

$$\text{Ex. 17.3 b) } \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 3 & 3 & 3 \\ 1 & 3 & 6 & 6 & 6 \\ 1 & 3 & 6 & 10 & 10 \\ 1 & 3 & 6 & 10 & 15 \end{vmatrix} = 120 (= 5!)$$

We start with subtracting Row 1 from each of the other Rows.

$$\begin{array}{l} \\ \\ \text{(R2-R1), (R3-R1), (R4-R1), (R5-R1) Yields:} \\ \\ \end{array} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 & 2 \\ 0 & 2 & 5 & 5 & 5 \\ 0 & 2 & 5 & 9 & 9 \\ 0 & 2 & 5 & 9 & 14 \end{vmatrix}$$

Next we subtract Row 2 from Rows 3, 4, and 5.

$$\begin{array}{l} \\ \\ \text{(R3-R2), (R4-R2), (R5-R2) Yields:} \\ \\ \end{array} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 3 & 7 & 7 \\ 0 & 0 & 3 & 7 & 12 \end{vmatrix}$$

Next we subtract Row 3 from Rows 4 and 5

$$\begin{array}{l} \\ \\ \text{(R4-R3), (R5-R3) Yields:} \\ \\ \end{array} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 4 & 9 \end{vmatrix}$$

Next we subtract Row 4 from Row 5

$$\begin{array}{l} \\ \\ \text{(R5-R4) Yields:} \\ \\ \end{array} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 0 & 5 \end{vmatrix} = (1 \cdot 2 \cdot 3 \cdot 4 \cdot 5) = 120 (= 5!)$$

Ex. 17.4) Find each determinant by first reducing to triangular form:

$$d.) \begin{array}{l} \left| \begin{array}{ccccc} 3 & 2 & 3 & 0 & 8 \\ 2 & 3 & 5 & 3 & 1 \\ 1 & 1 & 1 & 2 & 3 \\ 1 & 2 & 4 & 1 & 1 \\ 2 & 0 & -1 & -1 & 4 \end{array} \right| \\ \left( \begin{array}{l} \text{Row1} - \text{Row2} \\ \text{Row4} - \text{Row3} \\ \text{Row5} - \text{Row2} \end{array} \right) \Rightarrow \left| \begin{array}{ccccc} 1 & -1 & -2 & -3 & 7 \\ 2 & 3 & 5 & 3 & 1 \\ 1 & 1 & 1 & 2 & 3 \\ 0 & 1 & 3 & -1 & -2 \\ 0 & -3 & -6 & -4 & 3 \end{array} \right| \dots \text{Next} \dots \end{array}$$

$$\left( \begin{array}{l} \text{Row1} + \text{Row4} \\ \text{Row2} + (-2) * \text{Row3} \\ \text{Row5} + (3) * \text{Row4} \end{array} \right) \Rightarrow \left| \begin{array}{ccccc} 1 & 0 & 1 & -4 & 5 \\ 0 & 1 & 3 & -1 & -7 \\ 1 & 1 & 1 & 2 & 3 \\ 0 & 1 & 3 & -1 & -2 \\ 0 & 0 & 3 & -7 & -3 \end{array} \right| \dots \text{Next} \dots$$

$$\left( \begin{array}{l} \text{Row2} + (-1) * \text{Row4} \\ \text{Row3} + (-1) * \text{Row1} \end{array} \right) \Rightarrow \left| \begin{array}{ccccc} 1 & 0 & 1 & -4 & 5 \\ 0 & 0 & 0 & 0 & -5 \\ 0 & 1 & 0 & 6 & -2 \\ 0 & 0 & 3 & -7 & 0 \\ 0 & 0 & 3 & -7 & -3 \end{array} \right| \dots \text{Next} \dots$$

$$\left( \begin{array}{l} \text{Switch Row2 with Row3} \\ \text{Therefore } * (-1) \\ \text{Row5} + (-1) \text{Row4} \end{array} \right) (-1) * \left| \begin{array}{ccccc} 1 & 0 & 1 & -4 & 5 \\ 0 & 1 & 0 & 6 & -2 \\ 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 3 & -7 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{array} \right| \dots \text{Next} \dots$$

$$\left( \text{Switch Row3 with Row2} \therefore (-1) * (-1) \right) \Rightarrow (+1) * \left| \begin{array}{ccccc} 1 & 0 & 1 & -4 & 5 \\ 0 & 1 & 0 & 6 & -2 \\ 0 & 0 & 3 & -7 & 0 \\ 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 & -3 \end{array} \right| = (1 * 1 * 3 * 0 * (-3)) = 0$$