Ex 21.1 Determine whether each is a basis for the given space:

c.) $\left\{\begin{bmatrix}1\\2\\3\end{bmatrix},\begin{bmatrix}-2\\0\\1\end{bmatrix},\begin{bmatrix}1\\6\\10\end{bmatrix}\right\}$ **for** $\Re^3$ **Since** $\Re^3$ **then dim** ≤ 3, $\begin{vmatrix}1 & -2 & 1\\2 & 0 & 6\\3 & 1 & 10\end{vmatrix} = 0 \therefore$ **L.D.**

Columns are Linearly Dependent and not a basis for $\Re^3$**.**

d.) $\left\{\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}4\\2\\2\end{bmatrix}\right\}$ **for V = Sp** $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

$$A_{col} = \begin{bmatrix}1 & 4\\1 & 2\\1 & 2\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 0\\1 & -2\\1 & -2\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 0\\1 & 1\\1 & 1\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 0\\0 & 1\\0 & 1\end{bmatrix}$$ **Therefore, the original vectors**

$\left\{\begin{bmatrix}1\\1\\1\end{bmatrix},\begin{bmatrix}4\\2\\2\end{bmatrix}\right\}$ **are L. I. and a basis for** $A_{col} = Sp\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

Ex 21.2 Find a basis for V = Sp $\left\{\begin{bmatrix}1\\2\\2\end{bmatrix},\begin{bmatrix}3\\2\\1\end{bmatrix},\begin{bmatrix}11\\10\\7\end{bmatrix},\begin{bmatrix}7\\6\\4\end{bmatrix}\right\}$

Since $\Re^3$ **then dim** ≤ 3. $A_{col} = \begin{bmatrix}1 & 3 & 11 & 7\\2 & 2 & 10 & 6\\2 & 1 & 7 & 4\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 3 & 0 & 0\\2 & 2 & 0 & 0\\2 & 1 & 0 & 0\end{bmatrix}$

$\textit{Therefore}: Sp = \left\{\begin{bmatrix}1\\0\\-1/2\end{bmatrix},\begin{bmatrix}0\\1\\5/4\end{bmatrix}\right\}$ **will be the basis.**

<u>Ex. 21.3</u> Find the dimension of each vector space:**

a.) $Sp\left\{\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}2\\3\end{bmatrix},\begin{bmatrix}5\\8\end{bmatrix},\begin{bmatrix}4\\4\end{bmatrix}\right\}$ **Since this is an** $\Re^2$**, dim** ≤ 2. $A_{col} = \begin{bmatrix}1 & 2 & 5 & 4\\1 & 3 & 8 & 4\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\end{bmatrix}$ **Therefore,** $\left\{\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}2\\3\end{bmatrix}\right\}$ **are the basis.** $\begin{vmatrix}1 & 2\\1 & 3\end{vmatrix} = 1 \therefore L.I.$ **and dim = 2**

b.) $\left\{ \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix} \in \Re^4 : \text{x1} = 0 \text{ and x3} = \text{x4} \right\}.$

Since $\Re^4$**, dim** ≤ 4**. The only possible sets are:** $Sp = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\} \therefore L.I.$

Therefore, dim = 2.

Ex. 21.4 Give a basis for each space:

a.) $\left\{ \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} \in \Re^3 : \text{X1} + \text{X2} = 0 \right\}$

The Possibilities are:
If X1 = 0, then X2 = 0
If X1=1, then X2=-1
If X1= -1, then X2= -1
Also we need to consider if X3 is 0 or 1.

$\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ **but the last 2 are really the same, so we only use one of them.**

Since $\Re^3$ **then dim** ≤ 3**.** $Sp\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}$ **is the basis and L.I.**

****<u>Ex 21.5</u> Find a basis for** $\Re^3$ **containing** $\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} and \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}.$

$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 3 & 0 & 1 \end{vmatrix} = 1 \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} = 4 \neq 0 \therefore L.I.$ **Therefore,** $\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ **is the basis for** $\Re^3$**.**

Ex. 22. 1 Find bases for the row and column spaces of A= $\begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 2 & 2 & 1 \\ 3 & 1 & 4 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ by finding A_{row} and A_{col}. Also, verify that dimR(A) = dimC(A).

$$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 2 & 2 & 1 \\ 3 & 1 & 4 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{pmatrix} R2+(-2)*R1 \\ R3+(-3)*R1 \\ R4+(-1)*R1 \end{pmatrix} \Rightarrow \begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & -4 & 2 & -1 \\ 0 & -8 & 4 & -2 \\ 0 & -2 & 1 & 0 \end{bmatrix} \Rightarrow \begin{pmatrix} R2+(-2)*R4 \\ R3+(-4)*R4 \end{pmatrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -2 \\ 0 & 1 & -1/2 & 0 \end{bmatrix} \Rightarrow \begin{pmatrix} R1+(-3)*R4 \\ Switch \\ Rows \end{pmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 3/2 & 1 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = A_{row}$$

Basis for R(A)= $\begin{bmatrix} 1 & 0 & 3/2 & 1 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ **Therefore, these 3 rows dimR(A)= 3**

/**/

$$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 2 & 2 & 1 \\ 3 & 1 & 4 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{pmatrix} C1+(-1)*C4 \\ C2+(-3)*R4 \\ \end{pmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & -1 & 2 & 1 \\ 2 & -2 & 4 & 1 \\ 0 & -2 & 1 & 1 \end{bmatrix} \Rightarrow \begin{pmatrix} C2+C1 \\ C3+(-2)*C1 \end{pmatrix} \Rightarrow$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 0 & -2 & 1 & 1 \end{bmatrix} \Rightarrow \begin{pmatrix} C4+(-1)*C1 \\ C2+(2)*C3 \\ C4+(-1)*C3 \end{pmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{pmatrix} C4+(-1)*C1 \\ \end{pmatrix} \Rightarrow$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{pmatrix} Switch \\ Columns \end{pmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = A_{col}$$

Basis for C(A)= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **dimC(A)=3 Therefore: dimR(A)= dimC(A)**

****Ex 22.4) Find a canonical form for each of the matrices for Exercise 22.1 and 22.2 (take advantages of the work that you did to answer those questions.) Is either of these canonical forms unique?**

Ex22.1

$$\begin{bmatrix} 1 & 0 & 3/2 & 1 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow (C4 - C1) \Rightarrow \begin{bmatrix} 1 & 0 & 3/2 & 0 \\ 0 & 1 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{pmatrix} \textit{Switch} \\ \textit{Columns} \end{pmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 3/2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The first 3 columns are I and the last column is unique.

Ex22.2 $\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & -4 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ **the first 2 columns are I and the last 2 columns are not unique.**

Ex 22.8) Determine the rank of each matrix. Try to do this with minimum effort and maximum smarts.

b.) $A = \begin{bmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & 2 & 4 \\ 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix}$

Note: rows 1 & 3 are the same; also rows 2 & 4 are the same. Thus, R(A)=2
