

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \quad C(A) = |A - \lambda I| = \begin{vmatrix} 1 - \lambda & 2 \\ 3 & 2 - \lambda \end{vmatrix} = (1 - \lambda)(2 - \lambda) - 6$$

$$= 2 - 3\lambda + \lambda^2 - 6 = \lambda^2 - 3\lambda - 4 = (\lambda - 4)(\lambda + 1) \quad \text{Thus } \lambda = 4, -1 \quad \text{or } \lambda_1 = 4, \lambda_2 = -1$$



A, P non-singular matrix

A and PAP^{-1} have the same Eigenvalues. A and B are similar if $A = PBP^{-1}$

1. **Similar matrices have the same Characteristic Polynomials**
2. **Let λ have and Eigenvalue of A and B if A and B are similar, the $\dim(N(A - \lambda I)) = \dim(N(B - \lambda I))$.**

PROOF:

$\{V_1, \dots, V_k\}$ is a basis of $N(A - \lambda I)$ $\dim N(A - \lambda I) = K$

There is a P such that $A = PBP^{-1}$.

Since P is non-singular, PV_1, \dots, PV_k are Linearly Independent.

Then $PV_1, \dots, PV_k \in N(B - \lambda I)$

$$(B - \lambda I)PV_j = BPV_j - \lambda PV_j$$

Remember – from earlier-
 $A = PBP^{-1}$
 $B = P^{-1}AP$
 $AP = PB$

$$\dim N(A - \lambda I) \leq \dim N(B - \lambda I) \quad \text{and} \quad \dim N(B - \lambda I) \leq \dim N(A - \lambda I)$$

Therefore: They are equal.