appear higher than they actually are. Thus a castle or a house could be seen hovering in the air.

Q26.10 With the meniscus design, when you direct your gaze near the outer circumference of the lens you receive a ray that has passed through glass with more nearly parallel surfaces of entry and exit. Thus, the lens minimally distorts the direction to the object you are looking at. If you wear glasses, turn them around and look through them the wrong way to maximize this distortion.

Q26.11 The burning glass must make a real image of the distant Sun, making nearly parallel rays converge. Any converging lens will do. A large-diameter lens will gather more power to start the fire faster.

Q26.12 The focal point is defined as the location of the image formed by rays originally parallel to the axis. An object at a large but finite distance will radiate rays nearly but not exactly parallel. Infinite object distance describes the definite limiting case in which these rays become parallel. To measure the focal length of a converging lens, set it up to form an image of the farthest object you can see outside a window. The image distance will be equal to the focal length within one percent or better if the object distance is a hundred times larger or more.

Q26.13 For definiteness, we consider real objects ($p > 0$).

(a) For $M = -q/p$ to be negative, q must be positive.
This will happen in $1/q = 1/f - 1/p$ if $p > f$, if the object is farther than the focal point.

(b) For $M = -q/p$ to be positive, q must be negative.
From $1/q = 1/f - 1/p$ we need $p < f$.

(c) For a real image, q must be positive.
As in part (a), it is sufficient for p to be larger than f.

(d) For $q < 0$ we need $p < f$.

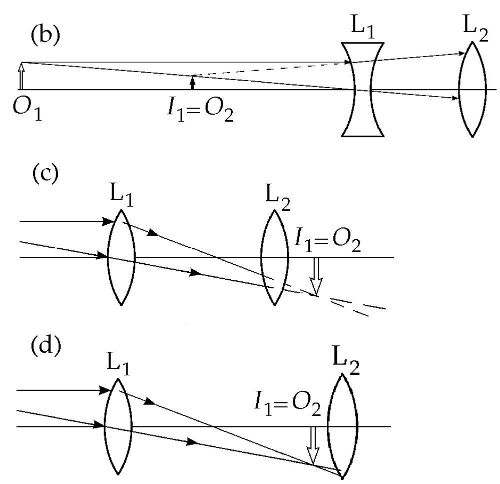
(e) For $|M| > 1$, we consider separately $M < -1$ and $M > 1$.
If $M = -q/p < -1$, we need $q/p > 1$ or $q > p$
or $1/q < 1/p$
From $1/p + 1/q = 1/f$, $1/p + 1/p > 1/f$ or $2/p > 1/f$
or $p/2 < f$ or $p < 2f$.

Now if $-q/p > 1$ or $-q > p$ or $q < -p$
we may require $q < 0$, since then $1/p = 1/f - 1/q$ with $1/f > 0$
gives $1/p > -1/q$ as required or $-p > q$.
For $q < 0$ in $1/q = 1/f - 1/p$ we need $p < f$.
Thus the overall condition for an enlarged image is simply $p < 2f$.

(f) For $|M| < 1$, we have the reverse of part (e), requiring $p > 2f$.

Q26.14 Use a converging lens as the projection lens in a slide projector. Place the brightly illuminated slide slightly farther than its focal length away from it, so that the lens will produce a real, inverted, enlarged image on the screen.

Q26.15 Refer to the diagram. For definiteness we show two lenses operating on light. The image of lens one is real if the rays leaving the lens are converging, and virtual if the rays are diverging. The object of lens two is real if the lens receives diverging rays, and virtual if it receives converging rays.



(a) False. Diverging rays from the first lens must be diverging as they fall on lens two. Thus lens two sees a virtual image as a real object, and (b) is true.

(c) and (d) are both true. Converging rays from the first lens can reach the second lens while still converging toward the first image, or after passing through the image location and starting to diverge.

Q26.16 (a) *Chromatic aberration* describes a sharp image formed by light of one color while light of another color forms a fuzzy image.

(b) If the image of a point on the axis is not quite a point, we have *spherical aberration*.

(c) If a flat screen cannot catch a sharp image of all of a flat extended object, we suspect *curvature of field*, in effect the mirror's or lens's failure to have a single well-defined focal length.

(d) *Distortion* is the name for variation in magnification across the image of an extended object. *Barrel distortion* denotes magnification decreasing with increasing distance from the axis.

Q26.17 Only absorbed light, not transmitted light, contributes internal energy to a transparent object. A clear lens can stay ice-cold as megajoules of light energy pass through it.

Q26.18 Make the mirror an efficient reflector (shiny). Make it reflect to the image even rays far from the axis, by giving it a parabolic shape. Most important, make it large in diameter to intercept a lot of solar power. And you get higher temperature if the image is smaller, as you get with shorter focal length; and if the furnace enclosure is an efficient absorber (black).

Q26.19 One point on the screen receives light only from the hole, only from a narrow range of directions in the scene beyond the hole. Altogether the screen faintly displays the scene outside the hole, with fuzziness measured by a small size of the hole. There is no definite image distance — you can place the screen at any distance large compared to the hole diameter, depending on the compromise you make between brightness and sharpness. The display on the screen looks like a real, inverted, diminished image.

Q26.20 Measure the distance q_n to your near point as follows: Without using the magnifying glass, move one fingertip as close as possible to your eye to get the clearest view of your fingerprint. Calculate $p = \dfrac{1}{\left(1/15\text{ cm} + 1/q_\text{n}\right)}$. Hold the converging lens close to your eye and the object at distance p beyond it. Then you will see the enlarged, virtual, upright image formed by the lens.

Q26.21 In contrast to a lens, a mirror is immune to chromatic aberration. The mirror has only one surface to shape and can be made of glass with inhomogeneities inside its volume. Thus your budget will buy a bigger mirror than a lens. Supported only at the edges, a lens larger than a meter in diameter tends to sag under its own weight, while a mirror can be supported all across the back side; motors placed there can actively correct the shape of the mirror. To maximize light throughput, an achromatic doublet needs antireflective coatings on several lens surfaces; a mirror needs a highly reflective coating on only one surface.

Q26.22 The invisible man would be blind. To form an image the materials of the eye must have index of refraction different from the surroundings. To register the image his retina must absorb light. The developing eye-spots of a tiny embryo are dark.

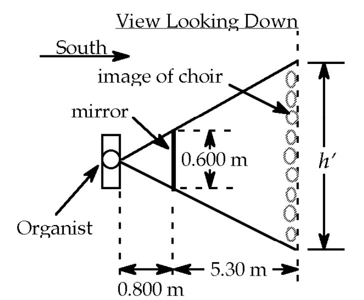
PROBLEM SOLUTIONS

26.1 I stand 40 cm from my bathroom mirror. I scatter light, which travels to the mirror and back to me in time

$$\frac{0.8\text{ m}}{3\times10^{8}\text{ m/s}}\ \boxed{\sim10^{-9}\text{ s}}$$

showing me a view of myself as I was at that look-back time. I'm no Dorian Gray!

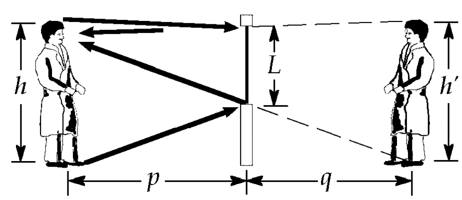
***26.2** The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror. The image of the choir is $0.800\text{ m}+5.30\text{ m}=6.10\text{ m}$ from the organist. Using similar triangles:



$$\frac{h'}{0.600\text{ m}}=\frac{6.10\text{ m}}{0.800\text{ m}}$$

or $$h'=(0.600\text{ m})\left(\frac{6.10\text{ m}}{0.800\text{ m}}\right)=\boxed{4.58\text{ m}}$$

26.3 The flatness of the mirror is described



by $R=\infty, f=\infty$

and $1/f=0$

By our general mirror equation,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=0$$

or $q=-p$

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

$$M=\frac{-q}{p}=1=\frac{h'}{h}$$

so $h'=h=70.0\text{ inches}$

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of his image, as shown. From the geometry of the triangle, we see that the mirror height must be:

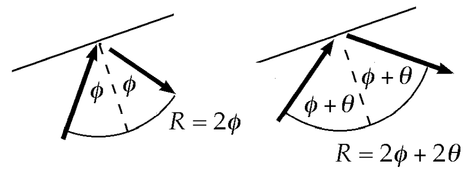
$$h'\left(\frac{p}{p-q}\right)=h'\left(\frac{p}{2p}\right)=\frac{h'}{2}$$

Thus, the mirror must be $\boxed{\text{at least 35.0 inches high}}$.

26.4 If the mirror or the incident beam is rotated by angle θ, then the angle between incident and reflected beams changes

from $R = 2\phi$

to $R = 2\phi + 2\theta$

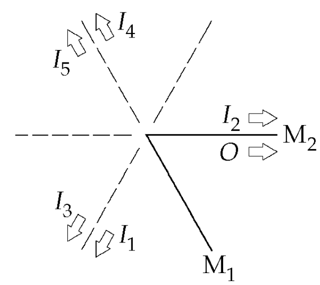


26.5 A graphical construction produces 5 images, with images I_1 and I_2 directly into the mirrors from the object O,

and $(O,\ I_3,\ I_4)$

and $(I_2,\ I_1,\ I_5)$

forming the vertices of equilateral triangles.



26.6 (1) The first image in the left mirror is 5.00 ft behind the mirror, or $\boxed{10.0\text{ ft}}$ from the position of the person.

(2) The first image in the right mirror is located 10.0 ft behind the right mirror, but this location is 25.0 ft from the left mirror. Thus, the second image in the left mirror is 25.0 ft behind the mirror, or $\boxed{30.0\text{ ft}}$ from the person.

(3) The first image in the left mirror forms an image in the right mirror. This first image is 20.0 ft from the right mirror, and, thus, an image 20.0 ft behind the right mirror is formed. This image in the right mirror also forms an image in the left mirror. The distance from this image in the right mirror to the left mirror is 35.0 ft. The third image in the left mirror is, thus, 35.0 ft behind the mirror, or $\boxed{40.0\text{ ft}}$ from the person.

***26.7** The flat mirrors have

$$R \to \infty$$

and $f \to \infty$

The upper mirror M_1 produces a virtual, actual sized image I_1 according to

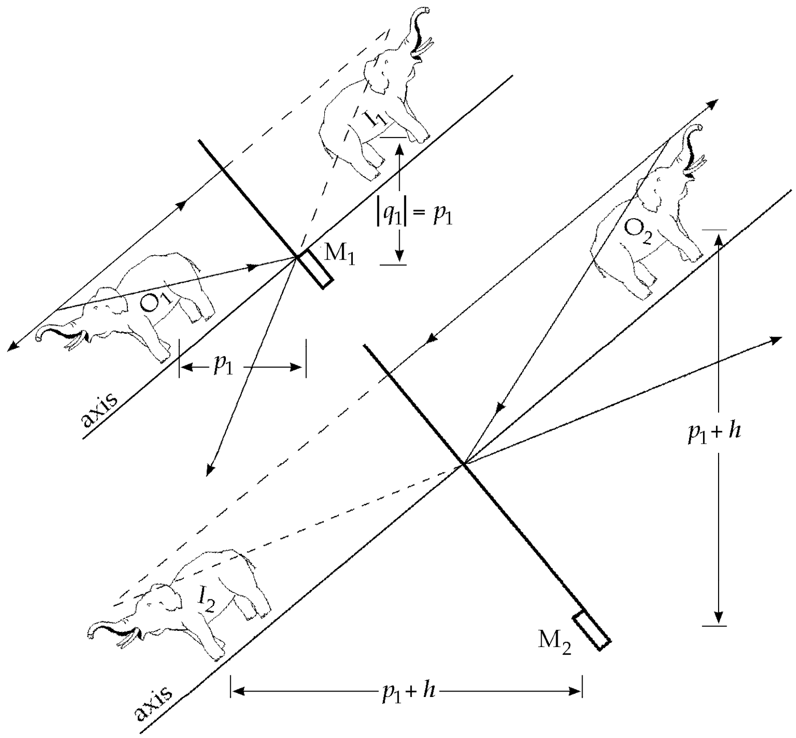
$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} = \frac{1}{\infty} = 0$$

$$q_1 = -p_1$$

with $M_1 = -\frac{q_1}{p_1} = +1$

As shown, this image is above the upper mirror. It is the object for mirror M_2, at object distance

$$p_2 = p_1 + h$$



The lower mirror produces a virtual, actual-size, right-side-up image according to

$$\frac{1}{p_2} + \frac{1}{q_2} = 0$$

$$q_2 = -p_2 = -(p_1 + h)$$

with $M_2 = -\frac{q_2}{p_2} = +1$ and $M_{\text{overall}} = M_1 M_2 = 1$

Thus the final image is at distance $\boxed{p_1 + h}$ behind the lower mirror.

(b) It is $\boxed{\text{virtual}}$

(c) $\boxed{\text{Upright}}$

(d) With magnification $\boxed{+1}$

(e) It $\boxed{\text{does not appear to be reversed}}$ left and right. In a top view of the periscope, parallel rays from the right and left sides of the object stay parallel and on the right and left.

26.8 For a concave mirror, both R and f are positive.

We also know that $$f = \frac{R}{2} = 10.0 \text{ cm}$$

(a) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{40.0 \text{ cm}} = \frac{3}{40.0 \text{ cm}}$$

and $$\boxed{q = 13.3 \text{ cm}}$$

$$M = \frac{q}{p} = -\frac{13.3 \text{ cm}}{40.0 \text{ cm}} = \boxed{-0.333}$$

The image is 13.3 cm in front of the mirror, is $\boxed{\text{real, and inverted}}$

(b) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}} = \frac{1}{20.0 \text{ cm}}$$

and $$\boxed{q = 20.0 \text{ cm}}$$

$$M = \frac{q}{p} = -\frac{20.0 \text{ cm}}{20.0 \text{ cm}} = \boxed{-1.00}$$

The image is 20.0 cm in front of the mirror, is $\boxed{\text{real, and inverted}}$

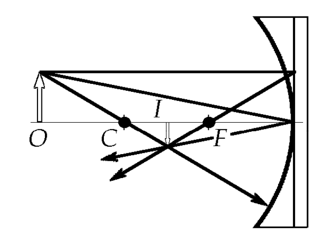
(c) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = 0$$

Thus, $q = \text{infinity.}$

$\boxed{\text{No image is formed}}$. The rays are reflected parallel to each other.

***26.9** (a) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ becomes $\frac{1}{q} = \frac{2}{60.0 \text{ cm}} - \frac{1}{90.0 \text{ cm}}$

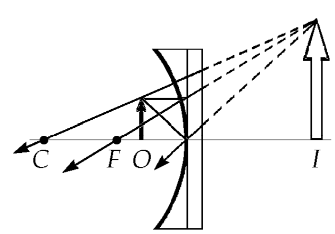
$q = \boxed{45.0 \text{ cm}}$ and $M = \frac{-q}{p} = -\frac{45.0 \text{ cm}}{90.0 \text{ cm}} = \boxed{-0.500}$



(a)

(b) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ becomes $\frac{1}{q} = \frac{2}{60.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}}$

$q = \boxed{-60.0 \text{ cm}}$ and $M = \frac{-q}{p} = -\frac{(-60.0 \text{ cm})}{(20.0 \text{ cm})} = \boxed{3.00}$



(b)

(c) The image (a) is real, inverted and diminished. That of (b) is virtual, upright, and enlarged. The ray diagrams are similar to Figure 26.12(a) and 26.12(b), respectively.

26.10 With radius 2.50 m, the cylindrical wall is a highly efficient mirror for sound, with focal length

$$f = \frac{R}{2} = 1.25 \text{ m}$$

In a vertical plane the sound disperses as usual, but that radiated in a horizontal plane is concentrated in a sound image at distance q from the back of the niche, where

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \text{so} \quad \frac{1}{2.00 \text{ m}} + \frac{1}{q} = \frac{1}{1.25 \text{ m}}$$

$$q = \boxed{3.33 \text{ m}}$$

26.11 (a) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ gives $\frac{1}{30.0 \text{ cm}} + \frac{1}{q} = \frac{2}{(-40.0 \text{ cm})}$

$$\frac{1}{q} = -\frac{2}{40.0 \text{ cm}} - \frac{1}{30.0 \text{ cm}} = -0.0833 \text{ cm}^{-1} \quad \text{so} \quad q = \boxed{-12.0 \text{ cm}}$$

$$M = \frac{-q}{p} = -\frac{(-12.0 \text{ cm})}{30.0 \text{ cm}} = \boxed{0.400}$$

(b) $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$ gives $\frac{1}{60.0 \text{ cm}} + \frac{1}{q} = \frac{2}{(-40.0 \text{ cm})}$

$$\frac{1}{q} = -\frac{2}{40.0 \text{ cm}} - \frac{1}{60.0 \text{ cm}} = -0.0666 \text{ cm}^{-1} \quad \text{so} \quad q = \boxed{-15.0 \text{ cm}}$$

$$M = \frac{-q}{p} = -\frac{(-15.0 \text{ cm})}{60.0 \text{ cm}} = \boxed{0.250}$$

(c) Since $M > 0$, the images are $\boxed{\text{upright}}$.

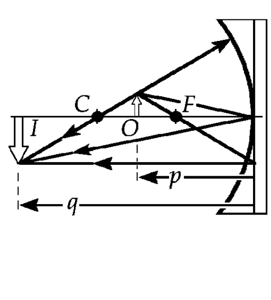
***26.12** (a) $M = -\frac{q}{p}$. For a real image, $q > 0$ so in this case $M = -4.00$

$$q = -pM = 120 \text{ cm}$$

and from $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$

$$R = \frac{2pq}{p+q} = \frac{2(30.0 \text{ cm})(120 \text{ cm})}{(150 \text{ cm})} = \boxed{48.0 \text{ cm}}$$

(b)



26.13 (a) $M = -4 = -\frac{q}{p}$ $\quad q = 4p$

$q - p = 0.60\text{ m} = 4p - p$ $\quad p = 0.2\text{ m}$ $\quad q = 0.8\text{ m}$

$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.2\text{ m}} + \frac{1}{0.8\text{ m}}$ $\quad f = \boxed{160\text{ mm}}$

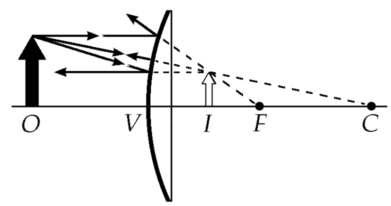
(b) $M = +\frac{1}{2} = -\frac{q}{p}$ $\quad p = -2p$

$|q| + p = 0.20\text{ m} = -q + p = -q - 2q$

$q = -66.7\text{ mm}$ $\quad p = 133\text{ mm}$

$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} = \frac{1}{0.133\text{ m}} + \frac{1}{-0.0667\text{ m}}$ $\quad R = \boxed{-267\text{ mm}}$

***26.14** $M = -\frac{q}{p}$



$q = -Mp = -0.013(30\text{ cm}) = -0.39\text{ cm}$

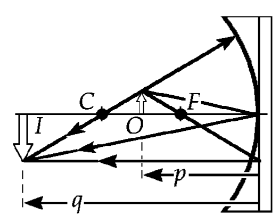
$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R}$

$\frac{1}{30\text{ cm}} + \frac{1}{-0.39\text{ cm}} = \frac{2}{R}$

$R = \frac{2}{-2.53\text{ m}^{-1}} = -0.790\text{ cm}$

The cornea is convex, with radius of curvature $\boxed{0.790\text{ cm}}$

26.15 (a) $q = (p + 5.00\text{ m})$ and, since the image must be real,



$M = -\frac{q}{p} = -5$ or $q = 5p$

Therefore, $p + 5.00 = 5p$

or $p = 1.25\text{ m}$ and $q = 6.25\text{ m}$

From $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$, $\quad R = \frac{2pq}{p+q} = \frac{2(1.25)(6.25)}{1.25 + 6.25} = \boxed{2.08\text{ m (concave)}}$

(b) From part (a), $p = 1.25$ m; the mirror should be $\boxed{1.25\text{ m}}$ in front of the object.

***26.16** Assume that the object distance is the same in both cases (i.e., her face is the same distance from the hubcap regardless of which way it is turned). Also realize that the near image ($q = -10.0$ cm) occurs when using the convex side of the hubcap. Applying the mirror equation to both cases gives:

(concave side: $R = |R|$, $q = -30.0\text{ cm}$)

$$\frac{1}{p} - \frac{1}{30.0} = \frac{2}{|R|}$$

or $$\frac{2}{|R|} = \frac{30.0\text{ cm} - p}{(30.0\text{ cm})p} \qquad (1)$$

(convex side: $R = -|R|$, $q = -10.0\text{ cm}$)

$$\frac{1}{p} - \frac{1}{10.0} = -\frac{2}{|R|}$$

or $$\frac{2}{|R|} = \frac{p - 10.0\text{ cm}}{(10.0\text{ cm})p} \qquad (2)$$

(a) Equating Equations (1) and (2) gives:

$$\frac{30.0\text{ cm} - p}{3.00} = p - 10.0\text{ cm}$$

or $$p = 15.0\text{ cm}$$

Thus, her face is $\boxed{15.0\text{ cm}}$ from the hubcap.

(b) Using the above result ($p = 15.0\text{ cm}$) in Equation [1] gives:

$$\frac{2}{|R|} = \frac{30.0\text{ cm} - 15.0\text{ cm}}{(30.0\text{ cm})(15.0\text{ cm})}$$

or $$\frac{2}{|R|} = \frac{1}{30.0\text{ cm}}$$

and $$|R| = 60.0\text{ cm}$$

The radius of the hubcap is $\boxed{60.0\text{ cm}}$

***26.17** (a) The flat mirror produces an image according to

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R} \qquad \frac{1}{24\text{ m}}+\frac{1}{q}=\frac{1}{\infty}=0 \qquad q=-24.0\text{ m}$$

The image is 24.0 m behind the mirror, distant from your eyes by

$$1.55\text{ m}+24.0\text{ m}=\boxed{25.6\text{ m}}$$

(b) The image is the same size as the object, so $\theta=\frac{h}{d}=\frac{1.50\text{ m}}{25.6\text{ m}}=\boxed{0.0587\text{ rad}}$

(c) $$\frac{1}{p}+\frac{1}{q}=\frac{2}{R} \qquad \frac{1}{24\text{ m}}+\frac{1}{q}=\frac{2}{(-2\text{ m})} \qquad q=\frac{1}{-(1/1\text{ m})-(1/24\text{ m})}=-0.960\text{ m}$$

This image is distant from your eyes by $1.55\text{ m}+0.960\text{ m}=\boxed{2.51\text{ m}}$

(d) The image size is given by $M=\frac{h'}{h}=-\frac{q}{p}$ $\qquad h'=-h\frac{q}{p}=-1.50\text{ m}\left(\frac{-0.960\text{ m}}{24\text{ m}}\right)=0.0600\text{ m}$

So its angular size at your eye is $\theta'=\frac{h'}{d}=\frac{0.06\text{ m}}{2.51\text{ m}}=\boxed{0.0239\text{ rad}}$

(e) Your brain assumes that the car is 1.50 m high and calculate its distance as

$$d'=\frac{h}{\theta'}=\frac{1.50\text{ m}}{0.0239}=\boxed{62.8\text{ m}}$$

26.18 When $R\to\infty$, Equation 26.8 for a spherical surface becomes $q=-p(n_2/n_1)$. We use this to locate the final images of the two surfaces of the glass plate. First, find the image the glass forms of the *bottom* of the plate.

$$q_{B1}=-\left(\frac{1.33}{1.66}\right)(8.00\text{ cm})=-6.41\text{ cm}$$

This virtual image is 6.41 cm below the top surface of the glass of 18.41 cm below the water surface. Next, use this image as an object and locate the image the water forms of the bottom of the plate.

$$q_{B2}=-\left(\frac{1.00}{1.33}\right)(18.41\text{ cm})=-13.84\text{ cm}$$ or 13.84 cm below the water surface.

Now find image the water forms of the *top* surface of the glass.

$$q_3=-\left(\frac{1}{1.33}\right)(12.0\text{ cm})=-9.02\text{ cm}$$ or 9.02 cm below the water surface.

Therefore, the apparent thickness of the glass is $\Delta t=13.84\text{ cm}-9.02\text{ cm}=\boxed{4.82\text{ cm}}$

26.19 $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}=0$ and $R\to\infty$

$$q=-\frac{n_2}{n_1}p=-\frac{1}{1.309}(50.0\text{ cm})=-38.2\text{ cm}$$

Thus, the virtual image of the dust speck is **38.2 cm below the top surface** of the ice.

26.20 $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ so $\frac{1.00}{\infty}+\frac{1.40}{21.0\text{ mm}}=\frac{1.40-1.00}{6.00\text{ mm}}$

and $0.0667=0.0667$

They agree. **The image is inverted, real and dimished**

26.21 From Equation 26.8 $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$

Solve for q to find $q=\frac{n_2Rp}{p(n_2-n_1)-n_1R}$

In this case, $n_1=1.50,\ n_2=1.00,\ R=-15.0\text{ cm}$

and $p=10.0\text{ cm}$

So $q=\frac{(1.00)(-15.0\text{ cm})(10.0\text{ cm})}{(10.0\text{ cm})(1.00-1.50)-(1.50)(-15.0\text{ cm})}=-8.57\text{ cm}$

Therefore, the **apparent depth is 8.57 cm**

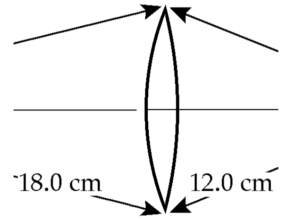
***26.22** $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ becomes $\frac{1.00}{p}+\frac{1.50}{q}=\frac{1.50-1.00}{6.00\text{ cm}}=\frac{1}{12.0\text{ cm}}$

(a) $\frac{1.00}{20.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}$ or $q=\frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}}-\frac{1.00}{20.0\text{ cm}}\right]}=$ **45.0 cm**

(b) $\frac{1.00}{10.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}$ or $q=\frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}}-\frac{1.00}{10.0\text{ cm}}\right]}=$ **–90.0 cm**

(c) $\frac{1.00}{3.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}$ or $q=\frac{1.50}{\left[\frac{1.00}{12.0\text{ cm}}-\frac{1.00}{3.0\text{ cm}}\right]}=$ **–6.00 cm**

26.23 (a) $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (0.440)\left[\frac{1}{12.0\text{ cm}} - \frac{1}{(-18.0\text{ cm})}\right]$



$f = \boxed{16.4\text{ cm}}$

(b) $\frac{1}{f} = (0.440)\left[\frac{1}{18.0\text{ cm}} - \frac{1}{(-12.0\text{ cm})}\right]$

$f = \boxed{16.4\text{ cm}}$

26.24 Let R_1 = outer radius and R_2 = inner radius

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.50-1)\left[\frac{1}{2.00\text{ m}} - \frac{1}{2.50\text{ cm}}\right] = 0.0500\text{ cm}^{-1}$$

so $f = \boxed{20.0\text{ cm}}$

26.25 For a converging lens, f is positive. We use $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

(a) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0\text{ cm}} - \frac{1}{40.0\text{ cm}} = \frac{1}{40.0\text{ cm}}$ $\boxed{q = 40.0\text{ cm}}$

$M = -\frac{q}{p} = -\frac{40.0}{40.0} = \boxed{-1.00}$

The image is $\boxed{\text{real, inverted}}$, and located 40.0 cm past the lens.

(b) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0\text{ cm}} - \frac{1}{20.0\text{ cm}} = 0$ $\boxed{q = \text{infinity}}$

$\boxed{\text{No image}}$ is formed. The rays emerging from the lens are parallel to each other.

(c) $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{20.0\text{ cm}} - \frac{1}{10.0\text{ cm}} = -\frac{1}{20.0\text{ cm}}$ $\boxed{q = -20.0\text{ cm}}$

$M = -\frac{q}{p} = -\frac{(-20.0)}{10.0} = \boxed{2.00}$

The image is $\boxed{\text{upright, virtual}}$ and 20.0 cm in front of the lens.

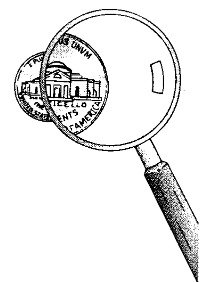
26.26 (a) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{25.0\text{ cm}}-\frac{1}{26.0\text{ cm}}$ $\boxed{q=650\text{ cm}}$

The image is $\boxed{\text{real, inverted, and enlarged}}$

(b) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{25.0\text{ cm}}-\frac{1}{24.0\text{ cm}}$ $\boxed{q=-600\text{ cm}}$

The image is $\boxed{\text{virtual, upright, and enlarged}}$

26.27 We are looking at an enlarged, upright, virtual image:



$M=\frac{h'}{h}=2=-\frac{q}{p}$ so $p=-\frac{q}{2}=-\frac{(-2.84\text{ cm})}{2}=+1.42\text{ cm}$

$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ gives $\frac{1}{1.42\text{ cm}}+\frac{1}{(-2.84\text{ cm})}=\frac{1}{f}$

$\boxed{f=2.84\text{ cm}}$

26.28 (a) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $\frac{1}{32.0\text{ cm}}+\frac{1}{8.00\text{ cm}}=\frac{1}{f}$

so $\boxed{f=6.40\text{ cm}}$

(b) $M=-\frac{q}{p}=-\frac{8.00\text{ cm}}{32.0\text{ cm}}=\boxed{-0.250}$

(c) Since $f>0$, the lens is $\boxed{\text{converging}}$

26.29 To use the lens as a magnifying glass, we form an upright, virtual image:

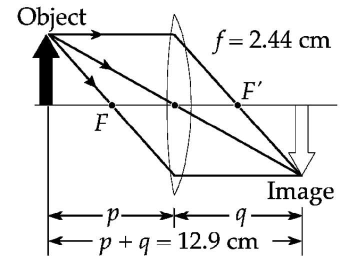
$M=+2.00=-\frac{q}{p}$ or $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$

We eliminate $q=-2.00p$: $\frac{1}{p}+\frac{1}{(-2.00p)}=\frac{1}{15.0\text{ cm}}$

or $\frac{-2.00+1.00}{-2.00p}=\frac{1}{15.0\text{ cm}}$

Solving, $p=\boxed{7.50\text{ cm}}$

26.30 (a) Note that $q = 12.9 \text{ cm} - p$ so $\frac{1}{p} + \frac{1}{12.9 - p} = \frac{1}{2.44}$

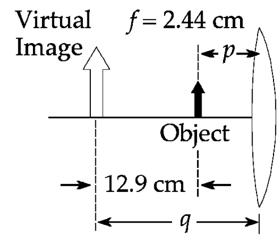


which yields a quadratic in p: $-p^2 + 12.9p = 31.5$

which has solutions $\boxed{p = 9.63 \text{ cm} \text{ or } p = 3.27 \text{ cm}}$

Both solutions are valid.

(b) For a virtual image, $-q = p + 12.9 \text{ cm}$



$\frac{1}{p} - \frac{1}{12.9 + p} = \frac{1}{2.44}$ or $p^2 + 12.9p = 31.8$

from which $\boxed{p = 2.10 \text{ cm}}$ or $p = -15.0 \text{ cm}$

We must have a real object so the –15.0 cm solution must be rejected.

26.31 $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$: $p^{-1} + q^{-1} = \text{constant}$

We may differentiate through with respect to p: $-1p^{-2} - 1q^{-2}\frac{dq}{dp} = 0$

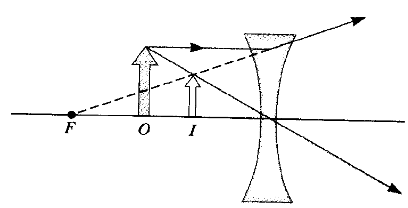
$$\frac{dq}{dp} = -\frac{q^2}{p^2} = -M^2$$

26.32 The image is inverted: $M = \frac{h'}{h} = \frac{-1.80 \text{ m}}{0.0240 \text{ m}} = -75.0 = \frac{-q}{p}$ $q = 75.0p$

(b) $q + p = 3.00 \text{ m} = 75.0p + p$ $p = \boxed{39.5 \text{ mm}}$

(a) $q = 2.96 \text{ m}$ $\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.0395 \text{ m}} + \frac{1}{2.96 \text{ m}}$ $f = \boxed{39.0 \text{ mm}}$

***26.33** (a) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\frac{1}{20.0 \text{ cm}} + \frac{1}{q} = \frac{1}{(-32.0 \text{ cm})}$



so $q = -\left(\frac{1}{20.0} + \frac{1}{32.0}\right)^{-1} = \boxed{-12.3 \text{ cm}}$

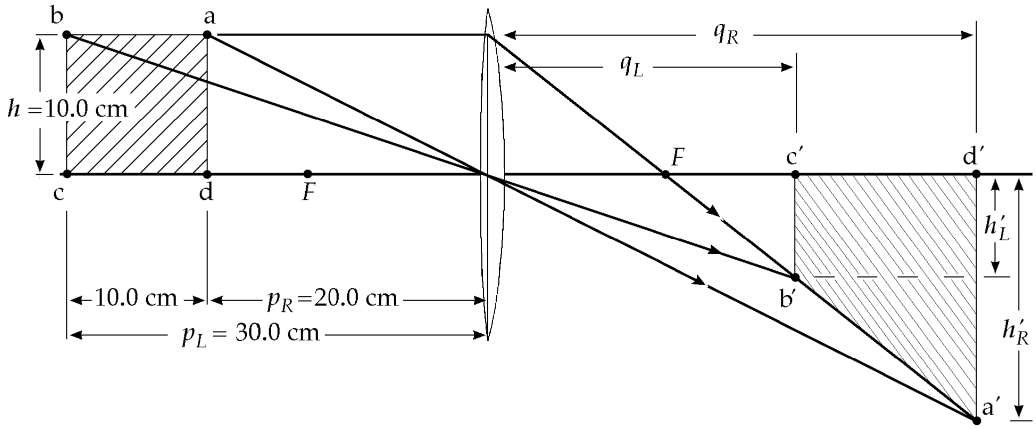
The image is 12.3 cm to the left of the lens.

(b) $M = -\frac{q}{p} = -\frac{(-12.3 \text{ cm})}{20.0 \text{ cm}} = \boxed{0.615}$

(c) See the ray diagram to the right.

***26.34** (a) $\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.50-1)\left[\frac{1}{15.0 \text{ cm}} - \frac{1}{(-12.0 \text{ cm})}\right]$ or $\boxed{f = 13.3 \text{ cm}}$

(b)



(c) To find the area, first find q_R and q_L, along with the heights h'_R and h'_L, using the thin lens equation.

$\frac{1}{p_R} + \frac{1}{q_R} = \frac{1}{f}$ becomes $\frac{1}{20.0 \text{ cm}} + \frac{1}{q_R} = \frac{1}{13.3 \text{ cm}}$ or $q_R = 40.0$ cm

$$h'_R = hM_R = h\left(\frac{-q_R}{p_R}\right) = (10.0 \text{ cm})(-2.00) = -20.0 \text{ cm}$$

$\frac{1}{30.0 \text{ cm}} + \frac{1}{q_L} = \frac{1}{13.3 \text{ cm}}$ or $q_L = 24.0$ cm

$$h'_L = hM_L = (10.0 \text{ cm})(-0.800) = -8.00 \text{ cm}$$

Thus, the area of the image is: $\text{Area} = |q_R - q_L||h'_L| + \frac{1}{2}|q_R - q_L||h'_R - h'_L| = \boxed{224 \text{ cm}^2}$

***26.35** (a) The image distance is: $q = d - p$

Thus, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ becomes $\frac{1}{p} + \frac{1}{d-p} = \frac{1}{f}$

This reduces to a quadratic equation: $p^2 + (-d)p + fd = 0$

which yields: $\boxed{p = \frac{d \pm \sqrt{d^2 - 4fd}}{2} = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - fd}}$

Since $f < d/4$, both solutions are meaningful and the two solutions are not equal to each other. Thus, there are two distinct lens positions that form an image on the screen.

(b) The smaller solution for p gives a larger value for q, with a $\boxed{\text{real, enlarged, inverted image}}$

The larger solution for p describes a $\boxed{\text{real, diminished, inverted image}}$

***26.36** To properly focus the image of a distant object, the lens must be at a distance equal to the focal length from the film ($q_1 = 65.0$ mm). For the closer object:

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f}$$

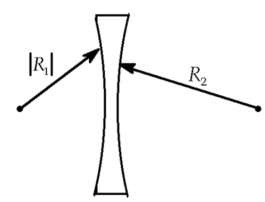
becomes
$$\frac{1}{2000 \text{ mm}} + \frac{1}{q_2} = \frac{1}{65.0 \text{ mm}}$$

and
$$q_2 = (65.0 \text{ mm})\left(\frac{2000}{2000 - 65.0}\right)$$

The lens must be moved **away from the film** by a distance

$$D = q_2 - q_1 = (65.0 \text{ mm})\left(\frac{2000}{2000 - 65.0}\right) - 65.0 \text{ mm} = \boxed{2.18 \text{ mm}}$$

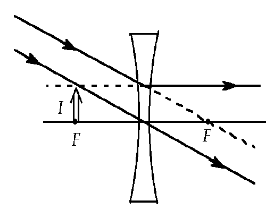
***26.37** (a) The focal length of the lens is given by



$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.53 - 1.00)\left(\frac{1}{-32.5 \text{ cm}} - \frac{1}{42.5 \text{ cm}}\right)$$

$f = -34.7$ cm

Note that R_1 is negative because the center of curvature of the first surface is on the virtual image side.



When $p = \infty$

the thin lens equation gives $q = f$

Thus, the violet image of a very distant object is formed

at $\boxed{q = -34.7 \text{ cm}}$

The image is **virtual, upright and diminished**

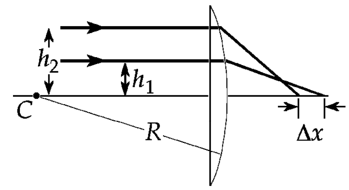
(b) The same ray diagram and image characteristics apply for red light.

Again, $q = f$

and now
$$\frac{1}{f} = (1.51 - 1.00)\left(\frac{1}{-32.5 \text{ cm}} - \frac{1}{42.5 \text{ cm}}\right)$$

giving $f = \boxed{-36.1 \text{ cm}}$

26.38 Ray h_1 is undeviated at the plane surface and strikes the second surface at angle of incidence given by



$$\theta_1 = \sin^{-1}\left(\frac{h_1}{R}\right) = \sin^{-1}\left(\frac{0.500 \text{ cm}}{20.0 \text{ cm}}\right) = 1.43°$$

Then, $$1.00 \sin\theta_2 = 1.60 \sin\theta_1 = (1.60)\left(\frac{0.500}{20.0 \text{ cm}}\right)$$

so $\theta_2 = 2.29°$

The angle this emerging ray makes with the horizontal is $\theta_2 - \theta_1 = 0.860°$

It crosses the axis at a point farther out by f_1

where $$f_1 = \frac{h_1}{\tan(\theta_2 - \theta_1)} = \frac{0.500 \text{ cm}}{\tan(0.860°)} = 33.3 \text{ cm}$$

The point of exit for this ray is distant axially from the lens vertex by

$$20.0 \text{ cm} - \sqrt{(20.0 \text{ cm})^2 - (0.500 \text{ cm})^2} = 0.00625 \text{ cm}$$

so ray h_1 crosses the axis at this distance from the vertex:

$$x_1 = 33.3 \text{ cm} - 0.00625 \text{ cm} = 33.3 \text{ cm}$$

Now we repeat this calculation for ray h_2:

$$\theta_1 = \sin^{-1}\left(\frac{12.0 \text{ cm}}{20.0 \text{ cm}}\right) = 36.9°$$

$$1.00 \sin\theta_2 = 1.60 \sin\theta_1 = (1.60)\left(\frac{12.00}{20.0}\right) \qquad \theta_2 = 73.7°$$

$$f_2 = \frac{h_2}{\tan(\theta_1 - \theta_2)} = \frac{12.0 \text{ cm}}{\tan 36.8°} = 16.0 \text{ cm}$$

$$x_2 = (16.0 \text{ cm})\left(20.0 \text{ cm} - \sqrt{(20.0 \text{ cm})^2 - (12.0 \text{ cm})^2}\right) = 12.0 \text{ cm}$$

Now $$\Delta x = 33.3 \text{ cm} - 12.0 \text{ cm} = \boxed{21.3 \text{ cm}}$$

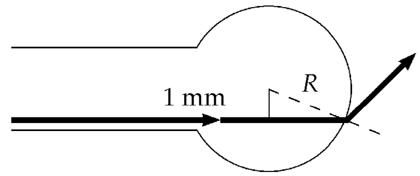
***26.39** The image will be inverted. With $h = 6$ cm, we require $h' = -1$ mm.

(a) $M = \frac{h'}{h} = -\frac{q}{p}$ $\qquad q = -p\frac{h'}{h} = -50 \text{ mm}\frac{(-1 \text{ mm})}{60 \text{ mm}} = \boxed{0.833 \text{ mm}}$

(b) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{50 \text{ mm}} + \frac{1}{0.833 \text{ mm}}$ $\qquad f = \boxed{0.820 \text{ mm}}$

***26.40** (a) Light leaving the sphere refracts away from the normal, so light that travels toward the upper right comes from the $\boxed{\text{bottom half}}$ of the sphere.

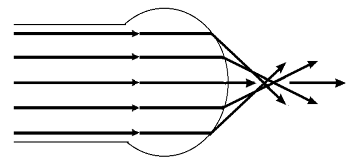
(b) $\sin\theta_1 = 1\text{ mm}/R$ $\qquad n_1\sin\theta_1 = n_2\sin\theta_2$



$$1.50\left(\frac{1\text{ mm}}{R}\right) = 1\sin 90° \qquad R = \boxed{1.50\text{ mm}}$$

(c) $$\delta = |\theta_2 - \theta_1| = \left|90° - \sin^{-1}\left(\frac{1}{1.5}\right)\right| = \boxed{48.2°}$$

(d) No ray can have an angle of refraction larger than 90°, so the ray considered in parts (b) and (c) has the largest possible angle of refraction and then the largest possible deviation.



All other rays, at distances from the axis of less than 1 mm, will leave the sphere at smaller angles with the axis than 48.2°. The angular diameter of the cone of diverging light is $2\times 48.2° = \boxed{96.4°}$.

(e) Light can be absorbed by a coating on the sphere and reradiated in any direction. The sphere deviates from a perfectly spherical shape, probably macroscopically and surely at the scale of the wavelength of light. Light rays enter the sphere along directions not parallel to the axis of the fiber. Inhomogeneities within the sphere scatter light. Light reflects from the interior surface of the sphere.

***26.41** Let I_0 represent the intensity of the light from the nebula and θ_o its angular diameter. With the first telescope, the image diameter h' on the film

is given by $\qquad \theta_o = -h'/f_o$ as $h' = -\theta_o(2000\text{ mm})$

The light power captured by the telescope aperture is $\qquad \mathcal{P}_1 = I_0A_1 = I_0\left[\pi(200\text{ mm})^2/4\right]$,

and the light energy focused on the film during the exposure is

$$E_1 = \mathcal{P}_1\Delta t_1 = I_0\left[\pi(200\text{ mm})^2/4\right](1.50\text{ min})$$

Likewise, the light power captured by the aperture of the second telescope is

$$\mathcal{P}_2 = I_0A_2 = I_0\left[\pi(60.0\text{ mm})^2/4\right]$$

and the light energy is $\qquad E_2 = I_0\left[\pi(60.0\text{ mm})^2/4\right]\Delta t_2$

Therefore, to have the same light energy per unit area, it is necessary that

$$\frac{I_0\left[\pi(60.0\text{ mm})^2/4\right]\Delta t_2}{\pi\left[\theta_o(900\text{ mm})^2/4\right]} = \frac{I_0\left[\pi(200\text{ mm})^2/4\right](1.50\text{ min})}{\pi\left[\theta_o(2000\text{ mm})^2/4\right]}$$

The required exposure time with the second telescope is

$$\Delta t_2 = \frac{(200\text{ mm})^2(900\text{ mm})^2}{(60.0\text{ mm})^2(2000\text{ mm})^2}(1.50\text{ min}) = \boxed{3.38\text{ min}}$$

26.42 If $M < 1$, the lens is diverging and the image is virtual. $d = p - |q| = p + q$

$M = -\dfrac{q}{p}$ so $q = -Mp$ and $d = p - Mp$

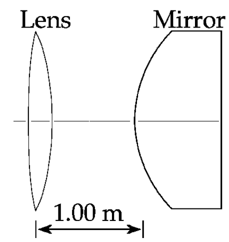
$p = \dfrac{d}{1-M}$: $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{(-Mp)} = \dfrac{(-M+1)}{-Mp} = \dfrac{(1-M)^2}{-Md}$ $\boxed{f = \dfrac{-Md}{(1-M)^2}}$

If $M > 1$, the lens is converging and the image is still virtual.

Now $d = -q - p$

We obtain in this case $\boxed{f = \dfrac{Md}{(M-1)^2}}$

26.43 Start with the first pass through the lens.



$\dfrac{1}{q_1} = \dfrac{1}{f_1} - \dfrac{1}{p_1} = \dfrac{1}{80.0\text{ cm}} - \dfrac{1}{100\text{ cm}}$ $q_1 = 400$ cm to right of lens

For the mirror, $p_2 = -300$ cm

$\dfrac{1}{q_2} = \dfrac{1}{f_2} - \dfrac{1}{p_2} = \dfrac{1}{(-50.0\text{ cm})} - \dfrac{1}{(-300\text{ cm})}$ $q_2 = -60.0$ cm

For the second pass through the lens, $p_3 = 160$ cm

$\dfrac{1}{q_3} = \dfrac{1}{f_1} - \dfrac{1}{p_3} = \dfrac{1}{80.0\text{ cm}} - \dfrac{1}{160\text{ cm}}$ $q_3 = \boxed{160\text{ cm to the left of lens}}$

$M_1 = -\dfrac{q_1}{p_1} = -\dfrac{400\text{ cm}}{100\text{ cm}} = -4.00$ $M_2 = -\dfrac{q_2}{p_2} = -\dfrac{(-60.0\text{ cm})}{(-300\text{ cm})} = -\dfrac{1}{5}$

$M_3 = -\dfrac{q_3}{p_3} = -\dfrac{160\text{ cm}}{160\text{ cm}} = -1$ $M = M_1M_2M_3 = \boxed{-0.800}$

Since $M < 0$ the final image is $\boxed{\text{inverted}}$.

26.44 The real image formed by the concave mirror serves as a real object for the convex mirror with $p = 50$ cm and $q = -10$ cm. Therefore,

$\dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{q}$ $\dfrac{1}{f} = \dfrac{1}{50\text{ cm}} + \dfrac{1}{(-10\text{ cm})}$

gives $f = -12.5$ cm and $R = 2f = \boxed{-25.0\text{ cm}}$

26.45 With $n_1 = 1$

and $n_2 = n$

for the lens material, the first refraction is described by

$$\frac{1}{\infty} + \frac{1}{q_1} = \frac{n-1}{R_1}$$

$$q_1 = \frac{R_1}{n-1}$$

The real image formed by the first surface is a virtual object seen by the second, at $p_2 = -q_1$.

Then for the second refraction

$$\frac{1}{-q_1} + \frac{1}{q_2} = \frac{1-n}{R_2}$$

$$\frac{1-n}{R_1} + \frac{1}{q_2} = \frac{1-n}{R_2}$$

$$\frac{1}{q_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

This image distance for an object at infinity is by definition the focal length.

26.46 $$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{10.0 \text{ cm}} - \frac{1}{12.5 \text{ cm}}$$

so $q_1 = 50.0 \text{ cm}$ (to left of mirror)

This serves as an object for the lens (a virtual object), so

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{(-16.7 \text{ cm})} - \frac{1}{(-25.0 \text{ cm})}$$

$q_2 = -50.3 \text{ cm}$ (to right of lens)

Thus, the final image is located **25.3 cm to right of mirror**.

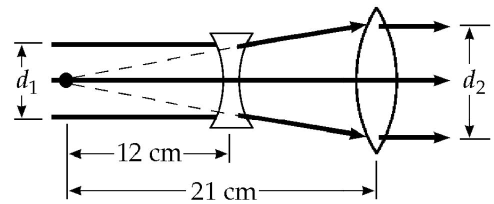
$$M_1 = -\frac{q_1}{p_1} = -\frac{50.0 \text{ cm}}{12.5 \text{ cm}} = -4.00$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-50.3 \text{ cm})}{(-25.0 \text{ cm})} = -2.01$$

$$M = M_1 M_2 = \boxed{8.05}$$

Thus, the final image is **virtual, upright**, 8.05 times the size of object, and 25.3 cm to right of the mirror.

***26.47** A telescope with an eyepiece decreases the diameter of a beam of parallel rays. When light is sent through the same device in the opposite direction, the beam expands. Send the light first through the diverging lens. It will then be diverging from a virtual image found like this:



$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \frac{1}{\infty}+\frac{1}{q}=\frac{1}{-12\text{ cm}}$$

$$q=-12\text{ cm}$$

Use this image as a real object for the converging lens, placing it at the focal point on the object side of the lens, at $p=21\text{ cm}$. Then

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \frac{1}{21\text{ cm}}+\frac{1}{q}=\frac{1}{21\text{ cm}}$$

$$q=\infty$$

The exiting rays will be parallel. The lenses must be $21.0\text{ cm}-12.0\text{ cm}=9.00\text{ cm}$ apart.

By similar triangles, $\frac{d_2}{d_1}=\frac{21\text{ cm}}{12\text{ cm}}=\boxed{1.75\text{ times}}$

***26.48** (a) $I=\frac{\mathcal{P}}{4\pi r^2}=\frac{4.50\text{ W}}{4\pi\left(1.60\times10^{-2}\text{ m}\right)^2}=\boxed{1.40\text{ kW/m}^2}$

(b) $I=\frac{\mathcal{P}}{4\pi r^2}=\frac{4.50\text{ W}}{4\pi(7.20\text{ m})^2}=\boxed{6.91\text{ mW/m}^2}$

(c) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $\qquad \frac{1}{7.20\text{ m}}+\frac{1}{q}=\frac{1}{0.350\text{ m}}$

so $\qquad q=0.368\text{ m}$

and $\qquad M=\frac{h'}{3.20\text{ cm}}=-\frac{q}{p}=-\frac{0.368\text{ m}}{7.20\text{ m}}$

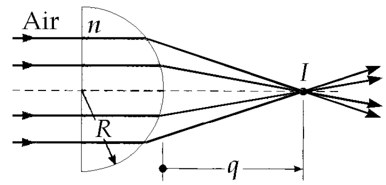
$$h'=\boxed{0.164\text{ cm}}$$

(d) The lens intercepts power given by $\qquad \mathcal{P}=IA=\left(6.91\times10^{-3}\text{ W/m}^2\right)\left[\frac{\pi}{4}(0.150\text{ m})^2\right]$

and puts it all onto the image where $\qquad I=\frac{\mathcal{P}}{A}=\frac{\left(6.91\times10^{-3}\text{ W/m}^2\right)\left[\pi(15.0\text{ cm})^2/4\right]}{\pi(0.164\text{ cm})^2/4}$

$$I=\boxed{58.1\text{ W/m}^2}$$

26.49 A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the second surface, for which $R = -6.00$ cm



The incident rays are parallel, so $p = \infty$

Then, $$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

becomes $$0 + \frac{1}{q} = \frac{1.00 - 1.56}{-6.00 \text{ cm}}$$

and $$\boxed{q = 10.7 \text{ cm}}$$

***26.50** (a) For the light the mirror intercepts,

$$\mathcal{P} = I_0 A = I_0 \pi R_a^{\,2}$$

$$350 \text{ W} = \left(1000 \text{ W/m}^2\right)\pi R_a^{\,2}$$

and $R_a = \boxed{0.334 \text{ m or larger}}$

(b) In $$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R}$$

we have $p \to \infty$

so $$q = \frac{R}{2}$$

$$M = \frac{h'}{h} = -\frac{q}{p}$$

so $$h' = -q\left(\frac{h}{p}\right) = -\left(\frac{R}{2}\right)\left[0.533°\left(\frac{\pi \text{ rad}}{180°}\right)\right] = -\left(\frac{R}{2}\right)(9.30 \text{ m rad})$$

where h/p is the angle the Sun subtends. The intensity at the image is

then $$I = \frac{\mathcal{P}}{\frac{\pi h'^2}{4}} = \frac{4 I_0 \pi R_a^{\,2}}{\pi h'^2} = \frac{4 I_0 R_a^{\,2}}{\left(\frac{R}{2}\right)^2 \left(9.30 \times 10^{-3} \text{ rad}\right)^2}$$

$$120 \times 10^3 \text{ W/m}^2 = \frac{16\left(1000 \text{ W/m}^2\right) R_a^{\,2}}{R^2 \left(9.30 \times 10^{-3} \text{ rad}\right)^2}$$

so $$\boxed{\frac{R_a}{R} = 0.0255 \text{ or larger}}$$

26.51 For the mirror, $f = R/2 = +1.50$ m. In addition, because the distance to the Sun is so much larger than any other distances, we can take $p = \infty$.

The mirror equation, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, then gives $q = f = \boxed{1.50\text{ m}}$

Now, in $M = -\frac{q}{p} = \frac{h'}{h}$

the magnification is nearly zero, but we can be more precise: h/p is the angular diameter of the object. Thus, the image diameter is

$$h' = -\frac{hq}{p} = (-0.533°)\left(\frac{\pi\text{ rad}}{180°}\right)(1.50\text{ m}) = -0.140\text{ m} = \boxed{-1.40\text{ cm}}$$

***26.52** The inverted real image is formed by the lens operating on light directly from the object, on light that has not reflected from the mirror.

For this we have $M = -1.50 = -\frac{q}{p}$ $\quad q = 1.50p$

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\quad \frac{1}{p} + \frac{1}{1.50p} = \frac{1}{10\text{ cm}} = \frac{2.50}{1.50p}$ $\quad p = 10\text{ cm}\left(\frac{2.5}{1.5}\right) = 16.7\text{ cm}$

Then the object is distant from the mirror by $40.0\text{ cm} - 16.7\text{ cm} = 23.3\text{ cm}$

The second image seen by the person is formed by light that first reflects from the mirror and then goes through the lens. For it to be in the same position as the inverted image, the lens must be receiving light from an image formed by the mirror at the same location as the physical object. The formation of this image is described by

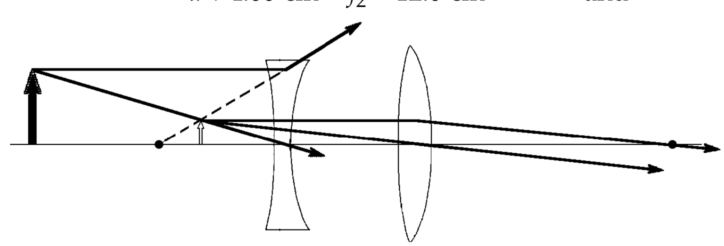
$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\quad \frac{1}{23.3\text{ cm}} + \frac{1}{23.3\text{ cm}} = \frac{1}{f}$ $\quad f = \boxed{11.7\text{ cm}}$

26.53 From the thin lens equation, $q_1 = \frac{f_1 p_1}{p_1 - f_1} = \frac{(-6.00\text{ cm})(12.0\text{ cm})}{12.0\text{ cm} - (-6.00\text{ cm})} = -4.00\text{ cm}$

When we require that $q_2 \to \infty$, the thin lens equation becomes $p_2 = f_2$.

In this case, $p_2 = d - (-4.00\text{ cm})$

Therefore, $d + 4.00\text{ cm} = f_2 = 12.0\text{ cm}$ and $d = \boxed{8.00\text{ cm}}$



***26.54** (a) The lens makers' equation,
$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

becomes: $$\frac{1}{5.00\text{ cm}}=(n-1)\left[\frac{1}{9.00\text{ cm}}-\frac{1}{(-11.0\text{ cm})}\right]$$ giving $\boxed{n = 1.99}$

(b) As the light passes through the lens for the first time, the thin lens equation
$$\frac{1}{p_1}+\frac{1}{q_1}=\frac{1}{f}$$

becomes: $$\frac{1}{8.00\text{ cm}}+\frac{1}{q_1}=\frac{1}{5.00\text{ cm}}$$

or $q_1 = 13.3$ cm, and $M_1=-\dfrac{q_1}{p_1}=-\dfrac{13.3\text{ cm}}{8.00\text{ cm}}=-1.67$

This image becomes the object for the concave mirror with:
$$p_m = 20.0\text{ cm}-q_1 = 20.0\text{ cm}-13.3\text{ cm}=6.67\text{ cm}$$

and $f=\dfrac{R}{2}=+4.00$ cm

The mirror equation becomes: $$\frac{1}{6.67\text{ cm}}+\frac{1}{q_m}=\frac{1}{4.00\text{ cm}}$$

giving $q_m = 10.0$ cm

and $M_2=-\dfrac{q_m}{p_m}=-\dfrac{10.0\text{ cm}}{6.67\text{ cm}}=-1.50$

The image formed by the mirror serves as a real object for the lens on the second pass of the light through the lens with:
$$p_3 = 20.0\text{ cm}-q_m=+10.0\text{ cm}$$

The thin lens equation yields: $$\frac{1}{10.0\text{ cm}}+\frac{1}{q_3}=\frac{1}{5.00\text{ cm}}$$

or $q_3 = 10.0$ cm

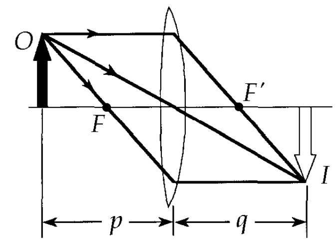
and $M_3=-\dfrac{q_3}{p_3}=-\dfrac{10.0\text{ cm}}{10.0\text{ cm}}=-1.00$

The final image is a real image located $\boxed{\text{10.0 cm to the left of the lens}}$

The overall magnification is $M_{\text{total}}=M_1M_2M_3=\boxed{-2.50}$

(c) Since the total magnification is negative, this final image is $\boxed{\text{inverted}}$

26.55 In the original situation, $p_1 + q_1 = 1.50 \text{ m}$



In the final situation, $p_2 = p_1 + 0.900 \text{ m}$

and $q_2 = q_1 - 0.900 \text{ m}$

Our lens equation is $$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} = \frac{1}{p_2} + \frac{1}{q_2}$$

Substituting, we have $$\frac{1}{p_1} + \frac{1}{1.50 \text{ m} - p_1} = \frac{1}{p_1 + 0.900} + \frac{1}{0.600 - p_1}$$

Adding the fractions, $$\frac{1.50 \text{ m} - p_1 + p_1}{p_1(1.50 \text{ m} - p_1)} = \frac{0.600 - p_1 + p_1 + 0.900}{(p_1 + 0.900)(0.600 - p_1)}$$

Simplified, this becomes $$p_1(1.50 \text{ m} - p_1) = (p_1 + 0.900)(0.600 - p_1)$$

(a) Thus, $$p_1 = \frac{0.540}{1.80} \text{ m} = \boxed{0.300 \text{ m}}$$

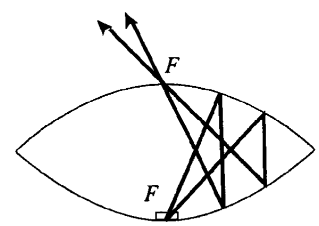
$$p_2 = p_1 + 0.900 = \boxed{1.20 \text{ m}}$$

(b) $$\frac{1}{f} = \frac{1}{0.300 \text{ m}} + \frac{1}{1.50 \text{ m} - 0.300 \text{ m}}$$ and $$f = \boxed{0.240 \text{ m}}$$

(c) The second image is **real, inverted, and diminished**

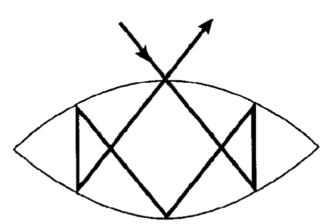
with $$M = -\frac{q_2}{p_2} = \boxed{-0.250}$$

26.56 The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror).



The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror.

Thus, the **image is real, inverted, and actual size**



For the upper mirror:

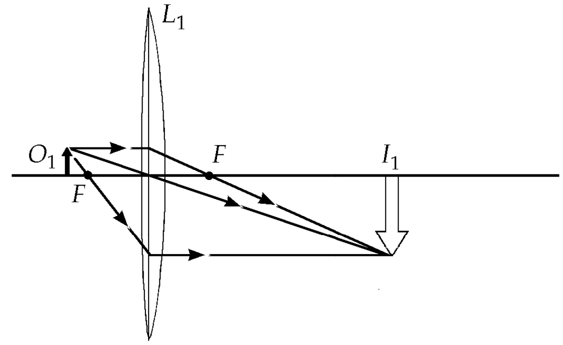
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}: \qquad \frac{1}{7.50 \text{ cm}} + \frac{1}{q_1} = \frac{1}{7.50 \text{ cm}} \qquad q_1 = \infty$$

For the lower mirror:

$$\frac{1}{\infty} + \frac{1}{q_2} = \frac{1}{7.50 \text{ cm}} \qquad q_2 = 7.50 \text{ cm}$$

Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.

26.57 (a) For lens one, as shown in the first figure,

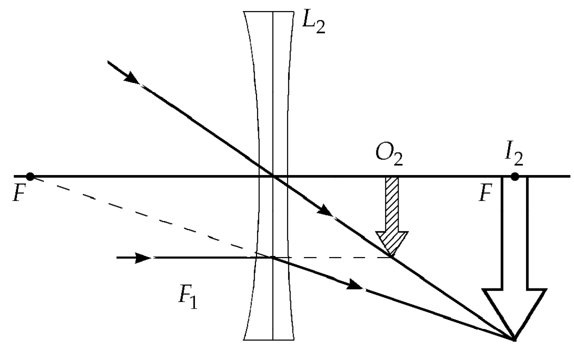


$$\frac{1}{40.0\text{ cm}}+\frac{1}{q_1}=\frac{1}{30.0\text{ cm}}$$

$$q_1 = 120\text{ cm}$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{120\text{ cm}}{40.0\text{ cm}} = -3.00$$

This real image $I_1 = O_2$ is a virtual object for the second lens. That is, it is *behind* the lens, as shown in the second figure. The object distance is



$$p_2 = 110\text{ cm} - 120\text{ cm} = -10.0\text{ cm}$$

$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{-20.0\text{ cm}}:$$

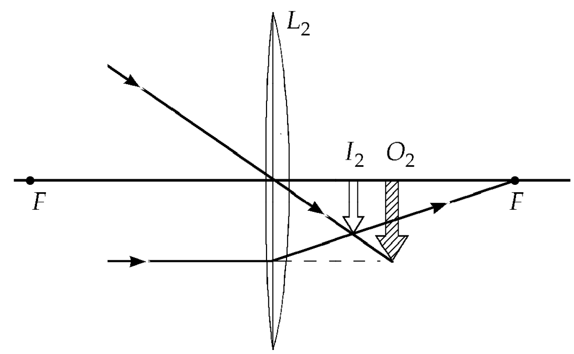
$$q_2 = \boxed{20.0\text{ cm}}$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{20.0\text{ cm}}{(-10.0\text{ cm})} = +2.00$$

$$M_{\text{overall}} = M_1 M_2 = \boxed{-6.00}$$

(b) $M_{\text{overall}} < 0$, so final image is $\boxed{\text{inverted}}$.

(c) If lens two is a converging lens (third figure):



$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{20.0\text{ cm}}$$

$$q_2 = \boxed{6.67\text{ cm}}$$

$$M_2 = -\frac{6.67\text{ cm}}{(-10.0\text{ cm})} = +0.667$$

$$M_{\text{overall}} = M_1 M_2 = \boxed{-2.00}$$

Again, $M_{\text{overall}} < 0$ and the final image is $\boxed{\text{inverted}}$

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 4.58 m

4. See the solution

6. 10.0 ft, 30.0 ft, 40.0 ft

8. (a) 13.3 cm, – 0.333, real and inverted
(b) 20.0 cm, –1.00, real and inverted
(c) no image is formed

10. Behind the worshiper, 3.33 m from the deepest point in the niche

12. (a) 48.0 cm (b) See the solution

14. 7.90 mm

16. (a) 15.0 cm (b) 60.0 cm

18. 4.82 cm

20. See the solution. The image is real, inverted, and diminished.

22. (a) 45.0 cm (b) –90.0 cm (c) –6.00 cm

24. 20.0 cm

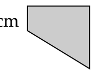
26. (a) 650 cm from the lens on the opposite side from the object; real, inverted, enlarged
(b) 600 cm from the lens on the same side as the object; virtual, upright, enlarged

28. (a) 6.40 cm (b) – 0.250 (c) converging

30. (a) either 9.63 cm or 3.27 cm (b) 2.10 cm

32. (a) $f = 39.0$ mm (b) $p = 39.5$ mm

34. (a) $f = 13.3$ cm (b) The image is a trapezoid as shown.
(c) 224 cm^2



36. 2.18 mm away from the film plane

38. 21.3 cm

40. (a) The bottom half (b) 1.50 mm (c) 48.2°
(d) See the solution; 96.4° (e) See the solution

42. $f = \frac{-Md}{(1-M)^2}$ if $M < 1$, $f = \frac{Md}{(M-1)^2}$ if $M > 1$

44. –25.0 cm

46. 25.3 cm to the right of the mirror; virtual; upright; enlarged 8.05 times

48. (a) 1.40 kW/m^2 (b) 6.91 mW/m^2
(c) 0.164 m (d) 58.1 W/m^2

50. (a) 0.334 m or larger (b) R_a must be at least 0.025 5 R

52. 11.7 cm

54. (a) 1.99 (b) 10.0 cm to the left of the lens, –2.50
(c) inverted

56. See the solution. The image is real, inverted, and actual size.