$A = \text{diag}(\lambda_1, \ldots, \lambda_n)$ $\quad\quad$ $\text{diag}(1, 2, 4) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$A^2 = \text{diag}(\lambda_1, \ldots, \lambda_n)^2$ $\quad\quad$ $\text{diag}(1, 2, 4)^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 16 \end{bmatrix}$ $\quad\quad$ $A^m = \text{diag}\ (\lambda_1{}^m, \ldots, \lambda_n{}^m)$

$A = PBP^{-1}$ (Where B is the $\text{diag}(\lambda_1, \ldots, \lambda_n)$)

$A^m = \text{diag}\ (\lambda_1{}^m, \ldots, \lambda_n{}^m) = PBP^{-1}$ $\quad\quad$ $PBP^{-1}, \ldots, PBP^{-1}, PBP^{-1} = PB^m P^{-1}$

$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 3 \end{bmatrix}$ Find A^{100} $\quad\quad$ $A^{100} = P^{-1} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{100} & 0 \\ 0 & 0 & 3^{100} \end{bmatrix} * P$

We already found P for $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 3 \end{bmatrix}$ in section 28.... $P = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$$A^{100} = \begin{bmatrix} 0 & -1 & 0 \\ 2 & 2 & 0 \\ -3 & -3 & 3 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{100} & 0 \\ 0 & 0 & 3^{100} \end{bmatrix} * \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

C polynomial $\quad$ C(A) $\quad$ $C(x) = x^2 - 2x + 5$ $\quad$ $C(A) = A^2 - 2A + 5I$

$C\ (\text{diag}(\lambda_1, \ldots, \lambda_n)) = \text{diag}(C(\lambda_1), C(\lambda_2), \ldots, C(\lambda_n))$

$C\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$ because $C(x) = x^2 - 2x + 5$ so $1^2 - 2(1) + 5 = 4$ and $3^2 - 2(3) + 5 = 8$

$\alpha \mathrm{diag}(\lambda_1,\ldots,\lambda_n) = \mathrm{diag}(\alpha\lambda_1,\ldots,\alpha\lambda_n)$

$\mathrm{diag}(\lambda_1,\ldots,\lambda_n) + \mathrm{diag}(G_1,\ldots,G_n) = \mathrm{diag}(\lambda_1+G_1,\ldots,\lambda_n+G_n)$

$\mathrm{diag}(\lambda_1,\ldots,\lambda_n)^m = \mathrm{diag}(\lambda_1{}^m,\ldots,\lambda_n{}^m)$

$A = PBP^{-1}$, **Bdiag,** $C(x) = \sum_{k=0}^{m} d_k X^k$, $C(A) = \sum_{k=0}^{m} \alpha_k A^k = C(A) = \sum_{k=0}^{m} \alpha_k A^k$

$= \sum_{k=0}^{m} P^{-1}\alpha_k B^k P = P^{-1}(\sum_{k=0}^{m} \alpha_k B^k)P = P^{-1}C(B)P$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $C(A) = A^2 - 2A + 5I$ yields $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

Therefore $C(A) = P^{-1} * \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 8 \end{bmatrix} * P$

Cayley-Hamilton Theorem (Page 199 Section 31)

Let A be a square matrix and let C(A) be the characteristic polynomial of A then C(A)= [0]

PROOF: For diagonalizable matrices:

$C(A)=C(P^{-1}BP) = P^{-1}C(B)P = P^{-1}\,\mathrm{diag}(C(\lambda_1),C(\lambda_2),\ldots,C(\lambda_n))P = P^{-1}[0]P = [0]$

Normed Space V vector space v ∈ V ||V|| ∈ ℜ "Norm of V"

1. $\| V \| = 0 \Leftrightarrow V = \vec{0}$
2. $\| \lambda V \| = |\lambda| * \| V \|$
3. $\| V + W \| \le \| V \| + \| W \|$