CHAPTER 30

ANSWERS TO QUESTIONS

Q30.1 Because of electrostatic repulsion between the + nucleus and the +2 alpha particle. To drive the α-particle into the nucleus would require extremely high kinetic energy.

Q30.2 An α-particle is a helium nucleus: ${}^4_2\text{He}$
A β-particle is an electron or a positron: either e^- or e^+.
A γ-ray is a high-energy photon emitted when a nucleus makes a downward transition between two states.

Q30.3 Although protons are held together by the short-range nuclear force, they are also repelled by the electrostatic force. Neutrons, without charge, allow the nucleus to exist with two or more protons by providing more nuclear attractive force between particles. As the number of protons in the nucleus increases, so does the electrostatic repulsion, and more and more neutrons are required to keep a nucleus together as the atomic number increases.

Q30.4 The proton precession frequency is given by $\omega_p = 2\mu B/\hbar$. Therefore, if the magnetic field doubles, so does the precession frequency.

Q30.5 Extra neutrons are required to overcome the increasing electrostatic repulsion of the protons.

Q30.6 If one half the number of radioactive nuclei decay in one year, then one half the remaining number will decay in the second year.

Q30.7 After one half-life, one half the radioactive atoms have decayed. After the second half-life, one half of the remaining atoms have decayed. Therefore $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ of the original radioactive atoms have decayed after two half-lives.

Q30.8 Since the samples are of the same radioactive nuclide, their half-lives are the same. When prepared, sample A has twice the activity (number of radioactive decays per second) of sample B. Therefore after 5 half-lives, the activity of sample A is decreased by a factor of 2^5, and after 5 half-lives the activity of sample B is decreased by a factor of 2^5. So after 5 half-lives, the ratio of activities is still 2 : 1.

Q30.9 The statement is false. Both patterns show monotonic decrease over time, but with very different shapes. For radioactive decay, maximum activity occurs at time zero. Cohorts of people now living will be dying most rapidly perhaps forty years from now. Everyone now living will be dead within less than two centuries, while the mathematical model of radioactive decay tails off exponentially forever. A radioactive nucleus never gets old. It has constant probability of decay however long it has existed.

Q30.10 The excitation energy comes from the binding energy of the extra nucleon.

Q30.11 Bullet and rifle carry equal amounts of momentum p. With a much smaller mass m, the bullet has much more kinetic energy $K = p^2/2m$. The daughter nucleus and alpha particle have equal momenta and the massive daughter nucleus, like the rifle, has a very small share of the energy released.

Q30.12 The alpha particle and the daughter nucleus carry equal and opposite amounts of momentum. Since kinetic energy can be written as $p^2/2m$, the small-mass alpha particle has much more of the decay energy than the recoiling nucleus.

Q30.13 In a heavy nucleus each nucleon is strongly bound to its momentary neighbors. Even if the nucleus could step down in energy by shedding an individual proton or neutron, one individual nucleon is never free to escape. Instead, the nucleus can decay when two protons and two neutrons, strongly bound to one another but not to their neighbors, happen momentarily to have a lot of kinetic energy, to lie at the surface of the nucleus, to be headed outward, and to tunnel successfully through the potential energy barrier they encounter.

Q30.14 From $\Sigma F = ma$, or $qvB = mv^2/r$, or $qBr = mv$, a charged particle fired into a magnetic field is deflected into a path with radius proportional to its momentum. If they have equal kinetic energies K, the much greater mass m of the alpha particle gives it more momentum $mv = \sqrt{2mK}$ than an electron. Thus the electron undergoes greater deflection. This conclusion remains true if one or both particles are moving relativistically.

Q30.15 The alpha particle stops in the wood, while many beta particles can make it through to deposit some or all of their energy in the film emulsion.

Q30.16 The samples would have started with more Carbon-14 than we first thought. We would increase our estimates of their ages.

Q30.17 Long-lived progenitors at the top of each of the three natural radioactive series are the sources of our radium. As an example, Uranium-235 with a half-life of 704 Myr produces Radium-223 at one stage in its series of decays, shown in Figure P30.27.

Q30.18 ^{4}He, ^{16}O, ^{40}Ca, and ^{208}Pb.

Q30.19 Sometimes the references are oblique indeed. Some must serve for more than one form of energy or mode of transfer. Here is one list:
kinetic: ocean currents.
rotational kinetic: Earth turning
gravitational: water lifted up
elastic: Elastic energy is necessary for sound, listed below.
internal: by contrast to a chilly night; or in forging a chain
chemical: flames
sound: thunder
electrical: lightning
electromagnetic radiation: heavens blazing; lightning
atomic electronic: In the blazing heavens, stars have different colors because of different predominant energy losses by atoms at their surfaces.
nuclear: The blaze of the heavens is produced by nuclear reactions in the cores of stars.

PROBLEM SOLUTIONS

30.1 An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons.

So protons and neutrons are nearly equally numerous in your body, each contributing mass (say) 35 kg:

$$35\text{ kg}\left(\frac{1\text{ nucleon}}{1.67\times10^{-27}\text{ kg}}\right)\boxed{\sim10^{28}\text{ protons}}$$

and $\boxed{\sim10^{28}\text{ neutrons}}$

The electron number is precisely equal to the proton number, $\boxed{\sim10^{28}\text{ electrons}}$

30.2 (a) $r = r_0A^{1/3} = \left(1.20\times10^{-15}\text{ m}\right)(4)^{1/3} = \boxed{1.90\times10^{-15}\text{ m}}$

(b) $r = r_0A^{1/3} = \left(1.20\times10^{-15}\text{ m}\right)(238)^{1/3} = \boxed{7.44\times10^{-15}\text{ m}}$

30.3 It must start with kinetic energy equal to $K_i = U_f = k_eqQ/r_f$. Here r_f stands for the sum of the radii of the ${}^{4}_{2}\text{He}$ and ${}^{197}_{79}\text{Au}$ nuclei, computed as

$$r_f = r_0A_1^{1/3} + r_0A_2^{1/3} = \left(1.20\times10^{-15}\text{ m}\right)\left(4^{1/3}+197^{1/3}\right) = 8.89\times10^{-15}\text{ m}$$

Thus, $$K_i = U_f = \frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times10^{-19}\text{ C}\right)^2}{8.89\times10^{-15}\text{ m}} = 4.09\times10^{-12}\text{ J} = \boxed{25.6\text{ MeV}}$$

30.4 $V = \frac{4}{3}\pi r^4 = \frac{4}{3}\pi(0.0215\text{ m})^3 = 4.16\times10^{-5}\text{ m}^3$

We take the nuclear density from Example 30.1

$$m = \rho V = \left(2.3\times10^{17}\text{ kg/m}^3\right)\left(4.16\times10^{-5}\text{ m}^3\right) = 9.57\times10^{12}\text{ kg}$$

and $$F = G\frac{m_1m_2}{r^2} = \left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(9.57\times10^{12}\text{ kg}\right)^2}{(1.00\text{ m})^2}$$

$F = \boxed{6.11\times10^{15}\text{ N}}$ toward the other ball.

30.5 The number of nucleons in a star of two solar masses is

$$A = \frac{2\left(1.99\times10^{30}\ \text{kg}\right)}{1.67\times10^{-27}\ \text{kg/nucleon}} = 2.38\times10^{57}\ \text{nucleons}$$

Therefore $$r = r_0 A^{1/3} = \left(1.20\times10^{-15}\ \text{m}\right)\left(2.38\times10^{57}\right)^{1/3} = \boxed{16.0\ \text{km}}$$

***30.6** The stable nuclei that correspond to magic numbers are:

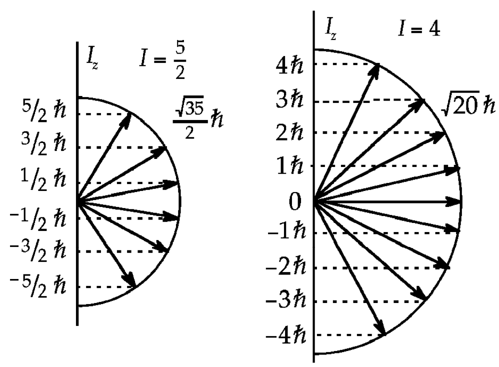
Z magic: $_{2}\text{He}$ $_{8}\text{O}$ $_{20}\text{Ca}$ $_{28}\text{Ni}$ $_{50}\text{Sn}$ $_{82}\text{Pb}$

An artificially produced nucleus with $Z = 126$ might be more stable than other nuclei with lower values for Z, since this number of protons is magic.

N magic: $^{3}_{1}\text{T}$, $^{4}_{2}\text{He}$, $^{15}_{7}\text{N}$, $^{16}_{8}\text{O}$, $^{37}_{17}\text{Cl}$, $^{39}_{19}\text{K}$, $^{40}_{20}\text{Ca}$, $^{51}_{23}\text{V}$, $^{52}_{24}\text{Cr}$, $^{88}_{38}\text{Sr}$, $^{89}_{39}\text{Y}$,

$^{90}_{40}\text{Zr}$, $^{136}_{54}\text{Xe}$, $^{138}_{56}\text{Ba}$, $^{139}_{57}\text{La}$, $^{140}_{58}\text{Ce}$, $^{141}_{59}\text{Pr}$, $^{142}_{60}\text{Nd}$, $^{208}_{82}\text{Pb}$, $^{209}_{83}\text{Bi}$, $^{210}_{84}\text{Po}$

***30.7**



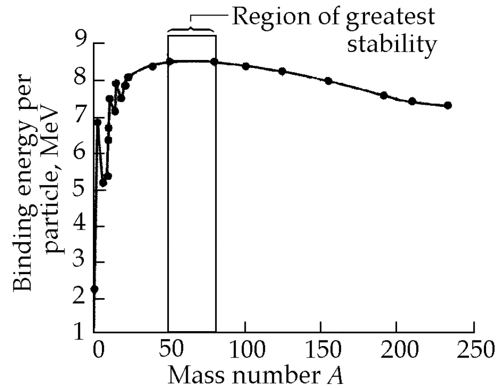
30.8 (a) $$f_n = \frac{|2\mu B|}{h} = \frac{2(1.9135)(5.05\times10^{-27}\ \text{J/T})(1.00\ \text{T})}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = \boxed{29.2\ \text{MHz}}$$

(b) $$f_p = \frac{2(2.7928)(5.05\times10^{-27}\ \text{J/T})(1.00\ \text{T})}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = \boxed{42.6\ \text{MHz}}$$

(c) In the Earth's magnetic field,

$$f_p = \frac{2(2.7928)(5.05\times10^{-27})(50.0\times10^{-6})}{6.626\times10^{-34}} = \boxed{2.13\ \text{kHz}}$$

***30.9** $\Delta E_b = E_{bf} - E_{bi}$



For $A = 200, \quad \dfrac{E_b}{A} = 7.4\text{ MeV}$

so $E_{bi} = 200(7.4\text{ MeV}) = 1480\text{ MeV}$

For $A \cong 100, \quad \dfrac{E_b}{A} \cong 8.4\text{ MeV}$

so $E_{bf} = 2(100)(8.4\text{ MeV}) = 1680\text{ MeV}$

$\Delta E_b = E_{bf} - E_{bi}$: $E_b = 1680\text{ MeV} - 1480\text{ MeV} = \boxed{200\text{ MeV}}$

30.10 Using atomic masses as given in Table A.3,

(a) For ${}^{2}_{1}\text{H}$: $\dfrac{-2.014102 + 1(1.008665) + 1(1.007825)}{2}$

$$E_b / A = (0.0011194\text{ u})\left(\frac{931.5\text{ MeV}}{\text{u}}\right) = \boxed{1.11\text{ MeV/nucleon}}$$

(b) For ${}^{4}_{2}\text{He}$: $\dfrac{2(1.008665) + 2(1.007825) - 4.002602}{4}$

$$E_b / A = 0.00759\text{ u}c^2 = \boxed{7.07\text{ MeV/nucleon}}$$

(c) For ${}^{56}_{26}\text{Fe}$: $30(1.008665) + 26(1.007825) - 55.934940 = 0.528\text{ u}$

$$E_b / A = \frac{0.528}{56} = 0.00944\text{ u}c^2 = \boxed{8.79\text{ MeV/nucleon}}$$

(d) For ${}^{238}_{92}\text{U}$: $146(1.008665) + 92(1.007825) - 238.050784 = 1.9342\text{ u}$

$$E_b / A = \frac{1.9342}{238} = 0.00813\text{ u}c^2 = \boxed{7.57\text{ MeV/nucleon}}$$

***30.11** The binding energy of a nucleus is $E_b(\text{MeV}) = \left[ZM(\text{H}) + Nm_n - M\left({}^{A}_{Z}\text{X}\right)\right](931.494\text{ MeV/u})$

For ${}^{15}_{8}\text{O}$: $E_b = \left[8(1.007\ 825\text{ u}) + 7(1.008\ 665\text{ u}) - 15.003\ 065\text{ u}\right](931.494\text{ MeV/u}) = 111.96\text{ MeV}$

For ${}^{15}_{7}\text{N}$: $E_b = \left[7(1.007\ 825\text{ u}) + 8(1.008\ 665\text{ u}) - 15.000\ 108\text{ u}\right](931.494\text{ MeV/u}) = 115.49\text{ MeV}$

Therefore, $\boxed{\text{the binding energy of } {}^{15}_{7}\text{N is larger by 3.54 MeV}}$

30.12 (a) The neutron-to-proton ratio $(A-Z)/Z$ is greatest for $\boxed{{}^{139}_{55}\mathrm{Cs}}$ and is equal to 1.53.

(b) $\boxed{{}^{139}\mathrm{La}}$ has the largest binding energy per nucleon of 8.378 MeV.

(c) ${}^{139}\mathrm{Cs}$ with a mass of 138.913 u. We locate the nuclei carefully on Figure 30.4, the neutron–proton plot of stable nuclei. $\boxed{\text{Cesium}}$ appears to be farther from the center of the zone of stability. Its instability means extra energy and extra mass.

30.13 $$\frac{dN}{dt} = -\lambda N$$

so $$\lambda = \frac{1}{N}\left(-\frac{dN}{dt}\right) = \left(1.00\times10^{-15}\right)\left(6.00\times10^{11}\right) = 6.00\times10^{-4}\ \mathrm{s}^{-1}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{1.16\times10^{3}\ \mathrm{s}}\ (= 19.3\ \mathrm{min})$$

30.14 $$R = R_0 e^{-\lambda t} = (6.40\ \mathrm{mCi})e^{-\left(\frac{\ln 2}{8.04\ \mathrm{d}}\right)(40.2\ \mathrm{d})} = (6.40\ \mathrm{mCi})\left(e^{-\ln 2}\right)^5 = (6.40\ \mathrm{mCi})\left(\frac{1}{2^5}\right) = \boxed{0.200\ \mathrm{mCi}}$$

30.15 (a) From $R = R_0 e^{-\lambda t}$,

$$\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right) = \left(\frac{1}{4.00\ \mathrm{h}}\right)\ln\left(\frac{10.0}{8.00}\right) = 5.58\times10^{-2}\ \mathrm{h}^{-1} = \boxed{1.55\times10^{-5}\ \mathrm{s}^{-1}}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{12.4\ \mathrm{h}}$$

(b) $$N_0 = \frac{R_0}{\lambda} = \frac{10.0\times10^{-3}\ \mathrm{Ci}}{1.55\times10^{-5}/\mathrm{s}}\left(\frac{3.70\times10^{10}/\mathrm{s}}{1\ \mathrm{Ci}}\right) = \boxed{2.39\times10^{13}\ \text{atoms}}$$

(c) $$R = R_0 e^{-\lambda t} = (10.0\ \mathrm{mCi})\exp\left(-5.58\times10^{-2}\times30.0\right) = \boxed{1.87\ \mathrm{mCi}}$$

30.16 $R = R_0 e^{-\lambda t}$ where $\lambda = \dfrac{\ln 2}{26.0\ \mathrm{h}} = 0.0266\ \mathrm{h}^{-1}$

$\dfrac{R}{R_0} = 0.100 = e^{-\lambda t}$ so $\ln(0.100) = -\lambda t$

$2.30 = \left(\dfrac{0.0266}{\mathrm{h}}\right)t$ $\qquad t = \boxed{86.4\ \mathrm{h}}$

30.17 The number of nuclei which decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$

First we find λ: $\lambda = \dfrac{\ln 2}{T_{1/2}}$

so $e^{-\lambda t} = e^{\ln 2\left(-t/T_{1/2}\right)} = 2^{-t/T_{1/2}}$

and $N_0 = \dfrac{R_0}{\lambda} = \dfrac{R_0 T_{1/2}}{\ln 2}$

Substituting in these values $N_1 - N_2 = \dfrac{R_0 T_{1/2}}{\ln 2}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right) = \boxed{\dfrac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)}$

***30.18** (a) $4.00\ \text{pCi/L} = \left(\dfrac{4.00\times 10^{-12}\ \text{Ci}}{1\ \text{L}}\right)\left(\dfrac{3.70\times 10^{10}\ \text{Bq}}{1\ \text{Ci}}\right)\left(\dfrac{1.00\times 10^{3}\ \text{L}}{1\ \text{m}^3}\right) = \boxed{148\ \text{Bq/m}^3}$

(b) $N = \dfrac{R}{\lambda} = R\left(\dfrac{T_{1/2}}{\ln 2}\right) = \left(148\ \text{Bq/m}^3\right)\left(\dfrac{3.82\ \text{d}}{\ln 2}\right)\left(\dfrac{86\,400\ \text{s}}{1\ \text{d}}\right) = \boxed{7.05\times 10^{7}\ \text{atoms/m}^3}$

(c) $\text{mass} = \left(7.05\times 10^{7}\ \text{atoms/m}^3\right)\left(\dfrac{1\ \text{mol}}{6.02\times 10^{23}\ \text{atoms}}\right)\left(\dfrac{222\ \text{g}}{1\ \text{mol}}\right) = 2.60\times 10^{14}\ \text{g/m}^3$

Since air has a density of 1.20 kg/ m^3, the fraction consisting of radon is

$\text{fraction} = \dfrac{2.60\times 10^{-14}\ \text{g/m}^3}{1.20\ \text{kg/m}^3} = \boxed{2.17\times 10^{-17}}$

30.19 $N_0 = \dfrac{\text{mass present}}{\text{mass of nucleus}} = \dfrac{5.00\ \text{kg}}{(89.9077\ \text{u})\left(1.66\times 10^{-27}\ \text{kg/u}\right)} = 3.35\times 10^{25}\ \text{nuclei}$

$\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{29.1\ \text{yr}} = 2.38\times 10^{-2}\ \text{yr}^{-1} = 4.52\times 10^{-8}\ \text{min}^{-1}$

$R_0 = \lambda N_0 = \left(4.52\times 10^{-8}\ \text{min}^{-1}\right)\left(3.35\times 10^{25}\right) = 1.52\times 10^{18}\ \text{counts/min}$

$\dfrac{R}{R_0} = e^{-\lambda t} = \dfrac{10.0\ \text{counts/min}}{1.52\times 10^{18}\ \text{counts/min}} = 6.60\times 10^{-18}$

and $\lambda t = -\ln\left(6.60\times 10^{-18}\right) = 39.6$

giving $t = \dfrac{39.6}{\lambda} = \dfrac{39.6}{2.38\times 10^{-2}\ \text{yr}^{-1}} = \boxed{1.66\times 10^{3}\ \text{yr}}$

30.20 (a) A gamma ray has zero charge and it contains no protons or neutrons. So for a gamma ray $Z=0$ and $A=0$. Keeping the total values of Z and A for the system conserved then requires $Z=28$ and $A=65$ for X. With this atomic number it must be nickel, and the nucleus must be in an exited state, so it is ${}^{65}_{28}\mathrm{Ni}^*$.

(b) $\alpha = {}^4_2$ He has $Z=2$ and $A=4$

so for X we require $Z=84-2=82$

for Pb and $A=215-4=211$, $X = {}^{211}_{82}\mathrm{Pb}$

(c) A positron $e^+ = {}^0_1e$ has charge the same as a nucleus with $Z=1$. A neutrino ${}^0_0\nu$ has no charge. Neither contains any protons or neutrons. So X must have by conservation $Z=26+1=27$. It is Co. And $A = 55 + 0 = 55$. It is ${}^{55}_{27}\mathrm{Co}$.

Similar reasoning about balancing the sums of Z and A across the reaction reveals:

(d) ${}^{0}_{-1}e$

(e) ${}^1_1$H (or p)

30.21

$$Q = \left(M_{\text{U-238}} - M_{\text{Th-234}} - M_{\text{He-4}}\right)(931.5\ \text{MeV/u})$$

$$Q = (238.050\,784 - 234.043\,593 - 4.002\,602)\text{u}\,(931.5\ \text{MeV/u}) = \boxed{4.27\ \text{MeV}}$$

30.22

$$N_C = \left(\frac{0.0210\ \text{g}}{12.0\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{molecules/mol}\right)$$

$\left(N_C = 1.05\times10^{21}\ \text{carbon atoms}\right)$ of which 1 in 7.70×10^{11} is a ^{14}C atom

$$(N_0)_{\text{C-14}} = 1.37\times10^9, \qquad \lambda_{\text{C}-14} = \frac{\ln 2}{5730\ \text{yr}} = 1.21\times10^{-4}\ \text{yr}^{-1} = 3.83\times10^{-12}\ \text{s}^{-1}$$

$$R = \lambda N = \lambda N_0 e^{-\lambda t}$$

At $t=0$, $$R_0 = \lambda N_0 = \left(3.83\times10^{-12}\ \text{s}^{-1}\right)\left(1.37\times10^9\right)\left[\frac{7(86400\ \text{s})}{1\ \text{week}}\right] = 3.17\times10^3\ \frac{\text{decays}}{\text{week}}$$

At time t, $$R = \frac{837}{0.88} = 951\ \text{decays/week}$$

Taking logarithms, $\ln\dfrac{R}{R_0} = -\lambda t$ so $t = \dfrac{-1}{\lambda}\ln\left(\dfrac{R}{R_0}\right)$

$$t = \frac{-1}{1.21\times10^{-4}\ \text{yr}^{-1}}\ln\left(\frac{951}{3.17\times10^3}\right) = \boxed{9.96\times10^3\ \text{yr}}$$

***30.23** $\frac{1}{2}mv^2 = q\Delta V$ and $\frac{mv^2}{r} = qvB$

$2m\Delta V = qr^2B^2$: $r = \sqrt{\frac{2m\Delta V}{qB^2}} = \sqrt{\frac{2(1000\text{ V})}{(1.60\times10^{-19}\text{ C})(0.200\text{ T})^2}}\sqrt{m}$

$$r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{m}$$

(a) For ^{12}C, $m = 12$ u and $r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{12(1.66\times10^{-27}\text{ kg})}$

$$r = 0.0789\text{ m} = \boxed{7.89\text{ cm}}$$

For ^{13}C: $r = \left(5.59\times10^{11}\ \frac{\text{m}}{\sqrt{\text{kg}}}\right)\sqrt{13(1.66\times10^{-27}\text{ kg})}$

$$r = 0.0821\text{ m} = \boxed{8.21\text{ cm}}$$

(b) With $r_1 = \sqrt{\frac{2m_1\Delta V}{qB^2}}$ and $r_2 = \sqrt{\frac{2m_2\Delta V}{qB^2}}$

the ratio gives $\boxed{\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}}}$

$$\frac{r_1}{r_2} = \frac{7.89\text{ cm}}{8.21\text{ cm}} = 0.961$$

and $\sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{12\text{ u}}{13\text{ u}}} = 0.961$

so they do agree.

***30.24** $^3_1\text{H nucleus} \to {}^3_2\text{He nucleus} + e^- + \bar{\nu}$

becomes $^3_1\text{H nucleus} + e^- \to {}^3_2\text{He nucleus} + 2e^- + \bar{\nu}$

Ignoring the slight difference in ionization energies,

we have $^3_1\text{H atom} \to {}^3_2\text{He atom} + \bar{\nu}$

$$3.016049\text{ u} = 3.016029\text{ u} + 0 + Q/c^2$$

$$Q = (3.016049\text{ u} - 3.016029\text{ u})(935\text{ MeV/u}) = 0.0186\text{ MeV} = \boxed{18.6\text{ keV}}$$

30.25 (a) $\boxed{e^- + p \to n + \nu}$

(b) For nuclei, $$^{15}O + e^- \to {}^{15}N + \nu$$

Add seven electrons to both sides to obtain $\boxed{{}^{15}_{8}O \text{ atom} \to {}^{15}_{7}N \text{ atom} + \nu}$

(c) From Table A.3, $m\left({}^{15}O\right) = m\left({}^{15}N\right) + \frac{Q}{c^2}$

$$\Delta m = 15.003\,065 \text{ u} - 15.000\,108 \text{ u} = 0.002\,957 \text{ u}$$

$$Q = (931.5 \text{ MeV/u})(0.002\,957 \text{ u}) = \boxed{2.75 \text{ MeV}}$$

30.26 (a) Because the reaction $p \to n + e^+ + \nu$ would violate the law of $\boxed{\text{conservation of energy}}$

$$m_p = 1.007\,276 \text{ u} \qquad m_n = 1.008\,665 \text{ u} \qquad m_{e^+} = 5.49 \times 10^{-4} \text{ u}$$

Note that $m_n + m_{e^+} > m_p$

(b) The $\boxed{\text{required energy can come from the electrostatic repulsion}}$ of protons in the nucleus.

(c) Add seven electrons to both sides of the reaction for nuclei ${}^{13}_{7}N \to {}^{13}_{6}C + e^+ + \nu$

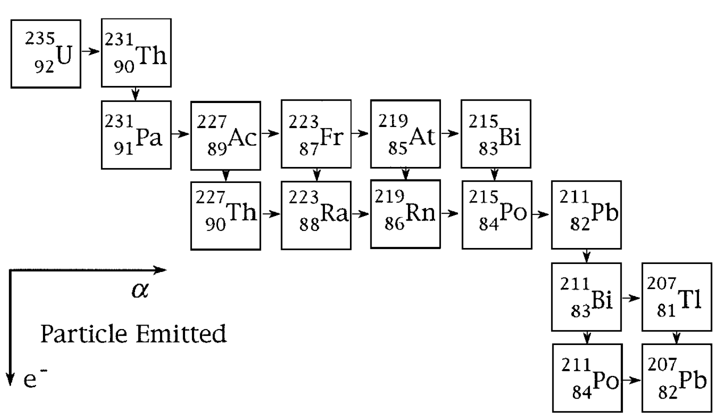
to obtain the reaction for neutral atoms ${}^{13}_{7}N \text{ atom} \to {}^{13}_{6}C \text{ atom} + e^+ + e^- + \nu$

$$Q = c^2\left[m\left({}^{13}N\right) - m\left({}^{13}C\right) - m_{e^+} - m_{e^-} - m_\nu\right]$$

$$Q = (931.5 \text{ MeV / u})\left[13.005\,738 - 13.003\,355 - 2\left(5.49 \times 10^{-4}\right) - 0\right]\text{u}$$

$$Q = (931.5 \text{ MeV/u})\left(1.285 \times 10^{-3} \text{ u}\right) = \boxed{1.20 \text{ MeV}}$$

30.27



***30.28** (a) For X, $A = 24 + 1 - 4 = 21$

and $Z = 12 + 0 - 2 = 10$, so X is $\boxed{{}^{21}_{10}\text{Ne}}$

(b) $A = 235 + 1 - 90 - 2 = 144$

and $Z = 92 + 0 - 38 - 0 = 54$, so X is $\boxed{{}^{144}_{54}\text{Xe}}$

(c) $A = 2 - 2 = 0$

and $Z = 2 - 1 = +1$, so X must be a positron.

As it is ejected, so is a neutrino: $\boxed{X = {}^{0}_{1}\text{e}^{+}}$ and $\boxed{X' = {}^{0}_{0}\nu}$

30.29 (a) $${}^{197}_{79}\text{Au} + {}^{1}_{0}\text{n} \to {}^{197}_{79}\text{Au}^{*} \to {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + \bar{\nu}$$

(b) Consider adding 79 electrons:

$${}^{197}_{79}\text{Au atom} + {}^{1}_{0}\text{n} \to {}^{198}_{80}\text{Hg atom} + \bar{\nu} + Q$$

$$Q = \left[M_{^{197}\text{Au}} + m_{\text{n}} - M_{^{198}\text{Hg}}\right]c^2$$

$$Q = [196.966\ 543 + 1.008\ 665 - 197.966\ 743]\text{u}\ (931.5\ \text{MeV/u}) = \boxed{7.89\ \text{MeV}}$$

30.30 Neglect recoil of product nucleus, (i.e., do not require momentum conservation for the system of colliding particles). The energy balance gives $K_{\text{emerging}} = K_{\text{incident}} + Q$. To find Q:

$$Q = \left[(M_{\text{H}} + M_{\text{Al}}) - (M_{\text{Si}} + m_n)\right]c^2$$

$$Q = \left[(1.007\ 825 + 26.981\ 538) - (26.986\ 721 + 1.008\ 665)\right]\text{u}\ (931.5\ \text{MeV/u}) = -5.61\ \text{MeV}$$

Thus, $K_{\text{emerging}} = 6.61\ \text{MeV} - 5.61\ \text{MeV} = \boxed{1.00\ \text{MeV}}$

***30.31** (a) $Q = (\Delta m)c^2 = [m_n + M_{\text{U-235}} - M_{\text{Ba-141}} - M_{\text{Kr-92}} - 3m_n]c^2$

$$\Delta m = \left[(1.008\ 665 + 235.043\ 924) - (140.913\ 9 + 91.897\ 3 + 3 \times 1.008\ 665)\right]\text{u} = 0.215\ 39\ \text{u}$$

$$Q = (0.215\ 39\ \text{u})(931.5\ \text{MeV/u}) = \boxed{201\ \text{MeV}}$$

(b) $f = \dfrac{\Delta m}{m_i} = \dfrac{0.215\ 39\ \text{u}}{236.052\ 59\ \text{u}} = 9.13 \times 10^{-4} = \boxed{0.0913\%}$

***30.32** The energy is

$$3.30\times10^{10}\ \text{J}\left(\frac{1\ \text{ev}}{1.60\times10^{-19}\ \text{J}}\right)\left(\frac{1\ \text{U-235 nucleus}}{208\ \text{MeV}}\right)\left(\frac{235\ \text{g}}{6.02\times10^{23}\ \text{nucleus}}\right)\left(\frac{\text{M}}{10^{6}}\right)=\boxed{0.387\ \text{g}}\ \text{of U-235.}$$

***30.33** The available energy to do work is 0.200 times the energy content of the fuel.

$$(1.00\ \text{kg fuel})\left(\frac{0.0340\ {}^{235}\text{U}}{\text{fuel}}\right)\left(\frac{1000\ \text{g}}{1\ \text{kg}}\right)\left(\frac{1\ \text{mol}}{235\ \text{g}}\right)\left(\frac{6.02\times10^{23}}{\text{mol}}\right)\left(\frac{(208)\left(1.60\times10^{-13}\ \text{J}\right)}{\text{fission}}\right)$$

$$\left(2.90\times10^{12}\ \text{J}\right)(0.200)=5.80\times10^{11}\ \text{J}=\left(1.00\times10^{5}\ \text{N}\right)\Delta r$$

$$\Delta r=5.80\times10^{6}\ \text{m}=\boxed{5.80\ \text{Mm}}$$

***30.34** (a) $V=\left(317\times10^{6}\ \text{mi}^{3}\right)\left(\frac{1609\ \text{m}}{1\ \text{mi}}\right)^{3}=1.32\times10^{18}\ \text{m}^{3}$

$$m_{\text{water}}=\rho V=\left(10^{3}\ \text{kg/m}^{3}\right)\left(1.32\times10^{18}\ \text{m}^{3}\right)=1.32\times10^{21}\ \text{kg}$$

$$m_{\text{H}_2}=\left(\frac{M_{\text{H}_2}}{M_{\text{H}_2\text{O}}}\right)m_{\text{H}_2\text{O}}=\left(\frac{2.016}{18.015}\right)\left(1.32\times10^{21}\ \text{kg}\right)=1.48\times10^{20}\ \text{kg}$$

$$m_{\text{Deuterium}}=(0.0300\%)m_{\text{H}_2}=\left(0.0300\times10^{-2}\right)\left(1.48\times10^{20}\ \text{kg}\right)=4.43\times10^{16}\ \text{kg}$$

The number of deuterium nuclei in this mass is

$$N=\frac{m_{\text{Deuterium}}}{m_{\text{Deuteron}}}=\frac{4.43\times10^{16}\ \text{kg}}{(2.014\ \text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)}=1.33\times10^{43}$$

Since two deuterium nuclei are used per fusion, ${}^{2}_{1}\text{H}+{}^{2}_{1}\text{H}\rightarrow{}^{4}_{2}\text{He}+Q$, the number of events is

$$N/2=6.63\times10^{42}$$

The energy released per event is

$$Q=\left[M_{{}^{2}\text{H}}+M_{{}^{2}\text{H}}-M_{{}^{4}\text{He}}\right]c^{2}=\left[2(2.014102)-4.002602\right]\text{u}\ (931.5\ \text{MeV/u})=23.8\ \text{MeV}$$

The total energy available is then

$$E=\left(\frac{N}{2}\right)Q=\left(6.63\times10^{42}\right)(23.8\ \text{MeV})\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right)=\boxed{2.51\times10^{31}\ \text{J}}$$

(b) The time interval over which this energy could possibly meet world requirements is $\Delta t=E/\mathcal{P}$:

$$\Delta t=\frac{2.52\times10^{31}\ \text{J}}{100\left(7.00\times10^{12}\ \text{J/s}\right)}=\left(3.61\times10^{16}\ \text{s}\right)\left(\frac{1\ \text{yr}}{3.16\times10^{7}\ \text{s}}\right)=\boxed{1.14\times10^{9}\ \text{yr}}\sim 1\ \text{billion years.}$$

***30.35** mass of ^{235}U available $\cong (0.007)\left(10^9 \text{ metric tons}\right)\left(\frac{10^6 \text{ g}}{1 \text{ metric ton}}\right) = 7\times10^{12} \text{ g}$

$$\text{number of nuclei} \sim \left(\frac{7\times10^{12}\text{ g}}{235\text{ g/mol}}\right)\left(6.02\times10^{23}\text{ nuclei/mol}\right) = 1.8\times10^{34}\text{ nuclei}$$

The energy available from fission (at 208 MeV/event) is

$$E \sim \left(1.8\times10^{34}\text{ events}\right)(208\text{ MeV / event})\left(1.60\times10^{-13}\text{ J / MeV}\right) = 6.0\times10^{23}\text{ J}$$

This would last for a time interval of

$$\Delta t = \frac{E}{\mathcal{P}} \sim \frac{6.0\times10^{23}\text{ J}}{7.0\times10^{12}\text{ J/s}} = \left(8.6\times10^{10}\text{ s}\right)\left(\frac{1\text{ yr}}{3.16\times10^{7}\text{ s}}\right) \sim \boxed{3000\text{ yr}}$$

***30.36** Add two electrons to both sides of the given reaction.

Then $4\,{}^1_1\text{H atom} \to {}^4_2\text{He atom} + Q$

where $Q = (\Delta m)c^2 = \left[4(1.007\,825) - 4.002\,602\right]\text{u}\,(931.5\text{ MeV/u}) = 26.7\text{ MeV}$

or $Q = (26.7\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right) = 4.28\times10^{-12}\text{ J}$

The proton fusion rate is then

$$\text{rate} = \frac{\text{power output}}{\text{energy per proton}} = \frac{3.77\times10^{26}\text{ J/s}}{\left(4.28\times10^{-12}\text{ J}\right)/(4\text{ protons})} = \boxed{3.53\times10^{38}\text{ protons/s}}$$

***30.37** (a) $Q_{\text{I}} = \left[M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}}\right]c^2$, and $Q_{\text{II}} = \left[M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}\right]c^2$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = \left[M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}} + M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}\right]c^2$$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = \left[M_{\text{A}} + M_{\text{B}} + M_{\text{D}} - M_{\text{E}} - M_{\text{F}} - M_{\text{G}}\right]c^2$$

Thus, reactions may be added. Any product like C used in a subsequent reaction does not contribute to the energy balance.

(b) Adding all five reactions gives

$${}^1_1\text{H} + {}^1_1\text{H} + {}^{\ 0}_{-1}\text{e} + {}^1_1\text{H} + {}^1_1\text{H} + {}^{\ 0}_{-1}\text{e} \to {}^4_2\text{He} + 2\nu + Q_{\text{net}}$$

or $4\,{}^1_1\text{H} + 2\,{}^{\ 0}_{-1}\text{e} \to {}^4_2\text{He} + 2\nu + Q_{\text{net}}$

Adding two electrons to each side $4\,{}^1_1\text{H atom} \to {}^4_2\text{He atom} + Q_{\text{net}}$

Thus, $Q_{\text{net}} = \left[4M_{{}^1_1\text{H}} - M_{{}^4_2\text{He}}\right]c^2 = \left[4(1.007\,825) - 4.002\,602\right]\text{ u }(931.5\text{ MeV/u}) = \boxed{26.7\text{ MeV}}$

*30.38 (a) The solar-core temperature of 15 MK gives particles enough kinetic energy to overcome the Coulomb-repulsion barrier to ${}^1_1\mathrm{H} + {}^3_2\mathrm{He} \to {}^4_2\mathrm{He} + \mathrm{e}^+ + \nu$, estimated as $k_e(e)(2e)/r$. The Coulomb barrier to Bethe's fifth and eight reactions is like $k_e(e)(7e)/r$, larger by $\frac{7}{2}$ times, so the required temperature can be estimated as $\frac{7}{2}\left(15\times10^6\ \mathrm{K}\right) \cong \boxed{5\times10^7\ \mathrm{K}}$.

(b) For ${}^{12}\mathrm{C} + {}^1\mathrm{H} \to {}^{13}\mathrm{N} + Q$,

$$Q_1 = (12.000\,000 + 1.007\,825 - 13.005\,738)(931.5\ \mathrm{MeV}) = \boxed{1.94\ \mathrm{MeV}}$$

For the second step, add seven electrons to both sides to have: ${}^{13}\mathrm{N}$ atom $\to$ ${}^{13}\mathrm{C}$ atom $+ \mathrm{e}^- + \mathrm{e}^+ + Q$

$$Q_2 = \left[13.005\,738 - 13.003\,355 - 2(0.000\,549)\right](931.5\ \mathrm{MeV}) = \boxed{1.20\ \mathrm{MeV}}$$

$$Q_3 = Q_7 = 2(0.000\,549)(931.5\ \mathrm{MeV}) = \boxed{1.02\ \mathrm{MeV}}$$

$$Q_4 = \left[13.003\,355 + 1.007\,825 - 14.003\,074\right](931.5\ \mathrm{MeV}) = \boxed{7.55\ \mathrm{MeV}}$$

$$Q_5 = \left[14.003\,074 + 1.007\,825 - 15.003\,065\right](931.5\ \mathrm{MeV}) = \boxed{7.30\ \mathrm{MeV}}$$

$$Q_6 = \left[15.003\,065 - 15.000\,108 - 2(0.000\,549)\right](931.5\ \mathrm{MeV}) = \boxed{1.73\ \mathrm{MeV}}$$

$$Q_8 = \left[15.000\,108 + 1.007\,825 - 12 - 4.002\,602\right](931.5\ \mathrm{MeV}) = \boxed{4.97\ \mathrm{MeV}}$$

The sum is $\boxed{26.7\ \mathrm{MeV}}$, the same as for the proton-proton cycle.

(c) Not all of the energy released appears as internal energy in the star. When a neutrino is created, it will likely fly directly out of the star without interacting with any other particle.

*30.39 (a) Add two electrons to both sides of the reaction to have it in energy terms:

$$4\ {}^1_1\mathrm{H}\ \text{atom} \to {}^4_2\mathrm{He}\ \text{atom} + Q \qquad Q = \Delta mc^2 = \left[4M_{{}^1_1\mathrm{H}} - M_{{}^4_2\mathrm{He}}\right]c^2$$

$$Q = \left[4(1.007\,825\ \mathrm{u}) - 4.002\,602\ \mathrm{u}\right](931.5\ \mathrm{MeV/u})\left(\frac{1.60\times10^{-13}\ \mathrm{J}}{1\ \mathrm{MeV}}\right) = \boxed{4.28\times10^{-12}\ \mathrm{J}}$$

(b) $$N = \frac{1.99\times10^{30}\ \mathrm{kg}}{1.67\times10^{-27}\ \mathrm{kg/atom}} = \boxed{1.19\times10^{57}\ \text{atoms}} = 1.19\times10^{57}\ \text{protons}$$

(c) The energy that could be created by this many protons in this reaction is:

$$\left(1.19\times10^{57}\ \text{protons}\right)\left(\frac{4.28\times10^{-12}\ \mathrm{J}}{4\ \text{protons}}\right) = 1.27\times10^{45}\ \mathrm{J}$$

$$\mathcal{P} = \frac{E}{\Delta t} \qquad \text{so} \qquad \Delta t = \frac{E}{\mathcal{P}} = \frac{1.27\times10^{45}\ \mathrm{J}}{3.77\times10^{26}\ \mathrm{W}} = 3.38\times10^{18}\ \mathrm{s} = \boxed{107\ \text{billion years}}$$

30.40 (a) At 6×10^8 K, the average kinetic energy of a carbon atom is

$$\frac{3}{2}k_B T = (1.5)\left(8.62\times10^{-5}\text{ eV/K}\right)\left(6\times10^{8}\text{ K}\right) = \boxed{8\times10^{4}\text{ eV}}$$

(b) The energy released is

$$E = \left[2m\left(\text{C}^{12}\right) - m\left(\text{Ne}^{20}\right) - m\left(\text{He}^{4}\right)\right]c^2$$

$$E = (24.000\,000 - 19.992\,435 - 4.002\,602)(931.5)\text{ MeV} = \boxed{4.62\text{ MeV}}$$

In the second reaction,

$$E = \left[2m\left(\text{C}^{12}\right) - m\left(\text{Mg}^{24}\right)\right](931.5)\text{MeV / u}$$

$$E = (24.000\,000 - 23.985\,042)(931.5)\text{ MeV} = \boxed{13.9\text{ MeV}}$$

(c) The energy released is the energy of reaction of the number of carbon nuclei in a 2.00-kg sample, which corresponds to

$$\Delta E = \left(2.00\times10^{3}\text{g}\right)\left(\frac{6.02\times10^{23}\text{ atoms/mol}}{12.0\text{ g/mol}}\right)\left(\frac{4.62\text{ MeV/fusion event}}{2\text{ nuclei/fusion event}}\right)\left(\frac{1\text{ kWh}}{2.25\times10^{19}\text{ MeV}}\right)$$

$$\Delta E = \frac{\left(1.00\times10^{26}\right)(4.62)}{2\left(2.25\times10^{19}\right)}\text{kWh} = \boxed{1.03\times10^{7}\text{ kWh}}$$

30.41 We have $$N_{235} = N_{0,\,235}e^{-\lambda_{235}t}$$

and $$N_{238} = N_{0,\,238}e^{-\lambda_{238}t}$$

$$\frac{N_{235}}{N_{238}} = 0.00725 = e^{\left(-(\ln 2)t/T_{h,\,235} + (\ln 2)t/T_{h,\,238}\right)}$$

Taking logarithms,

$$-4.93 = \left(-\frac{\ln 2}{0.704\times10^{9}\text{ yr}} + \frac{\ln 2}{4.47\times10^{9}\text{ yr}}\right)t$$

or $$-4.93 = \left(-\frac{1}{0.704\times10^{9}\text{ yr}} + \frac{1}{4.47\times10^{9}\text{ yr}}\right)(\ln 2)t$$

$$t = \frac{-4.93}{\left(-1.20\times10^{-9}\text{ yr}^{-1}\right)\ln 2} = \boxed{5.94\times10^{9}\text{ yr}}$$

***30.42** The number of nuclei in 0.155 kg of $^{210}\mathrm{Po}$ is

$$N_0 = \left(\frac{155\ \mathrm{g}}{209.98\ \mathrm{g/mol}}\right)\left(6.02\times10^{23}\ \mathrm{nuclei/mol}\right) = 4.44\times10^{23}\ \mathrm{nuclei}$$

The half-life of $^{210}\mathrm{Po}$ is 138.38 days, so the decay constant is given by

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(138.38\ \mathrm{d})\left(8.64\times10^{4}\ \mathrm{s/d}\right)} = 5.80\times10^{-8}\ \mathrm{s}^{-1}$$

The initial activity is

$$R_0 = \lambda N_0 = \left(5.80\times10^{-8}\ \mathrm{s}^{-1}\right)\left(4.44\times10^{23}\ \mathrm{nuclei}\right) = 2.58\times10^{16}\ \mathrm{Bq}$$

The energy released in each ${}^{210}_{84}\mathrm{Po}\rightarrow {}^{206}_{82}\mathrm{Pb} + {}^{4}_{2}\mathrm{He}$ reaction is

$$Q = \left[M_{{}^{210}_{84}\mathrm{Po}} - M_{{}^{206}_{82}\mathrm{Pb}} - M_{{}^{4}_{2}\mathrm{He}}\right]c^2:$$

$$Q = [209.982\,848 - 205.974\,440 - 4.002\,602]\ \mathrm{u}\ (931.5\ \mathrm{MeV/u}) = 5.41\ \mathrm{MeV}$$

Thus, assuming a conversion efficiency of 1.00%, the initial power output of the battery is

$$\mathcal{P} = (0.0100)R_0Q = (0.0100)\left(2.58\times10^{16}\ \frac{\mathrm{decays}}{\mathrm{s}}\right)\left(5.41\ \frac{\mathrm{MeV}}{\mathrm{decay}}\right)\left(1.60\times10^{-13}\ \frac{\mathrm{J}}{\mathrm{MeV}}\right) = \boxed{223\ \mathrm{W}}$$

30.43 (a) $$r = r_0 A^{1/3} = 1.20\times10^{-15} A^{1/3}\ \mathrm{m}.$$

When $A = 12$, $r = \boxed{2.75\times10^{-15}\ \mathrm{m}}$

(b) $$F = \frac{k_e(Z-1)e^2}{r^2} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)(Z-1)\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{r^2}$$

When $Z = 6$ and $r = 2.75\times 10^{-15}$ m, $F = \boxed{152\ \mathrm{N}}$

(c) $$U = \frac{k_e q_1 q_2}{r} = \frac{k_e(Z-1)e^2}{r} = \frac{\left(8.99\times10^{9}\right)(Z-1)\left(1.6\times10^{-19}\right)^2}{r}$$

When $Z = 6$ and $r = 2.75\times 10^{-15}$ m, $U = 4.19\times10^{-13}\ \mathrm{J} = \boxed{2.62\ \mathrm{MeV}}$

(d) $A = 238$; $Z = 92$, $r = \boxed{7.44\times10^{-15}\ \mathrm{m}}$ $F = \boxed{379\ \mathrm{N}}$

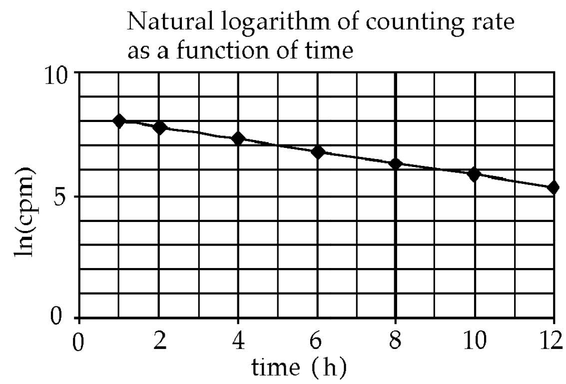
and $U = 2.82\times10^{-12}\ \mathrm{J} = \boxed{17.6\ \mathrm{MeV}}$

30.44 (a) A least-square fit to the graph yields:

$$\lambda = -\text{slope} = -\left(-0.250\ \text{h}^{-1}\right) = 0.250\ \text{h}^{-1}$$

and

$$\ln(\text{cpm})\big|_{t=0} = \text{intercept} = 8.30$$



(b) $\lambda = 0.250\ \text{h}^{-1}\left(\dfrac{1\ \text{h}}{60.0\ \text{min}}\right) = \boxed{4.17\times 10^{-3}\ \text{min}^{-1}}$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.17\times 10^{-3}\ \text{min}^{-1}} = 166\ \text{min} = \boxed{2.77\ \text{h}}$$

(c) From (a), $\quad \text{intercept} = \ln(\text{cpm})_0 = 8.30$

Thus, $\quad (\text{cpm})_0 = e^{8.30}\ \text{counts/min} = \boxed{4.02\times 10^{3}\ \text{counts/min}}$

(d) $N_0 = \dfrac{R_0}{\lambda} = \dfrac{1}{\lambda}\dfrac{(\text{cpm})_0}{\text{Eff}} = \dfrac{4.02\times 10^{3}\ \text{counts/min}}{\left(4.17\times 10^{-3}\ \text{min}^{-1}\right)(0.100)} = \boxed{9.65\times 10^{6}\ \text{atoms}}$

30.45 (a) If we assume all the ^{87}Sr came from ^{87}Rb,

then $\quad N = N_0 e^{-\lambda t}$

yields $\quad t = \dfrac{-1}{\lambda}\ln\left(\dfrac{N}{N_0}\right) = \dfrac{T_{1/2}}{\ln 2}\ln\left(\dfrac{N_0}{N}\right)$

where $\quad N = N_{\text{Rb-87}}$

and $\quad N_0 = N_{\text{Sr-87}} + N_{\text{Rb-87}}$

$$t = \frac{\left(4.75\times 10^{10}\ \text{yr}\right)}{\ln 2}\ln\left(\frac{1.82\times 10^{10} + 1.07\times 10^{9}}{1.82\times 10^{10}}\right) = \boxed{3.91\times 10^{9}\ \text{yr}}$$

(b) It could be $\boxed{\text{no older}}$. The rock could be younger if some ^{87}Sr were originally present.

30.46 (a) If ΔE is the energy difference between the excited and ground states of the nucleus of mass M, and hf is the energy of the emitted photon, conservation of energy for the nucleus-photon system gives

$$\Delta E = hf + E_r \qquad (1)$$

Where E_r is the recoil energy of the nucleus, which can be expressed as

$$E_r = \frac{Mv^2}{2} = \frac{(Mv)^2}{2M} \qquad (2)$$

Since system momentum must also be conserved, we have

$$Mv = \frac{hf}{c} \qquad (3)$$

Hence, E_r can be expressed as $\quad E_r = \dfrac{(hf)^2}{2Mc^2}$

When $\quad hf << Mc^2$

we can make the approximation that $\quad hf \cong \Delta E$

so $\quad E_r \cong \boxed{\dfrac{(\Delta E)^2}{2Mc^2}}$

(b) $E_r = \dfrac{(\Delta E)^2}{2Mc^2}$ where $\quad \Delta E = 0.0144\text{ MeV}$

and $\quad Mc^2 = (57\text{ u})(931.5\text{ MeV/u}) = 5.31\times 10^4\text{ MeV}$

Therefore, $\quad E_r = \dfrac{(1.44\times 10^{-2}\text{ MeV})^2}{2(5.31\times 10^4\text{ MeV})} = \boxed{1.94\times 10^{-3}\text{ eV}}$

***30.47** (a) One liter of milk contains this many ^{40}K nuclei:

$$N = (2.00\text{ g})\left(\frac{6.02\times 10^{23}\text{ nuclei/mol}}{39.1\text{ g/mol}}\right)\left(\frac{0.0117}{100}\right) = 3.60\times 10^{18}\text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.28\times 10^9\text{ yr}}\left(\frac{1\text{ yr}}{3.156\times 10^7\text{ s}}\right) = 1.72\times 10^{-17}\text{ s}^{-1}$$

$$R = \lambda N = \left(1.72\times 10^{-17}\text{ s}^{-1}\right)\left(3.60\times 10^{18}\right) = \boxed{61.8\text{ Bq}}$$

(b) For the iodine, $R = R_0 e^{-\lambda t}$ with $\quad \lambda = \dfrac{\ln 2}{8.04\text{ d}}$

$$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right) = \frac{8.04\text{ d}}{\ln 2}\ln\left(\frac{2000}{61.8}\right) = \boxed{40.3\text{ d}}$$

***30.48** (a) For cobalt–56,

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{77.1\ \text{d}}\left(\frac{365.25\ \text{d}}{1\ \text{yr}}\right) = 3.28\ \text{yr}^{-1}$$

The elapsed time from July 1054 to July 2001 is 947 yr.

$$R = R_0 e^{-\lambda t}$$

implies

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(3.28\ \text{yr}^{-1}\right)(947\ \text{yr})} = e^{-3110} = e^{-(\ln 10)1351} = \boxed{\sim 10^{-1351}}$$

(b) For carbon–14,

$$\lambda = \frac{\ln 2}{5730\ \text{yr}} = 1.21\times 10^{-4}\ \text{yr}^{-1}$$

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(1.21\times 10^{-4}\ \text{yr}^{-1}\right)(947\ \text{yr})} = e^{-0.115} = \boxed{0.892}$$

30.49 (a) Let us assume that the parent nucleus (mass M_p) is initially at rest, and let us denote the masses of the daughter nucleus and alpha particle by M_d and M_α, respectively. Applying the equations of conservation of momentum and energy for the alpha decay process gives

$$M_d v_d = M_\alpha v_\alpha \qquad (1)$$

$$M_p c^2 = M_d c^2 + M_\alpha c^2 + \tfrac{1}{2} M_\alpha v_\alpha^{\ 2} + \tfrac{1}{2} M_d v_d^{\ 2} \qquad (2)$$

The disintegration energy Q is given by

$$Q = (M_p - M_d - M_\alpha)c^2 = \tfrac{1}{2} M_\alpha v_\alpha^{\ 2} + \tfrac{1}{2} M_d v_d^{\ 2} \qquad (3)$$

Eliminating v_d from Equations (1) and (3) gives

$$Q = \tfrac{1}{2} M_d \left(\frac{M_\alpha}{M_d} v_\alpha\right)^2 + \tfrac{1}{2} M_\alpha v_\alpha^{\ 2} = \tfrac{1}{2}\frac{M_\alpha^{\ 2}}{M_d} v_\alpha^{\ 2} + \tfrac{1}{2} M_\alpha v_\alpha^{\ 2} = \tfrac{1}{2} M_\alpha v_\alpha^{\ 2}\left(1 + \frac{M_\alpha}{M_d}\right) = \boxed{K_a\left(1 + \frac{M_\alpha}{M_d}\right)}$$

(b) $$K_\alpha = \frac{Q}{1 + \left(M_\alpha / M_d\right)} = \frac{4.87\ \text{MeV}}{1 + (4/222)} = \boxed{4.78\ \text{MeV}}$$

***30.50** (a) At threshold, the particles have no kinetic energy relative to each other. That is, they move like two particles that have suffered a perfectly inelastic collision. Therefore, in order to calculate the reaction threshold energy, we can use the results of a perfectly inelastic collision. Initially, the projectile M_a moves with velocity v_a while the target M_X is at rest. We have from momentum conservation for the projectile-target system:

$$M_a v_a = (M_a + M_X) v_c$$

The initial energy is: $E_i = \frac{1}{2} M_a v_a^2$

The final kinetic energy is: $E_f = \frac{1}{2}(M_a + M_X) v_c^2 = \frac{1}{2}(M_a + M_X)\left[\frac{M_a v_a}{M_a + M_X}\right]^2 = \left[\frac{M_a}{M_a + M_X}\right] E_i$

From this, we see that E_f is always less than E_i and the change in energy, $E_f - E_i$, is given by

$$E_f - E_i = \left[\frac{M_a}{M_a + M_X} - 1\right] E_i = -\left[\frac{M_X}{M_a + M_X}\right] E_i$$

This loss of kinetic energy in the isolated system corresponds to an increase in mass-energy during the reaction. Thus, the absolute value of this kinetic energy change is equal to $-Q$ (remember that Q is negative in an endothermic reaction). The initial kinetic energy E_i is the threshold energy E_{th}.

Therefore, $$-Q = \left[\frac{M_X}{M_a + M_X}\right] E_{th}$$

or $$E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = \boxed{-Q\left[1 + \frac{M_a}{M_X}\right]}$$

(b) First, calculate the Q-value for the reaction: $Q = \left[M_{\text{N-14}} + M_{\text{He-4}} - M_{\text{O-17}} - M_{\text{H-1}}\right] c^2$

$$Q = [14.003\ 074 + 4.002\ 602 - 16.999\ 132 - 1.007\ 825]\,\text{u}\,(931.5\ \text{MeV/u}) = -1.19\ \text{MeV}$$

Then, $$E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = -(-1.19\ \text{MeV})\left[1 + \frac{4.002\ 602}{14.003\ 074}\right] = \boxed{1.53\ \text{MeV}}$$

***30.51** $R = R_0 \exp(-\lambda t)$ lets us write $\ln R = \ln R_0 - \lambda t$

which is the equation of a straight line with $|\text{slope}| = \lambda$

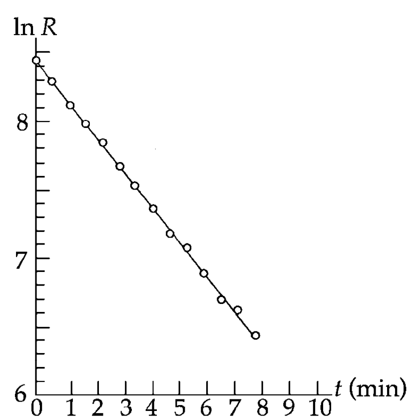
The logarithmic plot shown in Figure P30.51 is fitted by

$\ln R = 8.44 - 0.262 t$

If t is measured in minutes, then decay constant λ is 0.262 per minute. The half–life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.262/\text{min}} = \boxed{2.64\ \text{min}}$$

The reported half–life of ^{137}Ba is 2.55 min. The difference reflects experimental uncertainties.



30.52 The initial specific activity of $^{59}\mathrm{Fe}$ in the steel is

$$(R/m)_0 = \frac{20.0\ \mu\mathrm{Ci}}{0.200\ \mathrm{kg}} = \frac{100\ \mu\mathrm{Ci}}{\mathrm{kg}}\left(\frac{3.70\times10^4\ \mathrm{Bq}}{1\ \mu\mathrm{Ci}}\right) = 3.70\times10^6\ \mathrm{Bq/kg}$$

After 1000 h , $$\frac{R}{m} = (R/m)_0 e^{-\lambda t} = \left(3.70\times10^6\ \mathrm{Bq/kg}\right)e^{-\left(6.40\times10^{-4}\ \mathrm{h}^{-1}\right)(1000\ \mathrm{h})} = 1.95\times10^6\ \mathrm{Bq/kg}$$

The activity of the oil is $$R_{\mathrm{oil}} = \left(\frac{800}{60.0}\ \mathrm{Bq/liter}\right)(6.50\ \mathrm{liters}) = 86.7\ \mathrm{Bq}$$

Therefore, $$m_{\text{in oil}} = \frac{R_{\mathrm{oil}}}{R/m} = \frac{86.7\ \mathrm{Bq}}{1.95\times10^6\ \mathrm{Bq/kg}} = 4.45\times10^{-5}\ \mathrm{kg}$$

so that the wear rate is $$\frac{4.45\times10^{-5}\ \mathrm{kg}}{1000\ \mathrm{h}} = \boxed{4.45\times10^{-8}\ \mathrm{kg/h}}$$

30.53 (a) Starting with $N = 0$ radioactive atoms at $t = 0$, the rate of increase is (production – decay)

$$\frac{dN}{dt} = R - \lambda N \qquad \text{so} \qquad dN = (R - \lambda N)\,dt$$

The variables are separable.

$$\int_0^N \frac{dN}{R-\lambda N} = \int_0^t dt\ : \qquad -\frac{1}{\lambda}\ln\left(\frac{R-\lambda N}{R}\right) = t$$

so $$\ln\left(\frac{R-\lambda N}{R}\right) = -\lambda t \qquad \text{and} \qquad \left(\frac{R-\lambda N}{R}\right) = e^{-\lambda t}$$

Therefore, $$1 - \frac{\lambda}{R}N = e^{-\lambda t} \qquad N = \boxed{\frac{R}{\lambda}\left(1-e^{-\lambda t}\right)}$$

(b) The maximum number of radioactive nuclei would be $\boxed{R/\lambda}$.

***30.54** (a) Suppose each $^{235}\mathrm{U}$ fission releases 208 MeV of energy. Then, the number of nuclei that must have undergone fission is

$$N = \frac{\text{total release}}{\text{energy per nuclei}} = \frac{5\times10^{13}\ \mathrm{J}}{(208\ \mathrm{MeV})\left(1.60\times10^{-13}\ \mathrm{J/MeV}\right)} = \boxed{1.5\times10^{24}\ \text{nuclei}}$$

(b) $$\text{mass} = \left(\frac{1.5\times10^{24}\ \text{nuclei}}{6.02\times10^{23}\ \text{nuclei/mol}}\right)(235\ \mathrm{g/mol}) \cong \boxed{0.6\ \mathrm{kg}}$$

***30.55** The complete fissioning of 1.00 gram of U^{235} releases

$$Q = \frac{(1.00\text{ g})}{235\text{ grams/mol}}\left(6.02\times10^{23}\,\frac{\text{atoms}}{\text{mol}}\right)\left(200\,\frac{\text{MeV}}{\text{fission}}\right)\left(1.60\times10^{-13}\,\frac{\text{J}}{\text{MeV}}\right) = 8.20\times10^{10}\text{ J}$$

If all this energy could be utilized to convert m kilograms of 20.0°C water to 400°C steam (see Chapter 17 of text for values),

then $$Q = mc_w\,\Delta T + mL_v + mc_s\,\Delta T$$

$$Q = m\left[(4186\text{ J/kg °C})(80.0\text{ °C}) + 2.26\times10^6\text{ J/kg} + (2010\text{ J/kg °C})(300\text{ °C})\right]$$

Therefore $$m = \frac{8.20\times10^{10}\text{ J}}{3.20\times10^6\text{ J/kg}} = \boxed{2.56\times10^4\text{ kg}}$$

***30.56** (a) The number of molecules in 1.00 liter of water (mass = 1000 g) is

$$N = \left(\frac{1.00\times10^3\text{ g}}{18.0\text{ g/mol}}\right)\left(6.02\times10^{23}\text{ molecules/mol}\right) = 3.34\times10^{25}\text{ molecules}$$

The number of deuterium nuclei contained in these molecules is

$$N' = \left(3.34\times10^{25}\text{ molecules}\right)\left(\frac{1\text{ deuteron}}{3300\text{ molecules}}\right) = 1.01\times10^{22}\text{ deuterons}$$

Since 2 deuterons are consumed per fusion event, the number of events possible is

$$N'/2 = 5.07\times10^{21}$$

reactions, and the energy released is

$$E_{\text{fusion}} = \left(5.07\times10^{21}\text{ reactions}\right)\left(3.27\text{ MeV/reaction}\right) = 1.66\times10^{22}\text{ MeV}$$

$$E_{\text{fusion}} = \left(1.66\times10^{22}\text{ MeV}\right)\left(1.60\times10^{-13}\text{ J/MeV}\right) = \boxed{2.65\times10^9\text{ J}}$$

(b) In comparison to burning 1.00 liter of gasoline, the energy from the fusion of deuterium is

$$\frac{E_{\text{fusion}}}{E_{\text{gasoline}}} = \frac{2.65\times10^9\text{ J}}{3.40\times10^7\text{ J}} = \boxed{78.0\text{ times larger}}$$

30.57 $$\frac{N_1}{N_2} = \frac{N_0 - N_0e^{-\lambda T_h/2}}{N_0e^{-\lambda T_h/2} - N_0e^{-\lambda T_h}} = \frac{1 - e^{-\ln 2/2}}{e^{-\ln 2/2} - e^{-\ln 2}} = \frac{1 - 2^{-1/2}}{2^{-1/2} - 2^{-1}} = \boxed{\sqrt{2}}$$

***30.58** (a) $\Delta V = 4\pi r^2 \Delta r = 4\pi\left(14.0\times10^3 \text{ m}\right)^2(0.05 \text{ m}) = 1.23\times10^8 \text{ m}^3$ $\boxed{\sim10^8 \text{ m}^3}$

(b) The force on the next layer is determined by atmospheric pressure.

$$W = P\Delta V = \left(1.013\times10^5 \text{ N/m}^2\right)\left(1.23\times10^8 \text{ m}^3\right) = 1.25\times10^{13} \text{ J} \; \boxed{\sim10^{13} \text{ J}}$$

(c) $1.25\times10^{13} \text{ J} = \frac{1}{10}(\text{yield})$, so $\text{yield} = 1.25\times10^{14} \text{ J}$ $\boxed{\sim10^{14} \text{ J}}$

(d) $$\frac{1.25\times10^{14} \text{ J}}{4.2\times10^9 \text{ J/ton TNT}} = 2.97\times10^4 \text{ ton TNT} \;\sim10^4 \text{ ton TNT}$$

or $\boxed{\sim 10 \text{ kilotons}}$

***30.59** (a) The number of Pu nuclei in $1.00 \text{ kg} = \dfrac{6.02\times10^{23} \text{ nuclei/mol}}{239.05 \text{ g/mol}}(1000 \text{ g})$

The total energy = $(25.2\times10^{23}$ nuclei$)(200$ MeV$) = 5.04\times10^{26}$ MeV

$$E = \left(5.04\times10^{26} \text{ MeV}\right)\left(4.44\times10^{-20} \text{ kWh/MeV}\right) = \boxed{2.24\times10^7 \text{ kWh}}$$

or 22 million kWh

(b) $E = \Delta m c^2 =$ (3.016 049 u + 2.014 102 u – 4.002 602 u – 1.008 665 u) (931.5 MeV/u)

$E = \boxed{17.6 \text{ MeV for each D-T fusion}}$

(c) E_n = (Total number of D nuclei)(17.6)(4.44×10^{-20})

$$E_n = \left(6.02\times10^{23}\right)\left(\frac{1000}{2.014}\right)(17.6)\left(4.44\times10^{-20}\right) = \boxed{2.34\times10^8 \text{ kWh}}$$

(d) E_n = the number of C atoms in 1.00 kg × 4.20 eV

$$E_n = \left(\frac{6.02\times10^{26}}{12}\right)\left(4.20\times10^{-6} \text{ MeV}\right)\left(4.44\times10^{-20}\right) = \boxed{9.36 \text{ kWh}}$$

(e) Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. (a) 1.90 fm (b) 7.44 fm

4. 6.11×10^{15} N toward the other ball

6. *Z* magic: He, O, Ca, Ni, Sn, Pb.
N magic: isotopes of H, He, N, O, Cl, K, Ca, V, Cr, Sr, Y, Zr, Xe, Ba, La, Ce, Pr, Nd, Pb, Bi, Po.

8. (a) 29.2 MHz (b) 42.6 MHz (c) 2.13 kHz

10. (a) 1.11 MeV/nucleon (b) 7.07 MeV/nucleon
(c) 8.79 MeV/nucleon (d) 7.57 MeV/nucleon

12. (a) Cs (b) La (c) Cs

14. 0.200 mCi

16. 86.4 h

18. (a) 148 Bq/m^3 (b) $7.05 \times 10^7 \text{ atom/m}^3$ (c) 2.17×10^{-17}

20. (a) ${}^{65}_{28}\text{Ni}^*$ (b) ${}^{211}_{82}\text{Pb}$ (c) ${}^{55}_{27}\text{Co}$
(d) ${}^{0}_{-1}\text{e}$ (e) ${}^{1}_{1}\text{H}$ (or p)

22. 9.96×10^3 yr

24. ${}^{3}_{1}\text{H atom} \rightarrow {}^{3}_{2}\text{He atom} + \bar{\nu}$; 18.6 keV

26. (a) Conservation of energy (b) Electrostatic energy of the nucleus (c) 1.20 MeV

28. (a) ${}^{21}_{10}\text{Ne}$ (b) ${}^{144}_{54}\text{Xe}$ (c) ${}^{0}_{1}\text{e}^+ + {}^{0}_{0}\nu$

30. 1.00 MeV

32. 0.387 g

34. (a) 2.52×10^{31} J (b) 1.14×10^{9} yr

36. 3.53×10^{38} protons/s

38. (a) 5×10^{7} K
(b) 1.94 MeV, 1.20 MeV, 1.02 MeV, 7.55 MeV, 7.30 MeV, 1.73 MeV, 1.02 MeV, 4.97 MeV, 26.7 MeV
(c) Most of the neutrinos leave the star directly after their creation, without interacting with any other particles.

40. (a) 8×10^{4} eV (b) 4.62 MeV and 13.9 MeV (c) 1.03×10^{7} kWh

42. 223 W

44. (a) See the solution (b) 4.17×10^{-3} min^{-1}, 2.77 h
(c) 4.02×10^{3} counts/min (d) 9.65×10^{6} atoms

46. (a) See the solution (b) 1.94 meV

48. (a) $\sim10^{-1351}$ (b) 0.892

50. (a) See the solution (b) 1.53 MeV

52. 4.45×10^{-8} kg/h

54. (a) 1.5×10^{24} nuclei (b) 0.6 kg

56. (a) 2.65 GJ (b) The fusion energy is 78.0 times larger.

58. (a) $\sim10^{8}$ m^3 (b) $\sim10^{13}$ J
(c) $\sim10^{14}$ J (d) ~10 kilotons