CHAPTER 28

ANSWERS TO QUESTIONS

Q28.1 A particle is represented by a wave packet of nonzero width. The width necessarily introduces uncertainty in the position of the particle. The width of the wave packet can be reduced toward zero only by adding waves of all possible wavelengths together. Doing this, however, results in loss of all information about the momentum and, therefore, the speed of the particle.

Q28.2 Any object of macroscopic size — including a grain of sand — has an undetectably small wavelength and does not exhibit quantum behavior.

Q28.3 If we set $p^2/2m = q\Delta V$, which is the same for both particles, then we see that the electron has the smaller momentum and therefore the longer wavelength $(\lambda = h/p)$.

Q28.4 The *intensity* of electron waves in some small region of space determines the *probability* that there is an electron in that region.

Q28.5 High-intensity light (with $f > f_{\text{cutoff}}$, of course) ejects more electrons from the surface of the metal. Higher-frequency light ejects individual electrons of higher energy but not more of them.

Q28.6 Wave theory predicts that the photoelectric effect should occur at any frequency, provided the light intensity is high enough. However, as seen in the photoelectric experiments, the light must have a sufficiently high frequency for the effect to occur.

Q28.7 Light has both wave and particle characteristics. In single- and double-slit experiments light behaves like a wave. In the photoelectric effect light behaves like a particle. Light may be characterized as an electromagnetic wave with a particular wavelength or frequency, yet at the same time light may be characterized as a stream of photons, each carrying a discrete energy, *hf*. Since light displays *both* wave and particle characteristics, perhaps it would be fair to call light a "wavicle". It is customary to call a photon a quantum particle, different from a classical particle.

Q28.8 An electron has both wave and particle characteristics. In single- and double-slit diffraction and interference experiments, electrons behave like classical waves. An electron has mass and charge. It carries kinetic energy and momentum in parcels of definite size, as classical particles do. At the same time it has a particular wavelength and frequency. Since an electron displays characteristics of both classical waves and classical particles, it is neither a classical wave nor a classical particle. It is customary to call it a quantum particle, but another invented term, such as "wavicle", would serve equally well.

Q28.9 A few photons would only give a few dots of exposure, apparently randomly scattered.

Q28.10 Ultraviolet light has shorter wavelength and higher photon energy than visible light.

Q28.11 Most stars radiate nearly as blackbodies. Vega has a higher surface temperature than Arcturus. Vega radiates most intensely at shorter wavelengths.

Q28.12 The x-ray photon transfers some of its energy to the electron. Thus, its frequency must decrease.

Q28.13 Your skin is a good approximation to a blackbody. If a patch of your skin or clothing is at $30°\text{C} = 303\text{ K}$, you radiate with highest intensity at wavelength $2.898 \times 10^{-3}\text{ m}\cdot\text{K}/303\text{ K} = 9.6\ \mu\text{m}$. You glow in the infrared. Snakes called pit vipers search for this radiation to locate their prey; so does the army. To an infrared-detector security alarm, a big hot pizza would look very much like you.

Q28.14 The discovery of electron diffraction by Davisson and Germer was a fundamental advance in our understanding of the motion of material particles. Newton's laws fail to properly describe the motion of an object with small mass. It moves as a wave, not as a classical particle. Proceeding from this recognition, the development of quantum mechanics made possible describing the motion of electrons in atoms; understanding molecular structure and the behavior of matter at the atomic scale, including electronics, photonics, and engineered materials; accounting for the motion of nucleons in nuclei; and studying elementary particles.

Q28.15 A particle's wave function represents its state, containing all the information there is about its location and motion. The squared absolute value of its wave function tells where we would classically think of the particle as a spending most its time; $|\Psi|^2$ is the probability distribution function for the position of the particle.

Q28.16 The motion of the quantum particle does not consist of moving through successive points. The particle has no definite position. In can sometimes be found on one side of a node and sometimes on the other side, but never at the node itself. There is no contradiction here, for the quantum particle is moving as a wave. It is not a classical particle. In particular, the particle does not speed up to infinite speed to cross the node.

Q28.17 As Newton's laws are the rules which a particle of large mass follows in its motion, so the Schrödinger equation describes the motion of a quantum particle, a particle of small or large mass. In particular, the states of atomic electrons are confined-wave states with wave functions that are solutions to the Schrödinger equation.

Q28.18 No. The second metal may have a larger work function than the first, in which case the incident photons may not have enough energy to eject photoelectrons.

PROBLEM SOLUTIONS

***28.1** (a) Using $\lambda_{max} T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}$

we get $\lambda_{max} = \dfrac{2.898 \times 10^{-3}\ \text{m}}{2900\ \text{K}} = 9.99 \times 10^{-7}\ \text{m} = \boxed{999\ \text{nm}}$

(b) The $\boxed{\text{peak wavelength is in the infrared}}$ region of the electromagnetic spectrum, which is much wider than the visible region of the spectrum.

***28.2** (a) $\mathcal{P} = eA\sigma T^4$ so $T = \left(\dfrac{\mathcal{P}}{eA\sigma}\right)^{1/4} = \left[\dfrac{3.77 \times 10^{26}\ \text{W}}{1\left[4\pi\left(6.96 \times 10^{8}\ \text{m}\right)^2\right]\left(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\right)}\right]^{1/4}$

$T = \boxed{5.75 \times 10^{3}\ \text{K}}$

(b) $\lambda_{max} = \dfrac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{T} = \dfrac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{5.75 \times 10^{3}\ \text{K}} = 5.04 \times 10^{-7}\ \text{m} = \boxed{504\ \text{nm}}$

28.3 (a) $E = hf = \left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(620 \times 10^{12}\ \text{s}^{-1}\right)\left(\dfrac{1.00\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = \boxed{2.57\ \text{eV}}$

(b) $E = hf = \left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(3.10 \times 10^{9}\ \text{s}^{-1}\right)\left(\dfrac{1.00\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = \boxed{1.28 \times 10^{-5}\ \text{eV}}$

(c) $E = hf = \left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(46.0 \times 10^{6}\ \text{s}^{-1}\right)\left(\dfrac{1.00\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = \boxed{1.91 \times 10^{-7}\ \text{eV}}$

(d) $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}\ \text{m/s}}{620 \times 10^{12}\ \text{Hz}} = 4.84 \times 10^{-7}\ \text{m} = \boxed{484\ \text{nm, visible light (blue)}}$

$\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}\ \text{m/s}}{3.10 \times 10^{9}\ \text{Hz}} = 9.68 \times 10^{-2}\ \text{m} = \boxed{9.68\ \text{cm, radio wave}}$

$\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}\ \text{m/s}}{46.0 \times 10^{6}\ \text{Hz}} = \boxed{6.52\ \text{m, radio wave}}$

28.4 Energy of a single 500-nm photon:

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(500\times10^{-9}\ \text{m}\right)} = 3.98\times10^{-19}\ \text{J}$$

The energy entering the eye each second

$$E = \mathcal{P}\Delta t = IA\Delta t = \left(4.00\times10^{-11}\ \text{W/m}^2\right)\left[\frac{\pi}{4}\left(8.50\times10^{-3}\ \text{m}\right)^2\right](1.00\ \text{s}) = 2.27\times10^{-15}\ \text{J}$$

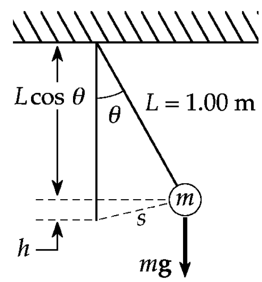
The number of photons required to yield this energy

$$n = \frac{E}{E_\gamma} = \frac{2.27\times10^{-15}\ \text{J}}{3.98\times10^{-19}\ \text{J/photon}} = \boxed{5.71\times10^{3}\ \text{photons}}$$

28.5 Each photon has an energy $E = hf = \left(6.626\times10^{-34}\right)\left(99.7\times10^{6}\right) = 6.61\times10^{-26}\ \text{J}$

This implies that there are $\dfrac{150\times10^{3}\ \text{J/s}}{6.61\times10^{-26}\ \text{J/photon}} = \boxed{2.27\times10^{30}\ \text{photons/s}}$

28.6 We take $\theta = 0.0300$ radians. Then the pendulum's total energy is



$$E = mgh = mg\left(L - L\cos\theta\right)$$

$$E = (1.00\ \text{kg})\left(9.80\ \text{m/s}^2\right)(1.00 - 0.9995) = 4.41\times10^{-3}\ \text{J}$$

The frequency of oscillation is $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}} = 0.498\ \text{Hz}$

The energy is quantized, $E = nhf$

Therefore, $n = \dfrac{E}{hf} = \dfrac{4.41\times10^{-3}\ \text{J}}{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(0.498\ \text{s}^{-1}\right)} = \boxed{1.34\times10^{31}}$

28.7 (a) $\lambda_c = \dfrac{hc}{\phi} = \dfrac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{(4.20\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)} = \boxed{296\ \text{nm}}$

$$f_c = \frac{c}{\lambda_c} = \frac{3.00\times10^{8}\ \text{m/s}}{296\times10^{-9}\ \text{m}} = \boxed{1.01\times10^{15}\ \text{Hz}}$$

(b) $\dfrac{hc}{\lambda} = \phi + e\Delta V_S$: $\dfrac{\left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\right)}{180\times10^{-9}} = (4.20\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right) + \left(1.60\times10^{-19}\right)\Delta V_S$

Therefore, $\boxed{\Delta V_S = 2.71\ \text{V}}$

28.8 $K_{max} = \frac{1}{2} m v_{max}^2 = \frac{1}{2}\left(9.11\times10^{-31}\right)\left(4.60\times10^{5}\right)^2 = 9.64\times10^{-20}\text{ J} = 0.602\text{ eV}$

(a) $\phi = E - K_{max} = \dfrac{1240\text{ eV}\cdot\text{nm}}{625\text{ nm}} - 0.602\text{ nm} = \boxed{1.38\text{ eV}}$

(b) $f_c = \dfrac{\phi}{h} = \dfrac{1.38\text{ eV}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\left(\dfrac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right) = \boxed{3.34\times10^{14}\text{ Hz}}$

***28.9** (a) $e\Delta V_S = \dfrac{hc}{\lambda} - \phi \rightarrow \phi = \dfrac{1240\text{ nm}\cdot\text{eV}}{546.1\text{ nm}} - 0.376\text{ eV} = \boxed{1.90\text{ eV}}$

(b) $e\Delta V_S = \dfrac{hc}{\lambda} - \phi = \dfrac{1240\text{ nm}\cdot\text{eV}}{587.5\text{ nm}} - 1.90\text{ eV} \rightarrow \boxed{\Delta V_S = 0.216\text{ V}}$

28.10 The energy needed is $E = 1.00\text{ eV} = 1.60\times10^{-19}\text{ J}$

The energy absorbed in time interval Δt is $E = \mathcal{P}\,\Delta t = IA\,\Delta t$

so $$\Delta t = \frac{E}{IA} = \frac{1.60\times10^{-19}\text{ J}}{\left(500\text{ J/s}\cdot\text{m}^2\right)\left[\pi\left(2.82\times10^{-15}\text{ m}\right)^2\right]} = 1.28\times10^{7}\text{ s} = \boxed{148\text{ days}}$$

The gross failure of the classical theory of the photoelectric effect contrasts with the success of quantum mechanics.

***28.11** Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $K_{max} = hf - \phi$,

or $$K_{max} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{200\times10^{-9}\text{ m}}\left(\frac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) - 4.70\text{ eV} = 1.51\text{ eV}$$

The sphere is left with positive charge and so with positive potential relative to $V = 0$ at $r = \infty$. As its potential approaches 1.51 V, no further electrons will be able to escape, but will fall back onto the sphere. Its charge is then given by

$$V = \frac{k_e Q}{r} \quad \text{or} \quad Q = \frac{rV}{k_e} = \frac{\left(5.00\times10^{-2}\text{ m}\right)\left(1.51\text{ N}\cdot\text{m/C}\right)}{8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2} = \boxed{8.41\times10^{-12}\text{ C}}$$

28.12 $E = \dfrac{hc}{\lambda} = \dfrac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{700\times10^{-9}\text{ m}} = 2.84\times10^{-19}\text{ J} = \boxed{1.78\text{ eV}}$

$p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{700\times10^{-9}\text{ m}} = \boxed{9.47\times10^{-28}\text{ kg}\cdot\text{m/s}}$

28.13 (a) $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta)$: $\Delta\lambda = \dfrac{6.626\times10^{-34}}{\left(9.11\times10^{-31}\right)\left(3.00\times10^{8}\right)}(1-\cos 37.0°) = \boxed{4.88\times10^{-13}\text{ m}}$

(b) $E_0 = hc/\lambda_0$: $\quad \left(300\times10^{3}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right) = \left(6.626\times10^{-34}\right)\left(3.00\times10^{8}\text{ m/s}\right)/\lambda_0$

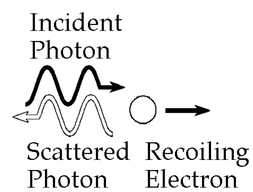
$$\lambda_0 = 4.14\times10^{-12}\text{ m}$$

and $\quad \lambda' = \lambda_0 + \Delta\lambda = 4.63\times10^{-12}\text{ m}$

$$E' = \frac{hc}{\lambda'} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{4.63\times10^{-12}\text{ m}} = 4.30\times10^{14}\text{ J} = \boxed{268\text{ keV}}$$

(c) $K_e = E_0 - E' = 300\text{ keV} - 268.5\text{ keV} = \boxed{31.5\text{ keV}}$

28.14 This is Compton scattering through 180°:



$$E_0 = \frac{hc}{\lambda_0} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(0.110\times10^{-9}\text{ m}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)} = 11.3\text{ keV}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = \left(2.43\times10^{-12}\text{ m}\right)(1-\cos 180°) = 4.86\times10^{-12}\text{ m}$$

$\lambda' = \lambda_0 + \Delta\lambda = 0.115\text{ nm}$ so $\quad E' = \dfrac{hc}{\lambda'} = 10.8\text{ keV}$

By conservation of momentum for the photon-electron system, $\quad \dfrac{h}{\lambda_0}\mathbf{i} = \dfrac{h}{\lambda'}(-\mathbf{i}) + p_e\,\mathbf{i}$

and $\quad p_e = h\left(\dfrac{1}{\lambda_0} - \dfrac{1}{\lambda'}\right)$

$$p_e = \left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(\frac{\left(3.00\times10^{8}\text{ m/s}\right)/c}{1.60\times10^{-19}\text{ J/eV}}\right)\left(\frac{1}{0.110\times10^{-9}\text{ m}} + \frac{1}{0.115\times10^{-9}\text{ m}}\right) = \boxed{22.1\text{ keV}/c}$$

By conservation of system energy, $\quad 11.3\text{ keV} = 10.8\text{ keV} + K_e$

so that $\quad \boxed{K_e = 478\text{ eV}}$

Check: $\quad E^2 = p^2c^2 + m_e^2c^4 \quad$ or $\quad (m_ec^2 + K_e)^2 = (pc)^2 + (m_ec^2)^2$

$$(511\text{ keV} + 0.478\text{ keV})^2 = (22.1\text{ keV})^2 + (511\text{ keV})^2$$

$$2.62\times10^{11} = 2.62\times10^{11}$$

28.15 With $K_e = E'$, $K_e = E_0 - E'$ gives $E' = E_0 - E'$

$E' = \dfrac{E_0}{2}$ and $\lambda' = \dfrac{hc}{E'}$ $\lambda' = \dfrac{hc}{E_0/2} = 2\dfrac{hc}{E_0} = 2\lambda_0$

$\lambda' = \lambda_0 + \lambda_C(1-\cos\theta)$: $2\lambda_0 = \lambda_0 + \lambda_C(1-\cos\theta)$

$1-\cos\theta = \dfrac{\lambda_0}{\lambda_C} = \dfrac{0.00160}{0.00243}$ $\theta = \boxed{70.0°}$

***28.16** (a) $K = \frac{1}{2}m_e v^2$: $K = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(1.40\times10^{6}\text{ m/s}\right)^2 = 8.93\times10^{-19}\text{ J} = 5.58\text{ eV}$

$$E_0 = \frac{hc}{\lambda_0} = \frac{1240\text{ eV}\cdot\text{nm}}{0.800\text{ nm}} = 1550\text{ eV}$$

$E' = E_0 - K$ and $\lambda' = \dfrac{hc}{E'} = \dfrac{1240\text{ eV}\cdot\text{nm}}{1550\text{ eV} - 5.58\text{ eV}} = 0.803\text{ nm}$

$$\Delta\lambda = \lambda' - \lambda_0 = 0.00288\text{ nm} = \boxed{2.88\text{ pm}}$$

(b) $\Delta\lambda = \lambda_C(1-\cos\theta)$: $\cos\theta = 1 - \dfrac{\Delta\lambda}{\lambda_C} = 1 - \dfrac{0.00288\text{ nm}}{0.00243\text{ nm}} = -0.189$,

so $\boxed{\theta = 101°}$

28.17 $$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\left(1.67\times10^{-27}\text{ kg}\right)\left(1.00\times10^{6}\text{ m/s}\right)} = \boxed{3.97\times10^{-13}\text{ m}}$$

28.18 (a) Electron: $\lambda = \dfrac{h}{p}$ and $K = \frac{1}{2}m_e v^2 = \dfrac{m_e^2 v^2}{2m_e} = \dfrac{p^2}{2m_e}$ so $p = \sqrt{2m_e K}$

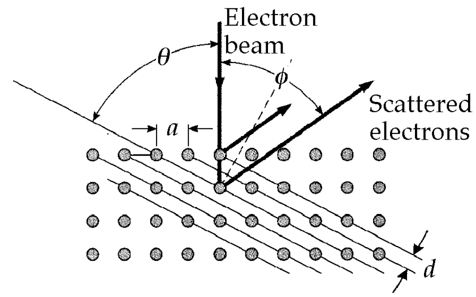
and $$\lambda = \frac{h}{\sqrt{2m_e K}} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)(3.00)\left(1.60\times10^{-19}\text{ J}\right)}}$$

$$\lambda = 7.09\times10^{-10}\text{ m} = \boxed{0.709\text{ nm}}$$

(b) Photon: $\lambda = c/f$ and $E = hf$ so $f = E/h$

and $$\lambda = \frac{hc}{E} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{3\left(1.60\times10^{-19}\text{ J}\right)} = 4.14\times10^{-7}\text{ m} = \boxed{414\text{ nm}}$$

28.19 From the Bragg condition (Eq 27.18),



$$m\lambda = 2d\sin\theta = 2d\cos\left(\frac{\phi}{2}\right)$$

But $$d = a\sin\left(\frac{\phi}{2}\right)$$

where a is the lattice spacing.

Thus, with $m = 1$, $$\lambda = 2a\sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right) = a\sin\phi$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} \qquad \lambda = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(54.0\times1.60\times10^{-19}\ \text{J}\right)}} = 1.67\times10^{-10}\ \text{m}$$

Therefore, the lattice spacing is $$a = \frac{\lambda}{\sin\phi} = \frac{1.67\times10^{-10}\ \text{m}}{\sin 50.0°} = 2.18\times10^{-10} = \boxed{0.218\ \text{nm}}$$

***28.20** (a) The wavelength of the student is $\lambda = h/p = h/mv$. If w is the width of the diffraction aperture,

then we need $$w \le 10.0\lambda = 10.0\left(h/mv\right)$$

so that $$v \le 10.0\frac{h}{mw} = 10.0\left(\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(80.0\ \text{kg})(0.750\ \text{m})}\right) = \boxed{1.10\times10^{-34}\ \text{m/s}}$$

(b) Using $\Delta t = \frac{d}{v}$ we get: $$\Delta t \ge \frac{0.150\ \text{m}}{1.10\times10^{-34}\ \text{m/s}} = \boxed{1.36\times10^{33}\ \text{s}}$$

(c) $\boxed{\text{No}}$. The minimum time to pass through the door is over 10^{15} times the age of the Universe.

28.21 (a) $\lambda \sim 10^{-14}$ m or less. $$p = \frac{h}{\lambda} \sim \frac{6.6\times10^{-34}\ \text{J}\cdot\text{s}}{10^{-14}\ \text{m}} = 10^{-19}\ \text{kg}\cdot\text{m/s or more.}$$

The energy of the electron is $$E = \sqrt{p^2c^2 + m_e{}^2c^4} \sim \sqrt{\left(10^{-19}\right)^2\left(3\times10^8\right)^2 + \left(9\times10^{-31}\right)^2\left(3\times10^8\right)^4}$$

or $$E \sim 10^{-11}\ \text{J} \sim 10^8\ \text{eV or more,}$$

so that $$K = E - m_ec^2 \sim 10^8\ \text{eV} - \left(0.5\times10^6\ \text{eV}\right)\ \boxed{\sim 10^8\ \text{eV}}\ \text{or more.}$$

(b) The electric potential energy of the electron-nucleus system would be

$$U_e = \frac{k_eq_1q_2}{r} \sim \frac{\left(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10^{-19}\ \text{C}\right)(-e)}{10^{-14}\ \text{m}} \sim -10^5\ \text{eV}$$

With its $K + U_e >> 0$, $\boxed{\text{the electron would immediately escape the nucleus}}$

28.22 (a) From $E = \gamma m_e c^2$ $\qquad \gamma = \frac{20.0 \times 10^3 \text{ MeV}}{0.511 \text{ MeV}} = \boxed{3.91 \times 10^4}$

(b) $p \approx \frac{E}{c}$ (for $m_e c^2 << pc$) $\qquad p = \frac{\left(2.00 \times 10^4 \text{ MeV}\right)\left(1.60 \times 10^{-13} \text{ J/MeV}\right)}{\left(3.00 \times 10^8 \text{ m/s}\right)} = \boxed{1.07 \times 10^{-17} \text{ kg} \cdot \text{m/s}}$

(c) $\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{1.07 \times 10^{-17} \text{ kg} \cdot \text{m/s}} = \boxed{6.22 \times 10^{-17} \text{ m}}$

Since the size of a nucleus is on the order of 10^{-14} m, the 20-GeV electrons would be small enough to go through the nucleus.

***28.23** As a bonus, we begin by proving that the phase speed $v_p = \omega / k$ is not the speed of the particle.

$$v_p = \frac{\omega}{k} = \frac{\sqrt{p^2c^2 + m^2c^4}\hbar}{\hbar \gamma m v} = \frac{\sqrt{\gamma^2 m^2 v^2 c^2 + m^2 c^4}}{\sqrt{\gamma^2 m^2 v^2}} = c\sqrt{1 + \frac{c^2}{\gamma^2 v^2}} = c\sqrt{1 + \frac{c^2}{v^2}\left(1 - \frac{v^2}{c^2}\right)} = c\sqrt{1 + \frac{c^2}{v^2} - 1} = \frac{c^2}{v}$$

In fact, the phase speed is larger than the speed of light. A point of constant phase in the wave function carries no mass, no energy, and no information.

Now for the group speed: $\qquad v_g = \frac{d\omega}{dk} = \frac{d\hbar\omega}{d\hbar k} = \frac{dE}{dp} = \frac{d}{dp}\sqrt{m^2c^4 + p^2c^2}$

$$v_g = \tfrac{1}{2}\left(m^2c^4 + p^2c^2\right)^{-1/2}\left(0 + 2pc^2\right) = \sqrt{\frac{p^2c^4}{p^2c^2 + m^2c^4}}$$

$$v_g = c\sqrt{\frac{\gamma^2 m^2 v^2}{\gamma^2 m^2 v^2 + m^2 c^2}} = c\sqrt{\frac{\frac{v^2}{1 - v^2/c^2}}{\frac{v^2}{1 - v^2/c^2} + c^2}} = c\sqrt{\frac{\frac{v^2}{1 - v^2/c^2}}{\frac{v^2 + c^2 - v^2}{1 - v^2/c^2}}} = v$$

It is this speed at which mass, energy, and momentum are transported.

28.24 Consider the first bright band away from the center:

$d\sin\theta = m\lambda \qquad \left(6.00 \times 10^{-8} \text{ m}\right)\sin\left(\tan^{-1}\left[\frac{0.400}{200}\right]\right) = (1)\lambda = 1.20 \times 10^{-10} \text{ m}$

$\lambda = \frac{h}{m_e v}$ so $\qquad m_e v = \frac{h}{\lambda}$

and $\qquad K = \tfrac{1}{2} m_e v^2 = \frac{m_e^2 v^2}{2m_e} = \frac{h^2}{2m_e \lambda^2} = e\Delta V$

$\Delta V = \frac{h^2}{2em_e\lambda^2} \qquad \Delta V = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)^2}{2\left(1.60 \times 10^{-19} \text{ C}\right)\left(9.11 \times 10^{-31} \text{ kg}\right)\left(1.20 \times 10^{-10} \text{ m}\right)^2} = \boxed{105 \text{ V}}$

28.25 (a) $\lambda = \dfrac{h}{mv} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(1.67\times10^{-27}\ \text{kg}\right)(0.400\ \text{m/s})} = \boxed{9.92\times10^{-7}\ \text{m}}$

(b) For destructive interference in a multiple-slit experiment, $d\sin\theta = \left(m+\frac{1}{2}\right)\lambda$, with $m=0$ for the first minimum.

Then, $\theta = \sin^{-1}\left(\dfrac{\lambda}{2d}\right) = 0.0284°$

so $\dfrac{y}{L} = \tan\theta$ $\quad y = L\tan\theta = (10.0\ \text{m})(\tan 0.0284°) = \boxed{4.96\ \text{mm}}$

(c) We cannot say the neutron passed through one slit. We can only say it passed through the slits.

28.26 $\lambda = \dfrac{h}{p}$ $\quad p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{1.00\times10^{-11}\ \text{m}} = 6.63\times10^{-23}\ \text{kg}\cdot\text{m/s}$

(a) electrons: $K_e = \dfrac{p^2}{2m_e} = \dfrac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\left(9.11\times10^{-31}\right)}\ \text{J} = \boxed{15.1\ \text{keV}}$

The relativistic answer is more precisely correct:

$$K_e = \sqrt{p^2c^2 + m_e{}^2c^4} - m_ec^2 = 14.9\ \text{keV}$$

(b) photons: $E_\gamma = pc = \left(6.63\times10^{-23}\right)\left(3.00\times10^{8}\right) = \boxed{124\ \text{keV}}$

28.27 For the electron, $\Delta p = m_e\Delta v = \left(9.11\times10^{-31}\ \text{kg}\right)(500\ \text{m/s})\left(1.00\times10^{-4}\right) = 4.56\times10^{-32}\ \text{kg}\cdot\text{m/s}$

$$\Delta x = \frac{h}{4\pi\,\Delta p} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(4.56\times10^{-32}\ \text{kg}\cdot\text{m/s}\right)} = \boxed{1.16\ \text{mm}}$$

For the bullet, $\Delta p = m\Delta v = (0.0200\ \text{kg})(500\ \text{m/s})\left(1.00\times10^{-4}\right) = 1.00\times10^{-3}\ \text{kg}\cdot\text{m/s}$

$$\Delta x = \frac{h}{4\pi\,\Delta p} = \boxed{5.28\times10^{-32}\ \text{m}}$$

28.28 (a) $\Delta p\Delta x = m\Delta v\Delta x \ge \hbar/2$

so $\Delta v \ge \dfrac{h}{4\pi m\Delta x} = \dfrac{2\pi\ \text{J}\cdot\text{s}}{4\pi(2.00\ \text{kg})(1.00\ \text{m})} = \boxed{0.250\ \text{m/s}}$

(b) The duck might move by $(0.25\ \text{m/s})(5\ \text{s}) = 1.25\ \text{m}$. With original position uncertainty of $1.00\ \text{m}$, we can think of Δx growing to $1.00\ \text{m} + 1.25\ \text{m} = \boxed{2.25\ \text{m}}$

***28.29** $\frac{\Delta y}{x} = \frac{\Delta p_y}{p_x}$ and $d\Delta p_y \geq h/4\pi$

Eliminate Δp_y and solve for x.

$x = 4\pi p_x(\Delta y)\frac{d}{h}$: $x = 4\pi\left(1.00 \times 10^{-3}\ \text{kg}\right)(100\ \text{m/s})\left(1.00 \times 10^{-2}\ \text{m}\right)\frac{\left(2.00 \times 10^{-3}\ \text{m}\right)}{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)}$

The answer , $x = \boxed{3.79 \times 10^{28}\ \text{m}}$, is 190 times greater than the diameter of the Universe!

***28.30** From the uncertainty principle $\Delta E \Delta t \geq \hbar/2$

or $\Delta\left(mc^2\right)\Delta t = \hbar/2$

Therefore, $\frac{\Delta m}{m} = \frac{h}{4\pi c^2(\Delta t)m} = \frac{h}{4\pi(\Delta t)E_R}$

$$\frac{\Delta m}{m} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(8.70 \times 10^{-17}\ \text{s}\right)(135\ \text{MeV})}\left(\frac{1\,\text{MeV}}{1.60 \times 10^{-13}\ \text{J}}\right) = \boxed{2.81 \times 10^{-8}}$$

28.31 (a) At the top of the ladder, the woman holds a pellet inside a small region Δx_i. Thus, the uncertainty principle requires her to release it with typical horizontal momentum $\Delta p_x = m\Delta v_x = \hbar/2\Delta x_i$. It falls to the floor in a travel time given by $H = 0 + \frac{1}{2}gt^2$ as $t = \sqrt{2H/g}$, so the total width of the impact points is

$$\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \left(\frac{\hbar}{2m\Delta x_i}\right)\sqrt{\frac{2H}{g}} = \Delta x_i + \frac{A}{\Delta x_i}$$

where $A = \frac{\hbar}{2m}\sqrt{\frac{2H}{g}}$

To minimize Δx_f, we require $\frac{d\left(\Delta x_f\right)}{d\left(\Delta x_i\right)} = 0$ or $1 - \frac{A}{\Delta x_i^2} = 0$

so $\Delta x_i = \sqrt{A}$

The minimum width of the impact points is

$$\left(\Delta x_f\right)_{\min} = \left(\Delta x_i + \frac{A}{\Delta x_i}\right)\Bigg|_{\Delta x_i = \sqrt{A}} = 2\sqrt{A} = \boxed{\sqrt{\frac{2\hbar}{m}}\left(\frac{2H}{g}\right)^{1/4}}$$

(b) $$\left(\Delta x_f\right)_{\min} = \left[\frac{2\left(1.0546 \times 10^{-34}\ \text{J}\cdot\text{s}\right)}{5.00 \times 10^{-4}\ \text{kg}}\right]^{1/2}\left[\frac{2(2.00\ \text{m})}{9.80\ \text{m/s}^2}\right]^{1/4} = \boxed{5.19 \times 10^{-16}\ \text{m}}$$

28.32 Probability $P = \int_{-a}^{a} |\psi(x)|^2 = \int_{-a}^{a} \frac{a}{\pi(x^2+a^2)} dx = \left(\frac{a}{\pi}\right)\left(\frac{1}{a}\right)\tan^{-1}\left(\frac{x}{a}\right)\Big|_{-a}^{a}$

$$P = \frac{1}{\pi}\left[\tan^{-1}1 - \tan^{-1}(-1)\right] = \frac{1}{\pi}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \boxed{\frac{1}{2}}$$

28.33 (a) $$\psi(x) = A\sin\left(\frac{2\pi x}{\lambda}\right) = A\sin\left(5.00\times10^{10}\,x\right)$$

so $$\frac{2\pi}{\lambda} = 5.00\times10^{10}\ \text{m}^{-1}$$

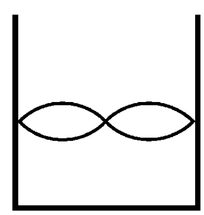
$$\lambda = \frac{2\pi}{5.00\times10^{10}} = \boxed{1.26\times10^{-10}\ \text{m}}$$

(b) $$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{1.26\times10^{-10}\ \text{m}} = \boxed{5.27\times10^{-24}\ \text{kg}\cdot\text{m/s}}$$

(c) $m = 9.11\times10^{-31}$ kg

$$K = \frac{p^2}{2m} = \frac{\left(5.27\times10^{-24}\ \text{kg}\cdot\text{m/s}\right)^2}{\left(2\times9.11\times10^{-31}\ \text{kg}\right)} = 1.52\times10^{-17}\ \text{J} = \frac{1.52\times10^{-17}\ \text{J}}{1.602\times10^{-19}\ \text{J/eV}} = \boxed{95.5\ \text{eV}}$$

28.34 For an electron wave to "fit" into an infinitely deep potential well, an integral number of half-wavelengths must equal the width of the well.



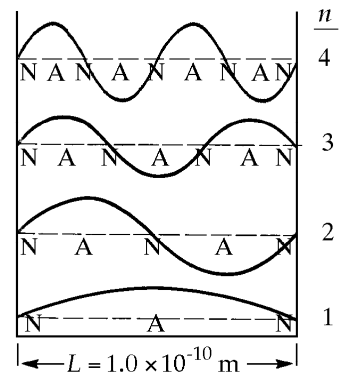
$$\frac{n\lambda}{2} = 1.00\times10^{-9}\ \text{m} \qquad \text{so} \qquad \lambda = \frac{2.00\times10^{-9}}{n} = \frac{h}{p}$$

(a) Since $$K = \frac{p^2}{2m_e} = \frac{\left(h^2/\lambda^2\right)}{2m_e} = \frac{h^2}{2m_e}\frac{n^2}{\left(2\times10^{-9}\right)^2} = \left(0.377n^2\right)\ \text{eV}$$

For $K \cong 6$ eV , $\boxed{n = 4}$

(b) With $n = 4$, $\boxed{K = 6.03\ \text{eV}}$

28.35 (a) We can draw a diagram that parallels our treatment of standing mechanical waves. In each state, we measure the distance d from one node to another (N to N), and base our solution upon that:

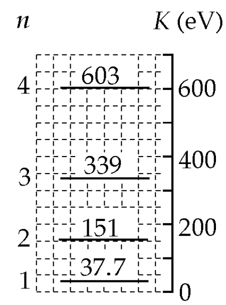


Since $d_{\text{N to N}} = \dfrac{\lambda}{2}$ and $\lambda = \dfrac{h}{p}$

$$p = \frac{h}{\lambda} = \frac{h}{2d}$$

Next, $K = \dfrac{p^2}{2m_e} = \dfrac{h^2}{8m_e d} = \dfrac{1}{d^2}\left[\dfrac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11 \times 10^{-31}\ \text{kg}\right)}\right]$

Evaluating, $K = \dfrac{6.02 \times 10^{-38}\ \text{J}\cdot\text{m}^2}{d^2}$ $K = \dfrac{3.77 \times 10^{-19}\ \text{eV}\cdot\text{m}^2}{d^2}$



In state 1, $d = 1.00 \times 10^{-10}$ m $K_1 = 37.7$ eV

In state 2, $d = 5.00 \times 10^{-11}$ m $K_2 = 151$ eV

In state 3, $d = 3.33 \times 10^{-11}$ m $K_3 = 339$ eV

In state 4, $d = 2.50 \times 10^{-11}$ m $K_4 = 603$ eV

(b) When the electron falls from state 2 to state 1, it puts out energy

$$E = 151\ \text{eV} - 37.7\ \text{eV} = 113\ \text{eV} = hf = \frac{hc}{\lambda}$$

into emitting a photon of wavelength

$$\lambda = \frac{hc}{E} = \frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00 \times 10^8\ \text{m/s}\right)}{(113\ \text{eV})\left(1.60 \times 10^{-19}\ \text{J/eV}\right)} = 11.0\ \text{nm}$$

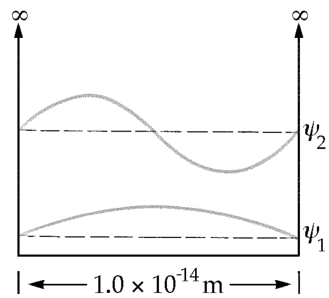
The wavelengths of the other spectral lines we find similarly:

Transition	$4 \to 3$	$4 \to 2$	$4 \to 1$	$3 \to 2$	$3 \to 1$	$2 \to 1$
E(eV)	264	452	565	188	302	113
λ(nm)	4.71	2.75	2.20	6.60	4.12	11.0

28.36 The confined proton can be described in the same way as a standing wave on a string. At level 1, the node–to–node distance of the standing wave is 1.00×10^{-14} m, so the wavelength is twice this distance: $h/p = 2.00\times10^{-14}$ m.

The proton's kinetic energy is

$$K = \tfrac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\left(1.67\times10^{-27}\ \text{kg}\right)\left(2.00\times10^{-14}\ \text{m}\right)^2} = \frac{3.29\times10^{-13}\ \text{J}}{1.60\times10^{-19}\ \text{J/eV}} = 2.05\ \text{MeV}$$



In the first excited state, level 2, the node–to–node distance is half as long as in state 1. The momentum is two times larger and the energy is four times larger: $K = 8.22$ MeV.

The proton has mass, has charge, moves slowly compared to light in a standing wave state, and stays inside the nucleus. When it falls from level 2 to level 1, its energy change is

2.05 MeV − 8.22 MeV = −6.16 MeV

Therefore, we know that a photon (a traveling wave with no mass and no charge) is emitted at the speed of light, and that it has an energy of $\boxed{+6.16\ \text{MeV}}$.

Its frequency is $$f = \frac{E}{h} = \frac{\left(6.16\times10^{6}\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = 1.49\times10^{21}\ \text{Hz}$$

And its wavelength is $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{1.49\times10^{21}\ \text{s}^{-1}} = \boxed{2.02\times10^{-13}\ \text{m}}$$

This is a gamma ray, according to Figure 24.13.

28.37 (a) $$\langle x\rangle = \int_0^L x\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \frac{2}{L}\int_0^L x\left(\tfrac{1}{2} - \tfrac{1}{2}\cos\frac{4\pi x}{L}\right)dx$$

$$\langle x\rangle = \frac{1}{L}\frac{x^2}{2}\bigg|_0^L - \frac{1}{L}\frac{L^2}{16\pi^2}\left[\frac{4\pi x}{L}\sin\frac{4\pi x}{L} + \cos\frac{4\pi x}{L}\right]_0^L = \boxed{\frac{L}{2}}$$

(b) $$\text{Probability} = \int_{0.490L}^{0.510L}\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \left[\frac{1}{L}x - \frac{1}{L}\frac{L}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.490L}^{0.510L}$$

$$\text{Probability} = 0.020 - \frac{1}{4\pi}\left(\sin 2.04\pi - \sin 1.96\pi\right) = \boxed{5.26\times10^{-5}}$$

(c) $$\text{Probability}\left[\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.240L}^{0.260L} = \boxed{3.99\times10^{-2}}$$

(d) In the $n = 2$ graph in Figure 28.21 (b), it is more probable to find the particle either near $x = \frac{L}{4}$ or $x = \frac{3L}{4}$ than at the center, where the probability density is zero.

Nevertheless, the symmetry of the distribution means that the average position is $L/2$.

28.38 Normalization requires

$$\int_{\text{all space}} |\psi|^2 dx = 1 \qquad \text{or} \qquad \int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = A^2\left(\frac{L}{2}\right) = 1 \qquad \text{or} \qquad \boxed{A = \sqrt{\frac{2}{L}}}$$

***28.39** The desired probability is $P = \int_0^{L/4} |\psi|^2 dx = \frac{2}{L}\int_0^{L/4} \sin^2\left(\frac{2\pi x}{L}\right) dx$

where $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

Thus, $P = \left(\frac{x}{L} - \frac{1}{4\pi}\sin\frac{4\pi x}{L}\right)\Big|_0^{L/4} = \left(\frac{1}{4} - 0 - 0 + 0\right) = \boxed{0.250}$

28.40 $\psi(x) = A\cos kx + B\sin kx$ $\qquad \frac{\partial\psi}{\partial x} = -kA\sin kx + kB\cos kx$

$$\frac{\partial^2\psi}{\partial x^2} = -k^2 A\cos kx - k^2 B\sin kx \qquad -\frac{2m}{\hbar^2}(E-U)\psi = -\frac{2mE}{\hbar^2}(A\cos kx + B\sin kx)$$

Therefore the Schrödinger equation is satisfied if

$$\frac{\partial^2\psi}{\partial x^2} = \left(-\frac{2m}{\hbar^2}\right)(E-U)\psi \qquad \text{or} \qquad -k^2(A\cos kx + B\sin kx) = \left(-\frac{2mE}{\hbar^2}\right)(A\cos kx + B\sin kx)$$

This is true as an identity (functional equality) for all x if $\boxed{E = \frac{\hbar^2 k^2}{2m}}$

28.41 We have $\psi = Ae^{i(kx-\omega t)}$ and $\frac{\partial^2\psi}{\partial x^2} = -k^2\psi$

Schrödinger's equation: $\frac{\partial^2\psi}{\partial x^2} = -k^2\psi = -\frac{2m}{\hbar^2}(E-U)\psi$

Since $k^2 = \frac{(2\pi)^2}{\lambda^2} = \frac{(2\pi p)^2}{h^2} = \frac{p^2}{\hbar^2}$ and $E - U = \frac{p^2}{2m}$

$\boxed{\text{Thus this equation balances}}$

***28.42** (a) With $\psi(x) = A\sin(kx)$,

$$\frac{d}{dx}A\sin kx = Ak\cos kx \qquad \text{and} \qquad \frac{d^2}{dx^2}\psi = -Ak^2\sin kx$$

Then $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = +\frac{\hbar^2k^2}{2m}A\sin kx = \frac{h^2(4\pi^2)}{4\pi^2(\lambda^2)(2m)}\psi = \frac{p^2}{2m}\psi = \frac{m^2v^2}{2m}\psi = \tfrac{1}{2}mv^2\psi = K\psi$$

(b) With $\psi(x) = A\sin\left(\frac{2\pi x}{\lambda}\right) = A\sin kx$, the proof given in part (a) applies again.

28.43 (a) $$\psi(x) = A\left(1-\frac{x^2}{L^2}\right) \qquad \frac{d\psi}{dx} = -\frac{2Ax}{L^2} \qquad \frac{d^2\psi}{dx^2} = -\frac{2A}{L^2}$$

Schrödinger's equation $$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}(E-U)\psi$$

becomes $$-\frac{2A}{L^2} = -\frac{2m}{\hbar^2}EA\left(1-\frac{x^2}{L^2}\right) + \frac{2m}{\hbar^2}\frac{(-\hbar^2x^2)\left(1-\frac{x^2}{L^2}\right)}{mL^2(L^2-x^2)}$$

$$-\frac{1}{L^2} = -\frac{mE}{\hbar^2} + \frac{mEx^2}{\hbar^2L^2} - \frac{x^2}{L^4}$$

This will be true for all x if both $$\frac{1}{L^2} = \frac{mE}{\hbar^2}$$

and $$\frac{mE}{\hbar^2L^2} - \frac{1}{L^4} = 0$$

both these conditions are satisfied for a particle of energy $$\boxed{E = \frac{\hbar^2}{L^2m}}$$

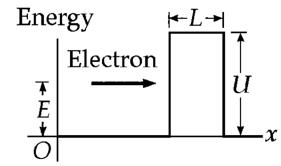
(b) For normalization, $$1 = \int_{-L}^{L} A^2\left(1-\frac{x^2}{L^2}\right)^2 dx = A^2\int_{-L}^{L}\left(1-\frac{2x^2}{L^2}+\frac{x^4}{L^4}\right)dx$$

$$1 = A^2\left[x - \frac{2x^3}{3L^2} + \frac{x^5}{5L^4}\right]_{-L}^{L} = A^2\left[L - \tfrac{2}{3}L + \frac{L}{5} + L - \tfrac{2}{3}L + \frac{L}{5}\right] = A^2\left(\frac{16L}{15}\right) \qquad \boxed{A = \sqrt{\frac{15}{16L}}}$$

(c) $$P = \int_{-L/3}^{L/3}\psi^2dx = \frac{15}{16L}\int_{-L/3}^{L/3}\left(1-\frac{2x^2}{L^2}+\frac{x^4}{L^4}\right)dx = \frac{15}{16L}\left[x - \frac{2x^3}{3L^2} + \frac{x^5}{5L^5}\right]_{-L/3}^{L/3} = \frac{30}{16L}\left[\frac{L}{3} - \frac{2L}{81} + \frac{L}{1215}\right]$$

$$P = \frac{47}{81} = \boxed{0.580}$$

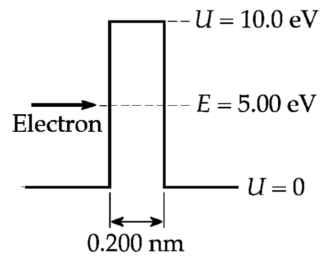
28.44 $$C = \frac{\sqrt{2\left(9.11\times10^{-31}\right)(5.00-4.50)\left(1.60\times10^{-19}\right)}\ \text{kg}\cdot\text{m/s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}} = 3.62\times10^{9}\ \text{m}^{-1}$$



$$T = e^{-2CL} = \exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)\left(950\times10^{-12}\ \text{m}\right)\right] = \exp(-6.88)$$

$$T = \boxed{1.03\times10^{-3}}$$

28.45 $T = e^{-2CL}$ (Use Equation 28.36)



$$2CL = \frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(8.00\times10^{-19}\right)}}{1.055\times10^{-34}}\left(2.00\times10^{-10}\right) = 4.58$$

(a) $T = e^{-4.58} = \boxed{0.0103}$, a 1% chance of transmission.

(b) $R = 1 - T = \boxed{0.990}$, a 99% chance of reflection.

***28.46** The radiation wavelength of $\lambda' = 500$ nm that is observed by observers on Earth is not the true wavelength, λ, emitted by the star because of the Doppler effect. The true wavelength is related to the observed wavelength using:

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-(v/c)}{1+(v/c)}}: \qquad \lambda = \lambda'\sqrt{\frac{1-(v/c)}{1+(v/c)}} = (500\ \text{nm})\sqrt{\frac{1-(0.280)}{1+(0.280)}} = 375\ \text{nm}$$

The temperature of the star is given by $\lambda_{\text{max}}T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$:

$$T = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{\lambda_{\text{max}}}: \qquad T = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{375\times10^{-9}} = \boxed{7.73\times10^{3}\ \text{K}}$$

***28.47** (a) Wien's law: $\lambda_{\text{max}}T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$

Thus, $$\lambda_{\text{max}} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{2.73\ \text{K}} = 1.06\times10^{-3}\ \text{m} = \boxed{1.06\ \text{mm}}$$

(b) This is a $\boxed{\text{microwave}}$.

***28.48** We suppose that the fireball of the Big Bang is a black body.

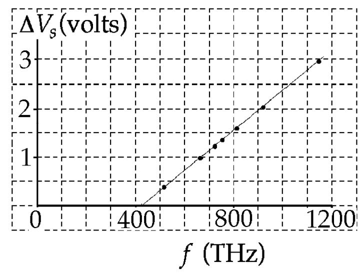
$$I = e\sigma T^4 = (1)\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)(2.73\ \text{K})^4 = \boxed{3.15\times10^{-6}\ \text{W/m}^2}$$

As a bonus, we can find the current power of direct radiation from the Big Bang in the section of the universe observable to us. If it is fifteen billion years old, the fireball is a perfect sphere of radius fifteen billion light years, centered at the point halfway between your eyes:

$$\mathcal{P} = IA = I(4\pi r^2) = \left(3.15\times10^{-6}\ \text{W/m}^2\right)(4\pi)\left(15\times10^9\ \text{ly}\right)^2\left(\frac{3\times10^8\ \text{m/s}}{1\ \text{ly/yr}}\right)^2\left(3.156\times10^7\ \text{s/yr}\right)^2$$

$$\mathcal{P} = 7.98\times10^{47}\ \text{W}$$

28.49 $$\Delta V_S = \left(\frac{h}{e}\right)f - \frac{\phi}{e}$$



From two points on the graph $\quad 0 = \left(\frac{h}{e}\right)\left(4.1\times10^{14}\ \text{Hz}\right) - \frac{\phi}{e}$

and $\quad 3.3\ \text{V} = \left(\frac{h}{e}\right)\left(12\times10^{14}\ \text{Hz}\right) - \frac{\phi}{e}$

Combining these two expressions we find:

(a) $\phi = \boxed{1.7\ \text{eV}}$

(b) $\dfrac{h}{e} = \boxed{4.2\times10^{-15}\ \text{V}\cdot\text{s}}$

(c) At the cutoff wavelength $\quad \dfrac{hc}{\lambda_c} = \phi = \left(\dfrac{h}{e}\right)\dfrac{ec}{\lambda_c}$

$$\lambda_c = \left(4.2\times10^{-15}\ \text{V}\cdot\text{s}\right)\left(1.6\times10^{-19}\ \text{C}\right)\frac{\left(3\times10^8\ \text{m/s}\right)}{(1.7\ \text{eV})\left(1.6\times10^{-19}\ \text{J/eV}\right)} = \boxed{730\ \text{nm}}$$

***28.50** From the path the electrons follow in the magnetic field, the maximum kinetic energy is seen to be:

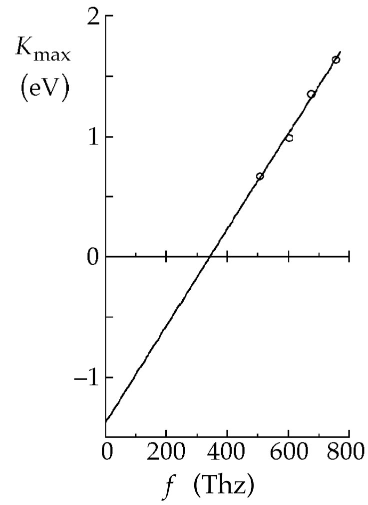
$$K_{\text{max}} = \frac{e^2B^2R^2}{2m_e}$$

From the photoelectric equation, $\quad K_{\text{max}} = hf - \phi = \dfrac{hc}{\lambda} - \phi$

Thus, the work function is $\quad \phi = \dfrac{hc}{\lambda} - K_{\text{max}} = \boxed{\dfrac{hc}{\lambda} - \dfrac{e^2B^2R^2}{2m_e}}$

28.51 We want an Einstein plot of $K_{\max}$ versus f

λ, nm	f, 10^{14} Hz	$K_{\max}$, eV
588	5.10	0.67
505	5.94	0.98
445	6.74	1.35
399	7.52	1.63



(a) $\text{slope} = \dfrac{0.402 \text{ eV}}{10^{14} \text{ Hz}} \pm 8\%$

(b) $e\Delta V_S = hf - \phi$

$$h = (0.402)\left(\frac{1.60\times10^{-19} \text{ J}\cdot\text{s}}{10^{14}}\right) = \boxed{6.4\times10^{-34} \text{ J}\cdot\text{s} \ \pm 8\%}$$

(c) $K_{\max} = 0$

at $f \approx 344\times10^{12}$ Hz

$$\phi = hf = 2.32\times10^{-19} \text{ J} = \boxed{1.4 \text{ eV}}$$

***28.52** Begin with momentum expressions: $p = \dfrac{h}{\lambda}$ and $p = \gamma mv = \gamma mc\left(\dfrac{v}{c}\right)$

Equating these expressions, $\gamma\left(\dfrac{v}{c}\right) = \left(\dfrac{h}{mc}\right)\dfrac{1}{\lambda} = \dfrac{\lambda_C}{\lambda}$

Thus, $\dfrac{(v/c)^2}{1-(v/c)^2} = \left(\dfrac{\lambda_C}{\lambda}\right)^2$

or $\left(\dfrac{v}{c}\right)^2 = \left(\dfrac{\lambda_C}{\lambda}\right)^2 - \left(\dfrac{\lambda_C}{\lambda}\right)^2\left(\dfrac{v}{c}\right)^2 = \dfrac{(\lambda_C/\lambda)^2}{1+(\lambda_C/\lambda)^2} = \dfrac{1}{(\lambda/\lambda_C)^2+1}$

giving $\boxed{v = \dfrac{c}{\sqrt{1+(\lambda/\lambda_C)^2}}}$

28.53 $p = mv = \sqrt{2mE} = \sqrt{2\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(0.0400\ \text{eV}\right)\left(1.60 \times 10^{-19}\ \text{J/eV}\right)}$

$$\lambda = \frac{h}{mv} = 1.43 \times 10^{-10}\ \text{m} = \boxed{0.143\ \text{nm}}$$

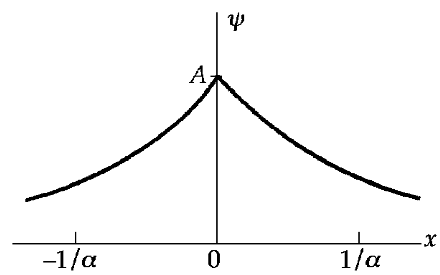
This is of the same order of magnitude as the spacing between atoms in a crystal so diffraction should appear.

28.54 (a) $\lambda = 2L = \boxed{2.00 \times 10^{-10}\ \text{m}}$

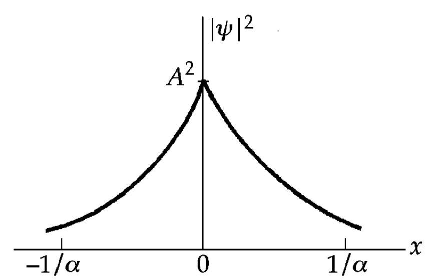
(b) $p = \dfrac{h}{\lambda} = \dfrac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{2.00 \times 10^{-10}\ \text{m}} = \boxed{3.31 \times 10^{-24}\ \text{kg}\cdot\text{m/s}}$

(c) $E = \dfrac{p^2}{2m} = \boxed{0.172\ \text{eV}}$

28.55 (a) $\boxed{\text{See the first figure to the right}}$.



(b) $\boxed{\text{See the second figure to the right}}$.



(c) ψ is continuous

and $\psi \to 0$

as $x \to \pm\infty$

(d) Since ψ is symmetric,

$$\int_{-\infty}^{\infty} |\psi|^2\, dx = 2\int_0^{\infty} |\psi|^2\, dx = 1$$

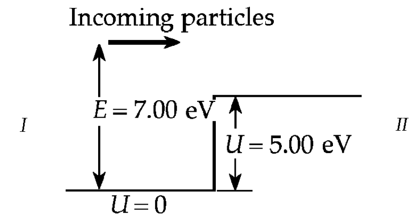
or $$2A^2 \int_0^{\infty} e^{-2\alpha x} dx = \left(\frac{2A^2}{-2\alpha}\right)\left(e^{-\infty} - e^0\right) = 1$$

This gives $A = \sqrt{\alpha}$

(e) $P_{(-1/2\alpha)\to(1/2\alpha)} = 2\left(\sqrt{\alpha}\right)^2 \int_0^{1/2\alpha} e^{-2\alpha x}\, dx = \left(\dfrac{2\alpha}{-2\alpha}\right)\left(e^{-2\alpha/2\alpha} - 1\right) = 1 - e^{-1} = \boxed{0.632}$

28.56 (a) Use Schrödinger's equation

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E-U)\psi$$



with solutions

$$\psi_1 = Ae^{ik_1x} + Be^{-ik_1x} \quad \text{[region } I\text{]}$$

$$\psi_2 = Ce^{ik_2x} \quad \text{[region } II\text{]}$$

Where $k_1 = \dfrac{\sqrt{2mE}}{\hbar}$

and $k_2 = \dfrac{\sqrt{2m(E-U)}}{\hbar}$

Then, matching functions and derivatives at $x = 0$:

$(\psi_1)_0 = (\psi_2)_0$ gives $A + B = C$

and $\left(\dfrac{d\psi_1}{dx}\right)_0 = \left(\dfrac{d\psi_2}{dx}\right)_0$ gives $k_1(A-B) = k_2C$

Then $B = \dfrac{1-k_2/k_1}{1+k_2/k_1}A$

and $C = \dfrac{2}{1+k_2/k_1}A$

Incident wave Ae^{ikx} reflects Be^{-ikx}, with probability $R = \dfrac{B^2}{A^2} = \dfrac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2} = \boxed{\dfrac{(k_1-k_2)^2}{(k_1+k_2)^2}}$

(b) With $E = 7.00 \text{ eV}$

and $U = 5.00 \text{ eV}$

$$\frac{k_2}{k_1} = \sqrt{\frac{E-U}{E}} = \sqrt{\frac{2.00}{7.00}} = 0.535$$

The reflection probability is $R = \dfrac{(1-0.535)^2}{(1+0.535)^2} = \boxed{0.0920}$

The probability of transmission is $T = 1 - R = \boxed{0.908}$

***28.57** (a) $mgy_i = \frac{1}{2}mv_f^{\,2}$

$$v_f = \sqrt{2gy_i} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(50.0\ \text{m})} = 31.3\ \text{m/s}$$

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(75.0\ \text{kg})(31.3\ \text{m/s})} = \boxed{2.82\times10^{-37}\ \text{m}} \quad \text{(not observable)}$$

(b) $$\Delta E\Delta t \ge \hbar/2$$

so $$\Delta E \ge \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(5.00\times10^{-3}\ \text{s}\right)} = \boxed{1.06\times10^{-32}\ \text{J}}$$

(c) $$\frac{\Delta E}{E} = \frac{1.06\times10^{-32}\ \text{J}}{(75.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)(50.0\ \text{m})} = \boxed{2.87\times10^{-35}\ \%}$$

***28.58** $$\left\langle x^2\right\rangle = \int_{-\infty}^{\infty} x^2|\psi|^2\,dx$$

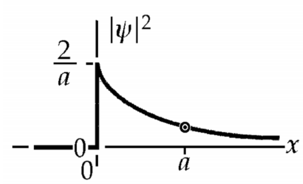
For a one-dimensional box of width L, $\psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$

Thus, $$\left\langle x^2\right\rangle = \frac{2}{L}\int_0^L x^2\sin^2\left(\frac{n\pi x}{L}\right)dx = \boxed{\frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}} \quad \text{(from integral tables)}$$

***28.59** For a particle with wave function

$$\psi(x) = \sqrt{\frac{2}{a}}e^{-x/a} \qquad \text{for } x > 0$$

and 0 for $x < 0$



(a) $\left|\psi(x)\right|^2 = 0,\ x < 0$ and $\left|\psi^2(x)\right| = \frac{2}{a}e^{-2x/a},\ x > 0$

(b) $$\text{Prob}(x<0) = \int_{-\infty}^{0}\left|\psi(x)\right|^2 dx = \int_{-\infty}^{0}(0)\,dx = \boxed{0}$$

(c) Normalization $$\int_{-\infty}^{\infty}\left|\psi(x)\right|^2 dx = \int_{-\infty}^{0}|\psi|^2 dx + \int_0^{\infty}|\psi|^2 dx = 1$$

$$\int_{-\infty}^{0} 0\,dx + \int_0^{\infty}(2/a)e^{-2x/a}dx = 0 - e^{-2x/a}\Big|_0^{\infty} = -\left(e^{-\infty} - 1\right) = 1$$

$$\text{Prob}(0<x<a) = \int_0^a|\psi|^2 dx = \int_0^a(2/a)e^{-2x/a}dx = -e^{-2x/a}\Big|_0^a = 1 - e^{-2} = \boxed{0.865}$$

ANSWERS TO EVEN NUMBERED PROBLEMS

2. (a) 5.75×10^{3} K (b) 504 nm

4. 5.71×10^{3} photons

6. 1.32×10^{31}

8. (a) 1.38 eV (b) 334 THz

10. 148 days; absurdly large

12. 1.78 eV, 9.47×10^{-28} kg·m/s

14. 22.1 keV/c, 478 eV

16. (a) 2.88 pm (b) 101°

18. (a) 0.709 nm (b) 414 nm

20. (a) 1.10×10^{-34} m/s (b) 1.36×10^{33} s
(c) No. The time is over 10^{15} times the age of the universe.

22. (a) 3.91×10^{4} (b) 1.07×10^{-17} kg·m/s
(c) 6.22×10^{-17} m, much smaller than 10^{-14} m

24. 105 V

26. (a) 15.1 keV (b) 124 keV

28. (a) 0.250 m/s (b) 2.25 m

30. 2.81×10^{-8}

32. 1/2

34. (a) $n = 4$ (b) 6.03 eV

36. 6.16 MeV, 202 fm, a gamma ray

38. See the solution

40. See the solution; $E = \hbar^2 k^2 / 2m$

42. See the solution

44. 1.03×10^{-3}

46. 7.73×10^3 K

48. $3.15\ \mu\text{W}/\text{m}^2$

50. $\dfrac{hc}{\lambda} - \dfrac{e^2 B^2 R^2}{2m_e}$

52. See the solution

54. (a) 2.00×10^{-10} m (b) 3.31×10^{-24} kg·m/s (c) 0.172 eV

56. (a) See the solution (b) $R = 0.0920$, $T = 0.908$

58. See the solution