CHAPTER 31

ANSWERS TO QUESTIONS

Q31.1 Strong Force — Mediated by gluons.
Electromagnetic Force — Mediated by photons.
Weak Force — Mediated by the $W^{\pm}$ and Z° bosons.
Gravitational Force — Mediated by gravitons.

Q31.2 In the quark model, all hadrons are composed of smaller units called quarks. Quarks have a fractional electric charge and a baryon number of $1/3$. There are 6 types of quarks: up, down, strange, charmed, top, and bottom. Further, all *baryons* contain 3 quarks, and all *mesons* contain one quark and one anti-quark. *Leptons* are thought to be fundamental particles.

Q31.3 Hadrons are massive particles with structure and size. There are two classes of hadron: mesons and baryons. Hadrons are composed of quarks. Hadrons interact via the strong force.
Leptons are light particles with no structure or size. It is believed that leptons are fundamental particles. Leptons interact via the weak force.

Q31.4 Baryons are heavy hadrons with spin $\frac{1}{2}$ or $\frac{3}{2}$, are composed of three quarks, and have long lifetimes. Mesons are light hadrons with spin 0 or 1, are composed of a quark and an antiquark, and have short lifetimes.

Q31.5 Resonances are hadrons. They decay into strongly interacting particles such as protons, neutrons, and pions, all of which are hadrons.

Q31.6 The baryon number of proton or neutron is one. Since baryon number is conserved, the baryon number of the kaon must be zero.

Q31.7 Decays by the weak interaction typically take 10^{-10} s or longer to occur. This is slow in particle physics.

Q31.8 The decays of the muon, tau, charged pion, kaons, neutron, lambda, charged sigmas, xis, and omega occur by the weak interaction. All have lifetimes longer than 10^{-13} s. Several produce neutrinos; none produce photons. Several violate strangeness conservation.

Q31.9 The decays of the neutral pion, eta, and neutral sigma occur by the electromagnetic interaction. These are the three shortest lifetimes in Table 31.2. All produce photons, which are the quanta of the electromagnetic force. All conserve strangeness.

Q31.10 Yes, protons interact via the weak interaction; but the strong interaction predominates.

Q31.11 You can think of a conservation law as a superficial regularity which we happen to notice, as a person who does not know the rules of chess might observe that one player's two bishops are always on squares of opposite colors. Alternatively, you can think of a conservation law as identifying some stuff of which the universe is made. In classical physics one can think of both matter and energy as fundamental constituents of the world. We buy and sell both of them. In classical physics you can also think of linear momentum, angular momentum, and electric charge as basic stuffs of which the universe is made. In relativity we learn that matter and energy are not conserved separately, but are both aspects of the conserved quantity *relativistic total energy*. Discovered more recently, four conservation laws appear equally general and thus equally fundamental: Conservation of baryon number, conservation of electron-lepton number, conservation of tau-lepton number, and conservation of muon-lepton number. Processes involving the strong force and the electromagnetic force follow conservation of strangeness, charm, bottomness, and topness, while the weak interaction can alter the total *S, C, B* and *T* quantum numbers of an isolated system.

Q31.12 No. Antibaryons have baryon number –1, mesons have baryon number 0, and baryons have baryon number +1. The reaction cannot occur because it would not conserve baryon number.

Q31.13 The Standard Model consists of quantum chromodynamics (to describe the strong interaction) and the electroweak theory (to describe the electromagnetic and weak interactions). The Standard Model is our most comprehensive description of nature. It fails to unify the two theories it includes, and fails to include the gravitational force. It pictures matter as made of six quarks and six leptons, interacting by exchanging gluons, photons, and W and Z bosons.

Q31.14 All baryons and antibaryons consist of three quarks. All mesons and antimesons consist of two quarks. Since quarks have spin quantum number 1/2 and can be spin-up or spin-down, it follows that the three-quark baryons must have a half-integer spin, while the two-quark mesons must have spin 0 or 1.

Q31.15 Each flavor of quark can have colors, designated as red, green and blue. Antiquarks are colored antired, antigreen, and antiblue. A baryon consists of three quarks, each having a different color. By analogy to additive color mixing we call it colorless. A meson consists of a quark of one color and antiquark with the corresponding anticolor, making it colorless as a whole.

Q31.16 In 1961 Gell-Mann predicted the omega-minus particle, with quark composition sss. Its discovery in 1964 confirmed the quark theory.

Q31.17 The xi-minus particle has, from Table 31.2, charge $-e$, spin $\frac{1}{2}$, $B = 1$, $L_e = L_\mu = L_\tau = 0$, and strangeness –2. All of these are described by its quark composition dss (Table 31.4). The properties of the quarks from Table 31.3 let us add up charge: $-\frac{1}{3}e - \frac{1}{3}e - \frac{1}{3}e = -e$; spin $+\frac{1}{2} - \frac{1}{2} + \frac{1}{2} = \frac{1}{2}$, supposing one of the quarks is spin-down relative to the other two; baryon number $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$; lepton numbers, charm, bottomness, and topness zero; and strangeness $0 - 1 - 1 = -2$.

Q31.18 The electroweak theory of Glashow, Salam, and Weinberg predicted the W^+, W^-, and Z particles. Their discovery in 1983 confirmed the electroweak theory.

PROBLEM SOLUTIONS

31.1 Assuming that the proton and antiproton are left nearly at rest after they are produced, the energy E of the photon must be

$$E = 2E_0 = 2(938.3 \text{ MeV}) = 1876.6 \text{ MeV} = 3.00 \times 10^{-10} \text{ J}$$

Thus, $E = hf = 3.00 \times 10^{-10}$ J

$$f = \frac{3.00 \times 10^{-10} \text{ J}}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = \boxed{4.53 \times 10^{23} \text{ Hz}}$$

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8} \text{ m/s}}{4.53 \times 10^{23} \text{ Hz}} = \boxed{6.62 \times 10^{-16} \text{ m}}$$

***31.2** The half-life of ^{14}O is 70.6 s, so the decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{70.6 \text{ s}} = 0.00982 \text{ s}^{-1}$$

The number of ^{14}O nuclei remaining after five minutes is

$$N = N_0 e^{-\lambda t} = \left(10^{10}\right) e^{-\left(0.009\,82 \text{ s}^{-1}\right)(300 \text{ s})} = 5.26 \times 10^{8}$$

The number of these in one cubic centimeter of blood is

$$N' = N\left(\frac{1.00 \text{ cm}^3}{\text{total vol. of blood}}\right) = \left(5.26 \times 10^{8}\right)\left(\frac{1.00 \text{ cm}^3}{2000 \text{ cm}^3}\right) = 2.63 \times 10^{5}$$

and their activity is

$$R = \lambda N' = \left(0.009\,82 \text{ s}^{-1}\right)\left(2.63 \times 10^{5}\right) = 2.58 \times 10^{3} \text{ Bq} \boxed{\sim 10^{3} \text{ Bq}}$$

31.3 In $\gamma \rightarrow \text{p}^{+} + \text{p}^{-}$,

we start with energy 2.09 GeV

we end with energy 938.3 MeV + 938.3 MeV + 95.0 MeV + K_2

where K_2 is the kinetic energy of the second proton.

Conservation of energy for the creation process gives $\boxed{K_2 = 118 \text{ MeV}}$

31.4 The minimum energy is released, and hence the minimum frequency photons are produced, when the proton and antiproton are at rest when they annihilate.

That is, $E = E_0$ and $K = 0$. To conserve momentum, each photon must carry away one-half the energy.

Thus, $$E_{min} = \frac{2E_0}{2} = E_0 = 938.3 \text{ MeV} = hf_{min}$$

Thus, $$f_{min} = \frac{(938.3 \text{ MeV})\left(1.60\times10^{-13} \text{ J/MeV}\right)}{\left(6.626\times10^{-34} \text{ J}\cdot\text{s}\right)} = \boxed{2.27\times10^{23} \text{ Hz}}$$

$$\lambda = \frac{c}{f_{min}} = \frac{3.00\times10^{8} \text{ m/s}}{2.27\times10^{23} \text{ Hz}} = \boxed{1.32\times10^{-15} \text{ m}}$$

31.5 The creation of a virtual Z^0 boson is an energy fluctuation $\Delta E = 93\times10^9$ eV. It can last no longer than $\Delta t = \hbar/2\Delta E$ and move no farther than

$$c(\Delta t) = \frac{hc}{4\pi\,\Delta E} = \frac{\left(6.626\times10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00\times10^{8} \text{ m/s}\right)}{4\pi\left(93\times10^{9} \text{ eV}\right)}\left(\frac{1 \text{ eV}}{1.60\times10^{-19} \text{ J}}\right) = 1.06\times10^{-18} \text{ m} = \boxed{\sim10^{-18} \text{ m}}$$

31.6 The reaction is $\mu^+ + e^- \to \nu + \nu$

muon-lepton number before reaction: $(-1) + (0) = -1$

electron-lepton number before reaction: $(0) + (1) = 1$

Therefore, after the reaction, the muon-lepton number must be –1. Thus, one of the neutrinos must be the anti-neutrino associated with muons, and one of the neutrinos must be the neutrino associated with electrons:

$\boxed{\bar{\nu}_\mu}$ and $\boxed{\nu_e}$

Then $\mu^+ + e^- \to \bar{\nu}_\mu + \nu_e$

31.7 A proton has rest energy 938.3 MeV. The time interval during which a virtual proton could exist is at most Δt in $\Delta E\Delta t = \hbar/2$. The distance it could move is at most

$$c\Delta t = \frac{\hbar c}{2\Delta E} = \frac{\left(1.055\times10^{-34} \text{ J}\cdot\text{s}\right)\left(3\times10^{8} \text{ m/s}\right)}{2(938.3)\left(1.6\times10^{-13} \text{ J}\right)} \boxed{\sim10^{-16} \text{ m}}$$

According to Yukawa's line of reasoning, this distance is the range of a force that could be associated with the exchange of virtual protons between high-energy particles.

31.8 The time interval for a particle traveling with the speed of light to travel a distance of 3×10^{-15} m is

$$\Delta t = \frac{d}{v} = \frac{3\times10^{-15}\text{ m}}{3\times10^{8}\text{ m/s}} = \boxed{\sim 10^{-23}\text{ s}}$$

31.9 By Table 31.2, $M_{\pi^0} = 135\text{ MeV}/c^2$

Therefore, $E_\gamma = \boxed{67.5\text{ MeV}}$ for each photon

$$p = \frac{E_\gamma}{c} = \boxed{67.5\ \frac{\text{MeV}}{c}}$$

and $f = \dfrac{E_\gamma}{h} = \boxed{1.63\times10^{22}\text{ Hz}}$

31.10 (a) $\Delta E = \left(m_n - m_p - m_e\right)c^2$

From Table A-3, $\Delta E = (1.008\,665 - 1.007\,825)(931.5) = \boxed{0.782\text{ MeV}}$

(b) Assuming the neutron at rest, momentum conservation for the decay process implies $p_p = p_e$

relativistic energy for the system is conserved $\sqrt{\left(m_pc^2\right)^2 + p_p{}^2c^2} + \sqrt{\left(m_ec^2\right)^2 + p_e{}^2c^2} = m_nc^2$

Since $p_p = p_e$, $\sqrt{(938.3)^2 + (pc)^2} + \sqrt{(0.511)^2 + (pc)^2} = 939.6\text{ MeV}$

Solving the algebra, $pc = 1.19\text{ MeV}$

If $p_ec = \gamma m_ev_ec = 1.19\text{ MeV}$, then $\dfrac{\gamma v_e}{c} = \dfrac{1.19\text{ MeV}}{0.511\text{ MeV}} = \dfrac{x}{\sqrt{1-x^2}} = 2.33$ where $x = \dfrac{v_e}{c}$

Solving, $x^2 = \left(1-x^2\right)5.43$ and $x = \dfrac{v_e}{c} = 0.919$

$$\boxed{v_e = 0.919c}$$

Then $m_pv_p = \gamma_em_ev_e$: $v_p = \dfrac{\gamma_em_ev_ec}{m_pc} = \dfrac{(1.19\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)}{\left(1.67\times10^{-27}\right)\left(3.00\times10^{8}\text{ m/s}\right)}$

$$v_p = 3.80\times10^{5}\text{ m/s} = \boxed{380\text{ km/s}}$$

(c) $\boxed{\text{The electron is relativistic, the proton is not.}}$

31.11 $\Omega^+ \to \overline{\Lambda}^0 + K^+$ $\quad\quad$ $\overline{K}_S^0 \to \pi^+ + \pi^-$ $\quad$ (or $\pi^0 + \pi^0$)

$\overline{\Lambda}^0 \to \overline{p} + \pi^+$ $\quad\quad$ $\overline{n} \to \overline{p} + e^+ + \nu_e$

31.12 In $? + p^+ \to n + \mu^+$, charge conservation requires the unknown particle to be neutral. Baryon number conservation requires baryon number = 0. The muon-lepton number of ? must be –1.

So the unknown particle must be $\boxed{\overline{\nu}_\mu}$.

31.13 (a) $p + \overline{p} \to \mu^+ + e^-$ $\quad \boxed{L_e}$ $\quad 0 + 0 \to 0 + 1$

and $\quad \boxed{L_\mu}$ $\quad 0 + 0 \to -1 + 0$

(b) $\pi^- + p \to p + \pi^+$ $\quad \boxed{\text{charge}}$ $\quad -1 + 1 \to +1 + 1$

(c) $p + p \to p + \pi^+$ $\quad \boxed{\text{baryon number}}$: $\quad 1 + 1 \to 1 + 0$

(d) $p + p \to p + p + n$ $\quad \boxed{\text{baryon number}}$: $\quad 1 + 1 \to 1 + 1 + 1$

(e) $\gamma + p \to n + \pi^0$ $\quad \boxed{\text{charge}}$ $\quad 0 + 1 \to 0 + 0$

31.14 (a) Baryon number and charge are conserved, with values of $\quad 0 + 1 = 0 + 1$

and $\quad 1 + 1 = 1 + 1$ in both reactions.

(b) $\boxed{\text{Strangeness is } \textit{not} \text{ conserved}}$ in the second reaction.

31.15 (a) $\pi^- \to \mu^- + \boxed{\overline{\nu}_\mu}$ $\quad L_\mu:\ 0 \to 1 - 1$

(b) $K^+ \to \mu^+ + \boxed{\nu_\mu}$ $\quad L_\mu:\ 0 \to -1 + 1$

(c) $\boxed{\overline{\nu}_e} + p^+ \to n + e^+$ $\quad L_e:\ -1 + 0 \to 0 - 1$

(d) $\boxed{\nu_e} + n \to p^+ + e^-$ $\quad L_e:\ 1 + 0 \to 0 + 1$

(e) $\boxed{\nu_\mu} + n \to p^+ + \mu^-$ $\quad L_\mu:\ 1 + 0 \to 0 + 1$

(f) $\mu^- \to e^- + \boxed{\overline{\nu}_e} + \boxed{\nu_\mu}$ $\quad L_\mu:\ 1 \to 0 + 0 + 1$ $\quad$ and $\quad L_e:\ 0 \to 1 - 1 + 0$

31.16 Baryon number conservation $\boxed{\text{allows the first and forbids the second}}$.

31.17 (a) $p^+ \to \pi^+ + \pi^0$ $\boxed{\text{Baryon number}}$ is violated: $1 \to 0 + 0$

(b) $p^+ + p^+ \to p^+ + p^+ + \pi^0$ This reaction $\boxed{\text{can occur}}$.

(c) $p^+ + p^+ \to p^+ + \pi^+$ $\boxed{\text{Baryon number}}$ is violated: $1 + 1 \to 1 + 0$

(d) $\pi^+ \to \mu^+ + \nu_\mu$ This reaction $\boxed{\text{can occur}}$.

(e) $n^0 \to p^+ + e^- + \bar{\nu}_e$ This reaction $\boxed{\text{can occur}}$.

(f) $\pi^+ \to \mu^+ + n$ Violates $\boxed{\text{baryon number}}$: $0 \to 0 + 1$

Violates $\boxed{\text{muon-lepton number}}$: $0 \to -1 + 0$

31.18 Momentum conservation for the decay requires the pions to have equal speeds.

The total energy of each is 497.7 MeV/2

so $E^2 = p^2c^2 + (mc^2)^2$ gives

$$(248.8\text{ MeV})^2 = (pc)^2 + (139.6\text{ MeV})^2$$

Solving,
$$pc = 206\text{ MeV} = \gamma mvc = \frac{mc^2}{\sqrt{1-(v/c)^2}}\left(\frac{v}{c}\right)$$

$$\frac{pc}{mc^2} = \frac{206\text{ MeV}}{139.6\text{ MeV}} = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{v}{c}\right) = 1.48$$

$$(v/c) = 1.48\sqrt{1-(v/c)^2}$$

and
$$(v/c)^2 = 2.18\left[1-(v/c)^2\right] = 2.18 - 2.18(v/c)^2$$

$$3.18(v/c)^2 = 2.18$$

so
$$\frac{v}{c} = \sqrt{\frac{2.18}{3.18}} = 0.828$$

and
$$\boxed{v = 0.828c}$$

***31.19** (a) $\mathrm{p} \to \mathrm{e}^+ + \gamma$

Baryon number: $+1 \to 0 + 0$

$\Delta B \neq 0$, so baryon number conservation is violated.

(b) From conservation of momentum for the decay: $p_e = p_\gamma$

Then, for the positron, $E_e^2 = (p_e c)^2 + E_{0,\,e}^2$

becomes $E_e^2 = (p_\gamma c)^2 + E_{0,\,e}^2 = E_\gamma^2 + E_{0,\,e}^2$

From conservation of energy for the system: $E_{0,\,p} = E_e + E_\gamma$

or $E_e = E_{0,\,p} - E_\gamma$

so $E_e^2 = E_{0,\,p}^2 - 2E_{0,\,p}E_\gamma + E_\gamma^2$

Equating this to the result from above gives $E_\gamma^2 + E_{0,\,e}^2 = E_{0,\,p}^2 - 2E_{0,p}E_\gamma + E_\gamma^2$

or $$E_\gamma = \frac{E_{0,\,p}^2 - E_{0,\,e}^2}{2E_{0,\,p}} = \frac{(938.3\text{ MeV})^2 - (0.511\text{ MeV})^2}{2(938.3\text{ MeV})} = \boxed{469\text{ MeV}}$$

Thus, $$E_e = E_{0,p} - E_\gamma = 938.3\text{ MeV} - 469\text{ MeV} = \boxed{469\text{ MeV}}$$

Also, $$p_\gamma = \frac{E_\gamma}{c} = \boxed{469\text{ MeV}/c}$$

and $$p_e = p_\gamma = \boxed{469\text{ MeV}/c}$$

(c) The total energy of the positron is $E_e = 469\text{ MeV}$

But, $$E_e = \gamma E_{0,\,e} = \frac{E_{0,\,e}}{\sqrt{1-(v/c)^2}}$$

so $$\sqrt{1-(v/c)^2} = \frac{E_{0,\,e}}{E_e} = \frac{0.511\text{ MeV}}{469\text{ MeV}} = 1.09 \times 10^{-3}$$

which yields: $\boxed{v = 0.9999994\,c}$

***31.20** The relevant conservation laws are: $\Delta L_e = 0$

$\Delta L_\mu = 0$

and $\Delta L_\tau = 0$

(a) $\pi^+ \to \pi^0 + e^+ + ?$ $L_e:\ 0 \to 0 - 1 + L_e$ implies $L_e = 1$ and we have a $\boxed{\nu_e}$

(b) $? + p \to \mu^- + p + \pi^+$ $L_\mu:\ L_\mu + 0 \to +1 + 0 + 0$ implies $L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

(c) $\Lambda^0 \to p + \mu^- + ?$ $L_\mu:\ 0 \to 0 + 1 + L_\mu$ implies $L_\mu = -1$ and we have a $\boxed{\bar{\nu}_\mu}$

(d) $\tau^+ \to \mu^+ + ? + ?$ $L_\mu:\ 0 \to -1 + L_\mu$ implies $L_\mu = 1$ and we have a $\boxed{\nu_\mu}$

$L_\tau:\ -1 \to 0 + L_\tau$ implies $L_\tau = -1$ and we have a $\boxed{\bar{\nu}_\tau}$

Conclusion for (d): $L_\mu = 1$ for one particle, and $L_\tau = -1$ for the other particle.

We have $\boxed{\nu_\mu}$

and $\boxed{\bar{\nu}_\tau}$

31.21 (a) $\Lambda^0 \to p + \pi^-$ Strangeness: $-1 \to 0 + 0$ (strangeness is $\boxed{\text{not conserved}}$)

(b) $\pi^- + p \to \Lambda^0 + K^0$ Strangeness: $0 + 0 \to -1 + 1$ (0 = 0 and strangeness is $\boxed{\text{conserved}}$)

(c) $\bar{p} + p \to \bar{\Lambda}^0 + \Lambda^0$ Strangeness: $0 + 0 \to +1 - 1$ (0 = 0 and strangeness is $\boxed{\text{conserved}}$)

(d) $\pi^- + p \to \pi^- + \Sigma^+$ Strangeness: $0 + 0 \to 0 - 1$ ($0 \neq -1$: strangeness is $\boxed{\text{not conserved}}$)

(e) $\Xi^- \to \Lambda^0 + \pi^-$ Strangeness: $-2 \to -1 + 0$ ($-2 \neq -1$ so strangeness is $\boxed{\text{not conserved}}$)

(f) $\Xi^0 \to p + \pi^-$ Strangeness: $-2 \to 0 + 0$ ($-2 \neq 0$ so strangeness is $\boxed{\text{not conserved}}$)

31.22 The $\rho^0 \to \pi^+ + \pi^-$ decay must occur via the strong interaction.

The $K_S^0 \to \pi^+ + \pi^-$ decay must occur via the weak interaction.

31.23 (a) $\mu^- \rightarrow e^- + \gamma$ L_e: $0 \rightarrow 1 + 0$,

and L_μ: $1 \rightarrow 0$

(b) $n \rightarrow p + e^- + \nu_e$ L_e: $0 \rightarrow 0 + 1 + 1$

(c) $\Lambda^0 \rightarrow p + \pi^0$ Strangeness: $-1 \rightarrow 0 + 0$,

and charge: $0 \rightarrow +1 + 0$

(d) $p \rightarrow e^+ + \pi^0$ Baryon number: $+1 \rightarrow 0 + 0$

(e) $\Xi^0 \rightarrow n + \pi^0$ Strangeness: $-2 \rightarrow 0 + 0$

***31.24** (a) $\pi^- + p \rightarrow 2\eta$ violates conservation of baryon number as $0 + 1 \rightarrow 0$, $\boxed{\text{not allowed}}$.

(b) $K^- + n \rightarrow \Lambda^0 + \pi^-$
Baryon number, $0 + 1 \rightarrow 1 + 0$
Charge, $-1 + 0 \rightarrow 0 - 1$
Strangeness, $-1 + 0 \rightarrow -1 + 0$
Lepton number, $0 \rightarrow 0$
The interaction may occur via the $\boxed{\text{strong interaction}}$ since all are conserved.

(c) $K^- \rightarrow \pi^- + \pi^0$
Strangeness, $-1 \rightarrow 0 + 0$
Baryon number, $0 \rightarrow 0$
Lepton number, $0 \rightarrow 0$
Charge, $-1 \rightarrow -1 + 0$
Strangeness is violated by one unit, but everything else is conserved. Thus, the reaction can occur via the $\boxed{\text{weak interaction}}$, but not the strong or electromagnetic interaction.

(d) $\Omega^- \rightarrow \Xi^- + \pi^0$
Baryon number, $1 \rightarrow 1 + 0$
Lepton number, $0 \rightarrow 0$
Charge, $-1 \rightarrow -1 + 0$
Strangeness, $-3 \rightarrow -2 + 0$
May occur by $\boxed{\text{weak interaction}}$, but not by strong or electromagnetic.

(e) $\eta \rightarrow 2\gamma$
Baryon number, $0 \rightarrow 0$
Lepton number, $0 \rightarrow 0$
Charge, $0 \rightarrow 0$
Strangeness, $0 \rightarrow 0$
No conservation laws are violated, but photons are the mediators of the electromagnetic interaction. Also, the lifetime of the η is consistent with the $\boxed{\text{electromagnetic interaction}}$.

***31.25** (a) $\Xi^- \to \Lambda^0 + \mu^- + \nu_\mu$

Baryon number:	$+1 \to +1+0+0$	Charge:	$-1 \to 0-1+0$
L_e:	$0 \to 0+0+0$	L_μ:	$0 \to 0+1+1$
L_τ:	$0 \to 0+0+0$	Strangeness:	$-2 \to -1+0+0$

Conserved quantities are: $\boxed{B,\ \text{charge},\ L_e,\ \text{and } L_\tau}$

(b) $K_S^0 \to 2\pi^0$

Baryon number:	$0 \to 0$	Charge:	$0 \to 0$
L_e:	$0 \to 0$	L_μ:	$0 \to 0$
L_τ:	$0 \to 0$	Strangeness:	$+1 \to 0$

Conserved quantities are: $\boxed{B,\ \text{charge},\ L_e,\ L_\mu\ \text{and } L_\tau}$

(c) $K^- + p \to \Sigma^0 + n$

Baryon number:	$0+1 \to 1+1$	Charge:	$-1+1 \to 0+0$
L_e:	$0+0 \to 0+0$	L_μ:	$0+0 \to 0+0$
L_τ:	$0+0 \to 0+0$	Strangeness:	$-1+0 \to -1+0$

Conserved quantities are: $\boxed{S,\ \text{charge},\ L_e,\ L_\mu\ \text{and } L_\tau}$

(d) $\Sigma^0 \to \Lambda^0 + \gamma$

Baryon number:	$+1 \to 1+0$	Charge:	$0 \to 0$
L_e:	$0 \to 0+0$	L_μ:	$0 \to 0+0$
L_τ:	$0 \to 0+0$	Strangeness:	$-1 \to -1+0$

Conserved quantities are: $\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu\ \text{and } L_\tau}$

(e) $e^+ + e^- \to \mu^+ + \mu^-$

Baryon number:	$0+0 \to 0+0$	Charge:	$+1-1 \to +1-1$
L_e:	$-1+1 \to 0+0$	L_μ:	$0+0 \to +1-1$
L_τ:	$0+0 \to 0+0$	Strangeness:	$0+0 \to 0+0$

Conserved quantities are: $\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu\ \text{and } L_\tau}$

(f) $\overline{p} + n \to \overline{\Lambda}^0 + \Sigma^-$

Baryon number:	$-1+1 \to -1+1$	Charge:	$-1+0 \to 0-1$
L_e:	$0+0 \to 0+0$	L_μ:	$0+0 \to 0+0$
L_τ:	$0+0 \to 0+0$	Strangeness:	$0+0 \to +1-1$

Conserved quantities are: $\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu\ \text{and } L_\tau}$

***31.26** (a) $K^+ + p \rightarrow \underline{?} + p$:

The strong interaction conserves everything.

Baryon number,	$0 + 1 \rightarrow B + 1$	so	$B = 0$
Charge,	$+1 + 1 \rightarrow Q + 1$	so	$Q = +1$
Lepton numbers,	$0 + 0 \rightarrow L + 0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$+1 + 0 \rightarrow S + 0$	so	$S = 1$

The conclusion is that the particle must be positively charged, a non-baryon, with strangeness of +1. Of particles in Table 31.2, it can only be the $\boxed{K^+}$. Thus, this is an elastic scattering process.

The weak interaction conserves all but strangeness, and $\Delta S = \pm 1$.

(b) $\Omega^- \rightarrow \underline{?} + \pi^-$:

Baryon number,	$+1 \rightarrow B + 0$	so	$B = 1$
Charge,	$-1 \rightarrow Q - 1$	so	$Q = 0$
Lepton numbers,	$0 \rightarrow L + 0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$-3 \rightarrow S + 0$	so	$\Delta S = 1$: $S = -2$

The particle must be a neutral baryon with strangeness of –2. Thus, it is the $\boxed{\Xi^0}$.

(c) $K^+ \rightarrow \underline{?} + \mu^+ + \nu_\mu$:

Baryon number,	$0 \rightarrow B + 0 + 0$	so	$B = 0$
Charge,	$+1 \rightarrow Q + 1 + 0$	so	$Q = 0$
Lepton numbers,	L_e, $0 \rightarrow L_e + 0 + 0$	so	$L_e = 0$
	L_μ, $0 \rightarrow L_\mu - 1 + 1$	so	$L_\mu = 0$
	L_τ, $0 \rightarrow L_\tau + 0 + 0$	so	$L_\tau = 0$
Strangeness,	$1 \rightarrow S + 0 + 0$	so	$\Delta S = \pm 1$ (for weak interaction): $S = 0$

The particle must be a neutral meson with strangeness $= 0 \Rightarrow \boxed{\pi^0}$.

31.27 Time-dilated lifetime:

$$T = \gamma T_0 = \frac{0.900 \times 10^{-10}\ \text{s}}{\sqrt{1 - v^2/c^2}} = \frac{0.900 \times 10^{-10}\ \text{s}}{\sqrt{1 - (0.960)^2}} = 3.214 \times 10^{-10}\ \text{s}$$

$$\text{distance} = 0.960\left(3.00 \times 10^8\ \text{m/s}\right)\left(3.214 \times 10^{-10}\ \text{s}\right) = \boxed{9.26\ \text{cm}}$$

31.28 (a)

	proton	u	u	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	e	$2e/3$	$2e/3$	$-e/3$	e

(b)

	neutron	u	d	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

31.29 (a) The number of protons $N_p = 1000\ \text{g}\left(\frac{6.02\times10^{23}\ \text{molecules}}{18.0\ \text{g}}\right)\left(\frac{10\ \text{protons}}{\text{molecule}}\right) = 3.34\times10^{26}\ \text{protons}$

and there are $N_n = (1000\ \text{g})\left(\frac{6.02\times10^{23}\ \text{molecules}}{18.0\ \text{g}}\right)\left(\frac{8\ \text{neutrons}}{\text{molecule}}\right) = 2.68\times10^{26}\ \text{neutrons}$

So there are for electric neutrality $\boxed{3.34\times10^{26}\ \text{electrons}}$

The up quarks have number $2\left(3.34\times10^{26}\right) + 2.68\times10^{26} = \boxed{9.36\times10^{26}\ \text{up quarks}}$

and there are $2\left(2.68\times10^{26}\right) + 3.34\times10^{26} = \boxed{8.70\times10^{26}\ \text{down quarks}}$

(b) Model yourself as 65 kg of water. Then you contain:

$$65\left(3.34\times10^{26}\right)\ \boxed{\sim10^{28}\ \text{electrons}}$$

$$65\left(9.36\times10^{26}\right)\ \boxed{\sim10^{29}\ \text{up quarks}}$$

$$65\left(8.70\times10^{26}\right)\ \boxed{\sim10^{29}\ \text{down quarks}}$$

Only these fundamental particles form your body. You have no strangeness, charm, topness or bottomness.

31.30 (a)

	K^0	d	$\bar{s}$	total
strangeness	1	0	1	1
baryon number	0	1/3	$-1/3$	0
charge	0	$-e/3$	$e/3$	0

(b)

	Λ^0	u	d	s	total
strangeness	-1	0	0	-1	-1
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

31.31 Quark composition of proton = uud and of neutron = udd.

Thus, if we neglect binding energies, we may write

$$m_p = 2m_u + m_d \quad (1)$$

and

$$m_n = m_u + 2m_d \quad (2)$$

Solving simultaneously,

we find $m_u = \frac{1}{3}\left(2m_p - m_n\right) = \frac{1}{3}\left[2\left(938\ \text{MeV}/c^2\right) - 939.6\ \text{meV}/c^2\right] = \boxed{312\ \text{MeV}/c^2}$

and from either (1) or (2), $m_d = \boxed{314\ \text{MeV}/c^2}$

***31.32** In the first reaction, $\pi^- + p \rightarrow K^0 + \Lambda^0$, the quarks in the particles are: $\bar{u}d + uud \rightarrow d\bar{s} + uds$. There is a net of 1 up quark both before and after the reaction, a net of 2 down quarks both before and after, and a net of zero strange quarks both before and after. Thus, the reaction conserves the net number of each type of quark.

In the second reaction, $\pi^- + p \rightarrow K^0 + n$, the quarks in the particles are: $\bar{u}d + uud \rightarrow d\bar{s} + udd$. In this case, there is a net of 1 up and 2 down quarks before the reaction but a net of 1 up, 3 down, and 1 anti-strange quark after the reaction. Thus, the reaction does not conserve the net number of each type of quark.

31.33 (a) $\pi^- + p \rightarrow K^0 + \Lambda^0$

In terms of constituent quarks: $\boxed{\bar{u}d + uud \rightarrow d\bar{s} + uds}$.

up quarks:	$-1 + 2 \rightarrow 0 + 1$,	or	$1 \rightarrow 1$
down quarks:	$1 + 1 \rightarrow 1 + 1$,	or	$2 \rightarrow 2$
strange quarks:	$0 + 0 \rightarrow -1 + 1$,	or	$0 \rightarrow 0$

(b) $\pi^+ + p \rightarrow K^+ + \Sigma^+$: $\boxed{\bar{u}d + uud \rightarrow u\bar{s} + uus}$.

up quarks:	$1 + 2 \rightarrow 1 + 2$,	or	$3 \rightarrow 3$
down quarks:	$-1 + 1 \rightarrow 0 + 0$,	or	$0 \rightarrow 0$
strange quarks:	$0 + 0 \rightarrow -1 + 1$,	or	$0 \rightarrow 0$

(c) $K^- + p \rightarrow K^+ + K^0 + \Omega^-$: $\boxed{\bar{u}s + uud \rightarrow u\bar{s} + d\bar{s} + sss}$.

up quarks:	$-1 + 2 \rightarrow 1 + 0 + 0$,	or	$1 \rightarrow 1$
down quarks:	$0 + 1 \rightarrow 0 + 1 + 0$,	or	$1 \rightarrow 1$
strange quarks:	$1 + 0 \rightarrow -1 - 1 + 3$,	or	$1 \rightarrow 1$

(d) $p + p \rightarrow K^0 + p + \pi^+ + \underline{?} \Rightarrow$ $uud + uud \rightarrow d\bar{s} + uud + u\bar{d} + \underline{?}$

The quark combination of ? must be such as to balance the last equation for up, down, and strange quarks.

up quarks:	$2 + 2 = 0 + 2 + 1 + ?$	(has 1 u quark)
down quarks:	$1 + 1 = 1 + 1 - 1 + ?$	(has 1 d quark)
strange quarks:	$0 + 0 = -1 + 0 + 0 + ?$	(has 1 s quark)

quark composition = uds = $\boxed{\Lambda^0 \text{ or } \Sigma^0}$

31.34 $\Sigma^0 + \mathrm{p} \to \Sigma^+ + \gamma + X$

dds + uud → uds + 0 + ?

The left side has a net 3d, 2u and 1s. The right-hand side has 1d, 1u, and 1s leaving 2d and 1u missing.

$\boxed{\text{The unknown particle is a neutron, udd.}}$

Baryon and strangeness numbers are conserved.

***31.35** Compare the given quark states to the entries in Table 31.4.

(a) suu = $\boxed{\Sigma^+}$

(b) $\overline{\mathrm{u}}\mathrm{d} = \boxed{\pi^-}$

(c) $\overline{\mathrm{s}}\mathrm{d} = \boxed{\mathrm{K}^0}$

(d) ssd = $\boxed{\Xi^-}$

***31.36** (a) $\overline{\mathrm{u}}\,\overline{\mathrm{u}}\,\overline{\mathrm{d}}$: charge $= \left(-\frac{2}{3}e\right)+\left(-\frac{2}{3}e\right)+\left(\frac{1}{3}e\right) = \boxed{-e}$. This is the $\boxed{\text{antiproton}}$.

(b) $\overline{\mathrm{u}}\,\overline{\mathrm{d}}\,\overline{\mathrm{d}}$: charge $= \left(-\frac{2}{3}e\right)+\left(\frac{1}{3}e\right)+\left(\frac{1}{3}e\right) = \boxed{0}$. This is the $\boxed{\text{antineutron}}$.

31.37 $v = HR$ (Equation 31.7) $H = \dfrac{\left(1.7\times10^{-2}\ \mathrm{m/s}\right)}{\mathrm{ly}}$

(a) $v\left(2.00\times10^{6}\ \mathrm{ly}\right) = 3.4\times10^{4}\ \mathrm{m/s}$ $\lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}} = 590(1.0001133) = \boxed{590.07\ \mathrm{nm}}$

(b) $v\left(2.00\times10^{8}\ \mathrm{ly}\right) = 3.4\times10^{6}\ \mathrm{m/s}$ $\lambda' = 590\sqrt{\dfrac{1+0.01133}{1-0.01133}} = \boxed{597\ \mathrm{nm}}$

(c) $v\left(2.00\times10^{9}\ \mathrm{ly}\right) = 3.4\times10^{7}\ \mathrm{m/s}$ $\lambda' = 590\sqrt{\dfrac{1+0.1133}{1-0.1133}} = \boxed{661\ \mathrm{nm}}$

31.38 (a) $\lambda_n' = \lambda_n\sqrt{\dfrac{1+v/c}{1-v/c}} = (Z+1)\lambda_n$ $\qquad \dfrac{1+v/c}{1-v/c} = (Z+1)^2$

$$1+\frac{v}{c} = (Z+1)^2 - \left(\frac{v}{c}\right)(Z+1)^2 \qquad \left(\frac{v}{c}\right)\left(Z^2+2Z+2\right) = Z^2+2Z$$

$$v = \boxed{c\left(\frac{Z^2+2Z}{Z^2+2Z+2}\right)}$$

(b) $R = \dfrac{v}{H} = \boxed{\dfrac{c}{H}\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)}$

***31.39** The density of the Universe is

$$\rho = 1.20\,\rho_c = 1.20\left(\frac{3H^2}{8\pi G}\right)$$

Consider a remote galaxy at distance r. The mass interior to the sphere below it is

$$M = \rho\left(\tfrac{4}{3}\pi r^3\right) = 1.20\left(\frac{3H^2}{8\pi G}\right)\left(\tfrac{4}{3}\pi r^3\right) = \frac{0.600\,H^2 r^3}{G}$$

both now and in the future when it has slowed to rest from its current speed $v = Hr$. The energy of this galaxy-sphere system is constant as the galaxy moves to apogee distance R:

$$\tfrac{1}{2}mv^2 - \frac{GmM}{r} = 0 - \frac{GmM}{R} \qquad \text{so} \qquad \tfrac{1}{2}mH^2r^2 - \frac{Gm}{r}\left(\frac{0.600\,H^2r^3}{G}\right) = 0 - \frac{Gm}{R}\left(\frac{0.600\,H^2r^3}{G}\right)$$

$$-0.100 = -0.600\frac{r}{R} \qquad \text{so} \qquad R = 6.00\,r$$

The Universe will expand by a factor of $\boxed{6.00}$ from its current dimensions.

***31.40** (a) $k_BT \cong 2m_pc^2$

so $$T \cong \frac{2m_pc^2}{k_B} = \frac{2(938.3\text{ MeV})}{\left(1.38\times10^{-23}\text{ J/K}\right)}\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right) \boxed{\sim 10^{13}\text{ K}}$$

(b) $k_BT \cong 2m_ec^2$

so $$T \cong \frac{2m_ec^2}{k_B} = \frac{2(0.511\text{ MeV})}{\left(1.38\times10^{-23}\text{ J/K}\right)}\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right) \boxed{\sim 10^{10}\text{ K}}$$

***31.41** (a) The Hubble constant is defined in $v = HR$. The distance R between any two far-separated objects opens at constant speed according to $R = vt$. Then the time t since the Big Bang is found from

$$v = Hvt \qquad 1 = Ht \qquad t = \frac{1}{H}$$

(b) $$\frac{1}{H} = \frac{1}{17\times10^{-3}\ \mathrm{m/s\cdot ly}}\left(\frac{3\times10^{8}\ \mathrm{m/s}}{1\ \mathrm{ly/yr}}\right) = \boxed{1.76\times10^{10}\ \mathrm{yr}} = 17.6 \text{ billion years}$$

***31.42** (a) Consider a sphere around us of radius R large compared to the size of galaxy clusters. If the matter M inside the sphere has the critical density, then a galaxy of mass m at the surface of the sphere is moving just at escape speed v according to

$$K + U_g = 0 \qquad \tfrac{1}{2}mv^2 - \frac{GMm}{R} = 0$$

The energy of the galaxy-sphere system is conserved, so this equation is true throughout the history of the Universe after the Big Bang, where $v = dR/dt$. Then

$$\left(\frac{dR}{dt}\right)^2 = \frac{2GM}{R} \qquad \frac{dR}{dt} = R^{-1/2}\sqrt{2GM} \qquad \int_0^R \sqrt{R}\,dR = \sqrt{2GM}\int_0^T dt$$

$$\left.\frac{R^{3/2}}{3/2}\right|_0^R = \sqrt{2GM}\,t\Big|_0^T \qquad \tfrac{2}{3}R^{3/2} = \sqrt{2GM}\,T$$

$$T = \tfrac{2}{3}\frac{R^{3/2}}{\sqrt{2GM}} = \tfrac{2}{3}\frac{R}{\sqrt{2GM/R}}$$

From above, $\sqrt{2GM/R} = v$

so $T = \tfrac{2}{3}\dfrac{R}{v}$

Now Hubble's law says $v = HR$

So $T = \tfrac{2}{3}\dfrac{R}{HR} = \dfrac{2}{3H}$

(b) $$T = \frac{2}{3\left(17\times10^{-3}\ \mathrm{m/s\cdot ly}\right)}\left(\frac{3\times10^{8}\ \mathrm{m/s}}{1\ \mathrm{ly/yr}}\right) = \boxed{1.18\times10^{10}\ \mathrm{yr}} = 11.8 \text{ billion years}$$

***31.43** In our frame of reference, Hubble's law is exemplified by $\mathbf{v}_1 = H\mathbf{R}_1$ and $\mathbf{v}_2 = H\mathbf{R}_2$. From these we may form the equations $-\mathbf{v}_1 = -H\mathbf{R}_1$ and $\mathbf{v}_2 - \mathbf{v}_1 = H(\mathbf{R}_2 - \mathbf{R}_1)$. These equations express Hubble's law as seen by the observer in the first galaxy cluster, as she looks at us to find $-\mathbf{v}_1 = H(-\mathbf{R}_1)$ and as she looks at cluster two to find $\mathbf{v}_2 - \mathbf{v}_1 = H(\mathbf{R}_2 - \mathbf{R}_1)$.

31.44 We find the number N of neutrinos:

$$10^{46}\text{ J} = N(6\text{ MeV}) = N\left(6\times 1.60\times 10^{-13}\text{ J}\right)$$

$$N = 1.0\times 10^{58}\text{ neutrinos}$$

The intensity at our location is

$$\frac{N}{A} = \frac{N}{4\pi r^2} = \frac{1.0\times 10^{58}}{4\pi\left(1.7\times 10^5\text{ ly}\right)^2}\left(\frac{1\text{ ly}}{\left(3.00\times 10^8\text{ m/s}\right)\left(3.16\times 10^7\text{ s}\right)}\right)^2 = 3.1\times 10^{14}\text{ m}^{-2}$$

The number passing through a body presenting $5000\text{ cm}^2 = 0.50\text{ m}^2$

is then $\left(3.1\times 10^{14}\ \frac{1}{\text{m}^2}\right)\left(0.50\text{ m}^2\right) = 1.5\times 10^{14}$

or $\boxed{\sim 10^{14}}$

***31.45** A photon travels the distance from the Large Magellanic Cloud to us in 170000 years. The hypothetical massive neutrino travels the same distance in 170000 years plus 10 seconds:

$$c(170000\text{ yr}) = v(170000\text{ yr} + 10\text{ s})$$

$$\frac{v}{c} = \frac{170000\text{ yr}}{170000\text{ yr} + 10\text{ s}} = \frac{1}{1+\dfrac{10\text{ s}}{\left(1.7\times 10^5\text{ yr}\right)\left(3.156\times 10^7\text{ s/yr}\right)}} = \frac{1}{1+1.86\times 10^{-12}}$$

For the neutrino we want to evaluate mc^2 in $E = \gamma mc^2$:

$$mc^2 = \frac{E}{\gamma} = E\sqrt{1-v^2/c^2} = 10\text{ MeV}\sqrt{1-\frac{1}{\left(1.86\times 10^{-12}\right)^2}} = 10\text{ MeV}\sqrt{\frac{\left(1+1.86\times 10^{-12}\right)^2-1}{\left(1+1.86\times 10^{-12}\right)^2}}$$

$$mc^2 \cong 10\text{ MeV}\sqrt{\frac{2\left(1.86\times 10^{-12}\right)}{1}} = 10\text{ MeV}\left(1.93\times 10^{-6}\right) = 19\text{ eV}$$

Then the upper limit on the mass is

$$m = \boxed{\frac{19\text{ eV}}{c^2}}$$

$$m = \frac{19\text{ eV}}{c^2}\left(\frac{\text{u}}{931.5\times 10^6\text{ eV}/c^2}\right) = 2.1\times 10^{-8}\text{ u}$$

31.46 (a) $\pi^- + \text{p} \to \Sigma^+ + \pi^0$ is forbidden by $\boxed{\text{charge conservation}}$

(b) $\mu^- \to \pi^- + \nu_e$ is forbidden by $\boxed{\text{energy conservation}}$

(c) $\text{p} \to \pi^+ + \pi^+ + \pi^-$ is forbidden by $\boxed{\text{baryon number conservation}}$

31.47 The total energy in neutrinos emitted per second by the Sun is:

$$(0.4)\left[4\pi\left(1.5\times10^{11}\right)^2\right] = 1.1\times10^{23}\text{ W}$$

Over 10^9 years, the Sun emits 3.6×10^{39} J in neutrinos. This represents an annihilated mass

$$mc^2 = 3.6\times10^{39}\text{ J}$$

$$m = 4.0\times10^{22}\text{ kg}$$

About $\boxed{\text{1 part in 50000000}}$ of the Sun's mass, over 10^9 years, has been lost to neutrinos.

31.48 $$\text{p} + \text{p} \to \text{p} + \pi^+ + X$$

We suppose the protons each have 70.4 MeV of kinetic energy. From conservation of momentum for the collision, particle X has zero momentum and thus zero kinetic energy. Conservation of system energy then requires

$$M_\text{p}c^2 + M_\pi c^2 + M_X c^2 = \left(M_\text{p}c^2 + K_\text{p}\right) + \left(M_\text{p}c^2 + K_\text{p}\right)$$

$$M_X c^2 = M_\text{p}c^2 + 2K_\text{p} - M_\pi c^2 = 938.3\text{ MeV} + 2(70.4\text{ MeV}) - 139.6\text{ MeV} = 939.5\text{ MeV}$$

X must be a neutral baryon of rest energy 939.5 MeV. Thus X is a $\boxed{\text{neutron}}$.

***31.49** $K = \frac{1}{2}mv^2$, so $$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(0.0400\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}{1.67\times10^{-27}\text{ kg}}} = 2.77\times10^3\text{ m/s}$$

The time for the trip is $$\Delta t = \frac{\Delta x}{v} = \frac{1.00\times10^4\text{ m}}{2.77\times10^3\text{ m/s}} = 3.61\text{ s}$$

The number of neutrons finishing the trip is given by $N = N_0 e^{-\lambda t}$.

The fraction decaying is $$1 - \frac{N}{N_0} = 1 - e^{-(\ln 2)t/T_{1/2}} = 1 - e^{-(\ln 2)(3.61\text{ s}/624\text{ s})} = 0.004\,00 = \boxed{0.400\%}$$

31.50 (a) $\Delta E \Delta t \cong \hbar$, and $\Delta t = \dfrac{r}{c} = \dfrac{1.4\times10^{-15}\ \text{m}}{3\times10^{8}\ \text{m/s}} = 4.7\times10^{-24}\ \text{s}$

$$\Delta E \cong \frac{\hbar}{\Delta t} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{4.7\times10^{-24}\ \text{s}} = \left(2.3\times10^{-11}\ \text{J}\right)\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) = 1.4\times10^{2}\ \text{MeV}$$

$$m = \frac{\Delta E}{c^2} \cong 1.4\times10^{2}\ \text{MeV}/c^2\ \boxed{\sim 10^{2}\ \text{MeV}/c^2}$$

(b) From Table 31.2, $m_\pi c^2 = 139.6\ \text{MeV}$, $\boxed{\text{a pi-meson}}$

31.51 $m_\Lambda c^2 = 1115.6\ \text{MeV}$ $\qquad \Lambda^0 \to \text{p} + \pi^-$

$m_p c^2 = 938.3\ \text{MeV}$ $\qquad m_\pi c^2 = 139.6\ \text{MeV}$

The difference between starting rest energy and final rest energy is the kinetic energy of the products.

$K_p + K_\pi = 37.7\ \text{MeV}$ and $p_p = p_\pi = p$

Applying conservation of relativistic energy to the decay process, we have

$$\left[\sqrt{(938.3)^2 + p^2c^2} - 938.3\right] + \left[\sqrt{(139.6)^2 + p^2c^2} - 139.6\right] = 37.7\ \text{MeV}$$

Solving the algebra yields

$$p_\pi c = p_p c = 100.4\ \text{MeV}$$

Then, $K_p = \sqrt{\left(m_p c^2\right)^2 + (100.4)^2} - m_p c^2 = \boxed{5.35\ \text{MeV}}$

$$K_\pi = \sqrt{(139.6)^2 + (100.4)^2} - 139.6 = \boxed{32.3\ \text{MeV}}$$

31.52 By relativistic energy conservation in the reaction, $E_\gamma + m_e c^2 = \dfrac{3m_e c^2}{\sqrt{1 - v^2/c^2}}$ (1)

By relativistic momentum conservation for the system, $\dfrac{E_\gamma}{c} = \dfrac{3m_e v}{\sqrt{1 - v^2/c^2}}$ (2)

Dividing (2) by (1), $X = \dfrac{E_\gamma}{E_\gamma + m_e c^2} = \dfrac{v}{c}$

Subtracting (2) from (1), $m_e c^2 = \dfrac{3m_e c^2}{\sqrt{1-X^2}} - \dfrac{3m_e c^2 X}{\sqrt{1-X^2}}$

Solving, $1 = \dfrac{3 - 3X}{\sqrt{1-X^2}}$ and $X = \dfrac{4}{5}$ so $E_\gamma = 4m_e c^2 = \boxed{2.04\ \text{MeV}}$

31.53 Momentum of proton is $qBr = \left(1.60\times10^{-19}\ \text{C}\right)\left(0.250\ \text{kg/C}\cdot\text{s}\right)\left(1.33\ \text{m}\right)$

$p_p = 5.32\times10^{-20}\ \text{kg}\cdot\text{m/s}$ $\quad cp_p = 1.60\times10^{-11}\ \text{kg}\cdot\text{m}^2/\text{s}^2 = 1.60\times10^{-11}\ \text{J} = 99.8\ \text{MeV}$

Therefore, $p_p = 99.8\ \text{MeV}/c$

The total energy of the proton is $E_p = \sqrt{E_0^2 + (cp)^2} = \sqrt{(938.3)^2 + (99.8)^2} = 944\ \text{MeV}$

For pion, the momentum qBr is the same (as it must be from conservation of momentum in a 2-particle decay).

$p_\pi = 99.8\ \text{MeV}/c$ $\quad E_{0\pi} = 139.6\ \text{MeV}$

$$E_\pi = \sqrt{E_0^2 + (cp)^2} = \sqrt{(139.6)^2 + (99.8)^2} = 172\ \text{MeV}$$

Thus, $E_{\text{total after}} = E_{\text{total before}} = \text{Rest energy}$

Rest Energy of unknown particle = 944 MeV + 172 MeV = 1116 MeV (This is a Λ^0 particle!)

Mass = $\boxed{1116\ \text{MeV}/c^2}$

31.54 $\Sigma^0 \to \Lambda^0 + \gamma$

From Table 31.2, $m_\Sigma = 1192.5\ \text{MeV}/c^2$ and $m_\Lambda = 1115.6\ \text{MeV}/c^2$

Conservation of energy in the decay requires

$$E_{0,\Sigma} = \left(E_{o,\Lambda} + K_\Lambda\right) + E_\gamma \quad \text{or} \quad 1192.5\ \text{MeV} = \left(1115.6\ \text{MeV} + \frac{{p_\Lambda}^2}{2m_\Lambda}\right) + E_\gamma$$

System momentum conservation gives $|p_\Lambda| = |p_\gamma|$, so the last result may be written as

$$1192.5\ \text{MeV} = \left(1115.6\ \text{MeV} + \frac{{p_\gamma}^2}{2m_\Lambda}\right) + E_\gamma$$

or

$$1192.5\ \text{MeV} = \left(1115.6\ \text{MeV} + \frac{{p_\gamma}^2c^2}{2m_\Lambda c^2}\right) + E_\gamma$$

Recognizing that $m_\Lambda c^2 = 1115.6\ \text{MeV}$ and $p_\gamma c = E_\gamma$,

we now have

$$1192.5\ \text{MeV} = 1115.6\ \text{MeV} + \frac{{E_\gamma}^2}{2(1115.6\ \text{MeV})} + E_\gamma$$

Solving this quadratic equation, $E_\gamma = \boxed{74.4\ \text{MeV}}$

31.55 $$p + p \rightarrow p + n + \pi^+$$

The total momentum is zero before the reaction. Thus, all three particles present after the reaction may be at rest and still conserve system momentum. This will be the case when the incident protons have minimum kinetic energy. Under these conditions, conservation of energy for the reaction gives

$$2\left(m_p c^2 + K_p\right) = m_p c^2 + m_n c^2 + m_\pi c^2$$

so the kinetic energy of each of the incident protons is

$$K_p = \frac{m_n c^2 + m_\pi c^2 - m_p c^2}{2} = \frac{(939.6 + 139.6 - 938.3)\text{ MeV}}{2} = \boxed{70.4\text{ MeV}}$$

31.56 $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$: From the conservation laws for the decay,

$$m_\pi c^2 = 139.5\text{ MeV} = E_\mu + E_\nu \qquad [1]$$

and $p_\mu = p_\nu$, $E_\nu = p_\nu c$: $E_\mu^2 = \left(p_\mu c\right)^2 + (105.7\text{ MeV})^2 = \left(p_\nu c\right)^2 + (105.7\text{ MeV})^2$

or $$E_\mu^2 - E_\nu^2 = (105.7\text{ MeV})^2 \qquad [2]$$

Since $$E_\mu + E_\nu = 139.5\text{ MeV} \qquad [1]$$

and $$\left(E_\mu + E_\nu\right)\left(E_\mu - E_\nu\right) = (105.7\text{ MeV})^2 \qquad [2]$$

then $$E_\mu - E_\nu = \frac{(105.7\text{ MeV})^2}{139.5\text{ MeV}} = 80.1 \qquad [3]$$

Subtracting [3] from [1], $2E_\nu = 59.4\text{ MeV}$ and $\boxed{E_\nu = 29.7\text{ MeV}}$

***31.57** The expression $e^{-E/k_B T}\,dE$ gives the fraction of the photons that have energy between E and $E + dE$. The fraction that have energy between E and infinity is

$$\frac{\int_E^\infty e^{-E/k_B T}\,dE}{\int_0^\infty e^{-E/k_B T}\,dE} = \frac{\int_E^\infty e^{-E/k_B T}\left(-dE/k_B T\right)}{\int_0^\infty e^{-E/k_B T}\left(-dE/k_B T\right)} = \frac{e^{-E/k_B T}\Big|_E^\infty}{e^{-E/k_B T}\Big|_0^\infty} = e^{-E/k_B T}$$

We require T when this fraction has a value of 0.0100 (i.e., 1.00%)

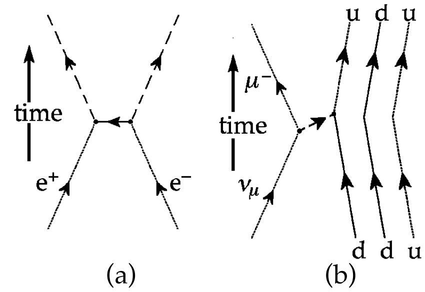
and $E = 1.00\text{ eV} = 1.60 \times 10^{-19}\text{ J}$

Thus, $0.0100 = e^{-\left(1.60\times10^{-19}\text{ J}\right)/\left(1.38\times10^{-23}\text{ J/K}\right)T}$

or $\ln(0.0100) = -\dfrac{1.60 \times 10^{-19}\text{ J}}{\left(1.38 \times 10^{-23}\text{ J/K}\right)T} = -\dfrac{1.16 \times 10^4\text{ K}}{T}$ giving $T = \boxed{2.52 \times 10^3\text{ K}}$

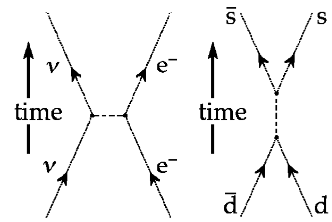
31.58 (a) This diagram represents the annihilation of an electron and an antielectron. From charge and lepton-number conservation at either vertex, the exchanged particle must be an electron, $\boxed{e^-}$.

(b) This is the tough one. A neutrino collides with a neutron, changing it into a proton with release of a muon. This is a weak interaction. The exchanged particle has charge $+e$ and is a $\boxed{W^+}$.



31.59 (a) The mediator of this weak interaction is a $\boxed{Z^0 \text{ boson}}$.

(b) The Feynman diagram shows a down quark and its antiparticle annihilating each other. They can produce a particle carrying energy, momentum, and angular momentum, but zero charge, zero baryon number, and, it may be, no color charge. In this case the product particle is a $\boxed{\text{photon}}$.



For conservation of both energy and momentum in the collision we would expect two photons; but momentum need not be strictly conserved, according to the uncertainty principle, if the photon travels a sufficiently short distance before producing another matter – antimatter pair of particles, as shown in Figure P31.59. Depending on the color charges of the d and $\bar{d}$ quarks, the ephemeral particle could also be a $\boxed{\text{gluon}}$, as suggested in the discussion of Figure 31.13(b).

ANSWERS TO EVEN NUMBERED PROBLEMS

2. $\sim 10^3$ Bq

4. 2.27×10^{23} Hz, 1.32×10^{-15} m

6. $\bar{\nu}_\mu$, ν_e

8. $\sim 10^{-23}$ s

10. (a) 0.782 MeV (b) $v_e = 0.919c$, $v_p = 380$ km/s
(c) The electron is relativistic, the proton is not.

12. $\bar{\nu}_\mu$

14. (a) See the solution (b) The second violates strangeness conservation

16. The second reaction does not conserve baryon number.

18. 0.828 c

20. (a) ν_e (b) ν_μ
(c) $\bar{\nu}_\mu$ (d) $\nu_\mu + \bar{\nu}_\tau$

22. The ρ^0 decays via the strong interaction; the $K_s{}^0$ must decay via the weak interaction.

24. (a) not allowed; violates conservation of baryon number (b) strong interaction
(c) weak interaction (d) weak interaction
(e) electromagnetic interaction

26. (a) K^+ (b) Ξ^0 (c) π^0

28. See the solution

30. See the solution

32. See the solution

34. a neutron, udd

36. (a) $-e$, antiproton (b) 0, antineutron

38. (a) $c\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)$ (b) $\dfrac{c}{H}\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)$

40. (a) $\sim 10^{13}$ K (b) $\sim 10^{10}$ K

42. (a) See the solution (b) 11.8 Gyr

44. $\sim 10^{14}$

46. (a) charge conservation (b) energy conservation
(c) baryon number conservation

48. a neutron

50. (a) $\sim 100\ \mathrm{MeV}/c^2$ (b) charged or neutral pion

52. 2.04 MeV

54. 74.4 MeV

56. 29.7 MeV

58. (a) Electron-positron annihilation; e^-
(b) A neutrino collides with a neutron; W^+