CORRECTIONS TO SOLUTIONS MANUAL

In the new edition, some chapter problems have been reordered and equations and figure references have changed. The solutions manual is based on the preview edition and therefore must be corrected to apply to the new edition. Below is a list reflecting those changes.

The "NEW" column contains the problem numbers in the new edition. If that problem was originally under another number in the preview edition, that number will be listed in the "PREVIEW" column on the same line. In addition, if a reference used in that problem has changed, that change will be noted under the problem number in quotes. Chapters and problems not listed are unchanged.

For example:

NEW	PREVIEW
4.18	4.5
"Fig. 4.38"	"Fig. 4.35"
"Fig. 4.39"	"Fig. 4.36"

The above means that problem 4.18 in the new edition was problem 4.5 in the preview edition. To find its solution, look up problem 4.5 in the solutions manual. Also, the problem 4.5 solution referred to "Fig. 4.35" and "Fig. 4.36" and should now be "Fig. 4.38" and "Fig. 4.39," respectively.

CHAPTER 3

NEW	PREVIEW
3.1	3.8
3.2	3.9
3.3	3.11
3.4	3.12
3.5	3.13
3.6	3.14
3.7	3.15
"From 3.6"	"From 3.14"
3.8	3.16
3.9	3.17
3.10	3.18
3.11	3.19
3.12	3.20
3.13	3.21
3.14	3.22
3.15	3.1

3.16	3.2
3.17	3.2’
3.18	3.3
3.19	3.4
3.20	3.5
3.21	3.6
3.22	3.7
3.23	3.10
3.24	3.23
3.25	3.24
3.26	3.25
3.27	3.26
3.28	3.27
3.29	3.28

CHAPTER 4

NEW	PREVIEW
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4.1	4.12
4.2	4.13
4.3	4.14
4.4	4.15
4.5	4.16
4.6	4.17
4.7	4.18
“p. 4.6”	“p. 4.17”
4.8	4.19
4.9	4.20
4.10	4.21
4.11	4.22
4.12	4.23
4.13	4.24
“p. 4.9”	“p. 4.20”
4.14	4.1
“(4.52)”	“(4.51)”
“(4.53)”	“(4.52)”
4.15	4.2
4.16	4.3
4.17	4.4
4.18	4.5
“Fig. 4.38”	“Fig. 4.35”
“Fig. 4.39”	“Fig. 4.36”
4.19	4.6
“Fig 4.39(c)”	“Fig 4.36(c)”

4.20	4.7
4.21	4.8
4.22	4.9
4.23	4.10
4.24	4.11
4.25	4.25
4.26	4.26
“p. 4.9”	“p. 4.20”

CHAPTER 5

NEW	PREVIEW
-----	---------
5.1	5.16
5.2	5.17
5.3	5.18
5.4	5.19
5.5	5.20
5.6	5.21
5.7	5.22
5.8	5.23
5.9	5.1
5.10	5.2
5.11	5.3
5.12	5.4
5.13	5.5
5.14	5.6
5.15	5.7
5.16	5.8
5.17	5.9
5.18	5.10
“Similar to 5.18(a)”	“Similar to 5.10(a)”
5.19	5.11
5.20	5.12
5.21	5.13
5.22	5.14
5.23	5.15

CHAPTER 6

NEW	PREVIEW
-----	---------
6.1	6.7
6.2	6.8

6.3	6.9
“from eq(6.23)”	“from eq(6.20)”
6.4	6.10
6.5	6.11
“eq (6.52)”	“eq (6.49)”
6.6	6.1
6.7	6.2
6.8	6.3
6.9	6.4
6.10	6.5
6.11	6.6
6.13	6.13
“eq (6.56)”	“eq (6.53)”
“problem 3”	“problem 9”
6.16	6.16
“to (6.23) & (6.80)”	“to (6.20) & (6.76)”
6.17	6.17
“equation (6.23)”	“equation (6.20)”

CHAPTER 7

NEW	PREVIEW
7.2	7.2
“eqn. (7.59)”	“eqn. (7.57)”
7.17	7.17
“eqn. (7.59)”	“eqn. (7.57)
7.19	7.19
“eqns 7.66 and 7.67”	“eqns 7.60 and 7.61”
7.21	7.21
“eqn. 7.66”	“eqn. 7.60”
7.22	7.22
“eqns 7.70 and 7.71”	“eqns. 7.64 and 7.65”
7.23	7.23
“eqn. 7.71”	“eqn. 7.65”
7.24	7.24
“eqn 7.79”	“eqn 7.73”

CHAPTER 8

NEW	PREVIEW
8.1	8.5
8.2	8.6

8.3	8.7
8.4	8.8
8.5	8.9
8.6	8.10
8.7	8.11
8.8	8.1
8.9	8.2
8.10	8.3
8.11	8.4
8.13	8.13
“problem 8.5”	“problem 8.9”

CHAPTER 13

NEW	PREVIEW
-----	---------
3.17	3.17
“Eq. (3.123)”	“Eq. (3.119)”

CHAPTER 14 - New Chapter, “Oscillators”

CHAPTER 15 - New Chapter, “Phase-Locked Loops”

CHAPTER 16 - Was Chapter 14 in Preview Ed.

Change all chapter references in solutions manual from 14 to 16.

CHAPTER 17 - Was Chapter 15 in Preview Ed.

Change all chapter references in solutions manual from 15 to 17.

CHAPTER 18 - Was Chapter 16 in Preview Ed.

NEW	PREVIEW
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18.3	16.3
“Fig. 18.12(c)”	“Fig. 16.13(c)”
18.8	16.8
“Fig. 18.33(a,b,c,d)”	“Fig. 16.34(a,b,c,d)”

Also, change all chapter references from 16 to 18.

Chapter 14
Oscillators

14.1 Open-Loop Transfer Function:

$$H(s) = \frac{-(g_m R_D)^2}{(1+\frac{s}{\omega_0})^2}, \quad \omega_0 = \frac{1}{R_D C_L}$$

The gain drops to unity at $\frac{g_m R_D}{(1+\frac{\omega_u^2}{\omega_0^2})^{1/2}} = 1$, which for $g_m R_D >> 1$, yields, $\omega_u >> \omega_0$ and $\omega_u \cong \omega_0 \cdot g_m R_D = \frac{g_m}{C_L}$. The phase changes from $-180°$ at $\omega \approx 0$ to $-2\tan^{-1}\frac{\omega_u}{\omega_0} - 180°$ at ω_u; i.e., the phase change at ω_u is $-2\tan^{-1}(g_m R_D)$ and the phase margin is equal to $180° - 2\tan^{-1}(g_m R_D)$.

14.2 (a) $g_m R_D \geq 2 \Rightarrow R_D \geq 400\,\Omega$.

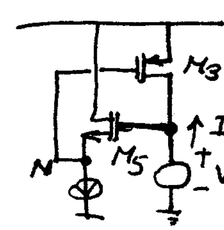
(b) $\begin{cases} \omega_{osc} = \sqrt{3}\,\omega_0 = \sqrt{3}/(R_D C_L) \\ \text{Total Gain} = (g_m R_D)^3 = 16 \Rightarrow R_D = 504\,\Omega \end{cases} \Rightarrow C_L = 0.547\,pF$

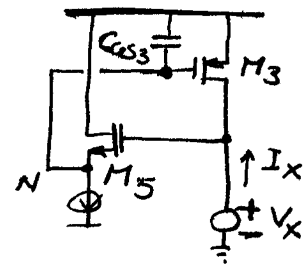
14.3 Each stage must provide a small-signal gain of 2. That is, $g_{m1} R_1 = 2$. With small swings, each transistor carries half of the tail current. For square-law devices, therefore, we have

$$g_{m1} R_1 = 2 = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}\, R_1 = 2 \Rightarrow$$

$$I_{SS} \geq \frac{4}{\mu_n C_{ox} \frac{W}{L} R_1^2}$$

14.4 Neglecting body effect of M_5, we have $V_N \approx V_X$. Thus, the gate and drain of M_3 experience equal voltage variations. That is, M_3 operates as a diode-connected device, providing an impedance of $1/g_{m3}$.





14.5 $\frac{V_N}{V_X} = \frac{\frac{1}{C_{GS3}s}}{\frac{1}{C_{GS3}s} + \frac{1}{g_{m5}}}$ $(\gamma = \lambda = 0)$

$$= \frac{g_{m5}}{g_{m5} + C_{GS3}s} \Rightarrow \frac{I_X}{V_X} = \frac{g_{m3}g_{m5}}{g_{m5} + C_{GS3}s}$$

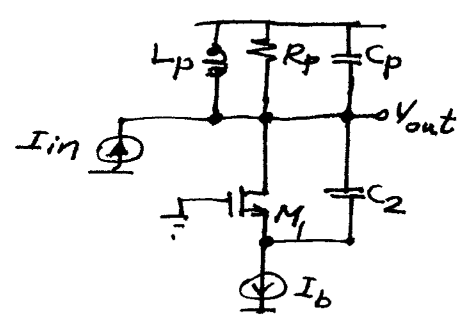
$\Rightarrow \frac{V_X}{I_X} = \frac{1}{g_{m3}} + \frac{C_{GS3}}{g_{m3}g_{m5}}s$ $\Rightarrow$ The impedance is always inductive.

14.6 To avoid latchup, $g_m R_S < 1 \Rightarrow R_S < \frac{1}{g_m}$.

14.7 The drain currents saturate near I_{SS} and 0 for a short while, creating a "squarish" waveform. The output voltages are the result of injecting the currents into the tanks. Since the tanks provide suppression at higher harmonics, V_X and V_Y are filtered versions of I_{D1} and I_{D2}.

14.8 For the circuit to oscillate, the loop gain must exceed unity: $g_m R_P > 1 \Rightarrow g_m > 1/R_P$. For square-law devices, $\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}} > \frac{1}{R_P}$. Thus, $I_{SS} > \frac{1}{\mu_n C_{ox}\frac{W}{L}R_P^2}$.

For M_1 and M_2 not to enter the triode region, the maximum value of V_X and the minimum value of V_Y must differ by no more than V_{TH}. That is, the peak-to-peak swing at X or Y must be less than V_{TH}. Since the peak-to-peak swing is $\approx I_{SS}R_P$, we must have $I_{SS}R_P < V_{TH}$.



14.9 Since the total current flowing thru M_1 and C_2 is equal to I_b, a constant value.

Thus, $\frac{V_{out}}{I_{in}} = (L_P s) \parallel R_P \parallel \frac{1}{C_P s}$.

14.10 Replace R_p with $R_p \| \frac{1}{C_p s} = \frac{R_p}{R_p C_p s + 1}$ in Eq. (14.40). The denominator then reduces to:

$$R_p C_1 C_2 L_p s^3 + (C_1 + C_2) L_p R_p C_p s^3 + (C_1 + C_2) L_p s^2 + [g_m L_p R_p C_p s + g_m L_p + R_p (C_1 + C_2)] s + g_m R_p$$

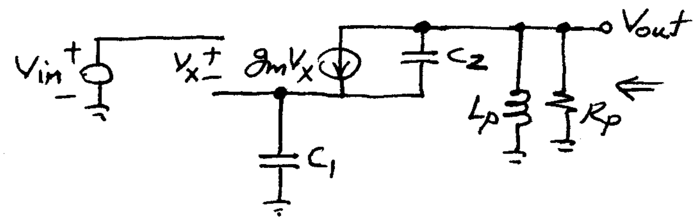
Grouping the imaginary terms and equating their sum to zero, we have

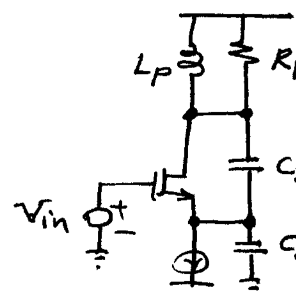
$$-R_p L_p \omega^3 [C_1 C_2 + (C_1 + C_2) C_p] + [g_m L_p + R_p (C_1 + C_2)] \omega = 0$$

Assuming $g_m L_p \ll R_p (C_1 + C_2)$, we obtain

$$\omega^2 = \frac{1}{L_p \left(\frac{C_1 C_2}{C_1 + C_2} + C_p \right)}.$$

14.11





The current thru $R_p \| (L_p s)$ is equal to $V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)$. The negative of this current flows thru C_1, generating a voltage $-V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}$ across it. Thus, $V_X = V_{in} + V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}$. Also, the current thru C_2 is equal to $\left[V_{out} + V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}\right] C_2 s$.

Adding $g_m V_X$ and the current thru C_2 and equating the result to $-V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)$, we have

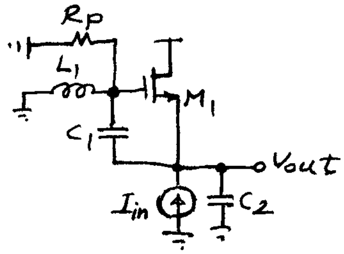
$$\left[V_{in} + V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}\right] g_{m1} + \left[V_{out} + V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}\right] C_2 s = -V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right).$$

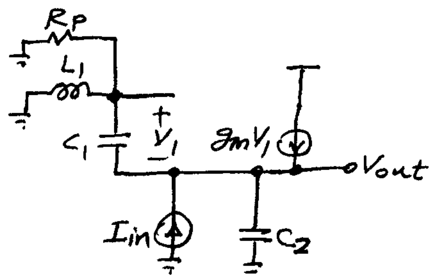
It follows that

$$\frac{V_{out}}{V_{in}} = \frac{-g_m L_p R_p C_1 s^2}{R_p L_p C_2 C_1 s^3 + L_p (C_1 + C_2) s^2 + [g_{m1} L_p + R_p (C_1 + C_2)] s + g_{m1} R_p}.$$

Note that the denomintor is the same as in Eq. (14.40).

14.12





$$V_1 = -(I_{in} - V_{out} C_2 s + g_m V_1)/C_1 s \Rightarrow V_1 (1 + g_m / C_1 s) = \frac{-I_{in} + V_{out} C_2 s}{C_1 s}$$

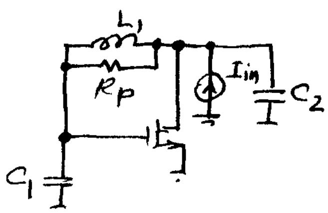
$$\Rightarrow V_1 = \frac{-I_{in} + V_{out} C_2 s}{g_m + C_1 s}$$

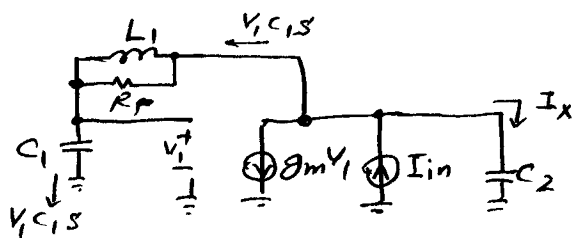
writing a KVL, we have $\quad -V_1 C_1 s \dfrac{R_p L_1 s}{R_p + L_1 s} = V_1 + V_{out}$.

It follows that

$$V_{out} = -\frac{I_{in} + V_{out} C_2 s}{g_m + C_1 s}\left[1 + \frac{C_1 s R_p L_1 s}{R_p + L_1 s}\right].$$

Simplifying and calculating the denominator of V_{out}/I_{in}, we have $R_p L_1 C_1 C_2 s^3 + L_1 (C_1 + C_2) s^2 + [R_p (C_1 + C_2) + g_m L_1] s + g_m R_p$, which is the same as Eq. (14.40). Thus, the oscillation conditions are the same as those of colpitts oscillator.





We can consider V_1 as the output because for oscillation to begin the gain from I_{in} to V_1 must be infinite as well. First, assume $R_p = \infty$:

$$I_X = +V_1 C_1 s \left(L_1 s + \frac{1}{C_1 s}\right) C_2 s = -g_m V_1 + I_{in} - V_1 C_1 s$$

$$\Rightarrow V_1 \left[C_1 C_2 s^2 \left(L_1 s + \frac{1}{C_1 s}\right) + g_m + C_1 s\right] = I_{in}$$

Now, include R_p: $\quad V_1 \left[C_1 C_2 s^2 \left(\dfrac{R_p L_1 s}{R_p + L_1 s} + \dfrac{1}{C_1 s}\right) + g_m + C_1 s\right] = I_{in}$

$$\Rightarrow V_1 \left[\frac{C_1 C_2 s^2 (R_p C_1 L_1 s^2 + R_p + L_1 s) + (g_m + C_1 s)(C_1 s)(R_p + L_1 s)}{C_1 s (R_p + L_1 s)}\right] = I_{in}$$

$\Rightarrow$ denominator of V_1 / I_{in} is $\qquad$ ($C_1 s$ is factored from numerator & denominator.)

$$R_p C_1 C_2 L_1 s^3 + R_p C_2 s + L_1 C_2 s^2 + g_m R_p + g_m L_1 s + C_1 R_p s + C_1 L_1 s^2$$

$$= R_p C_1 C_2 L_1 s^3 + L_1 (C_1 + C_2) s^2 + [R_p (C_1 + C_2) + g_m L_1] s + g_m R_p,$$

the same as that in Eq. (14.40).

14.13 $I_T = 1mA$, $(\frac{W}{L})_{1,2} = 50/0.5$

(a) For a three-stage ring, the minimum gain per stage at low freqs must be 2. Thus, $g_{m1,2} R_{1,2} = 2$ (when no current flows thru M_3 and M_4). $\Rightarrow R_{1,2} = 2/g_{m1,2}$. ($g_{m1,2} = \sqrt{\mu_n C_{ox} (\frac{W}{L})_{1,2} I_T}$.)

(b) $g_{m3,4} R = 0.5$ with $I_{D3,4} = 0.5$ mA.

$$g_{m3,4} = \sqrt{\mu_n C_{ox} (\frac{W}{L})_{3,4} I_T} = \underbrace{g_{m1,2}}_{=\frac{2}{R}} \sqrt{\frac{(W/L)_{3,4}}{(W/L)_{1,2}}}$$

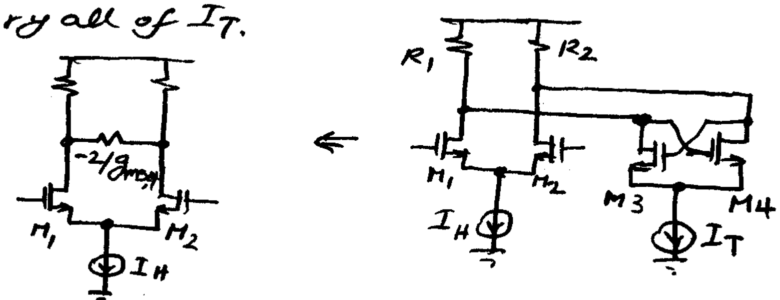
$$\Rightarrow \frac{2}{R} \sqrt{\frac{(W/L)_{3,4}}{(W/L)_{1,2}}} R = 0.5$$

$$\Rightarrow (W/L)_{3,4} = 0.25^2 (W/L)_{1,2}.$$

(c) The voltage gain must be equal to 2 with a diff pair tail current of I_H while M_3 and M_4 carry all of I_T.

$$|A_v| = g_{m1,2} (R_{1,2} \| \frac{-1}{g_{m3,4}})$$

$$= g_{m1,2} \frac{R_{1,2}}{1 - g_{m3,4} R_{1,2}}$$



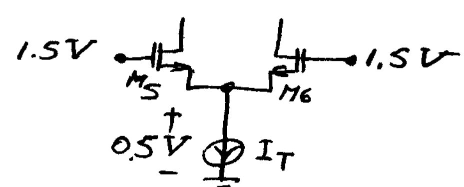
If $g_{m3,4} R_{1,2} < 1$ (to avoid latch-up), then

$$g_{m1,2} R_{1,2} > 2(1 - g_{m3,4} R_{1,2})$$

$$\Rightarrow \sqrt{2 \frac{I_H}{2} \mu_n C_{ox} (\frac{W}{L})_{1,2}} R_{1,2} > 2(1 - \sqrt{2 \frac{I_T}{2} \mu_n C_{ox} (\frac{W}{L})_{3,4}} R_{1,2})$$

Thus, I_H can be determined.

(d) Neglecting body effect for simplicity, we have



$$\frac{I_T}{2} = \frac{1}{2} \mu_n C_{ox} (\frac{W}{L})_{5,6} (V_{GS5,6} - V_{TH5,6})^2$$

$$\Rightarrow (\frac{W}{L})_{5,6} = \frac{I_T}{\mu_n C_{ox} (V_{GS5} - V_{TH5,6})^2}$$ and $V_{GS5,6} + 0.5V = 1.5V$.

14.14 If each inductor contributes a cap of C_1, then

$$f_{osc,min} = \frac{1}{2\pi\sqrt{L(C_0+C_1)}}, \quad f_{osc,max} = \frac{1}{2\pi\sqrt{L(0.62C_0+C_1)}}$$

Thus, the tuning range is given by $\frac{f_{osc,max}}{f_{osc,min}} = \sqrt{\frac{C_0+C_1}{0.62C_0+C_1}}$,
which is less than 27%. For example, if $C_1 = 0.2C_0$, then,
$f_{osc,max}/f_{osc,min} \cong 1.21$.

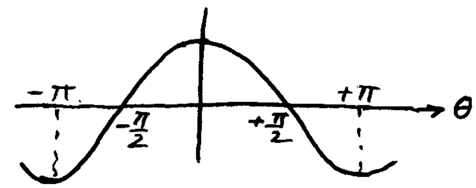
14.15 (a) $L_p = 5$ nH, $C_X = 0.5$ pF $f_{osc} = 1\text{ GHz} = \frac{1}{2\pi\sqrt{5\text{nH}\times(C_X+C_D)}}$
$\Rightarrow C_D = 4.566$ pF.

(b) $Q = \frac{L\omega}{R_p} = 4 \Rightarrow R_p = 125.7\ \Omega \Rightarrow$
With a 1-mA tail current, the peak-to-peak swing on each side is approximately equal to 126 mV.

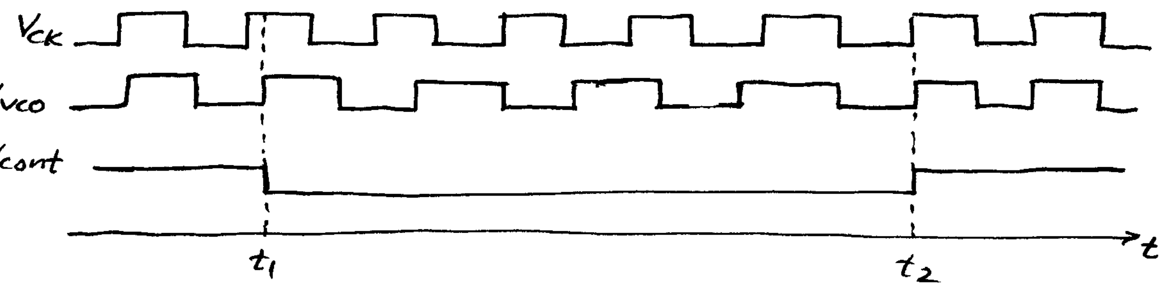
Chapter 15
Phase-Locked Loops

15.1 With two signals $V_1 \cos\omega t$ and $V_2 \cos(\omega t + \theta)$, the product is $V_{out} = \frac{1}{2} V_1 V_2 [\cos(2\omega t + \theta) + \cos\theta]$. If the high-freq. component is filtered out, $\overline{V_{out}} \propto \cos\theta$.

· The phase detector is linear only for a small neighborhood around $\theta = \pm\frac{\pi}{2}$.



15.2



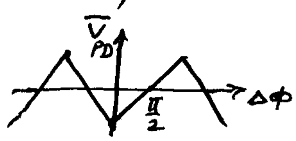
The difference between the two frequencies is integrated between t_1 and t_2 to accumulate a difference of ϕ_0:

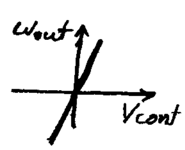
$$(f_H - f_L)(t_2 - t_1) = \frac{\phi_0}{2\pi}$$

$$\Rightarrow \quad t_2 - t_1 = \frac{\phi_0}{2\pi(f_H - f_L)}$$

15.3 The VCO still requires a dc voltage that defines the frequency of operation. A high-pass filter would not provide the dc component.

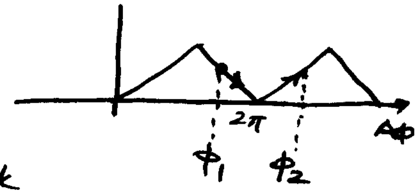
15.4 The loop must lock such that the phase difference is away from zero because the PD gain drops to zero at $\Delta\phi = 0$. With a large loop gain, the PD output settles around half of its full scale. This point can be better seen in a fully-differential implementation:



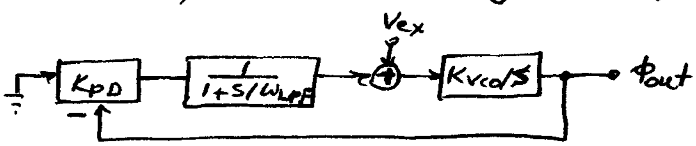


$\Rightarrow V_{cont}$ is small $\Rightarrow \Delta\phi \approx 90°$.

15.5 Suppose the loop begins with $\Delta\phi = \phi_1$. If the feedback is positive, the loop accumulates so much phase to drive the PD toward ϕ_2, where the feedback is negative and the loop can settle.



15.6 Note: ϕ_{ex} should be changed to V_{ex}.



$$\left(-\phi_{out} \cdot K_{PD} \cdot \frac{1}{1+\frac{s}{\omega_{LPF}}} + V_{ex}\right)\frac{K_{VCO}}{s} = \phi_{out}$$

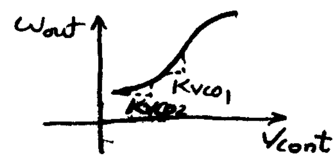
$$\Rightarrow \phi_{out}\left(1 + \frac{K_{PD}K_{VCO}}{s(1+\frac{s}{\omega_{LPF}})}\right) = V_{ex}\frac{K_{VCO}}{s} \Rightarrow$$

$$\frac{\phi_{out}}{V_{ex}} = \frac{K_{VCO}}{s + \frac{K_{PD}K_{VCO}}{1+\frac{s}{\omega_{LPF}}}} = \frac{K_{VCO}(1+\frac{s}{\omega_{LPF}})}{\frac{s^2}{\omega_{LPF}} + s + K_{PD}K_{VCO}}$$

15.7 $$\zeta = \frac{1}{2}\sqrt{\frac{\omega_{LPF}}{K_{PD}K_{VCO}}} \qquad \sqrt{\frac{K_{VCO1}}{K_{VCO2}}} = 1.5$$

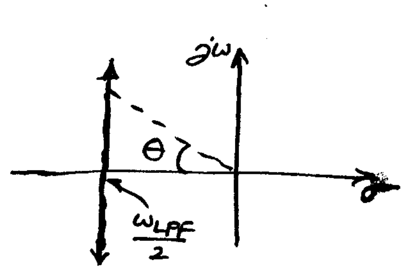
$$\Rightarrow \frac{K_{VCO1}}{K_{VCO2}} = 2.25$$

The slope can vary by a factor of 2.25.



15.8 $$\tan\varphi = \frac{Im(pole)}{-Re(pole)} = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

This is indeed as if $\zeta = \cos\varphi$ and $\sqrt{1-\zeta^2} = \sin\varphi$.



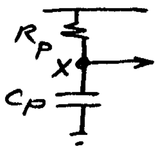
15.9 $K_{VCO} = 100\ MHz/V$, $K_{PD} = 1\ V/rad$, $\omega_{LPF} = 2\pi(1\ MHz)$

$$\Rightarrow \zeta = \frac{1}{2}\sqrt{\frac{1\ MHz}{(1\ V/rad)(100\ MHz/V)}} = 0.05 \qquad \frac{\omega_n}{2\pi} = \sqrt{(1 MHz)(1 V/rad)(100 MHz/V)} = 10\ MHz.$$

The loop is heavily underdamped. $\tau = 318\ ns$

Step response $\simeq [1 - e^{-t/318\,ns}\sin(2\pi \times 10\ MHz \times t + \theta)]\,u(t)$, $\theta \cong 90°$.

15.10

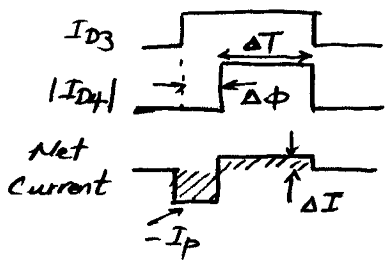


If the control voltage is sensed at node X, then R_P appears in series with the current sources in the charge pump, failing to provide a zero.

15.11 From (15.40), $\frac{I_{out}}{\Delta\phi}(s) = \frac{I_P}{2\pi}$. Since I_{out} is multiplied by the series combination of R_P and C_P:

$$\frac{V_{out}}{\Delta\phi}(s) = \frac{I_P}{2\pi}\left(R_P + \frac{1}{C_P s}\right).$$

15.12



$\Delta\phi$ must be such that the net current is zero. If the current mismatch equals ΔI and the width of $|I_{D4}|$ pulses is ΔT, then

$$\left(\frac{\Delta\phi}{2\pi}\cdot T_P\right) I_P = \Delta T.\Delta I, \text{ where } T_P \text{ is the period.}$$

$$\Rightarrow \Delta\phi = 2\pi\,\frac{\Delta T}{T_P}\cdot\frac{\Delta I}{I_P}$$

15.13 $\omega_{out} = \omega_0 + K_{VCO}V_{cont}$, $V_{cont} = V_m\cos\omega_m t$. The VCO output is

$$V_{out} = V_0\cos\left[\int\omega_{out}\,dt\right] = V_0\cos\left[\omega_0 t + K_{VCO}V_m\int\cos\omega_m t\,dt\right]$$

$$= V_0\cos\omega_0 t\cos\left(K_{VCO}\frac{V_m}{\omega_m}\sin\omega_m t\right) - V_0\sin\omega_0 t\sin\left(K_{VCO}\frac{V_m}{\omega_m}\sin\omega_m t\right).$$

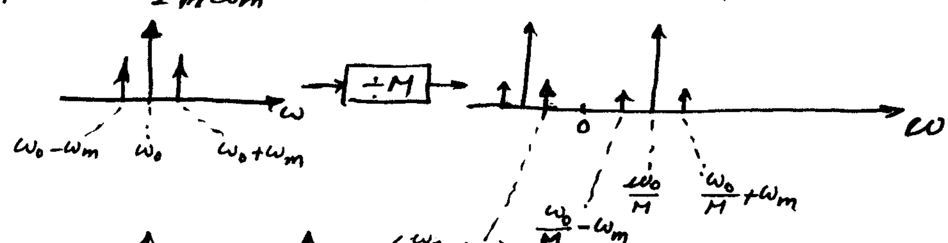
For small V_m, $V_{out}(t) \approx V_0\cos\omega_0 t - \frac{K_{VCO}V_mV_0}{2\omega_m}\left[\cos(\omega_0-\omega_m)t - \cos(\omega_0+\omega_m)t\right]$.

The divider output is expressed as

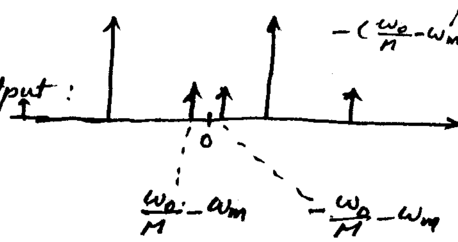
$$V_{out,M} = V_0\cos\left[\frac{\omega_0 t}{M} + \frac{K_{VCO}V_m}{M}\int\cos\omega_m t\,dt\right]$$

$$\approx V_0\cos\frac{\omega_0}{M}t - \frac{K_{VCO}V_mV_0}{2M\omega_m}\left[\cos\left(\frac{\omega_0}{M}-\omega_m\right)t - \cos\left(\frac{\omega_0}{M}+\omega_m\right)t\right].$$

If $\frac{\omega_0}{M} > \omega_m$,



If $\frac{\omega_0}{M} > \omega_m$, output:



(aliasing)

15.14 $S_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$ $\xi \propto \sqrt{I_p K_{vco}}$

$\omega_n \propto \sqrt{I_p K_{vco}}$

As $I_p K_{vco}$ starts from small values, $S_{1,2}$ are complex:

$Re\{S_{1,2}\} = -\xi\omega_n$ $Im\{S_{1,2}\} = \pm\omega_n\sqrt{1-\xi^2}$.

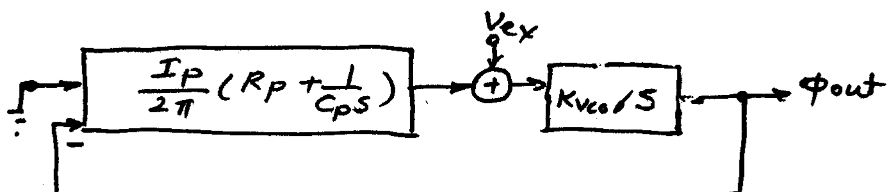
Noting that $\omega_n = \frac{2\xi}{R_pC_p}$, we can write $\omega_n^2 - \frac{2\xi\omega_n}{R_pC_p} = 0$

Adding $(\frac{1}{R_pC_p})^2$ to both sides and subtracting and adding $-\xi^2\omega_n^2$, we obtain $(-\xi\omega_n + \frac{1}{R_pC_p})^2 + \omega_n^2(1-\xi^2) = (\frac{1}{R_pC_p})^2$, which is a circle centered at $-\frac{1}{R_pC_p}$ with a radius equal to $\frac{1}{R_pC_p}$.

For $\xi \geq 1$, the poles become real and move away from each other: $-\xi\omega_n + \omega_n\sqrt{\xi^2-1}$ and $-\xi\omega_n - \omega_n\sqrt{\xi^2-1}$. If $\xi \to \infty$, then $-\xi\omega_n + \omega_n\sqrt{\xi^2-1} = \omega_n(-\xi+\sqrt{\xi^2-1}) = \omega_n\xi(-1+\sqrt{1-\frac{1}{\xi^2}})$

$\approx \omega_n\xi(-1+(1-\frac{1}{2\xi^2})) \approx \frac{-\omega_n}{2\xi} = \frac{-1}{R_pC_p}$.

15.15 Note: ϕ_{ex} should be changed to V_{ex}.



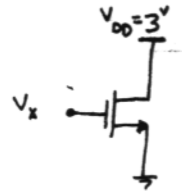
$$\left[-\phi_{out} \cdot \frac{I_p}{2\pi}\left(\frac{R_pC_pS+1}{C_pS}\right) + V_{ex}\right]\frac{K_{vco}}{S} = \phi_{out}$$

$$\Rightarrow \phi_{out}\left[1 + \frac{I_pK_{vco}(R_pC_pS+1)}{2\pi C_pS^2}\right] = V_{ex}\frac{K_{vco}}{S} \Rightarrow$$

$$\frac{\phi_{out}}{V_{ex}} = \frac{K_{vco}(2\pi C_pS^2)}{2\pi C_pS^2 + I_pK_{vco}R_pC_pS + 1}$$

15.16 When the VCO frequency is far from the input frequency, the PFD operates as a frequency detector, comparing the VCO and input frequencies. Thus, the VCO transfer function must relate the output frequency to the control voltage:
$\Delta\omega_{out} = K_{VCO}\,\Delta V_{cont}$ ⇒ the order of the system falls by one (compared to when the VCO phase is of interest: K_{VCO}/s.)

2.1) a) NMOS :

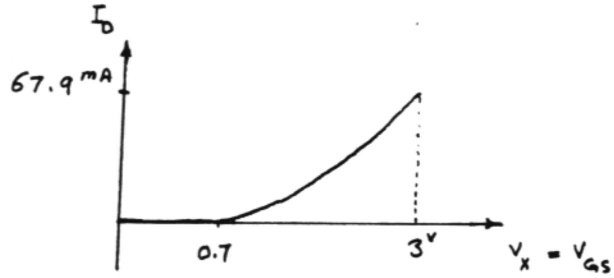


for $V_x < V_{TH}\ (=0.7)$ device is off , $I_D \simeq 0$

for $V_x \geq 0.7$

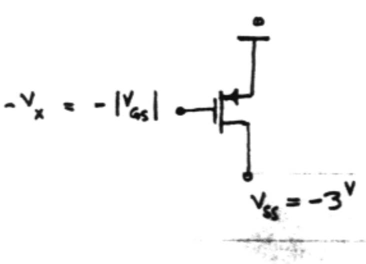
$$I_D \simeq \frac{1}{2}\mu_n C_{ox}\frac{W}{L_{eff}}(V_x - 0.7)^2(1+\lambda \cdot 3^V) \quad (L_{eff} = 0.5^{\mu} - 2L_D)$$

$$I_D = 12.8\ \left(\frac{mA}{V^2}\right)\cdot(V_x - 0.7)^2$$



b) PMOS :

Solution is the same

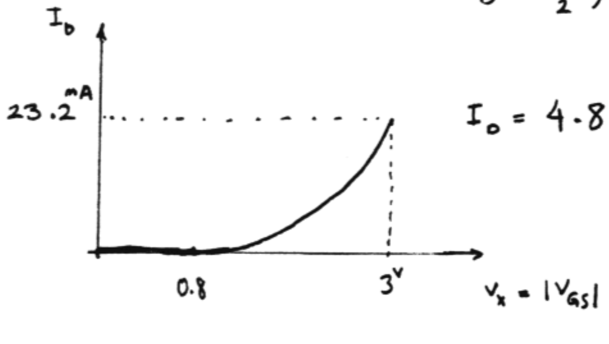


for $|V_{GS}| < V_{TH}\ (=0.8)$ $I_D \simeq 0$

for $|V_{GS}| \geq 0.8$

$$I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L_{eff}}(V_x - 0.8)^2(1+\lambda \cdot 3^V)$$

$$I_D = 4.8\ \left(\frac{mA}{V^2}\right)\cdot(V_x - 0.8)^2$$



2.2) a) Nmos

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 3.66 \frac{mA}{V} \quad \text{(Neglecting } L_D\text{)}$$

$$r_o = \frac{1}{\lambda I_D} = 20^{K\Omega}$$

$$\text{Intrinsic gain} = g_m r_o = 73.3 \frac{V}{V}$$

b) PMOS

$$g_m = \sqrt{2\mu_p C_{ox} \frac{W}{L} I_D} = 1.96 \frac{mA}{V}$$

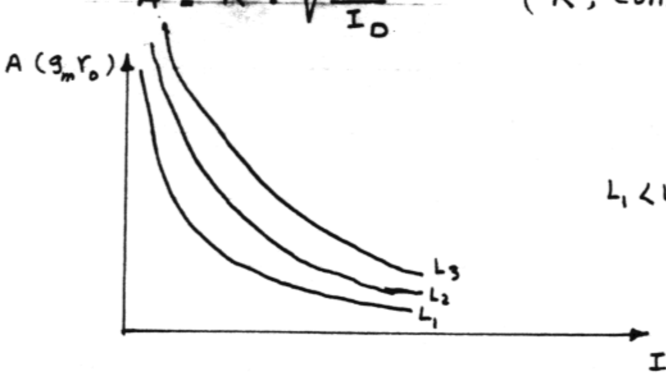
$$r_o = \frac{1}{\lambda I} = \frac{1}{0.2 \cdot 0.5^{mA}} = 10^{K\Omega}$$

$$g_m r_o = 19.6 \frac{V}{V}$$

2.3) $g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D}$ $\qquad r_o = \frac{1}{\lambda I_D}$ $\qquad$ Assume $\lambda = \frac{\alpha}{L}$

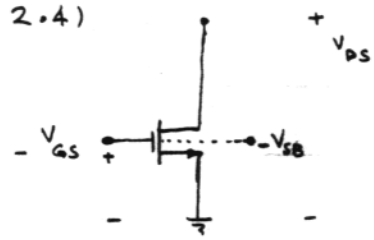
$$A = g_m r_o = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} \cdot \frac{L}{\alpha I_D}$$

$$A = K \cdot \sqrt{\frac{WL}{I_D}} \quad \text{(K ; Constant)}$$



$L_1 < L_2 < L_3$

2.4)



I_D versus V_{GS} : (for NMOS)

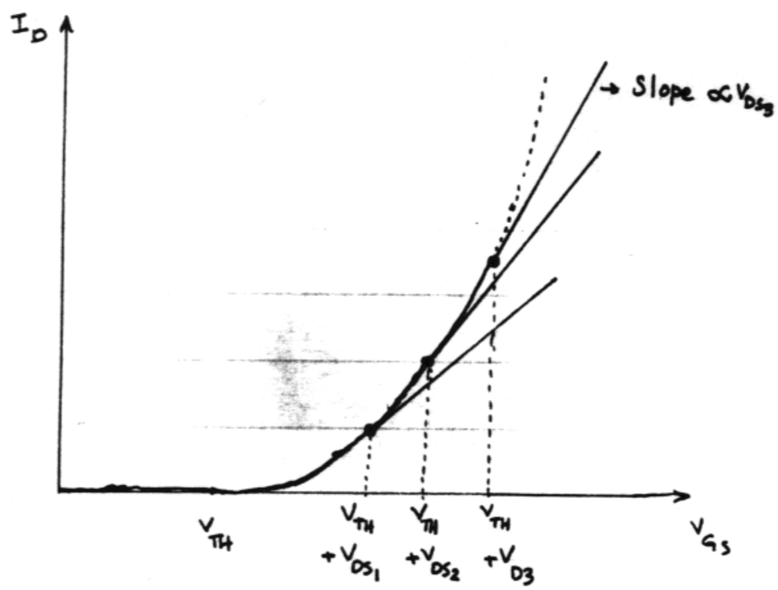
I) for $V_{GS} < V_{TH}$, $I_D \approx 0$

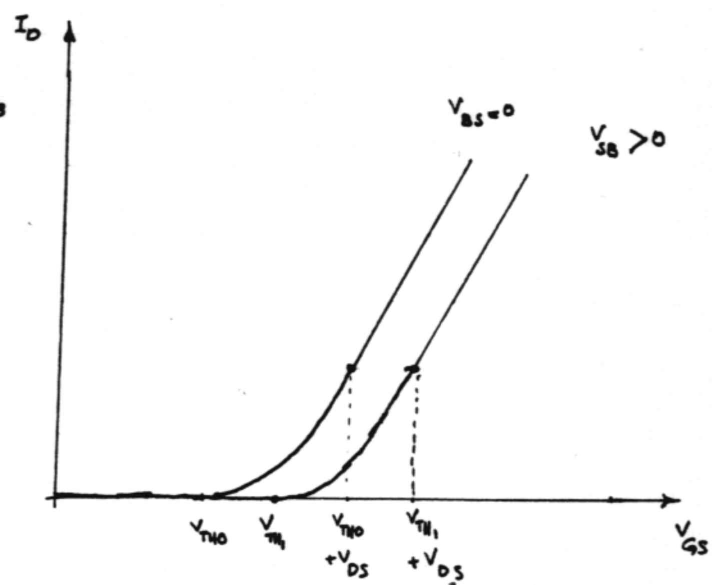
II) for $V_{TH} < V_{GS} < V_{TH} + V_{DS}$ ⇒ Device is in the Saturation region

$$I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$$

III) for $V_{GS} > V_{TH} + V_{DS}$ ⇒ Device operates in the triode region

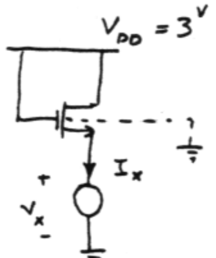
$$I_D \approx \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$





Changing V_{SB} just shifts the curve to the right for $V_{SB} > 0$ or to the left for $V_{SB} < 0$

2.5) a)



$\lambda = 0.1$, $\gamma = 0.45$, $2\phi_F = 0.9$, $V_{TH0} = 0.7$

$V_{GS} = 3 - V_x$, $V_{DS} = 3 - V_x$, $V_{SB} = V_x$

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$$

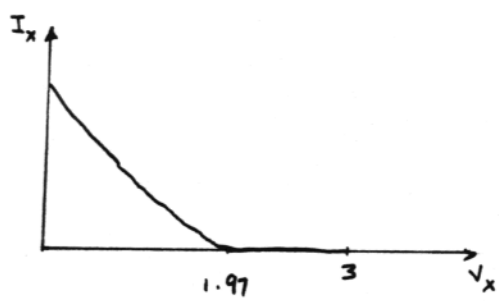
So, $$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(3 - V_x - 0.7 - 0.45\left(\sqrt{0.9 + V_x} - \sqrt{0.9}\right)\right)^2\left(1 + \lambda(3 - V_x)\right)$$

The above equation is valid for

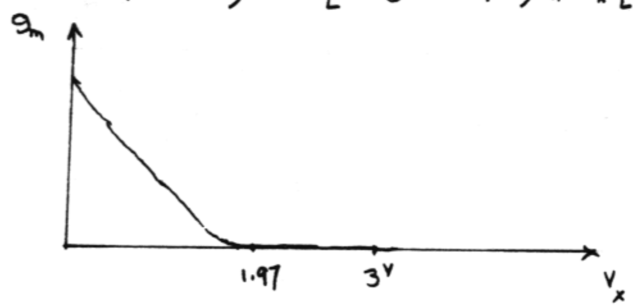
$3 - V_x - 0.7 - 0.45\left(\sqrt{0.9 + V_x} - \sqrt{0.9}\right) > 0$, i.e. $V_x < 1.97^V$

So, $$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2.727 - V_x - 0.45\sqrt{0.9 + V_x}\right)^2\left(1.3 - 0.1 V_x\right)$$

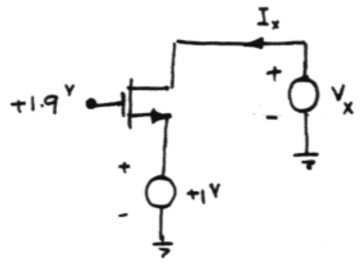
and $I_x = 0$ for $1.97 < V_x$



$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_x}$$



2.5) b,



$\lambda = \gamma = 0 \qquad V_{TH} = 0.7$

for $0 < V_x < 1$, S and D exchang their roles.

$$V_{GS} = 1.9 - V_X \qquad V_{DS} = 1 - V_X \quad , V_{OD} = 1.2 - V_X$$

$$I_X = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(1.2 - V_X)\times 2\times(1 - V_X) - (1 - V_X)^2\right]$$

$$I_X = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(1 - V_X)(1.4 - V_X).$$

$$g_m = \mu_n C_{ox}\frac{W}{L}V_{DS} = \mu_n C_{ox}\frac{W}{L}(1 - V_X) \quad \text{(absolute Value)}$$

The above equations are valid for $V_x < 1$

Then the direction of current is reversed.

$$V_{GS} = 1.9 - 1 = 0.9 \qquad V_{DS} = V_X - 1 \quad , V_{OD} = 0.9 - 0.7 = 0.2$$

for $V_X < 1.2$, device operates in the triode region.

$$I_X = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\times 0.2\times(V_X - 1) - (V_X - 1)^2\right]$$

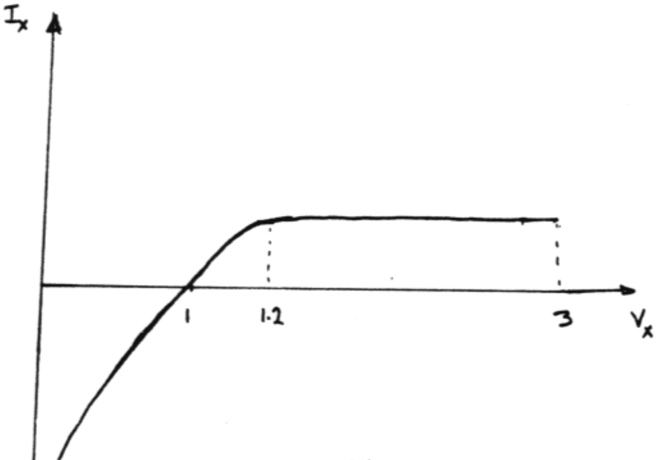
$$g_m = \mu_n C_{ox}\frac{W}{L}(V_X - 1)$$

for $V_X > 1.2$, Device goes into Saturation region

2.5) b Cont

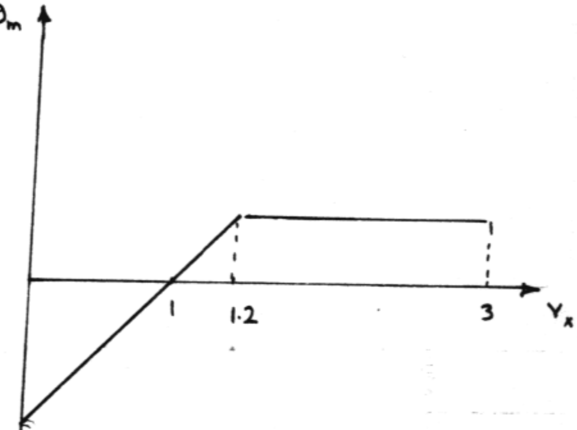
So, $I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.2)^2$,

$g_m = \mu_n C_{ox} \frac{W}{L} (0.2)$



$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_x - 1)(1.4 - V_x)$; $x < 1.2$

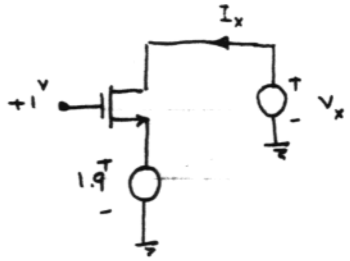
$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.2)^2$; $x > 1.2$



$g_m = \mu_n C_{ox} \frac{W}{L} (V_x - 1)$; $x < 1.2$

$g_m = \mu_n C_{ox} \frac{W}{L} (0.2)$; $x > 1.2$

2.5) c



$\lambda = \gamma = 0$ $\qquad V_{TH} = 0.7$

S and D exchange their roles.

$V_{GS} = 1 - V_x \qquad V_{DS} = 1.9 - V_x \qquad V_{OD} = V_{GS} - V_{TH} = 0.3 - V_x$

Device is in Saturation region, So, $I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)^2$

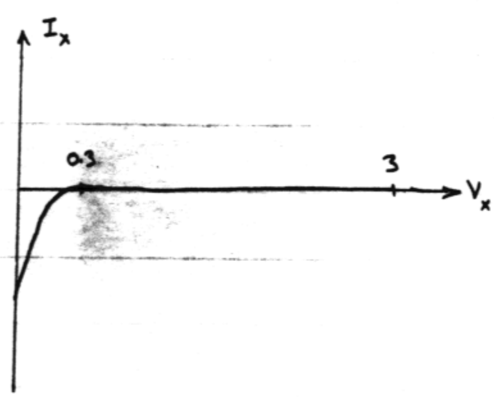
Device turns off when $V_x = 0.3$ and never turns on again.

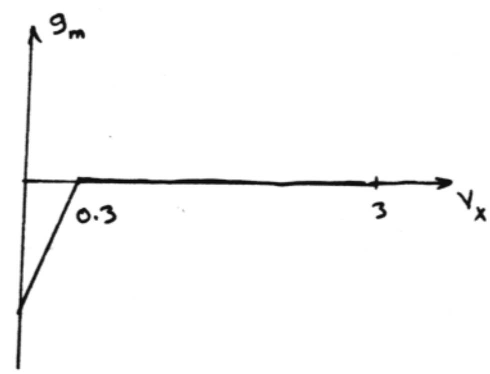
So, $I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)^2 \quad ; x < 0.3$

$I_x = 0 \quad$; other wise

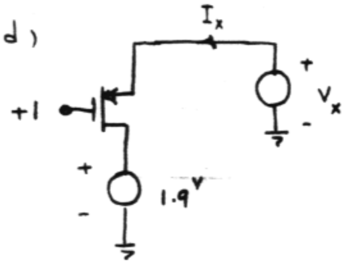
Then $g_m = -\mu_n C_{ox}\frac{W}{L}(0.3 - V_x) \quad ; x < 0.3$

$g_m = 0 \quad$; 0.ther wise





2.5) d)



$V_{TH} = -0.8$ $\gamma = 0$

D and S exchang their roles.

$V_{GS} = -0.9$ $V_{DS} = V_x - 1.9$

for $V_x < 1.8$:

$$I_x = -\frac{1}{2} \mu_p C_{ox} \frac{W}{L} (0.1)^2$$

$$g_m = -\mu_p C_{ox} \frac{W}{L} (0.1)$$

Device remains in the saturation region until

$V_x = 1.9 - 0.1 = 1.8$, then device goes into the triode

region.

for $1.8 < V_x < 1.9$:

$$I_x = -\mu_p C_{ox} \frac{W}{L} \left[(-0.1)(V_x - 1.9) - \frac{1}{2} (V_x - 1.9)^2 \right]$$

$$g_m = +\mu_p C_{ox} \frac{W}{L} (V_x - 1.9)$$

for $V_x > 1.9$:

S and D exchange their roles again, when $V_x = 1.9$

for $V_x > 1.9$, Device operates in the triode region.

$V_{GS} = 1 - V_x$, $V_{DS} = 1.9 - V_x$

$$I_x = +\mu_p C_{ox} \frac{W}{L} \left[(1.8 - V_x)(1.9 - V_x) - \frac{1}{2} (1.9 - V_x)^2 \right]$$

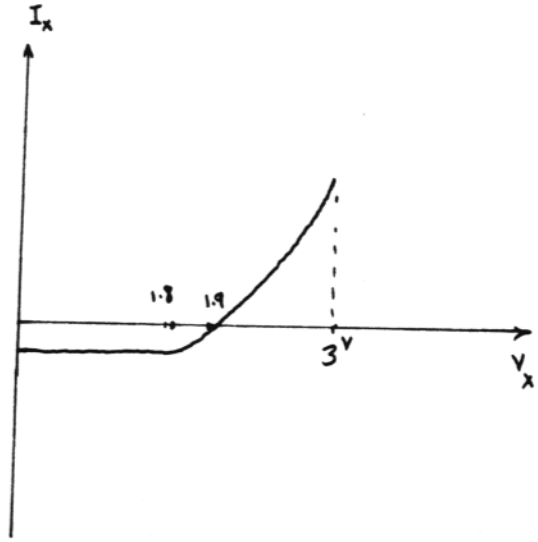
$$g_m = -\mu_p C_{ox} \frac{W}{L} (1.9 - V_x)$$

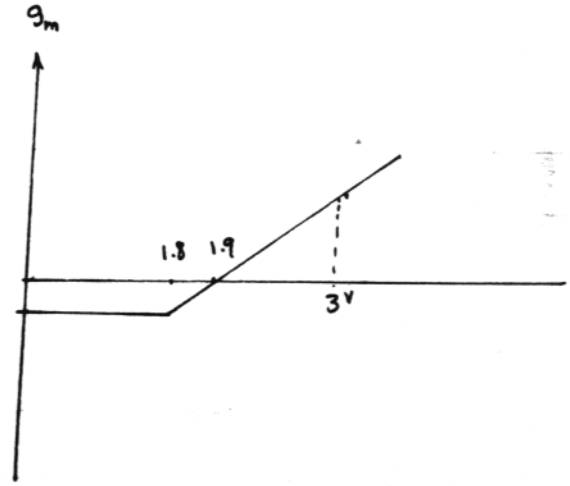
2.5)d So, $0 < V_x < 1.8$ $\quad I_x = -\frac{1}{2}\mu_p C_{ox}\frac{W}{L}(0.1)^2$

$$g_m = -\mu_p C_{ox}\frac{W}{L}(0.1)$$

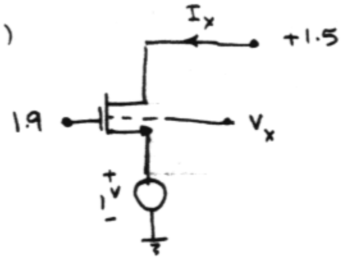
$1.8 < V_x < 3$ $\quad I_x = +\mu_p C_{ox}\frac{W}{L}\times\frac{1}{2}(V_x-1.9)(V_x-1.7)$

$$g_m = \mu_p C_{ox}\frac{W}{L}(V_x-1.9)$$





2.5) e)



$V_{TH0} = 0.7$ $\gamma = 0.45$ $2\phi_F = 0.9$, $\lambda = 0$

$V_{SB} = 1 - V_x$

$V_{TH} = 0.7 + 0.45\left(\sqrt{0.9 + 1 - V_x} - \sqrt{0.9}\right)$

$V_{GS} = 0.9$ $V_{DS} = 0.5$

for $V_x = 0$, $V_{TH} = 0.893$ So device is in saturation region.

So $$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right)\right)^2$$

$$g_m = \mu_n C_{ox}\frac{W}{L}\left(0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right)\right)$$

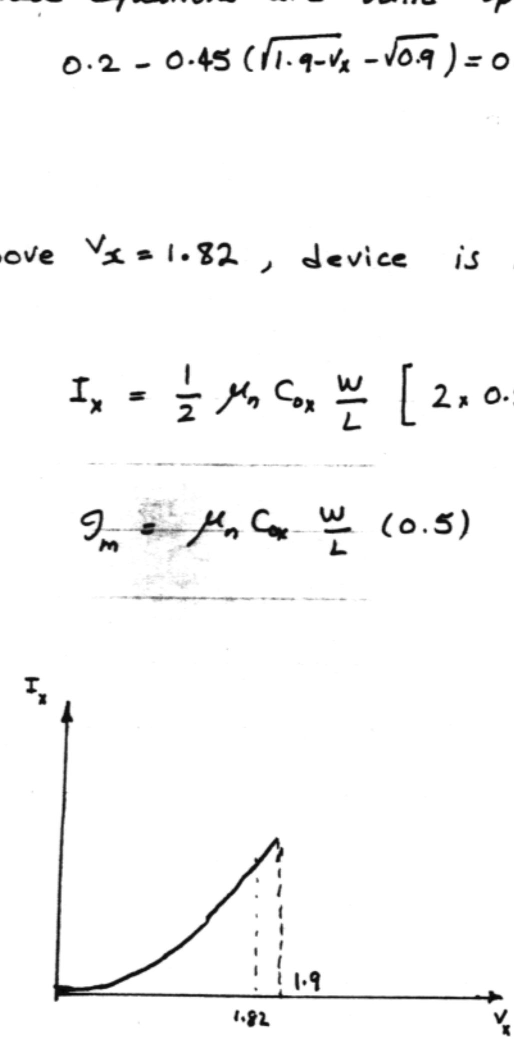
These equations are valid upto the edge of triode region, i.e.

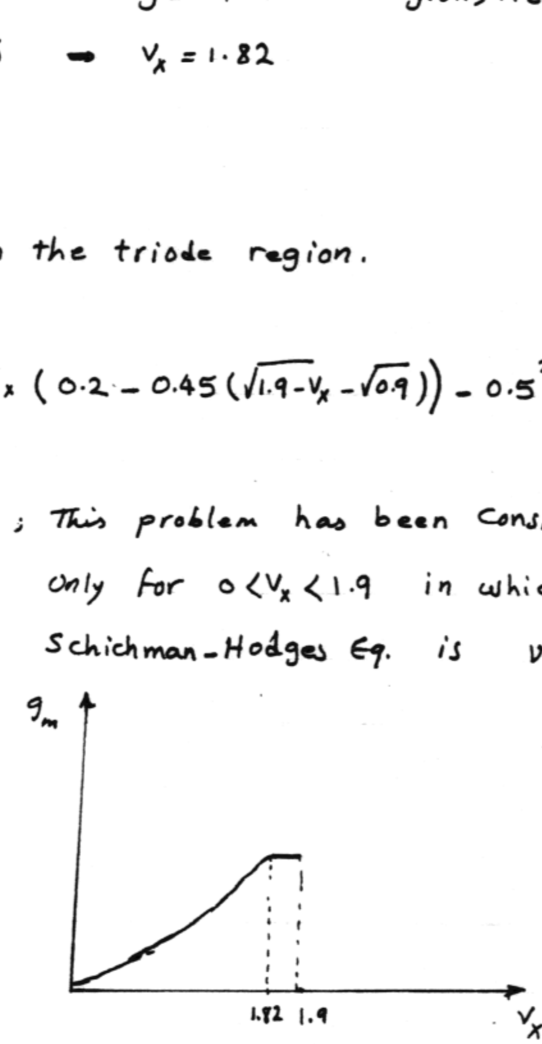
$0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right) = 0.5 \Rightarrow V_x = 1.82$

Above $V_x = 1.82$, device is in the triode region.

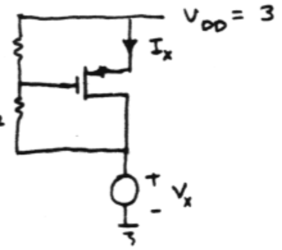
$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2 \times 0.5 \times \left(0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right)\right) - 0.5^2\right]$$

$g_m = \mu_n C_{ox}\frac{W}{L}(0.5)$; This problem has been considered only for $0 < V_x < 1.9$ in which Schichman-Hodges Eq. is valid for V_{TH}.





2.6) a)



$\gamma = 0$

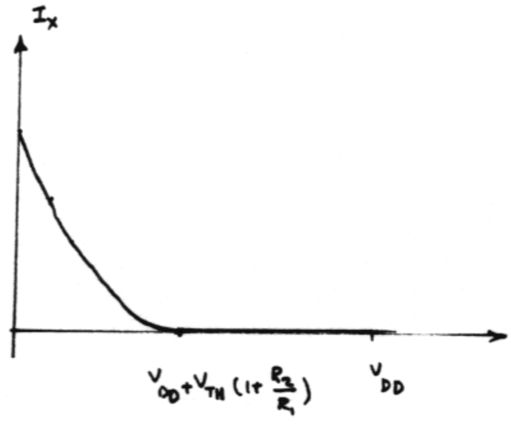
$$V_{SG} = (V_{DD} - V_X)\frac{R_1}{R_1 + R_2} \qquad V_{SD} = V_{DD} - V_X$$

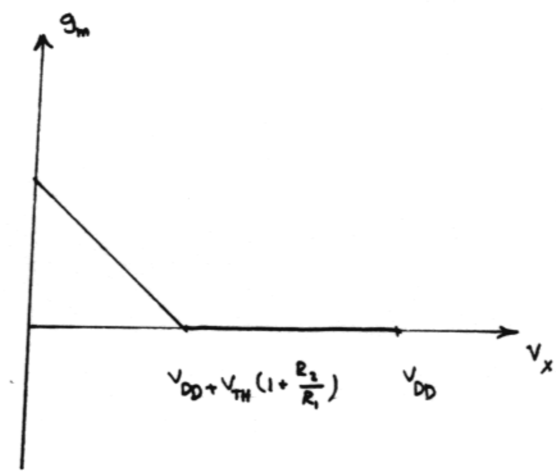
for $V_{SG} > |V_{TH}|$ Device is in the Saturation region (Device is off; otherwise)

$$(V_{DD} - V_X)\frac{R_1}{R_1+R_2} > -V_{TH}$$

$$V_X < V_{DD} + V_{TH}\left(1 + \frac{R_2}{R_1}\right) \Rightarrow I_X = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{DD} - V_X)\frac{R_1}{R_1+R_2} + V_{TH}\right]^2$$

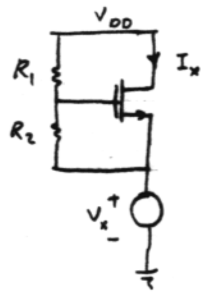
$$g_m = \mu_p C_{ox}\frac{W}{L}\left[(V_{DD} - V_X)\frac{R_1}{R_1+R_2} + V_{TH}\right]$$





If $V_{DD} + V_{TH}\left(1 + \frac{R_2}{R_1}\right) < 0$ (e.g. for small value of R_1), device never turns on!

2.6) b)



$\gamma = 0$

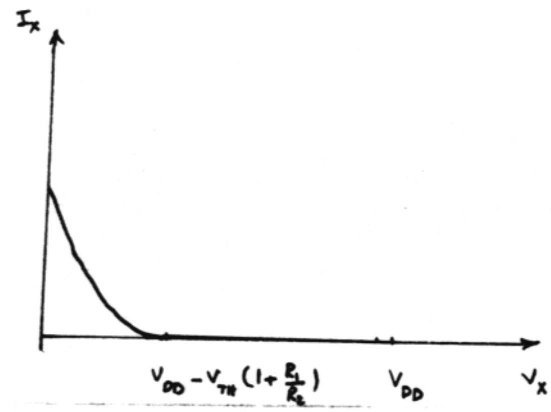
$$V_{GS} = (V_{DD} - V_x)\frac{R_2}{R_1 + R_2} \qquad V_{DS} = V_{DD} - V_x$$

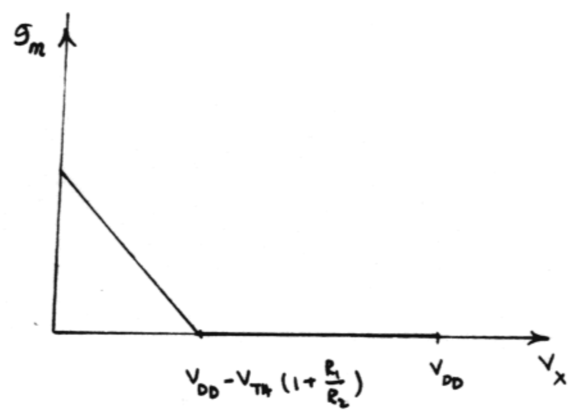
for $V_{GS} > V_{TH}$, Device is in the saturation region and

$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(V_{DD} - V_x)\frac{R_2}{R_1 + R_2} - V_{TH}\right]^2$$

$$g_m = \mu_n C_{ox}\frac{W}{L}\left[(V_{DD} - V_x)\frac{R_2}{R_1 + R_2} - V_{TH}\right]$$

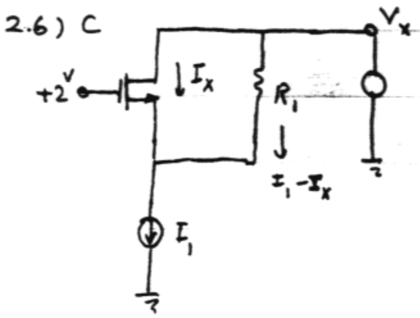
for $V_x < V_{DD} - V_{TH}\left(1 + \frac{R_1}{R_2}\right)$ (i.e. $V_{GS} > V_{TH}$)





If $V_{DD} - V_{TH}\left(1 + \frac{R_2}{R_1}\right) < 0$ device doesn't turn on.

2.6) C



I_x and $I_R = I_1 - I_x$ have the same polarity

So, $0 \leqslant I_x \leqslant I_1$

for $0 < V_x < 2 - V_{TH}$ (1.3) Device is in the triode.

$$V_{GS} = 2 - V_x + R_1 (I_1 - I_x) \quad , \quad V_{DS} = R_1 (I_1 - I_x)$$

$$I_x = I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) - V_{DS} \right] V_{DS}$$

$$\Rightarrow (*) \; I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[R_1 (I_1 - I_x) + 2(2 - V_{TH} - V_x) \right] \left\{ R_1 (I_1 - I_x) \right\}$$

The above equation peresents $I_x - V_x$ characteristics in this region.

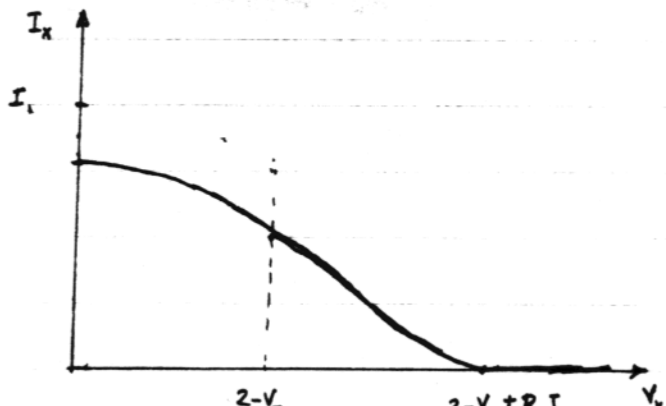
In this region $g_m = \mu_n C_{ox} V_{DS} = \mu_n C_{ox} R_1 (I_1 - I_x)$

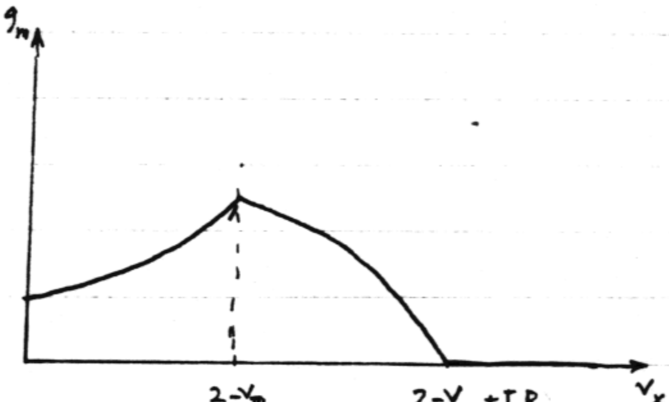
Then device enters the Saturation region; $V_{GS} = 2 - V_x + R_1 (I_1 - I_x)$

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 - V_x + R_1 (I_1 - I_x) - V_{TH} \right]^2$$

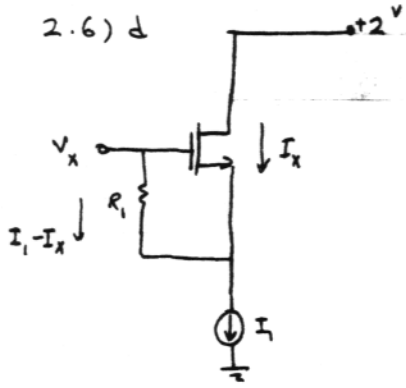
$$g_m = \mu_n C_{ox} \frac{W}{L} \left[2 - V_x + R_1 (I_1 - I_x) - V_{TH} \right]$$

Then device turns off when $V_x = 2 - V_{TH} + R_1 I_1$





2.6) d



Assumption : $R_1 I_1 > V_{TH}$

for $0 < V_x < 2 + V_{TH}$: Device is in the saturation region

$$V_{GS} = R_1 (I_1 - I_x)$$

$$I_D = I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[R_1 (I_1 - I_x) - V_{TH} \right]^2$$

I_x is a constant that can be derived by solving the above equation.

Then device enters the triode region for $V_x > 2 + V_{TH}$

In this case $V_{GS} = R_1 (I_1 - I_x)$ $\quad V_{DS} = 2 - [V_x - R_1 (I_1 - I_x)] = 2 - V_x + R_1 (I_1 - I_x)$

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 (V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right] = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \Big[2 \left[R(I_1 - I_x) - V_{TH} \right] - 2 + V_x - R_1 (I_1 - I_x) \Big] \times (2 - V_x + R_1 (I_1 - I_x))$$

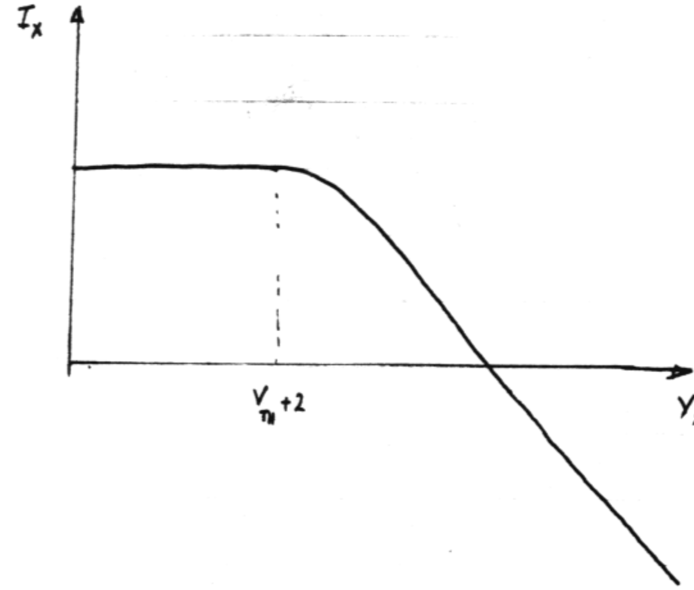
$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \Big[\big(R_1 (I_1 - I_x) - V_{TH} \big) + (V_x - 2 - V_{TH}) \Big] \Big[\big(R_1 (I_1 - I_x) - V_{TH} \big) - (V_x - 2 - V_{TH}) \Big]$$

$$(*) \quad I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \Big[\big(R_1 (I_1 - I_x) - V_{TH} \big)^2 - (V_x - 2 - V_{TH})^2 \Big]$$

The second term shows that I_x decreases when we increase V_x

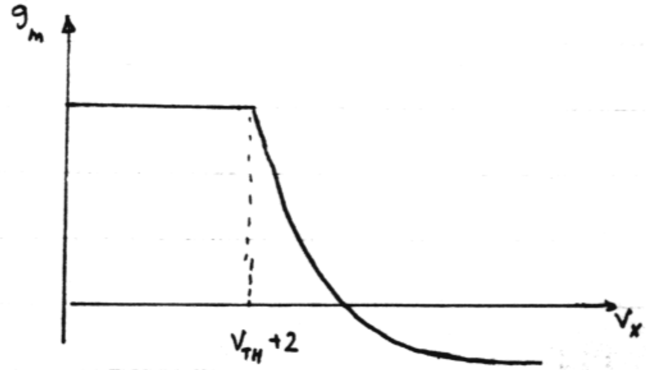
The polarity of I_x changes for higher V_x (Device still is in triode)

$(*)$ presents $I_x - V_x$ relationship in this region.

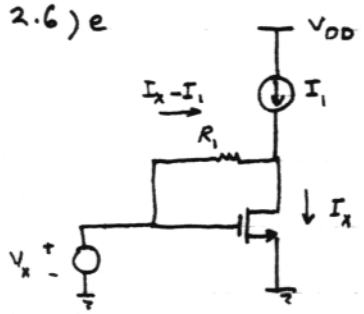


$$g_m = \mu_n C_{ox}\frac{W}{L}\left[R_1(I_1 - I_X) - V_{TH}\right] \qquad ; V_X < 2 + V_{TH}$$

$$g_m = \mu_n C_{ox}\frac{W}{L} V_{DS} = \mu_n C_{ox}\frac{W}{L}\left[R_1(I_1 - I_X) + 2 - V_X\right] \qquad V_X > 2 + V_{TH}$$



2.6) e



for $0 < V_X < V_{TH}$ Device is off $I_X = 0$ $g_m = 0$

Then device turns on (in the saturation region)

$$I_X = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_X - V_{TH})^2$$

Transistor is in the saturation until $V_{GD} = R_1(I_X - I_1) = V_{TH}$, Then device enters the triode region. (when $I_X = I_1 + \frac{V_{TH}}{R_1}$, i.e. $V_X = V_{TH} + \sqrt{\frac{2I_1 + 2V_{TH}/R_1}{\mu_n C_{ox}\frac{W}{L}}}$)

So, $V_{TH} < V_X < V_{TH} + \sqrt{\frac{2I_1 + 2V_{TH}/R_1}{\mu_n C_{ox}\frac{W}{L}}}$ $\qquad I_X = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_X - V_{TH})^2$

$$g_m = \mu_n C_{ox}\frac{W}{L}(V_X - V_{TH})$$

2.6) e Cont.

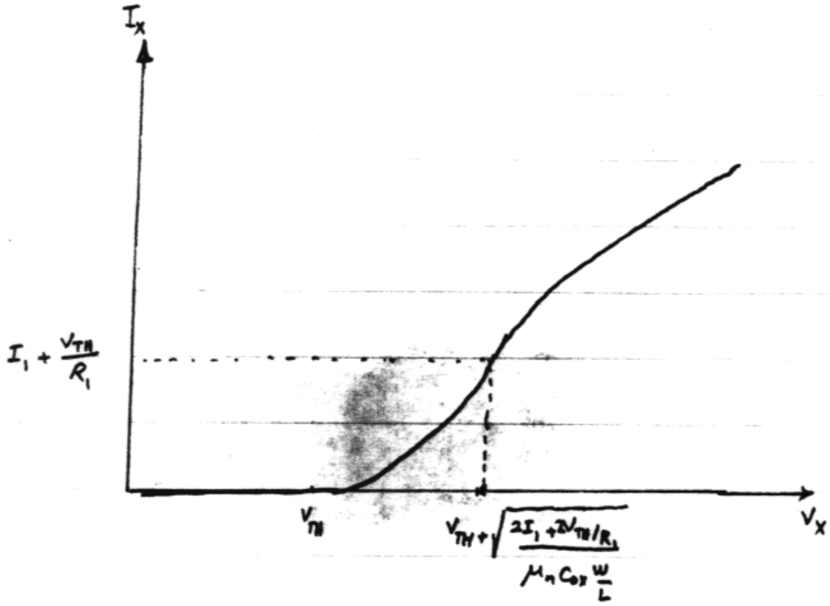
Then device enters the triode region. $V_{GS} = V_X$ $\quad V_{DS} = V_X - R_1(I_X - I_1)$

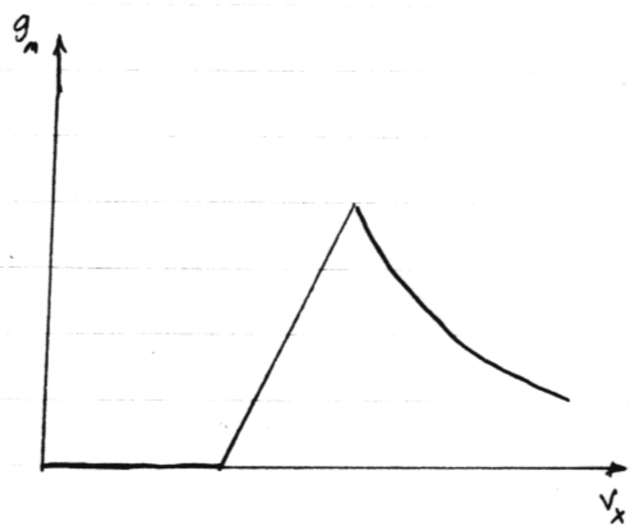
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})-V_{DS}\right]V_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_X-V_{TH})-V_X+R_1(I_X-I_1)\right]\times (V_X - R_1(I_X-I_1))$$

$$(*)\quad I_X = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_X + R_1(I_X-I_1) - 2V_{TH}\right)\left(V_X - R_1(I_X-I_1)\right)$$

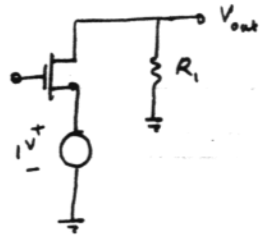
The above equation presents $I_X - V_X$ relationship in triode region.

In this region, $g_m = \mu_n C_{ox}\frac{W}{L}V_{DS} = \mu_n C_{ox}\frac{W}{L}\left(V_X - R_1(I_X-I_1)\right)$





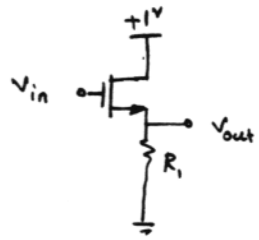
2.7) a



$\lambda = \gamma = 0$ $V_{TH} = 0.7$

Drain and source exchange their roles.

⇒



for $0 < V_{in} < 0.7$ device is off $V_{out} = 0$

for $0.7 < V_{in} < 1.7$ device is in the saturation region

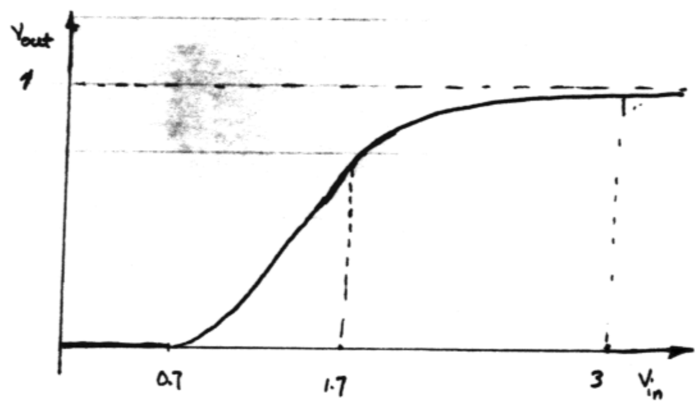
(*) $I_D = \dfrac{V_{out}}{R_1} = \dfrac{1}{2}\mu_n C_{ox}\dfrac{W}{L}\left(V_{in} - V_{out} - 0.7\right)^2$ ⇒ Input - Output relationship

for $1.7 < V_{in} < 3$ device is in the triode region

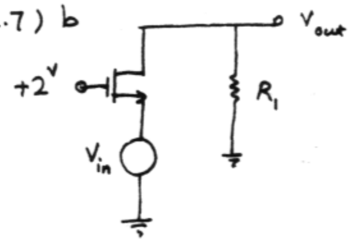
$V_{GS} = V_{in} - V_{out}$ $V_{DS} = 1 - V_{out}$

(*) $I_D = \dfrac{V_{out}}{R_1} = \dfrac{1}{2}\mu_n C_{ox}\dfrac{W}{L}\left[2\left(V_{in} - V_{out} - 0.7\right)\left(1 - V_{out}\right) - \left(1 - V_{out}\right)^2\right]$

⇒ Input - output relationship

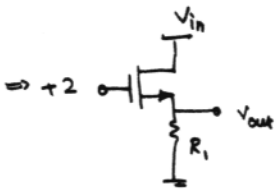


2.7) b



$\gamma = \lambda = 0 \qquad V_{TH} = 0.7$

Drain and source exchange their roles!



for $0 < V_{in} < 1.3$ device is in triode

$V_{GS} = 2 - V_{out} \qquad V_{DS} = V_{in} - V_{out}$

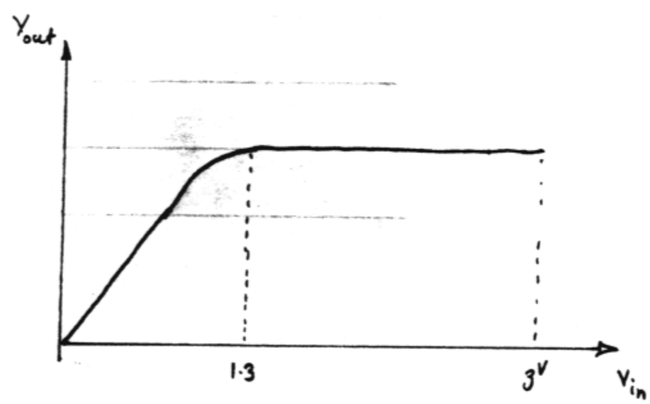
$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(2 - V_{out} - 0.7)(V_{in} - V_{out}) - (V_{in} - V_{out})^2 \right]$$

Input output relationship is presented by the above equation.

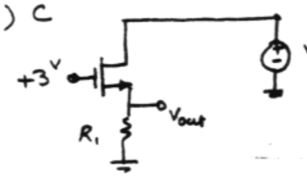
for $1.3 < V_{in} < 3$ device is in the saturation region

$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2 - V_{out} - 0.7)^2$$

V_{out} doesn't depend on V_{in} and it is constant for $V_{in} > 1.3$



2.7) c



$\gamma = \lambda = 0 \qquad V_{TH} = 0.7$

for $0 < V_{in} < 2.3$ device is in triode

$$V_{GS} = 3 - V_{out} \qquad V_{DS} = V_{in} - V_{out}$$

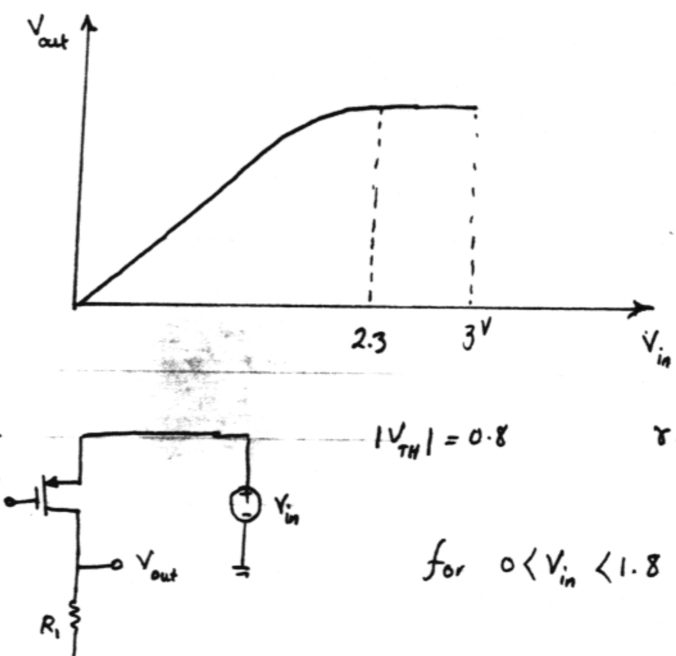
$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3 - V_{out} - 0.7)(V_{in} - V_{out}) - (V_{in} - V_{out})^2\right]$$

Input-output relationship is presented by the above equation.

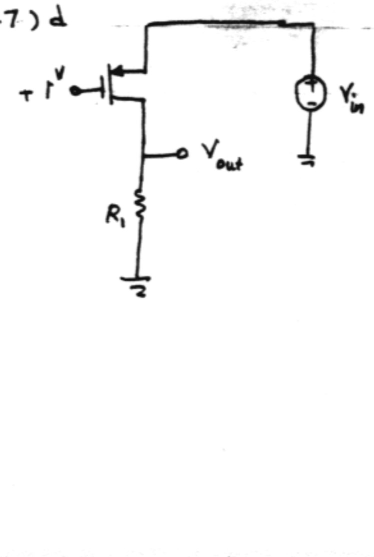
for $2.3 < V_{in} < 3$ device is in the saturation region

$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3 - V_{out} - 0.7)^2$$

V_{out} is constant for $V_{in} > 2.3$ (It doesn't depend on V_{in})



2.7) d



$|V_{TH}| = 0.8 \qquad \gamma = \lambda = 0$

for $0 < V_{in} < 1.8$ device is off $\Rightarrow V_{out} = 0$

Then device turns on (in sat.) and V_{out} goes up

until $V_{out} = 1.8$, then device enters the triode region for $V_{in} > 1.8$ and $V_{out} < 1.8$

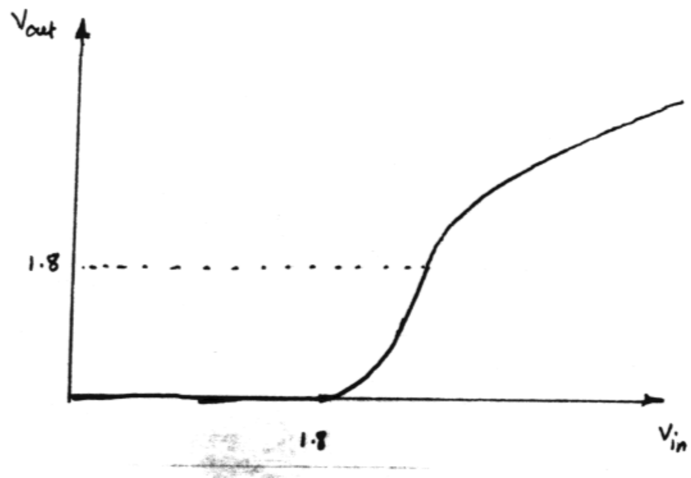
$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{in} - 1.8)^2 \Rightarrow V_{out} = \frac{1}{2}\mu_p C_{ox} R_1 \frac{W}{L}(V_{in} - 1.8)^2$$

This is good for $1.8 < V_{in} < 1.8 + \sqrt{\dfrac{2 \times 1.8^V}{\mu_p C_{ox}\frac{W}{L}R_1}}$

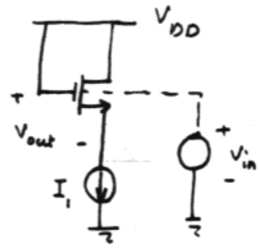
for $V_{in} > 1.8 + \sqrt{\dfrac{2\times 1.8}{\mu_p C_{ox}\frac{W}{L}R_1}}$

$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[2(V_{in} - 1.8)(V_{in} - V_{out}) - (V_{in} - V_{out})^2\right].$$

Input-output relationship is presented by the above equation.



2.8) a



$V_S = V_{DD} - V_{out}$ $\qquad V_B = V_{in}$ $\quad , V_{SB} = V_{DD} - V_{out} - V_{in}$

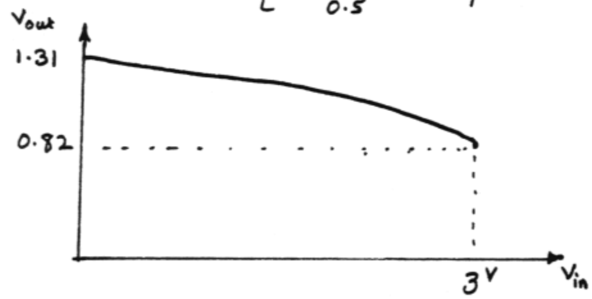
$$I_D = I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{out} - V_{TH})^2$$

$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F} \right)$$

$$\Rightarrow I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{out} - V_{TH0} - \gamma \left(\sqrt{2\phi_F + V_{DD} - V_{out} - V_{in}} - \sqrt{2\phi_F} \right) \right)^2$$

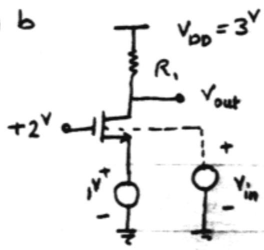
for each V_{in}, the above equation should be solved to obtain V_{out}

for $\frac{W}{L} = \frac{50}{0.5}$ $\qquad I_1 = 1^{mA}$



Assumption: $2\phi_F + V_{DD} - V_{out} - V_{in} > 0$

2.8) b



$V_{SB} = 1 - V_{in}$ $\qquad V_{GS} = 1$

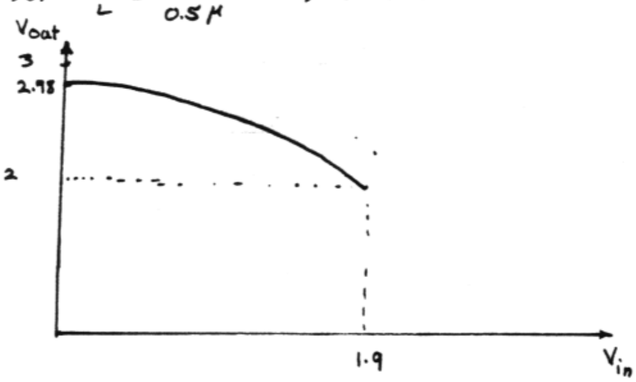
$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F} \right)$$

$$V_{TH} = 0.7 + 0.45 \left(\sqrt{1.9 - V_{in}} - \sqrt{0.9} \right)$$

Assumption: V_{in} varies from 0 to 1.9 and R_1 is small enough to guarantee that the device remains in the saturation region.

$$V_{out} = 3 - R_1 \cdot \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(0.3 - 0.45 \left(\sqrt{1.9 - V_{in}} - \sqrt{0.9} \right) \right)^2$$

for $\frac{W}{L} = \frac{50\mu}{0.5\mu}$, $R = 0.2\,K$



2.8) c Drain and Source exchange their roles, $V_{THO} = 0.7$ $\gamma = 0.45$ $2\phi_F = 0.9$

Assumption : $V_{SB} > -2\phi_F$ $(V_{out} - V_{in} > -2\phi_F)$ ⇒ Device is in the Saturation

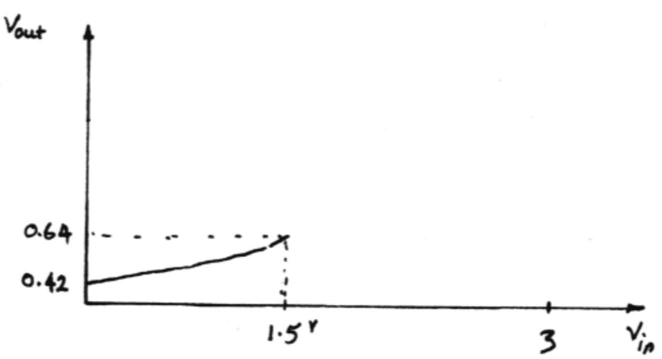
$$V_{TH} = 0.7 + 0.45\left(\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}\right) \qquad V_{GS} = 2 - V_{out}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2 - V_{out} - 0.7 - 0.45\left(\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}\right)\right)^2$$

$$I_D = \frac{V_{out}}{R_1}$$

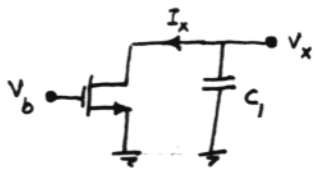
$$(*) \quad \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2 - V_{out} - 0.7 - 0.45\left(\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}\right)\right)^2$$

Input – Output relationship is presented by the above equation.



$\frac{W}{L} = \frac{50}{0.5}$ $R = 100^{\Omega}$

2.9) a



$\gamma = \lambda = 0$ $\qquad V_{TH} = 0.7$

Assume $V_b > V_{TH}$

for $V_b - 0.7 < V_x < 3$ device is in saturation

$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_b - V_{TH}\right)^2$$

$$V_x = -\frac{1}{C_1}\int I_x\,dt + 3^V = 3 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_b - V_{TH}\right)^2 t$$

Then device goes into triode , for $0 < V_x < V_b - 0.7$

$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\left(V_b - 0.7\right)V_x - V_x^2\right] = -\frac{dV_x}{dt}\times C_1$$

$$\Rightarrow \quad -dt\,\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{C_1}}_{\alpha} = \frac{dV_x}{V_x\left[2(V_b - 0.7) - V_x\right]}$$

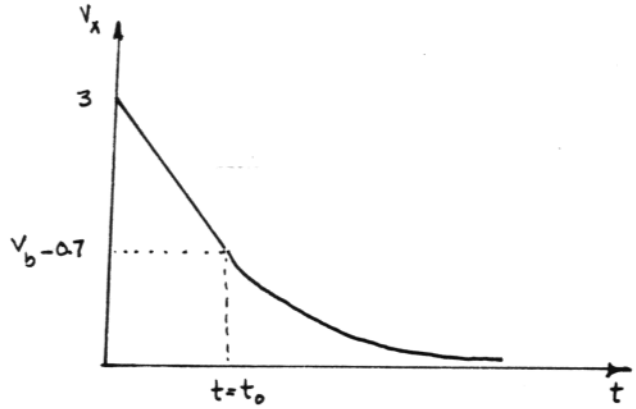
$$-\alpha\,dt = \left[\frac{1}{V_x} + \frac{1}{2(V_b - 0.7) - V_x}\right]\times\frac{1}{2(V_b - 0.7)}$$

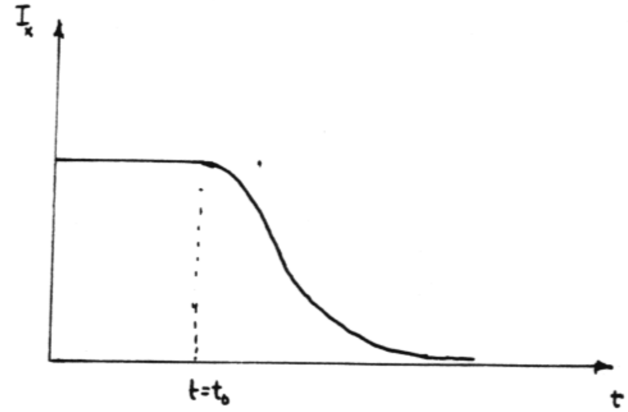
$$\Rightarrow \quad -\alpha(t - t_0) = \left[\ln\frac{V_x}{2(V_b - 0.7) - V_x}\right]\frac{1}{2(V_b - 0.7)} \qquad @\,t = t_0\,,\ V_x = V_b - 0.7$$

$$\Rightarrow \quad \frac{2(V_b - 0.7) - V_x}{V_x} = e^{2\alpha(V_b - 0.7)(t - t_0)}$$

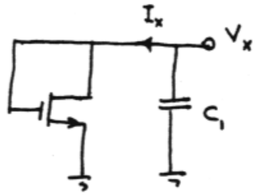
$$\Rightarrow \quad V_x = \frac{2(V_b - 0.7)}{1 + e^{2\alpha(V_b - 0.7)(t - t_0)}}$$

$$I_x = -C_1\frac{dV_x}{dt} = \frac{4\alpha C_1 (V_b - 0.7)^2\, e^{2\alpha(V_b - 0.7)(t - t_0)}}{\left(1 + e^{2\alpha(V_b - 0.7)(t - t_0)}\right)^2}$$





2.9) b



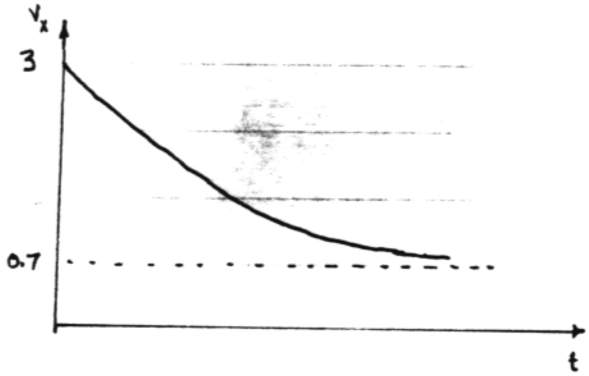
Device is always in the saturation region.

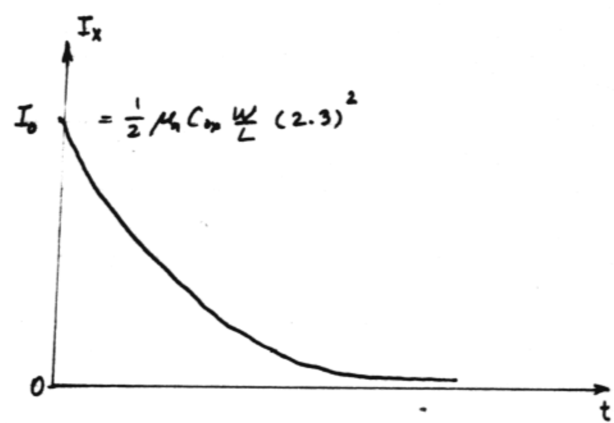
$$I_x = -C_1 \frac{dV_x}{dt} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_x - 0.7)^2$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox} \frac{W}{L} \frac{1}{C_1}}_{\alpha} dt = -\frac{dV_x}{(V_x - 0.7)^2} \quad \Rightarrow \alpha t = \frac{1}{V_x - 0.7} + K$$

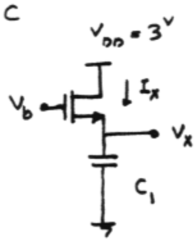
$$@\ t = 0,\ V_x = 3 \Rightarrow \alpha t = \frac{1}{V_x - 0.7} - \frac{1}{2.3} \Rightarrow V_x = 0.7 + \frac{1}{\alpha t + 1/2.3}$$

$$I_x = -C_1 \frac{dV_x}{dt} = \frac{\alpha C_1}{\left(\alpha t + \frac{1}{2.3}\right)^2}$$



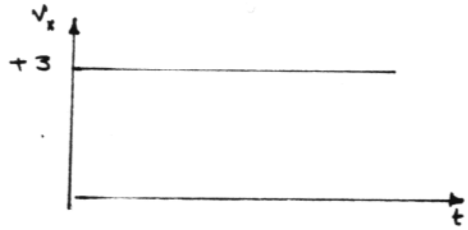


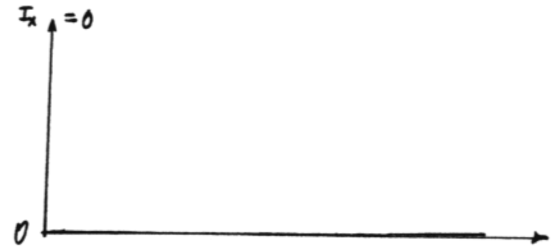
2.9) c



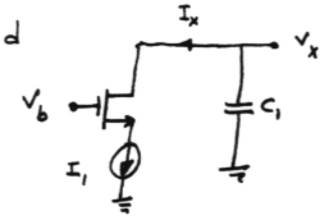
@ $t=0$ $\quad V_x = 3$, $V_{DD}=3^V \Rightarrow V_{DS}=0 \Rightarrow I_x = 0$

And the Circuit remains in this state





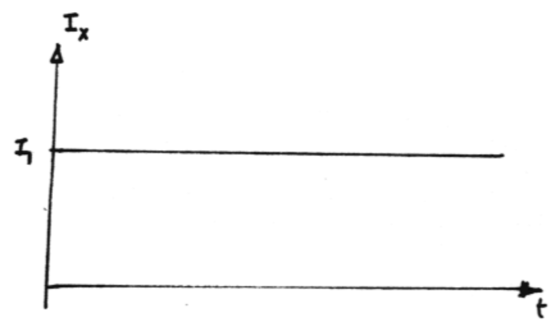
2.9) d



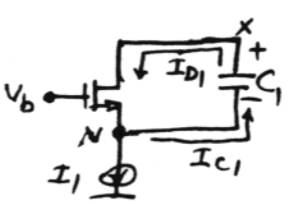
$I_x = I_1$

$$-C_1 \frac{dV_x}{dt} = I_1 \Rightarrow V_x = 3 - \frac{I_1}{C} t$$

Infact these Equations are valid until I_1 is no longer an ideal current Source.



2.9)e *Initially, the current thru $M_1 = I_1 \Rightarrow$ certain V_{GS} is developed and $V_X = V_b - V_{GS1} + 3V$ and $I_X = I_1$. However, at $t=0^+$, the drain current of M_1 flows from C_1: $I_{D1} - I_{C1} = I_1$. But,*

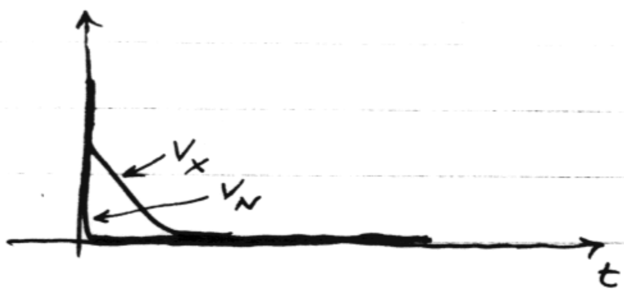


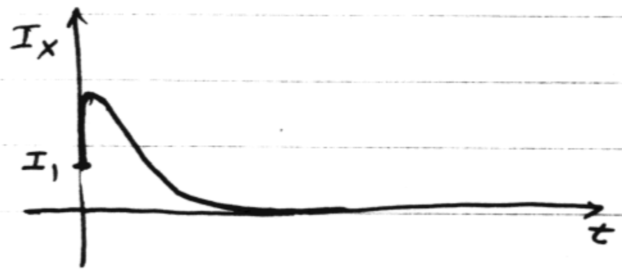
$I_{D1} = I_{C1} \Rightarrow I_1 = 0$. If the current source is ideal, V_N jumps to $-\infty$ (actually about 0.6V below 0, where the S-B diode turns on.)

If I_1 is not ideal, V_N jumps to zero and C_1 discharges

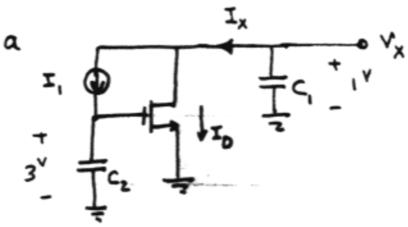
2.9) e (cnt'd)

through M_1:





2.10) a



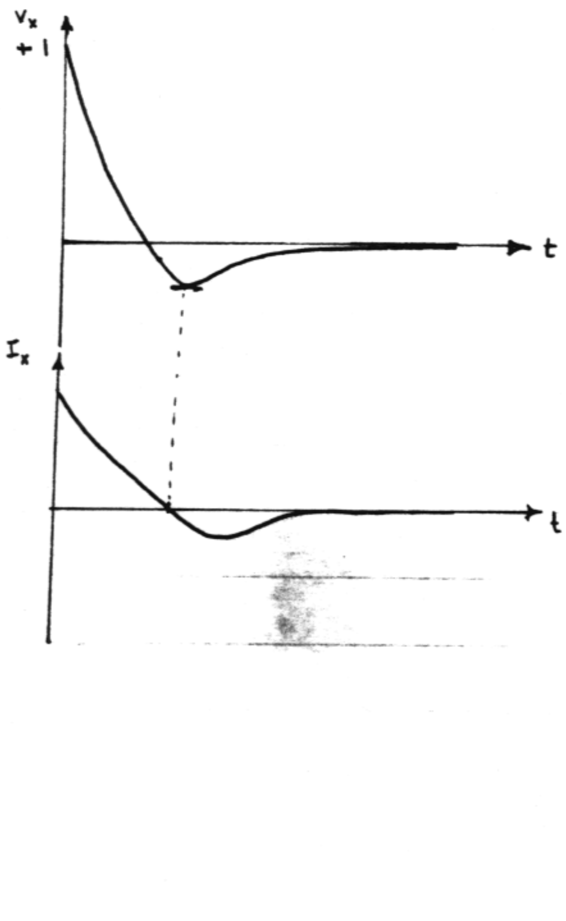
$V_G = 3 + \frac{I_1 t}{C}$

This circuit settles at $t=\infty$, when $V_G=\infty$

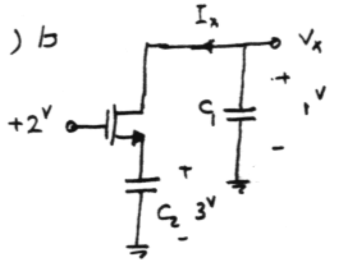
$I_D = -I_1$, $V_{DS} = 0$ (Actually, Drain and Source exchange their roles after a specific time at which $I_x = I_1$ and afterward V_x becomes negative) However, transistor always operates in the triode region.

$$I_x = I_1 + \frac{1}{2}\mu_n C_{ox} \frac{W}{L}\left[2(3+\frac{I_1}{C_2}t - 0.7)V_x - V_x^2\right] = -C_1 \frac{dV_x}{dt}$$

The values of V_x can be obtained by numerical methods



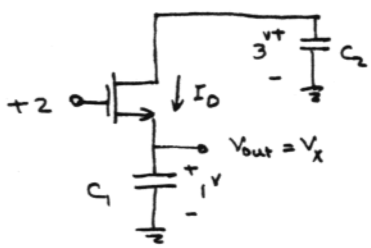
2.10) b



Drain and source exchange their roles.

$(\gamma = \lambda = 0)$ $V_{TH} = 0.7$

⇛



$\int I_D \, dt = q$ $V_x = 1 + \frac{q}{C_1}$, $V_D = V_{C_2} = 3 - \frac{q}{C_2}$

V_x goes up until transistor turns off when $V_x = 1.3$

Assumption: Transistor is in saturation.

This assumption is correct if: $V_D = 3 - \frac{q}{C_2} > 1.3 \quad (2 - 0.7)$

$V_x(\infty) = 1 + \frac{q(\infty)}{C_1} = 1.3$ $V_D(\infty) = 3 - \frac{q(\infty)}{C_2} = 3 - 0.3\frac{C_1}{C_2} > 1.3$

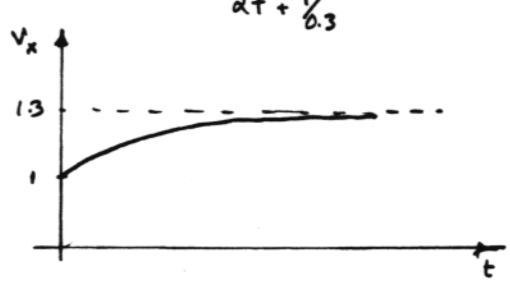
$0.3\frac{C_1}{C_2} < 1.7$ $C_1 < 5.67\, C_2$

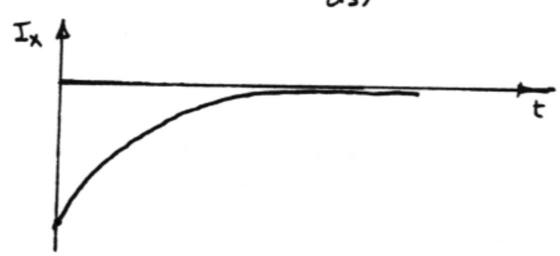
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2 - 1 - \frac{q}{C_1} - 0.7\right)^2 = \frac{dq}{dt}$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{1}{C_1}}_{\alpha} dt = \frac{dq/C_1}{(0.3 - q/C_1)^2} \Rightarrow \alpha t = \frac{1}{0.3 - q/C_1} + K \quad (t=0, q=0)$$

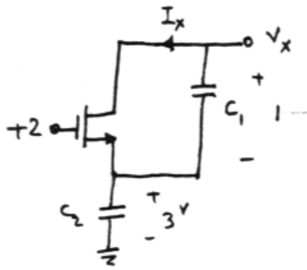
$$\Rightarrow \alpha t = \frac{1}{0.3 - q/C_1} - \frac{1}{0.3} \Rightarrow \frac{q}{C_1} = 0.3 - \frac{1}{\alpha t + \frac{1}{0.3}} \qquad V_x = 1 + \frac{q}{C_1}$$

$$\Rightarrow V_x = 1.3 - \frac{1}{\alpha t + 1/0.3} \qquad I_x = -C_1\frac{dV_x}{dt} = \frac{-\alpha C_1}{(\alpha t + 1/0.3)^2}$$





2.10) c

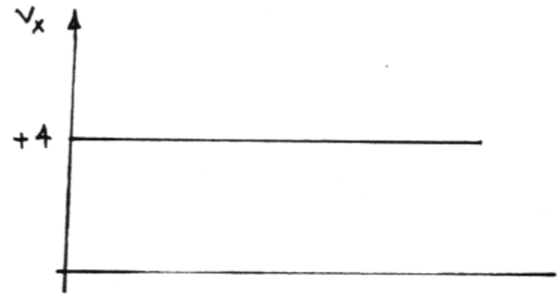


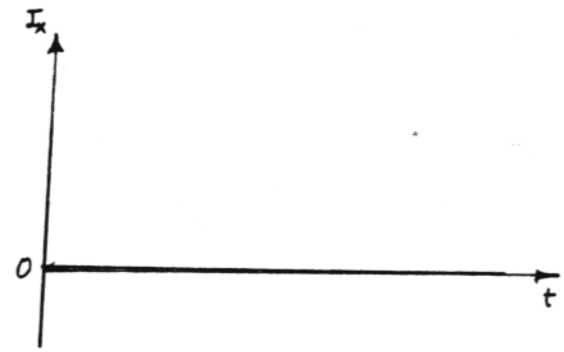
At $t = 0$ $V_G = 2$ $V_S = 3$ $V_D = 4$

Device is off and doesn't turn on.

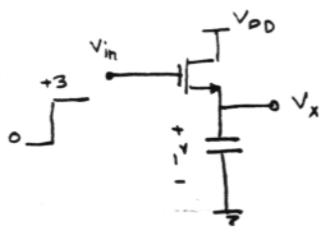
The Circuit remains in this state.

So, $V_x = 4$ $I_x = 0$





2.11) a



$\gamma = \lambda = 0$ $\quad V_{TH} = 0.7$

At $t = 0^+$, device turns on (in Sat) and starts charging the capacitor, until device turns off when; $V_x = V_{in} - V_{TH} = 3 - 0.7 = 2.3$

$$I_c = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2.3 - V_x)^2 \quad ; V_{GS} = 3 - V_x - 0.7$$

$$I_c = C_1 \frac{dV_x}{dt}$$

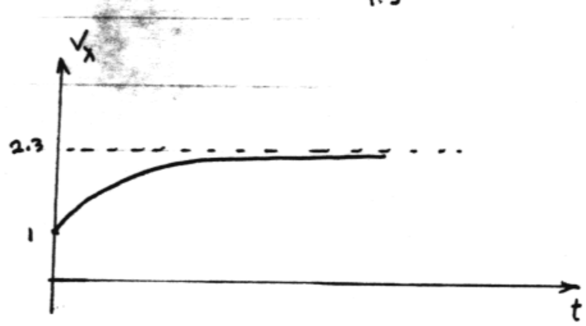
$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L} \times \frac{1}{C_1}}_{\alpha}(2.3 - V_x)^2 = \frac{dV_x}{dt}$$

$$\Rightarrow \alpha\, dt = \frac{dV_x}{(2.3 - V_x)^2} \quad \Rightarrow \alpha t + K_0 = \frac{1}{2.3 - V_x}$$

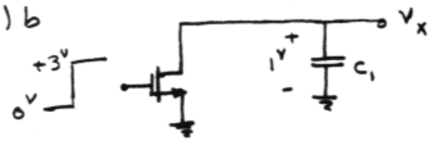
$(t = 0, V_x = 1)$ $\quad \alpha \times 0 + K_0 = \frac{1}{2.3 - 1} \quad \Rightarrow K_0 = 1/1.3$

$$\Rightarrow \frac{1}{1.3} + \alpha t = \frac{1}{2.3 - V_x} \quad \Rightarrow 2.3 - V_x = \frac{1}{\alpha t + 1/1.3}$$

$$\Rightarrow V_x = 2.3 - \frac{1}{\alpha t + 1/1.3}$$



2.11) b



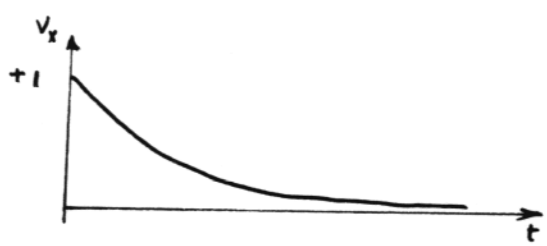
Transistor turns on at $t=0$, and discharges C_1 until $V_x = 0$, (device always operats in triode)

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3-0.7)V_x - V_x^2\right] = -C_1\frac{dV_x}{dt}$$

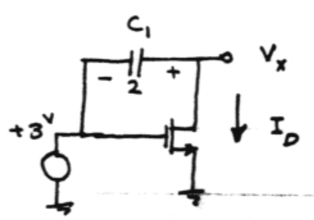
$$\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{C_1}}_{\alpha}\left[4.6V_x - V_x^2\right] = -\frac{dV_x}{dt} \quad \Rightarrow -\alpha\, dt = \frac{dV_x}{V_x(4.6-V_x)}$$

$$\Rightarrow -\alpha t = \left(\frac{1}{V_x} + \frac{1}{4.6-V_x}\right)\frac{1}{4.6} + K \quad , \quad @\, t=0\ , V_x = 1$$

$$\frac{1}{3.6}e^{-\alpha t} = \frac{V_x}{4.6-V_x} \quad \longrightarrow \quad V_x = \frac{4.6}{1+3.6\, e^{4.6\alpha t}}$$



2.11) c



At $t=0^+$, $V_x = 5$, device is in Saturation region

$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3-0.7)^2$, V_x decreases until

$V_x = 2.3$ at $t=t_0$, then device enters triode region

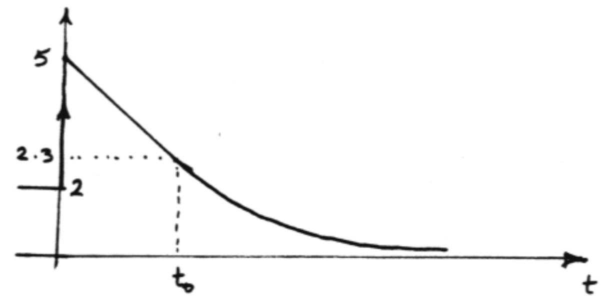
for $t < t_0$ $(V_x > 2.3)$ $\quad V_x = 5 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2.3)^2\, t/C_1$

for $t > t_0$ $\quad I_D = -C_1\frac{dV_x}{dt} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3-0.7)V_x - V_x^2\right]$

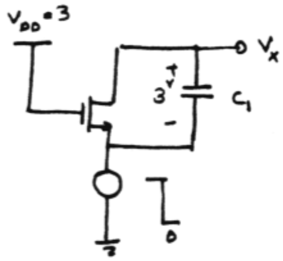
$$\Rightarrow \frac{dV_x}{V_x(4.6-V_x)} = -\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot\frac{1}{C_1}}_{\alpha}\, dt$$

2.11) c, Cont. $-\alpha(t-t_o) = \left[\ln \frac{V_x}{4.6-V_x}\right] \cdot \frac{1}{4.6}$ $t=t_o$, $V_x = 2.3$

$$\Rightarrow V_x = \frac{4.6}{1+e^{4.6\alpha(t-t_o)}}$$



2.11) d



At $t=0^+$, $V_x = 3$ device is in saturation

$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3-0.7)^2$, V_x decreases until

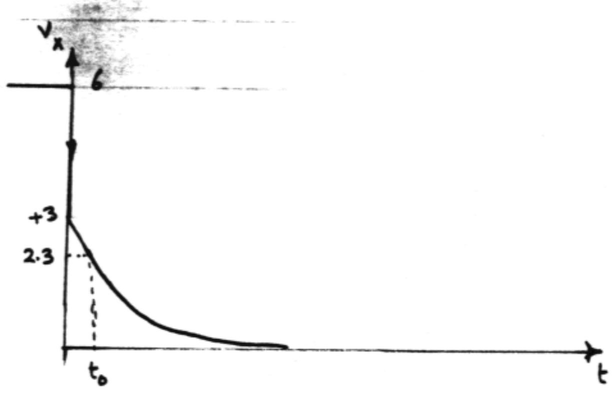
$V_x = 2.3$ at $t=t_o$, then device enters triode region.

for $t<t_o$ $V_x = 3 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2.3)^2\frac{t}{C_1}$; $2.3 < V_x < 3$

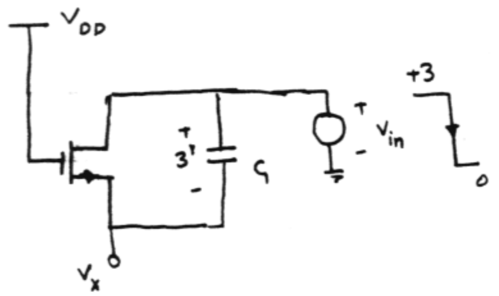
for $t>t_o$ $I_D = -C_1\frac{dV_x}{dt} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3-0.7)V_x - V_x^2\right]$

$$\frac{dV_x}{V_x(4.6-V_x)} = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{1}{C_1}dt \qquad ,(t=t_o\ ,\ V_x=2.3)$$

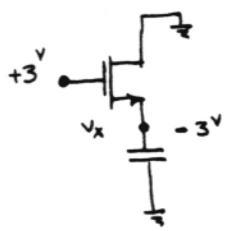
$$-\alpha(t-t_o) = \left[\ln\frac{V_x}{4.6-V_x}\right]\frac{1}{4.6} \quad\Rightarrow\quad V_x = \frac{4.6}{1+e^{4.6\alpha(t-t_o)}}$$



2.12) a)



$t \geq 0^+$



Device is in the triode region.

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\,(2.3 - V_x)(-V_x) - V_x^2\right]$$

$$\begin{cases} V_{GS} = 3 - V_x \\ V_{DS} = -V_x \end{cases}$$

$$I_D = C_1 \frac{dV_x}{dt}$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{C_1}}_{\alpha}\left[V_x^2 - 4.6\,V_x\right] = \frac{dV_x}{dt}$$

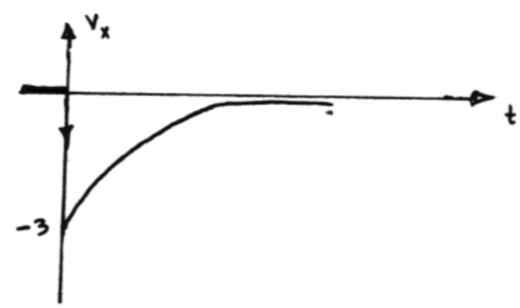
$$\Rightarrow \alpha\, dt = \frac{dV_x}{V_x^2 - 4.6\,V_x} = dV_x\left(\frac{1}{V_x - 4.6} + \frac{-1}{V_x}\right)\times\frac{1}{4.6}$$

$$\Rightarrow 4.6\,\alpha t + K_0 = \ln\left(\frac{V_x - 4.6}{V_x}\right) \quad ; \quad V_x(0^+) = -3$$

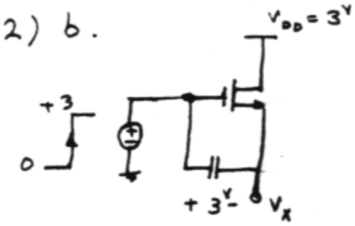
$$\Rightarrow K_0 = \ln\frac{7.6}{3} \quad \Rightarrow \quad \frac{V_x - 4.6}{V_x} = \frac{7.6}{3}e^{4.6\alpha t}$$

$$\Rightarrow \frac{4.6}{V_x} = 1 - \frac{7.6}{3}e^{4.6\alpha t}$$

$$\Rightarrow V_x = \frac{-4.6}{\frac{7.6}{3}e^{4.6\alpha t} - 1}$$

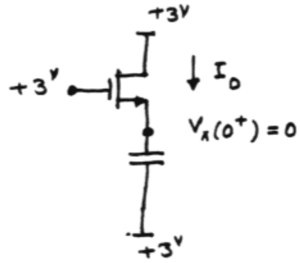


2.12) b.



Device is in saturation region

$t = 0^+$

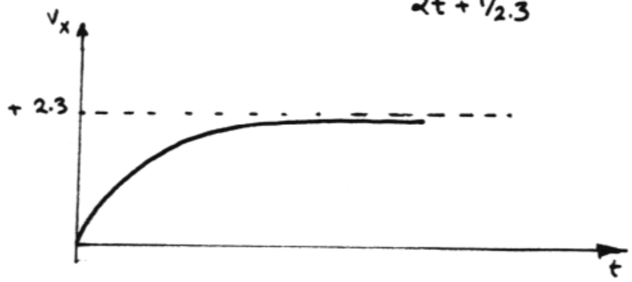


$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3 - V_x - 0.7)^2 = C_1\frac{dV_x}{dt}$$

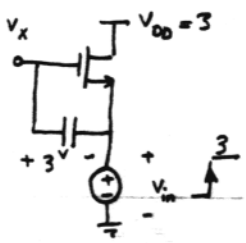
$$\frac{dV_x}{(2.3 - V_x)^2} = \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot\frac{1}{C_1}}_{\alpha}\,dt$$

$$\Rightarrow \frac{1}{2.3 - V_x} = \alpha t + K \qquad (t = 0, V_x = 0) \quad \Rightarrow \frac{1}{2.3 - V_x} - \frac{1}{2.3} = \alpha t$$

$$\Rightarrow V_x = 2.3 - \frac{1}{\alpha t + 1/2.3}$$

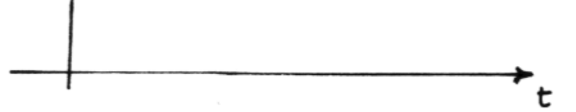


2.12) c

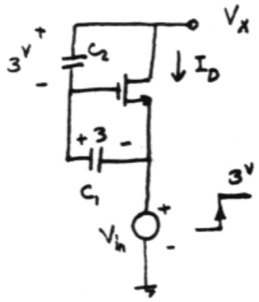


At $t = 0^+$ $\quad V_D = 3 \quad V_S = 3 \quad V_G = 6$

So, $V_{DS} = 0$ and $I_C = I_D = 0$ And circuit remains in this state. $\quad V_x(0^-) = 3 \quad , V_x(t) = 6$



2.12) d



Assume that the device remains in the saturation region until it turns off when $V_{gs} = 0.7$

$$V_{C_1} = V_{gs} = 3 - \frac{1}{C_1}\int I_D\,dt \qquad V_{C_2} = V_{dg} = 3 - \frac{1}{C_2}\int I_D\,dt$$

This assumption is correct if $V_{dg} > -0.7$ when $V_{gs} = 0.7$

$$\int I_D\,dt = q(t) \qquad V_{gs} = 3 - \frac{q}{C_1} = 0.7 \Rightarrow \frac{q}{C_1} = 2.3 \qquad V_{dg} = 3 - \frac{q}{C_2} > -0.7$$

$$\Rightarrow \frac{q}{C_2} < 3.7 \qquad 2.3\frac{C_1}{C_2} < 3.7 \qquad \Rightarrow C_1 < 1.61\,C_2$$

With this assumption,

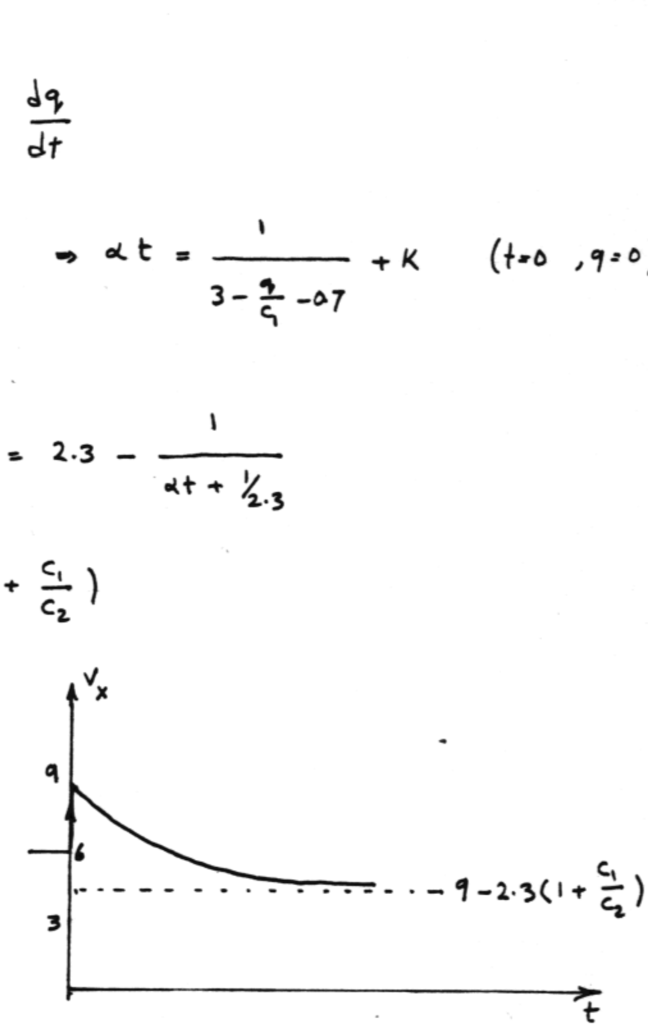
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(3 - \frac{q}{C_1} - 0.7\right)^2 = \frac{dq}{dt}$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot\frac{1}{C_1}}_{\alpha}\,dt = \frac{dq/C_1}{\left(3 - \frac{q}{C_1} - 0.7\right)^2} \Rightarrow \alpha t = \frac{1}{3 - \frac{q}{C_1} - 0.7} + K \qquad (t=0\ ,q=0)$$

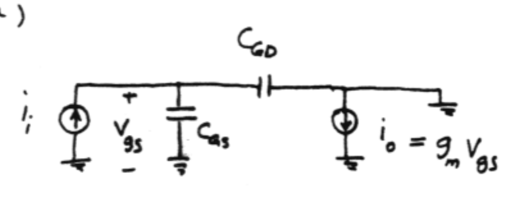
$$\Rightarrow \alpha t = \frac{1}{2.3 - \frac{q}{C_1}} - \frac{1}{2.3} \qquad \Rightarrow \frac{q}{C_1} = 2.3 - \frac{1}{\alpha t + 1/2.3}$$

$$V_X = 3 + 3 - \frac{q}{C_1} + 3 - \frac{q}{C_2} = 9 - \frac{q}{C_1}\left(1 + \frac{C_1}{C_2}\right)$$

$$V_X(t) = 9 - \left(1 + \frac{C_1}{C_2}\right)\frac{2.3\,\alpha t}{\alpha t + 1/2.3}$$



2.13) a)



$$i_i = (C_{GS} + C_{GD}) s \, V_{gs}$$

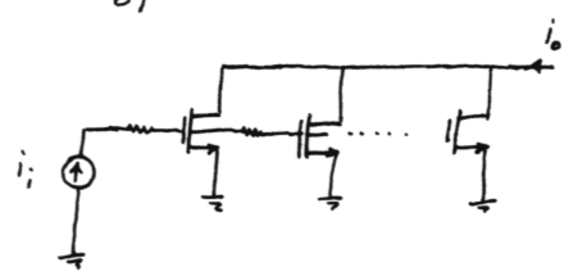
$$i_o = g_m V_{gs}$$

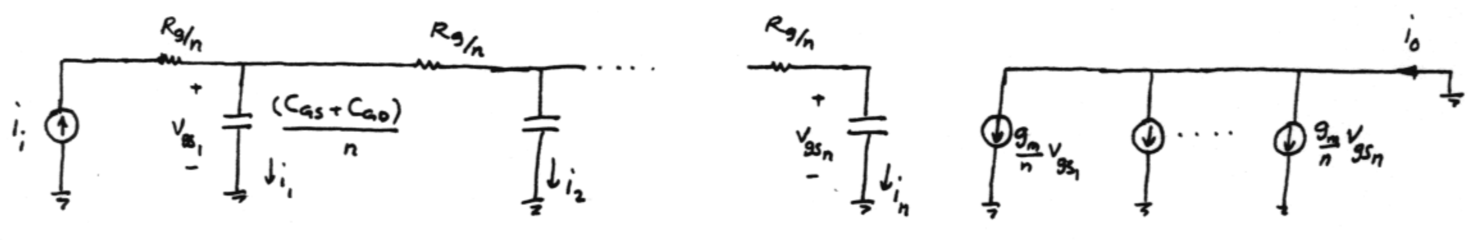
$$\beta = \frac{i_o}{i_i} = \frac{g_m}{(C_{GS} + C_{GD}) s} \quad ; \quad |\beta| = 1 \quad \Rightarrow \quad \frac{g_m}{(C_{GS} + C_{GD}) \omega_T} = 1$$

$$\Rightarrow \omega_T = \frac{g_m}{(C_{GS} + C_{GD})} \quad \rightarrow \quad f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi (C_{GS} + C_{GD})}$$

Approximation : $g_m v_{gs}$ is the output current.

b)





$$i_K = \frac{1}{n} (C_{GS} + C_{GD}) s \, V_{gsK} \qquad K = 1 \ldots n$$

$$(*) \quad i_i = i_1 + i_2 + \cdots + i_n = \frac{1}{n} (C_{GS} + C_{GD}) s \, (V_{gs_1} + V_{gs2} + \cdots + V_{gsn})$$

$$(**) \quad i_o = \frac{g_m}{n} V_{gs1} + \cdots + \frac{g_m}{n} V_{gsn} = \frac{g_m}{n} (V_{gs1} + V_{gs2} + \cdots + V_{gsn})$$

$$(*), (**) \Rightarrow \beta = \frac{i_o}{i_i} = \frac{g_m}{(C_{GD} + C_{GS}) s} \quad ; \quad |\beta| = 1 \quad \Rightarrow f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi (C_{GS} + C_{GD})}$$

c) $$f_T = \frac{g_m}{2\pi(C_{GS} + C_{GD})}$$

$$g_m = \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$C_{GS} + C_{GD} \simeq C_{ox} WL$$

$$\Longrightarrow \quad f_T = \frac{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}{2\pi C_{ox} WL} \simeq \frac{\mu}{2\pi} \frac{(V_{GS} - V_{TH})}{L^2}$$

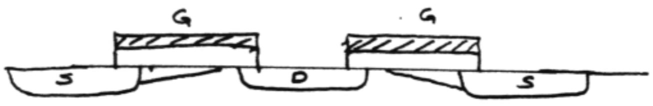
2.14)

$$f_T = \frac{g_m}{2\pi(C_{GS} + C_{GD})} \quad ; \quad g_m = \frac{I_D}{\xi V_T}$$

In the subthreshold $C_{GS} = C_{GD} = W C_{ov}$ (Fig 2.?33)

So, $$f_T = \frac{I_D / \xi V_T}{4\pi W C_{ov}} = \frac{I_D}{4\pi \xi V_T W L_D C_{ox}}$$

2.15)



$$\begin{cases} C_{j0} = 0.56e{-3}\ F/m^2 & C_{jsw0} = 0.35e{-11}\ F/m \\ m_j = 0.6 & m_{jsw} = 0.2 \end{cases}$$

$$C_{DB} = \frac{W}{2} E C_j + 2\left(\frac{W}{2} + E\right) C_{jsw} \qquad C_j = \frac{C_{j0}}{\left(1 + \frac{V_R}{2\varphi_F}\right)^m}$$

$$C_{SB} = 2\left[\frac{W}{2} E C_j' + 2\left(\frac{W}{2} + E\right) C_{jsw}'\right]$$

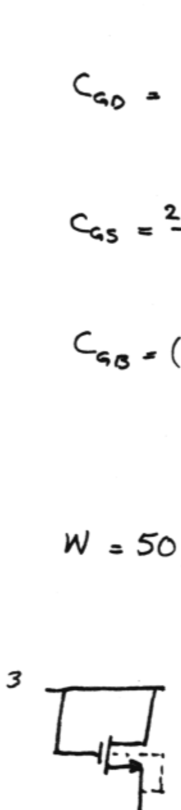
$$C_{GD} = 2\left(\frac{W}{2} C_{ov}\right) \qquad C_{ov} = L_D C_{ox}$$

$$C_{GS} = \frac{2WLC_{ox}}{3} + WC_{ov}$$

$$C_{GB} = (WLC_{ox})C_d / (WLC_{ox} + C_d) \ ; \ C_d = WL\sqrt{q\epsilon_{si} N_{sub} / 2\varphi_F}$$

$W = 50\mu \qquad L = 0.5\mu \qquad E = 1.5^{\mu}$

3



$$I = \frac{1}{2}\mu_n C_{ox} \frac{W}{L - 2L_D}(V_{GS} - V_{TH})^2 \ , \ 1^{mA} = \frac{1}{2} \times 0.13429 \times \frac{50}{0.5 - 0.16}(V_{GS} - 0.7)^2$$

$$V_{GS} = 1.0182 \qquad g_m = \frac{2I_D}{V_{GS} - V_{TH}} = 6.285\ mA/V \ , \quad V_{DB} = 1.0182$$

$\lambda = 0$, $L_D = 0.08\ \mu m$

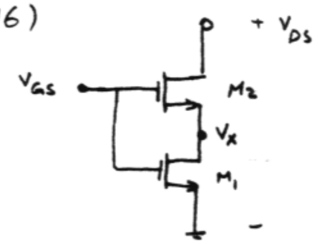
$\frac{W}{L} = \frac{50\mu}{0.5\mu}$, $V_{TH} = 0.7$ $\qquad C_{GD} = 15.4\ fF \qquad C_{GS} = 79.36\ fF$

$\mu_n C_{ox} = 134.29\ \mu A/V^2$

$C_{ox} = 3.84 \times 10^{-3}\ F/m^2 \qquad C_{SB} = 42.4\ fF \qquad C_{DB} = 13.5\ fF$

$$f_T = \frac{g_m}{2\pi(C_{GD} + C_{GS})} = 10.6\ GHz$$

2.16)



CASE I, M_1: Triode M_2: Triode

$$V_{OD1} = V_{GS} - V_{TH} \qquad V_{OD2} = V_{GS} - V_X - V_{TH}$$

$$V_{DS1} = V_X \qquad V_{DS2} = V_{DS} - V_X$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_X - V_X^2\right] \qquad (*)$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH}-V_X)(V_{DS}-V_X) - (V_{DS}-V_X)^2\right]$$

$$I_{D1} = I_{D2} \Rightarrow 2(V_{GS}-V_{TH})V_X - V_X^2 = 2(V_{GS}-V_{TH})V_{DS} + 2V_X^2 - 2V_X(V_{GS}-V_{TH}) - 2V_XV_{DS} - V_{DS}^2 - V_X^2 + 2V_XV_{DS}$$

$$\Rightarrow 2\left[2(V_{GS}-V_{TH})V_X - V_X^2\right] = 2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2 \qquad (**)$$

$$(*),(**) \Rightarrow I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{2}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right] \quad \left(\frac{W}{2L} \text{ in Triode}\right)$$

CASE II, M_1: Triode, M_2: Sat

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_X - V_X^2\right] \qquad (*)$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_X-V_{TH})^2$$

$$I_{D1} = I_{D2} \Rightarrow V_X^2 - 2V_X(V_{GS}-V_{TH}) + (V_{GS}-V_{TH})^2 = 2(V_{GS}-V_{TH})V_X - V_X^2$$

$$\Rightarrow (V_{GS}-V_{TH})^2 = 2\left[2(V_{GS}-V_{TH})V_X - V_X^2\right] \qquad (**)$$

2.16) Cont. (*), (**) $\Rightarrow$ $I_{D_1} = I_{D_2} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \times \frac{1}{2} (V_{GS} - V_{TH})^2$ $\left(\frac{W}{2L} \text{ in Sat}\right)$

Note that M_1 is always in triode, because V_{OD2} is always positive

i.e. $V_{GS2} - V_{TH} > 0 \Rightarrow V_{GS} - V_X - V_{TH} > 0 \Rightarrow V_{GS} - V_{TH} > V_X$

$\Rightarrow V_{GS_1} - V_{TH} > V_{DS_1} \Rightarrow$ M_1 is in the triode region.

Saturation – triode transition edge of M_2:

We show that the transition point the saturation and triode region of the equivalent transitor is the same as that of M_2.

$V_{OD2} = V_{GS} - V_X - V_{TH}$ $\quad V_{DS2} = V_{DS} - V_X$

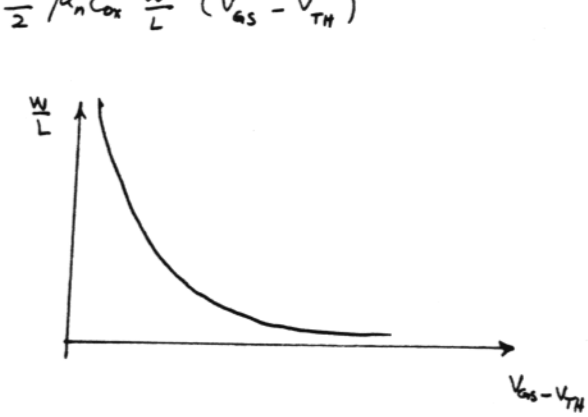
for $V_{OD2} > V_{DS2}$, M_2 is in the triode region, i.e. $V_{GS} - V_{TH} > V_{DS}$

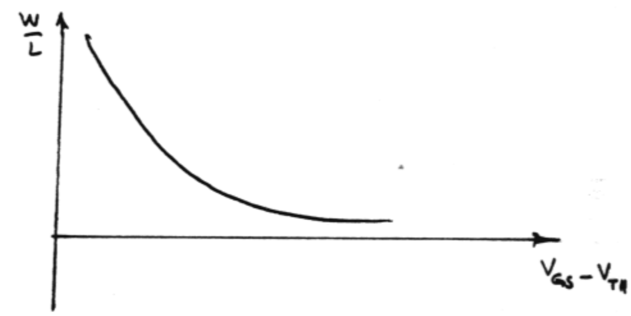
It means that when M_2 is in the saturation, then the equivalent transistor is in the saturation, and vice versa.

2.17)

In Saturation region, $I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$

$$\Rightarrow \frac{W}{L} = \frac{2 I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2}$$

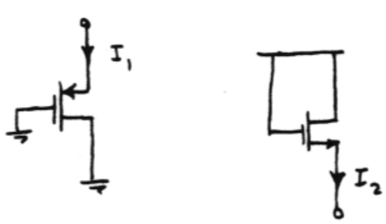




$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$\Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox} (V_{GS} - V_{TH})}$$

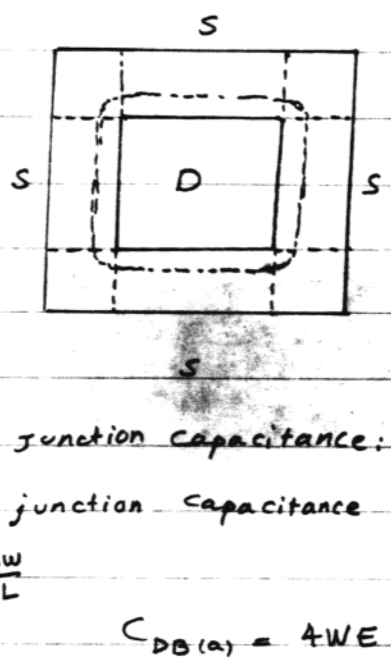
2.18)



These structures cannot operate as current sources, because their currents strongly depend on source voltages, but an ideal current source should provide a constant current, independent of its voltage.

2.19) From Eq. (2.1) we know that $V_{TH} = \varphi_{MS} + 2\varphi_F + \frac{Q_{dep}}{C_{ox}}$, where φ_{MS} and φ_F are constant values, So any changes in V_{TH} Come from the third term, infact $\Delta V_{TH} = \frac{\Delta Q_{dep}}{C_{ox}}$ and From Eq (2.22), we have $\Delta V_{TH} = \gamma(\sqrt{2\varphi_F + V_{SB}} - \sqrt{2\varphi_F})$ (infact, this is definition of γ). from pn junction theory we know that Q_{dep} is proportional to $\sqrt{N_{sub}}$, So γ is directly proportional to $\sqrt{N_{sub}}$ and inversely proportional to C_{ox}.

2.20)



This structure operates as a traditional device does, infact if we neglect edges we have four Mosfets in parallel, Where the aspect ratio of each is $\frac{W}{L}$. So the overal aspect ratio is almost $\frac{4W}{L}$

Drain junction capacitance: $C_{DB} = W^2 C_J + 4W C_{JSW}$

Drain junction capacitance of devices shown in fig 2.32 a, b for the aspect ratio of $\frac{4W}{L}$

$$C_{DB(a)} = 4WE\,C_J + (8W + 2E)C_{jsw}$$

$$C_{DB(b)} = 2WE\,C_J + (4W + 2E)C_{jsw}$$

The value of side wall capacitance in the ring structure is less than that in folded and traditional structures, but the bottom capacitance of ring structure is higher than that of the other two structures. (for $W > 4E$)

2.21) We first check the terminals of the device with a multimeter in order to find BS or BD junctions. There are 12 experiments in total of which two lead to conduction and remaining ones show no conduction. If we find one of those two conductions then we are done. Finding B and S (or D), we need to do one other experiment between B (Cathode of junction) and one of the two remaining terminals; In case of no Connection, the terminal under test is G, otherwise it is D (or S). In worst case with a maximum of 8 experiments, each terminal can be specified. It is as follows:

Assume, the two selected terminals do not Conduct in both directions and this is the case for the other two terminals. Upto this point, four experiments have been done while not yet encountering any Conduction. It is clear that one group Consists of

G and B and the other Comprises from D and S, Because at least One Conduction should be observed if B were in the same group with one of the source or Drain. In the next step, we pick up one terminal from each group to undergo the conductivity test. Assume, no Conduction happens in either direction (Worst case). It means that we had chosen G from (G B) group. Thus for, we have done six experiments. We change both of terminals and now we have chosen B for sure. and in worst case, we will find a connection in 8th experiment. Now, we know B and S (D), Bulk's groupmate is Gate and Source's (Drain's) groupmate is Drain (Source).

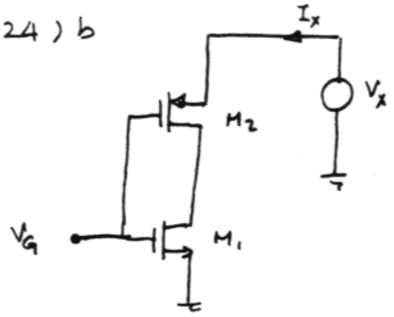
2.22) If we don't know the type of device, In eight experiment we cannot distinguish between B and S (D) and we should perform another experiment, which is exchanging one of

2.22) Cont. terminals with its groupmate. If we still had the Conduction then the exchanged terminal and its groupmate are Source and Drain, otherwise the exchanged terminal is Bulk.

2.23) a) NO, Because in DC model equations of MOSFET, we always have the product of $\mu_n C_{ox}$ and $\frac{W}{L}$.

b) NO, Because we cannot obtain as many independent equations as the unknown quantities. But if the difference between the aspect ratios is known, then $\mu_n C_{ox}$ and both $\frac{W}{L}$, are attainable.

2.24) b



CASE I : $V_G < V_{THN} \Rightarrow M_1$: off $I_x = 0$

$g_m = 0$

CASE II : $V_G > V_{THN}$

for $0 < V_X < V_G + |V_{THP}| \Rightarrow I_x = 0$ (M_2 : off) $g_m = \frac{\partial I_x}{\partial V_G} = 0$

Then M_2 turns on (in sat) , M_1 still is in triode region

$$I_x = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_X - V_G - |V_{THP}|)^2$$

This is correct until M_1 , goes into saturation , when

$$\frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_X - V_G - |V_{THP}|)^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_N (V_G - V_{THN})^2$$

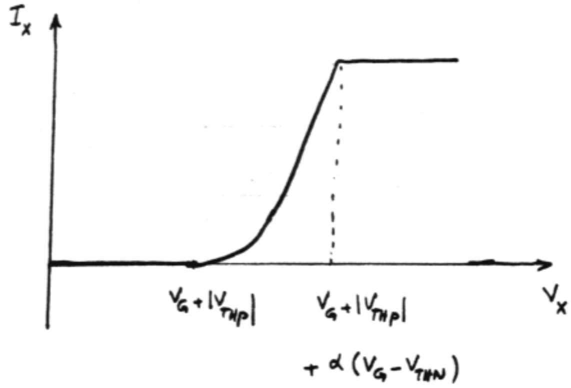
i.e. $$V_X = V_G + |V_{THP}| + \underbrace{\sqrt{\frac{\mu_n}{\mu_p}\frac{(W/L)_n}{(W/L)_p}}}_{\alpha} (V_G - V_{THN})$$

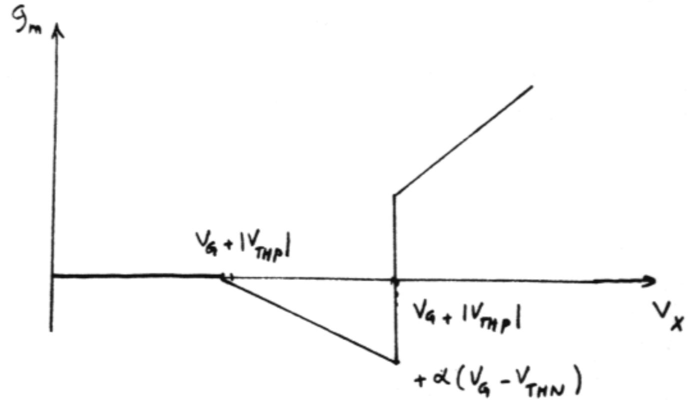
And afterward , M_2 goes into triode region and $I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_N (V_G - V_{THN})^2$

So, $0 < V_X < V_G + |V_{THP}| \Rightarrow I_x = 0$ $g_m = \frac{\partial I_x}{\partial V_G} = 0$

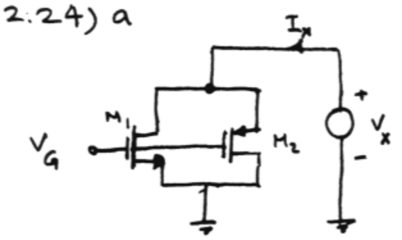
$V_G + |V_{THP}| < V_X < V_G + |V_{THP}| + \alpha (V_G - V_{THN})$ $I_x = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_X - V_G - |V_{THP}|)^2$ $g_m = \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_G + |V_{THP}| - V_X)$

$V_G + |V_{THP}| + \alpha (V_G - V_{THN}) < V_X$ $I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_N (V_G - V_{THN})^2$ $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_N (V_G - V_{THN})$





2.24) a

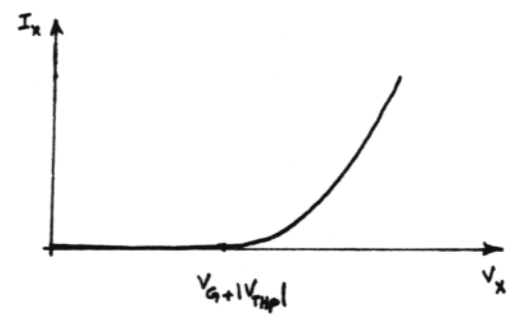


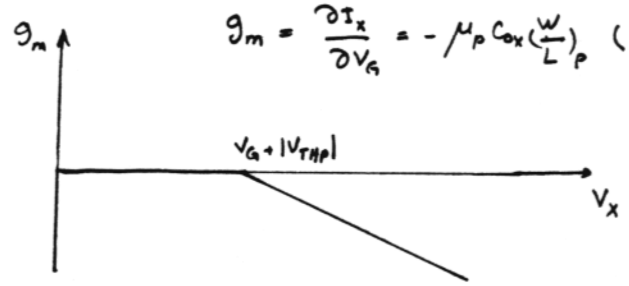
CASE I : $V_G < V_{THN}$ $\quad M_1$: off

for $0 < V_x < V_G + |V_{THP}|$ $\quad I_x = 0$, $g_m = \frac{\partial I_x}{\partial V_G} = 0$

for $V_G + |V_{THP}| < V_x$ $\Rightarrow I_x = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)^2$

$$g_m = \frac{\partial I_x}{\partial V_G} = -\mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)$$





CASE II : $V_G > V_{THN}$

for $0 < V_x < V_G - V_{THN}$ $\quad$ (M_2 : off $\quad M_1$: triode)

$$I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n \left[2 (V_G - V_{THN}) V_x - V_x^2\right] \qquad g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_n V_x$$

for $V_G - V_{THN} < V_x < V_G + |V_{THP}|$ $\quad$ (M_2 : off $\quad M_1$: Sat)

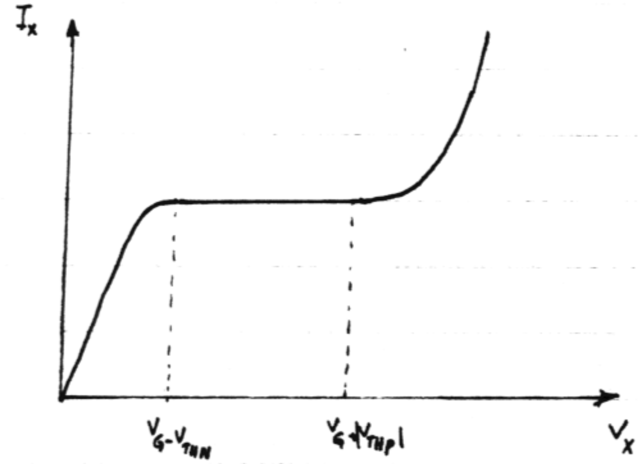
$$I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_G - V_{THN})^2 \qquad g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_G - V_{THN})$$

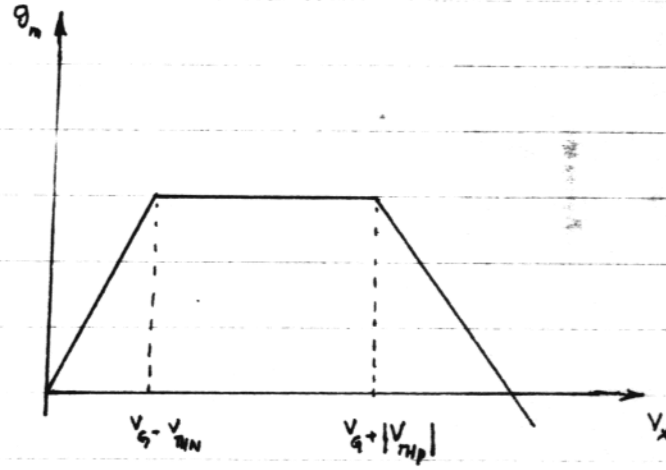
for $V_G + |V_{THP}| < V_x$ $\quad$ (M_2 : Sat $\quad M_1$: Sat)

2.24) a Cont.

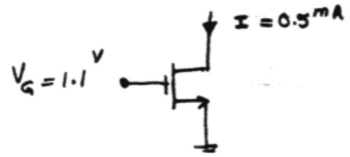
$$I_x = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_G - V_{THN})^2 + \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)^2$$

$$g_m = \frac{\partial I_x}{\partial V_G} = \mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_G - V_{THN}) - \mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)$$





2.25) $V_{TH} = 0.7$ $\lambda = 0.1$ (for $L = 0.5^{\mu}$)



for $L = 0.5^{\mu}$ $\lambda = 0.1$ $\rightarrow r_o = \frac{1}{\lambda I_D} = 20^{K\Omega}$

$$V_{OD} = V_{GS} - V_{TH} = 0.4 \Rightarrow V_{GS} = 1.1^{V}$$

Calculating W, $I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2$

$$0.5^{mA} = \frac{1}{2} \times 0.1343 \frac{mA}{V^2} \times \frac{W}{0.5^{\mu} - 0.16^{\mu}} \times (0.4)^2$$

$\frac{W}{L_{eff}} \simeq 47$ $\Rightarrow$ $W = 15.82\ \mu m$

$$C_{gs} = \frac{2}{3} WL C_{ox} + W C_{ov} \simeq 25\ fF$$

$$C_{GD} = W C_{ov} = 4.85\ fF$$

$$C_{DB} = \frac{W}{2} E C_j + 2\left(\frac{W}{2} + E\right) C_{jsw} \quad (@\ V_D = 0.4) = 10.7\ fF$$

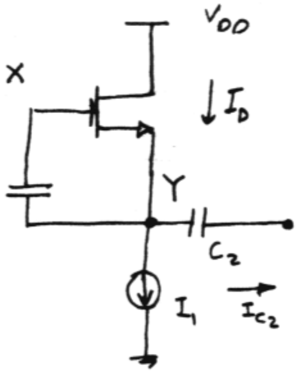
(for folded structure)

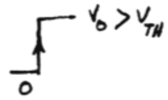
$$\left(C_j = \frac{C_{j0}}{\left(1 + \frac{V_{DB}}{2\varphi_F}\right)^{m_j}} = 0.449 \times 10^{-3} \frac{F}{m^2} \quad , \quad C_{jsw} = \frac{C_{jsw0}}{\left(1 + \frac{V_{DB}}{2\varphi_F}\right)^{m_{jsw}}} = 0.325 \times 10^{-11} F/m \right)$$

$C_{ox} = 3.84 \times 10^{-3}\ F/m$ $C_{j0} = 0.56 \times 10^{-3}$ $m_j = 0.6$

$C_{jsw_0} = 0.35 \times 10^{-11}$ $m_{jsw_0} = 0.2$

2.26) a





Before applying the pulse

$$X(0^-) = V_{DD}$$

$$Y(0^-) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}}$$

After Applying the Pulse

$$X(0^+) = V_{DD} + V_0$$

$$Y(0^+) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} + V_0$$

for $t > 0$

$$\begin{cases} X(t) = V_{DD} + \alpha(t) \\ Y(t) = V_{DD} - V_{TH} - \sqrt{\dfrac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} + \alpha(t) \end{cases}$$

$\alpha(0^+) = V_0$, Device is in triode

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right] = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} - \left(V_{TH} + \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} - \alpha(t) \right) \right] \left(V_{TH} + \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} - \alpha(t) \right)$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}} - (\alpha(t) - V_{TH})^2 \right] = I_1 - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (\alpha(t) - V_{TH})^2$$

$$I_{C_2} = I_D - I_1 = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (\alpha(t) - V_{TH})^2 = C_2 \frac{dV_{C_2}}{dt} = C_2 \frac{d\alpha(t)}{dt}$$

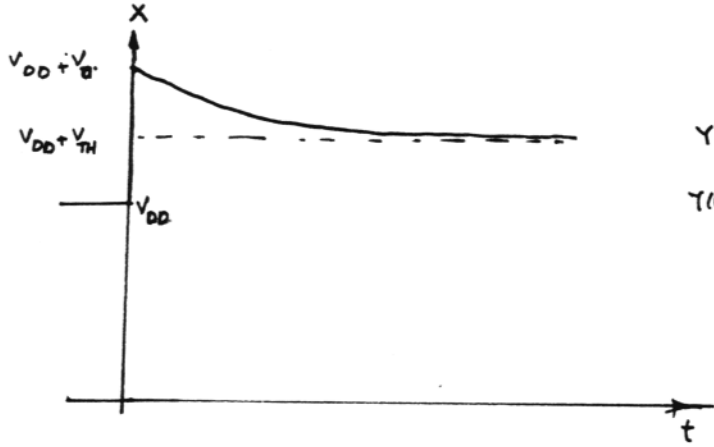
$$\underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot \frac{1}{C_2}}_{K} dt = \frac{-d\alpha}{(\alpha - V_{TH})^2} \Rightarrow Kt = \frac{1}{\alpha - V_{TH}} - \frac{1}{V_0 - V_{TH}}$$

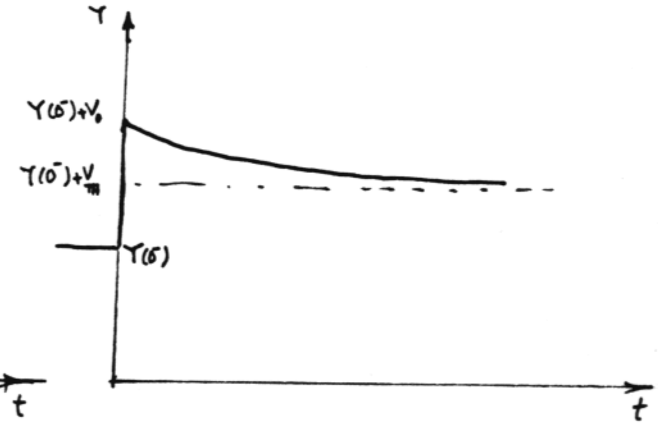
$$\Rightarrow \alpha(t) = V_{TH} + \frac{1}{Kt + \frac{1}{V_0 - V_{TH}}} \qquad \alpha(\infty) = V_{TH}$$

2.26) a Cont.

$$X(\infty) = V_{DD} + V_{TH}$$

$$Y(\infty) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} + V_{TH} = V_{DD} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}}$$





2.26) b

Before applying the pulse.

$$X(0^-) = V_{DD}$$

$$Y(0^-) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}}$$

After applying the pulse

$$X(0^+) = V_{DD} - V_{TH}$$

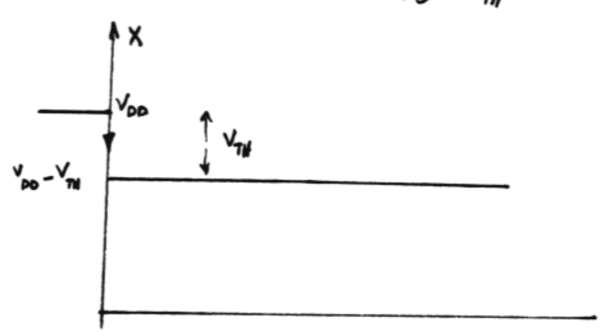
$$Y(0^+) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} - V_{TH}$$

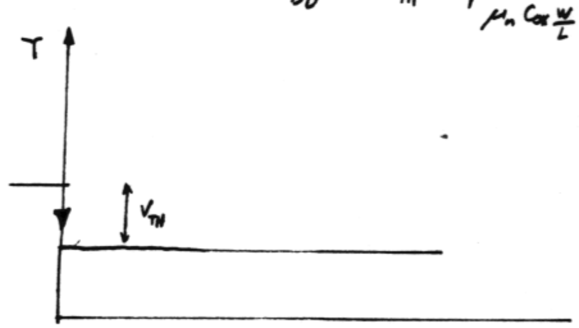
After applying the pulse, device remains in the saturation region, and its current doesn't chang, So, $I_{C_1} = I_{C_2} = 0$

Therefore, the circuit keeps its state.

$$X(t) = X(0^+) = V_{DD} - V_{TH}$$

$$Y(t) = Y(0^+) = V_{DD} - 2V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}}$$





2.27)

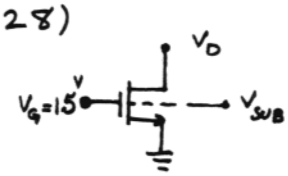
$$I_D = I_0 \exp \frac{V_{GS}}{\xi V_T}$$

$$\frac{I_{D2}}{I_{D1}} = \exp \frac{V_{GS2} - V_{GS1}}{\xi V_T} \qquad \frac{I_{D2}}{I_1} = 10 \quad \Rightarrow \quad \Delta V_{GS} = \xi V_T \ln 10$$

$$\Delta V_{GS} = 1.5 \times \ln 10 \times 26^{mV} = 89.8 \, mV$$

$$g_m = \frac{I_D}{\xi V_T} = \frac{10^{\mu A}}{1.5 \times 26^{mV}} = 0.26 \, mA/V$$

2.28)



a) If we decrease V_D below zero, Source and drain exchange their roles and device operates in the triode region.

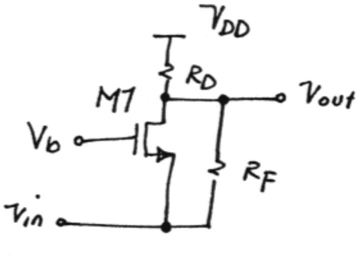
b) If we increase V_B, V_{TH} decreases, because

$$\Delta V_{TH} = \gamma \left(\sqrt{2\varphi_F - V_B} - \sqrt{2\varphi_F} \right) \text{ is negative.}$$

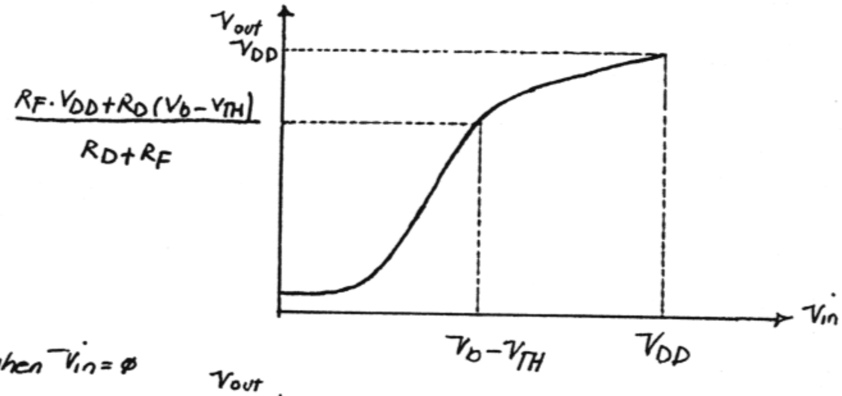
Therefore, I_D increases.

Chapter 3

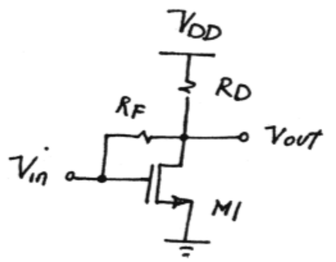
3.1.



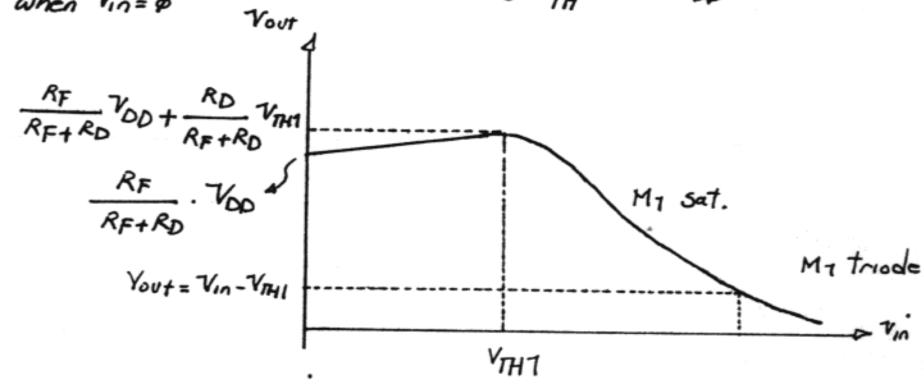
(a)



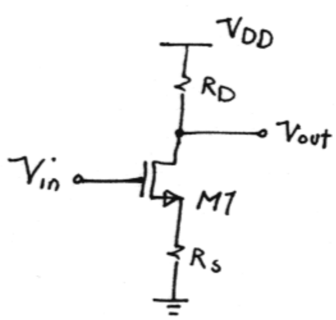
We assume that M_1 is saturated when $V_{in} = \phi$



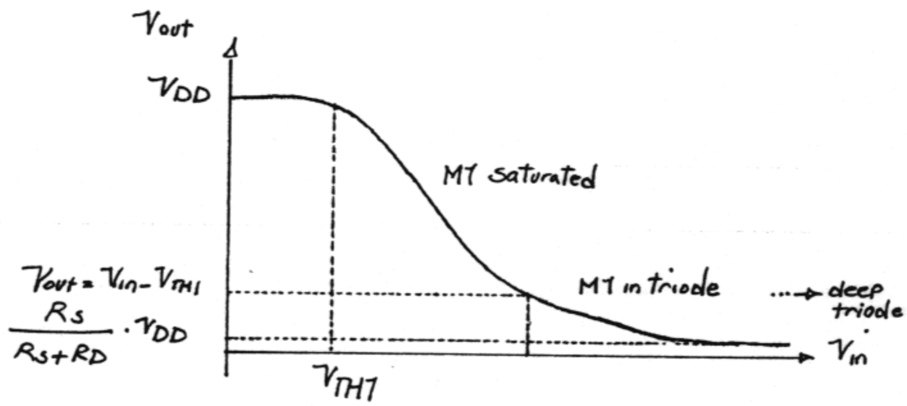
(b)

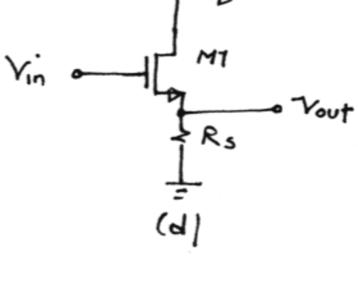


if $V_{in} < V_{TH1} \rightarrow V_{out} = \frac{R_F}{R_F+R_D} V_{DD} + \frac{R_D}{R_F+R_D} V_{in}$

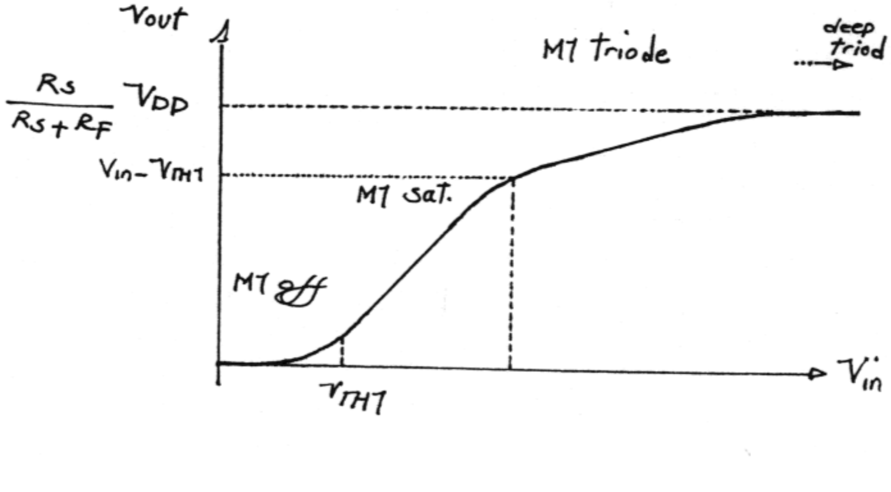


(c)

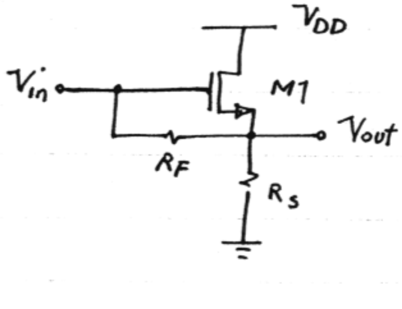




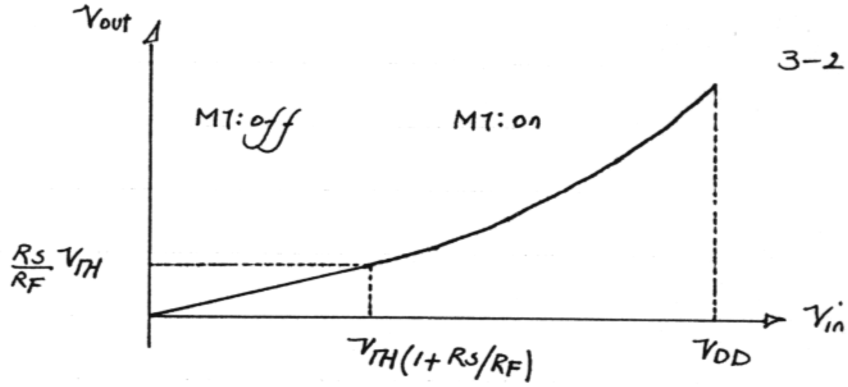
(d)



3-2

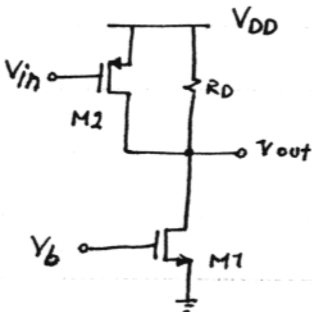


(e)

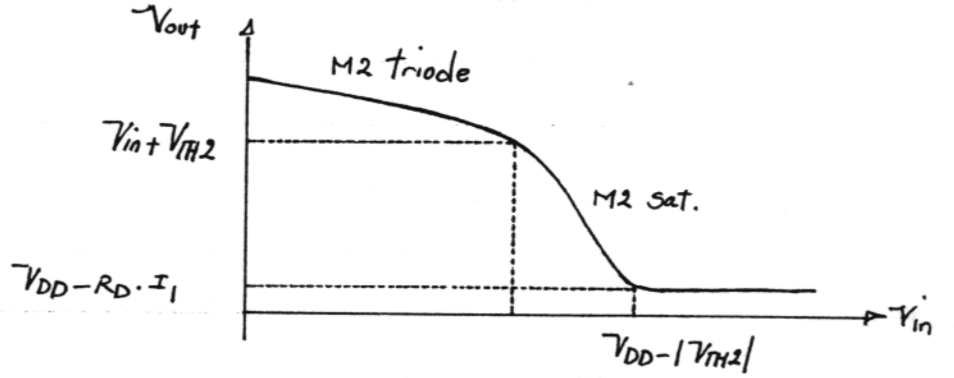


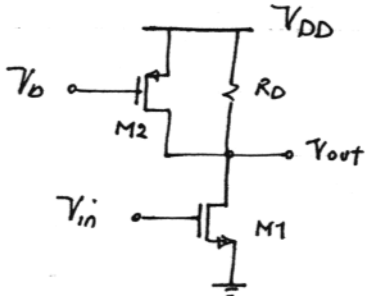
if $V_{in} < (1+R_S/R_F)V_{TH} \rightarrow V_o = \frac{R_S}{R_S+R_F} V_{in}$

3.2.

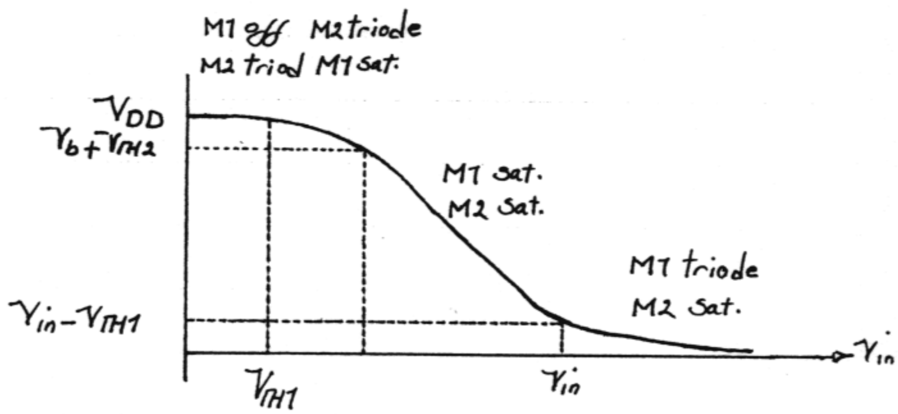


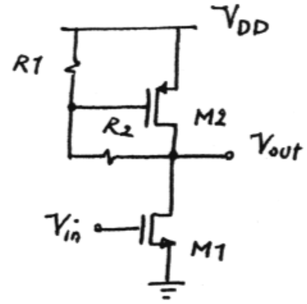
(b)





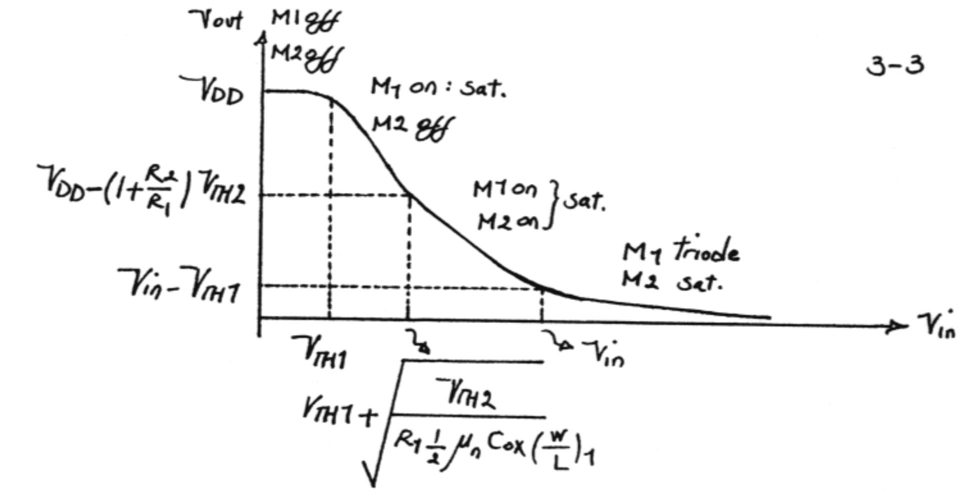
(a)



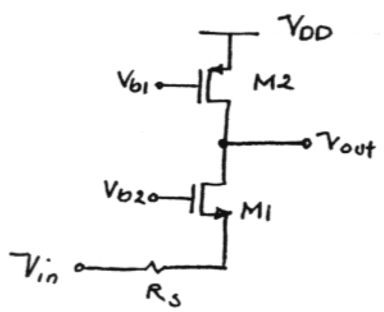


VDD
R1
R2
M2
Vout
Vin
M1

(c)

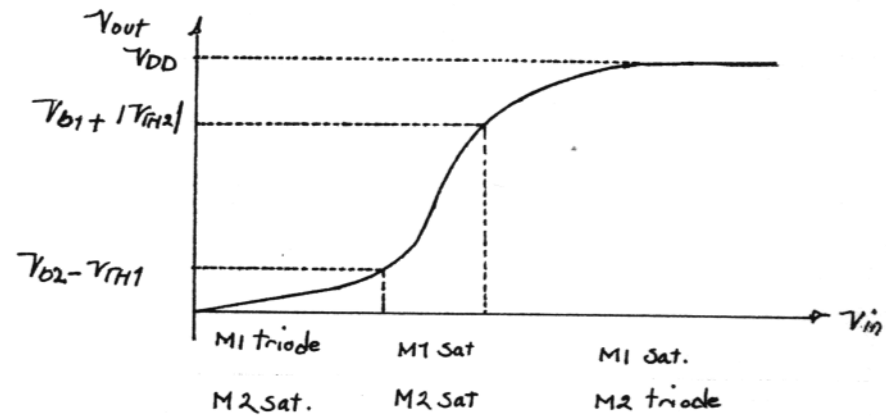


3-3
Vout
M1 off
M2 off
VDD
M1 on : Sat.
M2 off
VDD-(1+R2/R1) VTH2
M1 on
M2 on
Sat.
M1 triode
M2 Sat.
Vin-VTH1
Vin
VTH1
Vin
VTH1 + √(VTH2 / (R1 1/2 μn Cox (W/L)1))



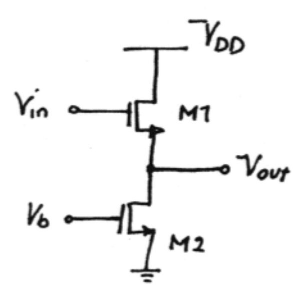
VDD
Vb1
M2
Vout
Vb2
M1
Vin
Rs

(d)



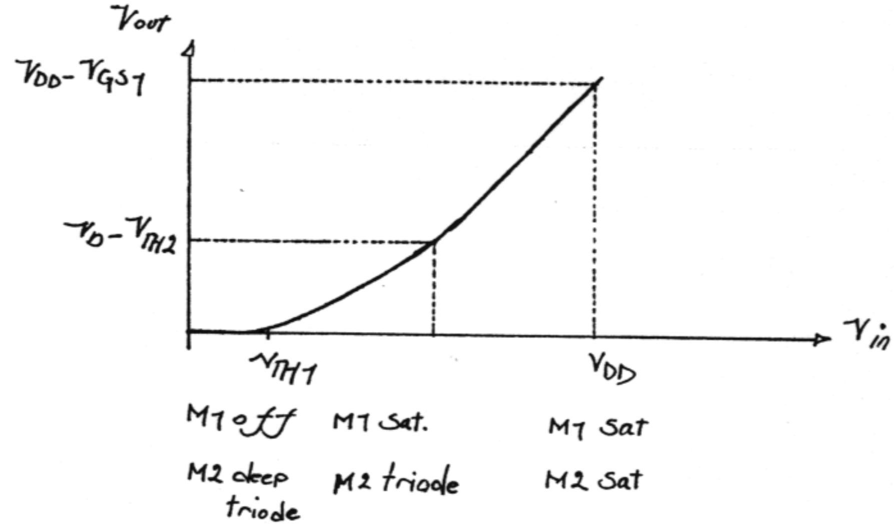
Vout
VDD
Vb1 + |VTH2|
Vb2 - VTH1
Vin
M1 triode
M2 Sat.
M1 Sat
M2 Sat
M1 Sat.
M2 triode

3.2:

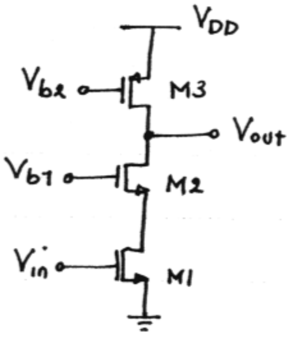


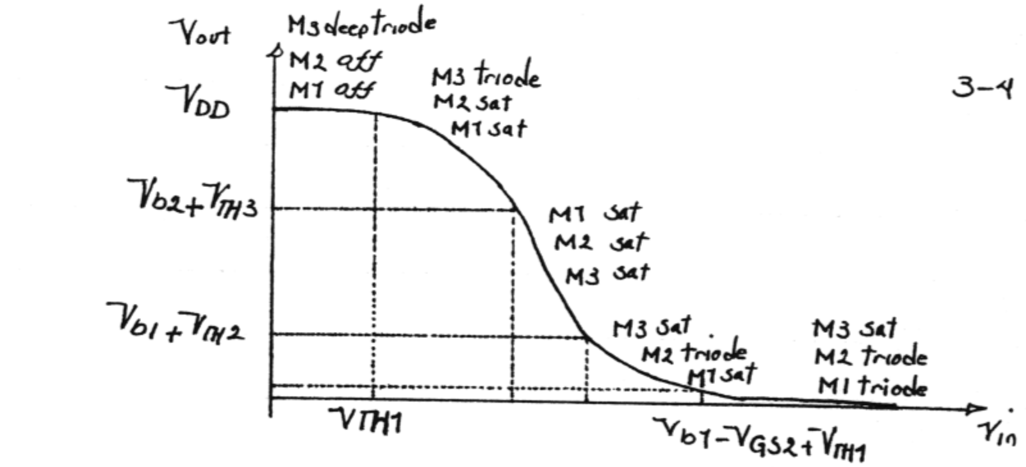
VDD
Vin
M1
Vout
Vb
M2

(a)

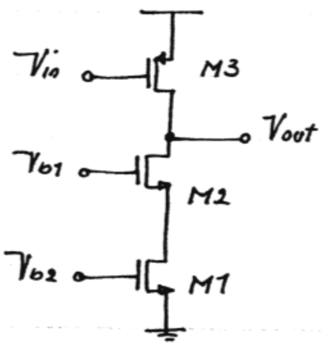


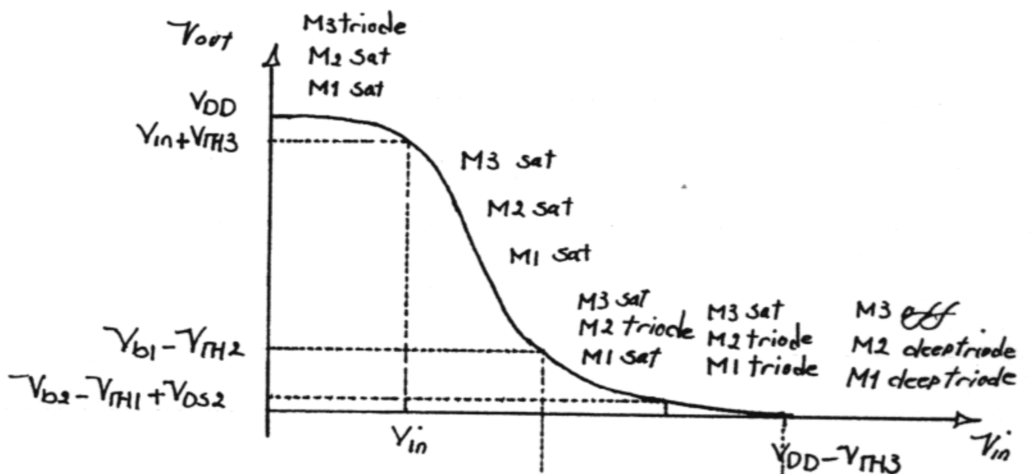
Vout
VDD - VGS1
Vb - VTH2
Vin
VTH1
VDD
M1 off
M2 deep triode
M1 Sat.
M2 triode
M1 Sat
M2 Sat





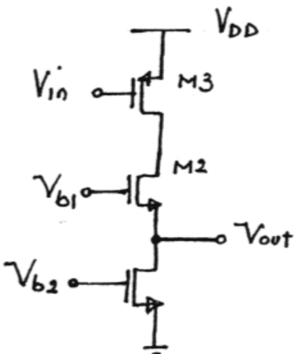
(b)

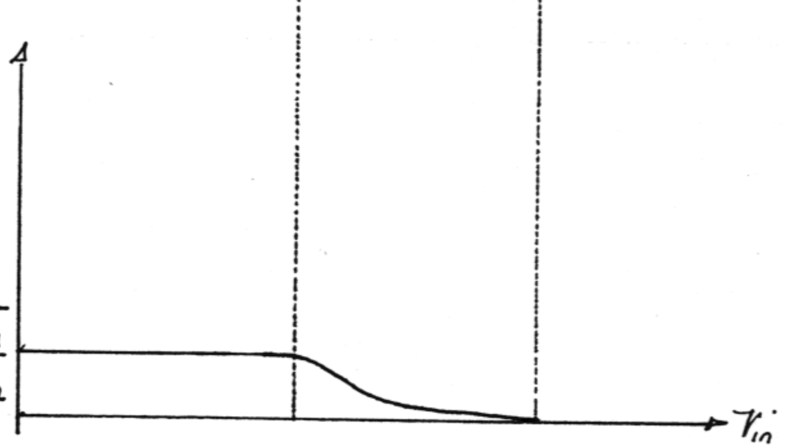




(C)

V_{DS2} is obtained from $\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b2}-V_{TH1})^2 = \mu_n C_{ox}\left(\frac{W}{L}\right)_2\left[(V_{b1}-V_{b2}+V_{TH1}-V_{TH2})V_{DS2}-\frac{V_{DS2}^2}{2}\right]$

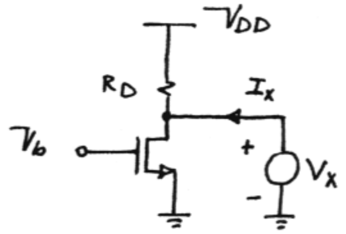




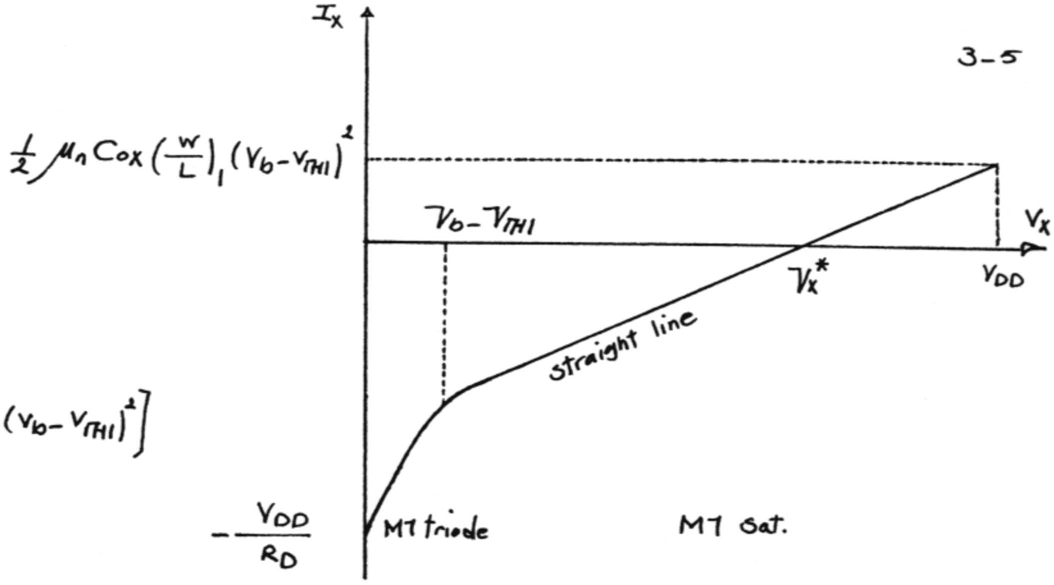
$(V_{b1}-V_{TH2})-(V_{b2}-V_{TH1})\sqrt{\frac{(W/L)_1}{(W/L)_2}}$

(d)

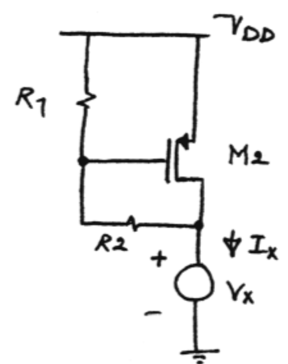
3.3.



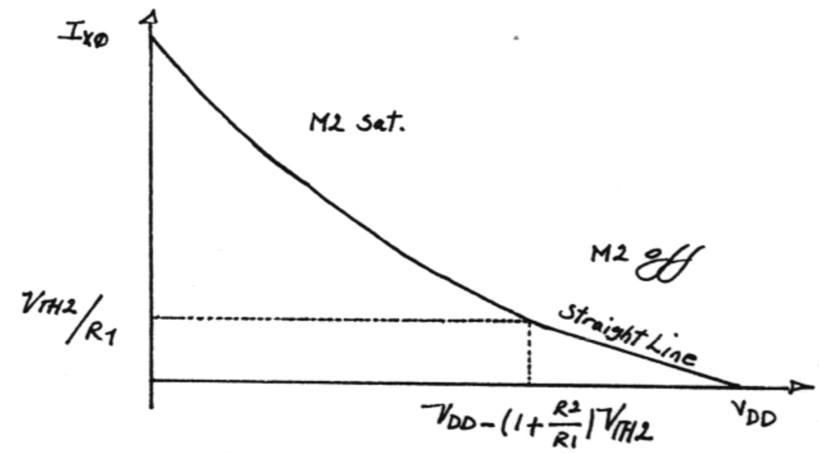
(a)



$$V_X^* = V_{DD} - R_D\left[\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_b - V_{TH1})^2\right]$$

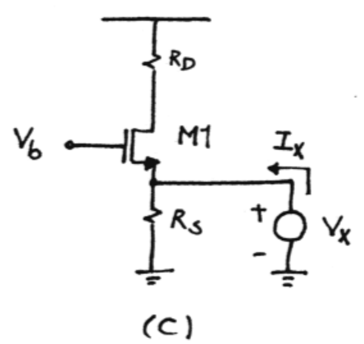


(b)

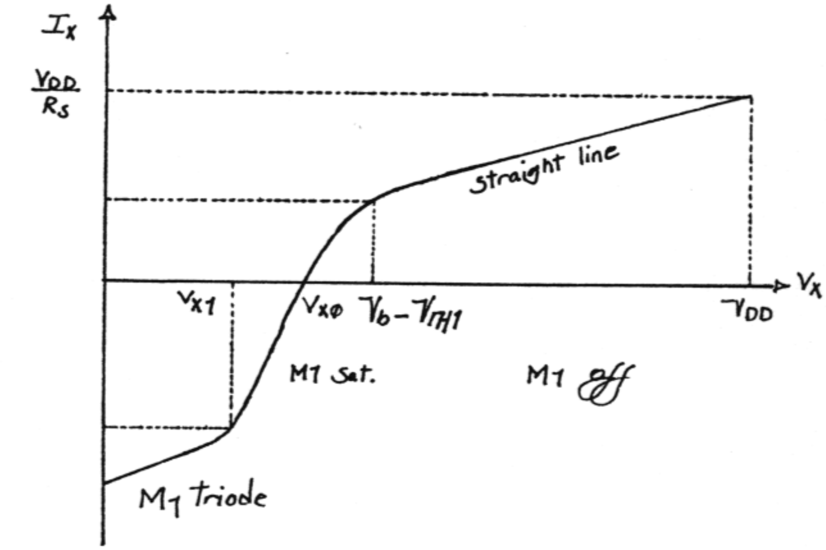


If $V_X < V_{DD} - (1+\frac{R_2}{R_1})V_{TH2}$ $\quad I_X = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 \left(\frac{(V_{DD}-V_X)}{R_1+R_2}\cdot R_1 - V_{TH2}\right)^2 + \frac{V_{DD}-V_X}{R_1+R_2}$

If $V_X > V_{DD} - (1+\frac{R_2}{R_1})V_{TH2}$ $\quad I_X = \frac{V_{DD}-V_X}{R_1+R_2}$



(C)

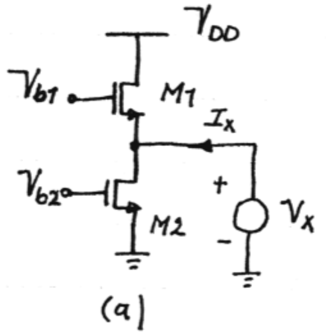


$$V_{X0} = V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_b - V_{TH1})^2 \cdot R_D$$

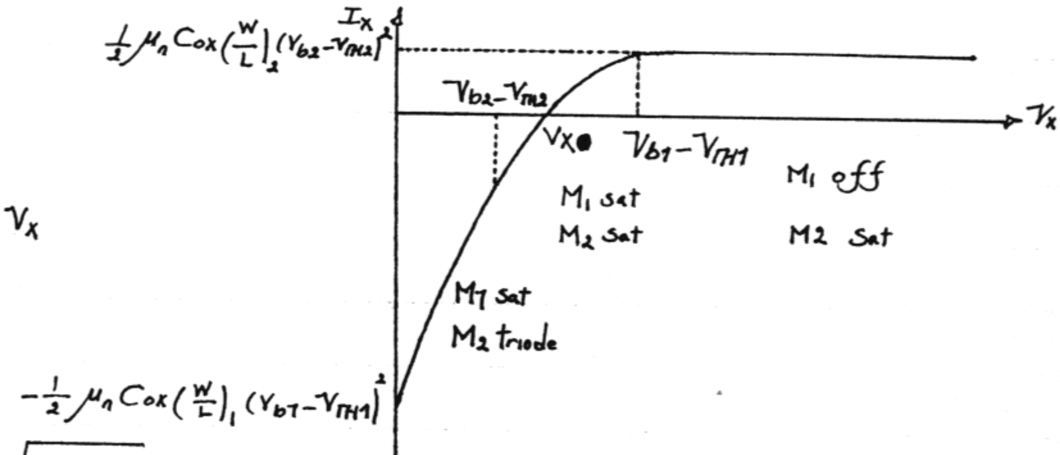
3-6

$$V_{X1} = V_b - V_{TH1} - \left(\frac{2(V_{DD} - V_b + V_{TH1})}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot R_D}\right)^{1/2}$$

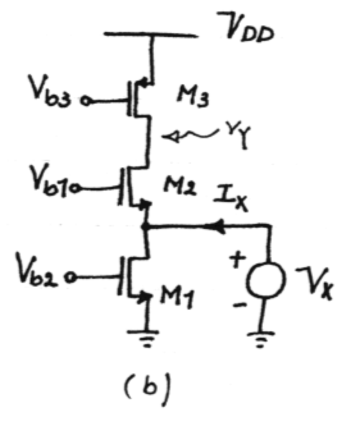
3.4.



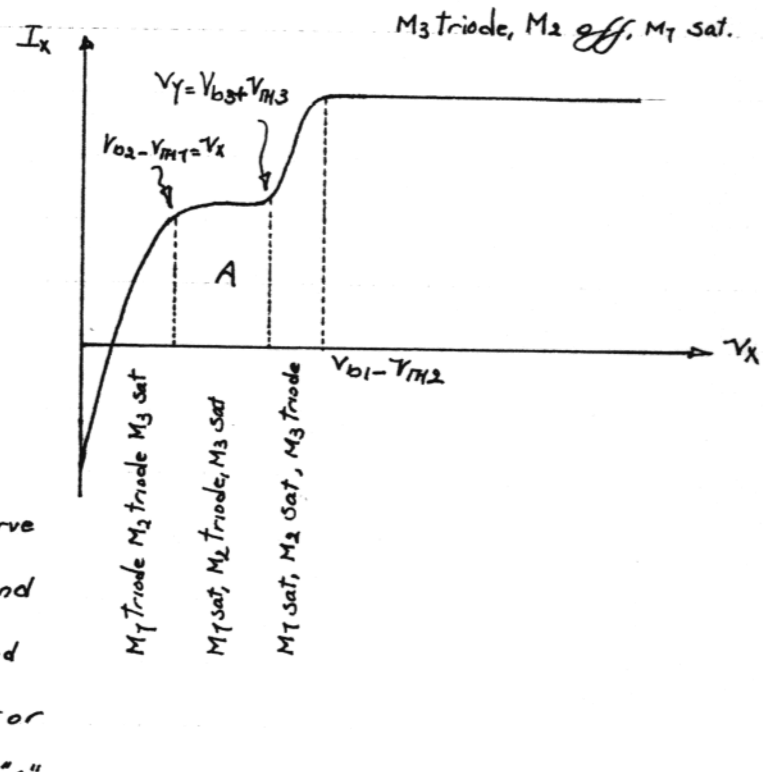
(a)



$$V_{X0} = V_{b1} - V_{TH1} - \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\,(V_{b2} - V_{TH2})$$

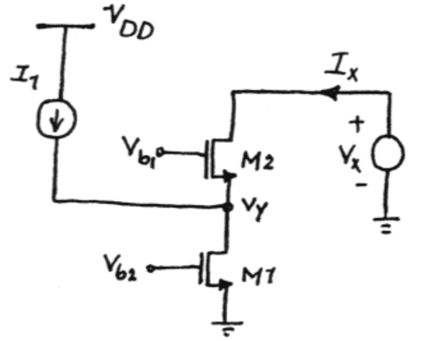


(b)



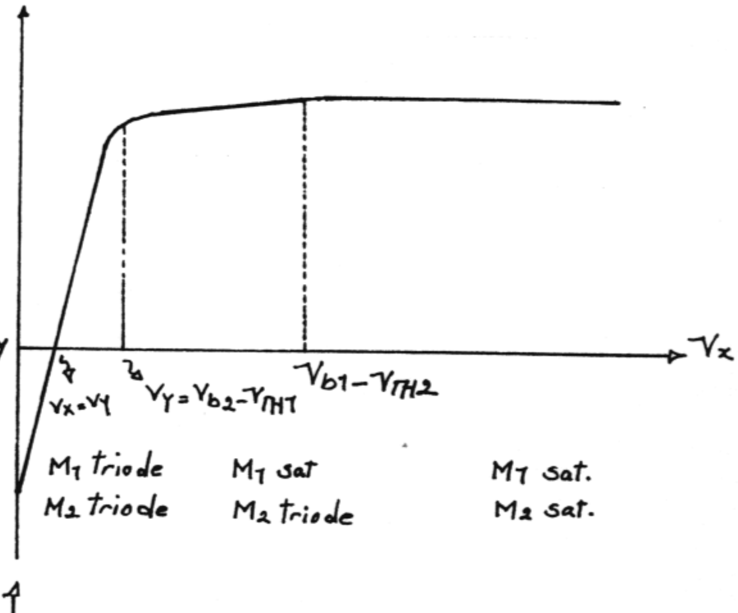
It's worth mentioning that the I_X/V_X Curve varies with the value of bias voltages and aspect ratios, therefore, some region(s), based on the aforementioned parameters, gets wider or narrower, especially the region Called "A" in the above figure.

3-7

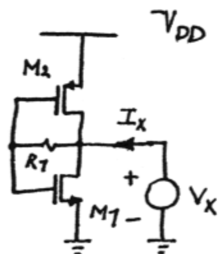


(c)

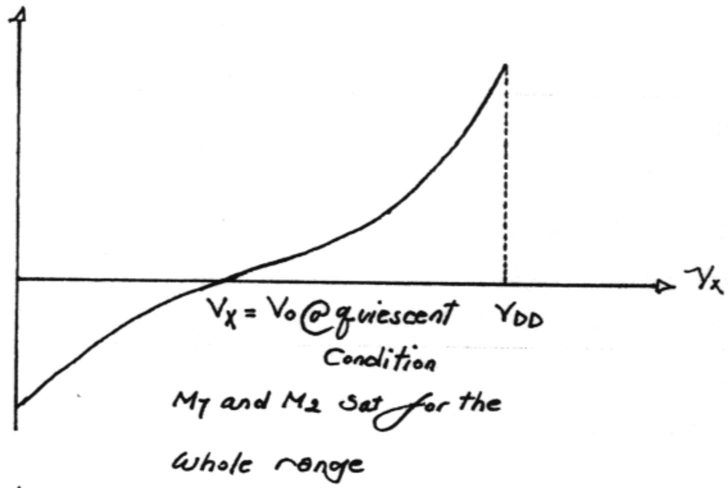
we assume $V_{b1} > V_{b2}$ and both M_1 and M_2 operate in Saturation region if $V_X = V_{DD}$

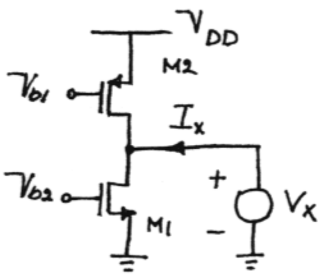


Below V_X, for which $V_X = V_Y$, drain current of M_2 flows in opposit direction, revealing the fact the drain and source terminals of M_2 are reversed. As expected, most of I_1 flow through M_2 when $V_X = 0$, because we assume that $V_{b1} > V_{b2}$.

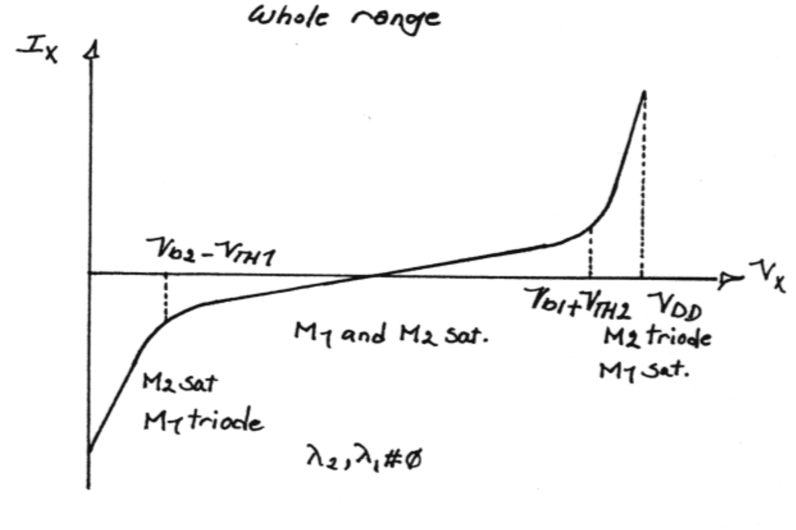


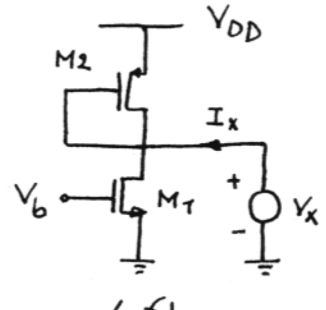
(d)



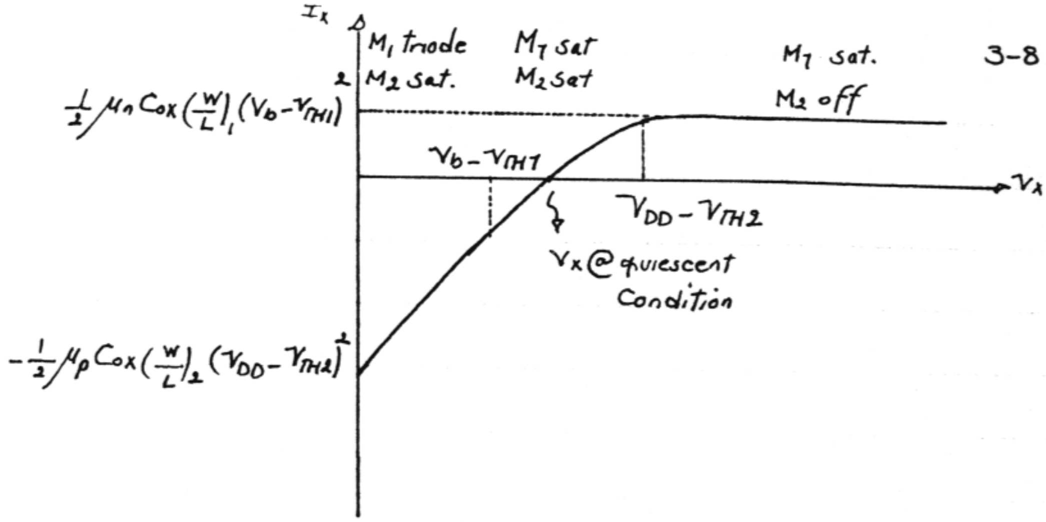


(e)



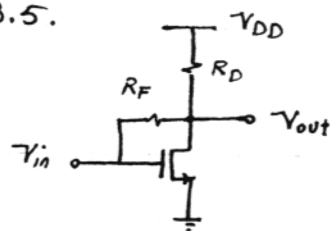


(f)



3-8

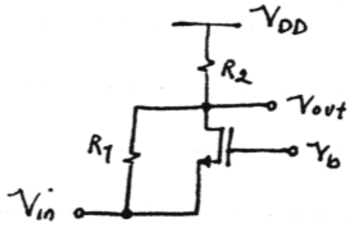
3.5.



(a)

$$\frac{V_o - V_{in}}{R_F} + g_{m1} V_{in} + \frac{V_o}{r_{o1}} + \frac{V_o}{R_D} = 0$$

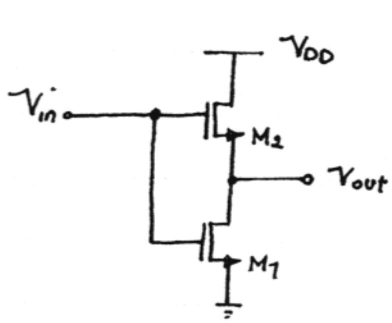
$$A_V = \frac{V_o}{V_{in}} = -\frac{g_{m1} - 1/R_F}{\frac{1}{R_F} + \frac{1}{r_{o1}} + \frac{1}{R_D}}$$



(b)

$$\frac{V_o}{R_2} + (V_o - V_{in})\left(\frac{1}{R_1} + \frac{1}{r_{o1}}\right) - g_{m1} V_{in} = 0$$

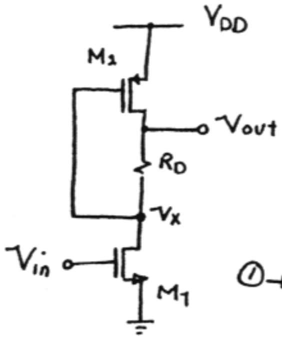
$$\frac{V_o}{V_{in}} = \frac{g_{m1} + \frac{1}{R_1} + \frac{1}{r_{o1}}}{\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{r_{o1}}}$$



(c)

$$g_{m2}(V_{in} - V_{out}) + \frac{-V_{out}}{r_{o2}} = g_{m1} V_{in} + \frac{V_{out}}{r_{o1}}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1} - g_{m2}}{g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}}}$$



(d)

① $\left(g_{m1}V_{in} + \frac{V_X}{r_{o1}}\right)R_D + V_X = V_{out}$, ② $-\left(g_{m2}V_X + \frac{V_{out}}{r_{o2}}\right) = g_{m1}V_{in} + \frac{V_X}{r_{o1}}$,

② → $V_X\left(-g_{m2} - \frac{1}{r_{o1}}\right) = g_{m1}V_{in} + \frac{V_{out}}{r_{o2}}$ ② → $V_X = -\frac{g_{m1}\cdot V_{in} + V_{out}/r_{o2}}{g_{m2} + \frac{1}{r_{o1}}}$ ③

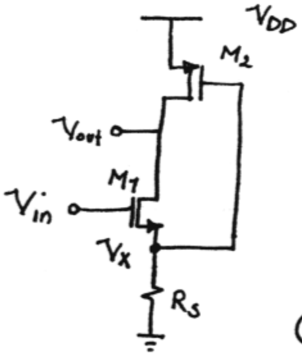
① → $g_{m1}R_D V_{in} + \left(1 + \frac{R_D}{r_{o1}}\right)V_X = V_o$ ④

③, ④ → $$g_m R_D V_{in} - \frac{\left(1+\frac{R_D}{r_{o1}}\right)\left(g_{m1}V_{in} + \frac{V_{out}}{r_{o2}}\right)}{g_{m2} + \frac{1}{r_{o1}}} = V_{out}$$

$$\left[g_{m1}\cdot R_D - \frac{g_{m1}(1 + R_D/r_{o1})}{g_{m2} + \frac{1}{r_{o1}}}\right]V_{in} = \left[1 + \frac{\frac{1}{r_{o2}}(1 + R_D/r_{o1})}{g_{m2} + \frac{1}{r_{o1}}}\right]V_{out}$$

$$\left[g_{m1}\cdot R_D\left(g_{m2} + \frac{1}{r_{o1}}\right) - g_{m1}\left(1 + \frac{R_D}{r_{o1}}\right)\right]V_{in} = \left[g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}\left(1 + \frac{R_D}{r_{o1}}\right)\right]V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1}(g_{m2}R_D - 1)}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}\left(1 + \frac{R_D}{r_{o1}}\right)}$$



(e)

$$-\left(\frac{V_{out}}{r_{o2}} + g_{m2}V_X\right) \overset{①}{=} \frac{V_{out} - V_X}{r_{o1}} + g_{m1}(V_{in} - V_X) \overset{②}{=} \overset{③}{\frac{V_X}{R_S}}$$

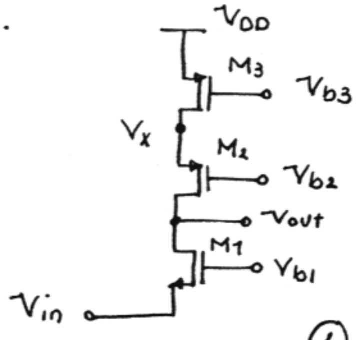
①, ③ → $$V_X = -\frac{V_{out}}{r_{o2}\left(g_{m2} + \frac{1}{R_S}\right)}$$

②, ③ $$\frac{V_{out}}{r_{o1}} + g_{m1}V_{in} = -\frac{V_{out}}{r_{o2}\left(g_{m2} + \frac{1}{R_S}\right)}\left(\frac{1}{R_S} + g_{m1} + \frac{1}{r_{o1}}\right)$$

$$\frac{V_{out}}{r_{o1}}\cdot r_{o2}\left(g_{m2} + \frac{1}{R_S}\right) + g_{m1}\cdot V_{in}\cdot r_{o2}\left(g_{m2} + \frac{1}{R_S}\right) = -V_{out}\left(\frac{1}{R_S} + g_{m1} + \frac{1}{r_{o1}}\right)$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}(g_{m2} + 1/R_S)r_{o2}}{g_{m1} + \frac{1}{R_S} + \frac{1}{r_{o1}}\left[1 + r_{o2}\left(g_{m2} + \frac{1}{R_S}\right)\right]}$$

3.6.



(a)

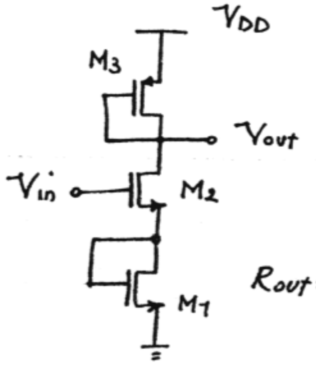
$$\overset{①}{-\frac{V_X}{r_{o3}}} = \overset{②}{g_{m2}V_X + \frac{V_X - V_{out}}{r_{o2}}} = \overset{③}{\frac{V_{out} - V_{in}}{r_{o1}} - g_{m1}V_{in}}$$

$$①, ② \rightarrow V_X = \frac{V_{out}}{1 + r_{o2}\left(g_{m2} + \frac{1}{r_{o3}}\right)}$$

$$①, ③ \rightarrow \frac{V_{out} - V_{in}}{r_{o1}} - g_{m1}V_{in} = -\frac{V_{out}}{r_{o3} + r_{o2}(1 + g_{m2}\cdot r_{o3})}$$

$$V_{out}\left[\frac{1}{r_{o1}} + \frac{1}{r_{o3} + r_{o2}(1 + g_{m2}\cdot r_{o3})}\right] = \left(g_{m1} + \frac{1}{r_{o1}}\right)V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{(1 + g_{m1}\cdot r_{o1})\left[r_{o3} + r_{o2}(1 + g_{m2}\cdot r_{o3})\right]}{r_{o1} + r_{o3} + r_{o2}(1 + g_{m2}r_{o3})}$$

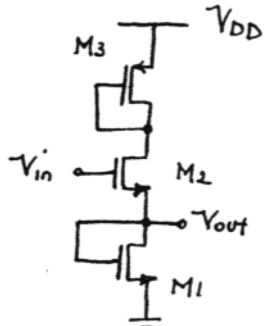


$$G_m = \frac{g_{m2}\cdot r_{o2}}{\left(\frac{1}{g_{m1}} \| r_{o1}\right) + \left[1 + g_{m2}\left(\frac{1}{g_{m1}} \| r_{o1}\right)\right] r_{o2}}$$

$$R_{out} = \left(\frac{1}{g_{m3}} \| r_{o3}\right) \| \left\{\left[1 + g_{m2}\left(\frac{1}{g_{m1}} \| r_{o1}\right)\right] r_{o2} + \left(\frac{1}{g_{m1}} \| r_{o1}\right)\right\}$$

(b)

$$A_V = -G_m\cdot R_{out} = -\frac{g_{m2}r_{o2}\left(\frac{1}{g_{m3}} \| r_{o3}\right)}{\left(\frac{1}{g_{m3}} \| r_{o3}\right) + \left\{\left[1 + g_{m2}\left(\frac{1}{g_{m1}} \| r_{o1}\right)\right] r_{o2} + \left(\frac{1}{g_{m1}} \| r_{o1}\right)\right\}}$$

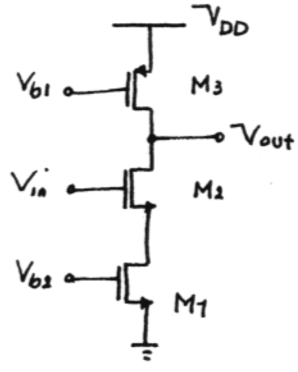


(C)

resistance seen looking up at the source of M_2, $\frac{\left(\frac{1}{g_{m3}} \| r_{o3}\right) + r_{o2}}{1 + g_{m2}\cdot r_{o2}}$

$$\frac{V_{out}}{V_{in}} = \frac{\left(\frac{1}{g_{m1}} \| r_{o1}\right)}{\left(\frac{1}{g_{m1}} \| r_{o1}\right) + \frac{\left(\frac{1}{g_{m3}} \| r_{o3}\right) + r_{o2}}{1 + g_{m2}\cdot r_{o2}}}$$

3-11

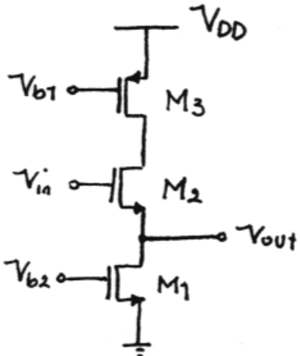


(d)

$$G_m = \frac{g_{m2} \cdot r_{o2}}{r_{o1} + \left[1 + g_{m2} \cdot r_{o1}\right] r_{o2}}$$

$$R_{out} = r_{o3} \,\|\, \left[(1 + g_{m2} \cdot r_{o1}) r_{o2} + r_{o1}\right]$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m2} \cdot r_{o2} \cdot r_{o3}}{r_{o3} + (1 + g_{m2} \cdot r_{o1}) r_{o2} + r_{o1}}$$

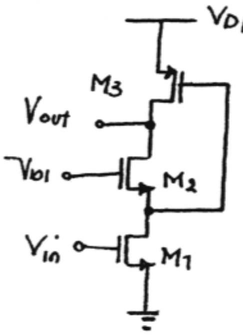


(e)

resistance seen looking up at the source of M_2

$$R_{in} = \frac{r_{o3} + r_{o2}}{1 + g_{m2} \cdot r_{o2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{r_{o1}}{r_{o1} + \dfrac{r_{o3} + r_{o2}}{1 + g_{m2} r_{o2}}} = \frac{r_{o1}(1 + g_{m2} \cdot r_{o2})}{r_{o1}(1 + g_{m2} \cdot r_{o2}) + r_{o2} + r_{o3}}$$



$$\overset{①}{-\left(\frac{V_{out}}{r_{o3}} + g_{m3} V_X\right)} = \overset{②}{\left(\frac{V_{out} - V_X}{r_{o2}} - g_{m2} V_X\right)} = \overset{③}{\frac{V_X}{r_{o1}} + g_{m1} V_{in}}$$

①, ② $\rightarrow$ $$\frac{V_X}{r_{o2}} + g_{m2} V_X - g_{m3} V_X = \frac{V_{out}}{r_{o2}} + \frac{V_{out}}{r_{o3}} \rightarrow V_X = \frac{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}} V_{out}$$

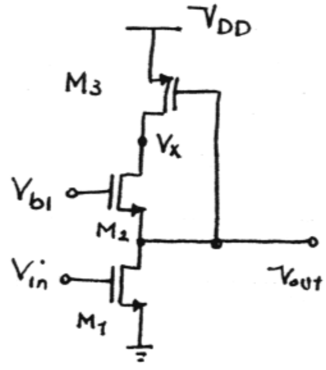
①, ③ $$-\frac{V_{out}}{r_{o3}} - g_{m3} V_X = \frac{V_X}{r_{o1}} + g_{m1} V_{in}$$

$$-\frac{V_{out}}{r_{o3}} - \left(g_{m3} + \frac{1}{r_{o1}}\right) \frac{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}} V_{out} = g_{m1} \cdot V_{in}$$

$$-V_{out}\left[\frac{1}{r_{o3}} + \frac{\left(g_{m3} + \frac{1}{r_{o1}}\right)\left(\frac{1}{r_{o2}} + \frac{1}{r_{o3}}\right)}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}}\right] = g_{m1} \cdot V_{in}$$

$$-V_{out}\left[\frac{1}{r_{o3}}+\frac{(1+g_{m3}r_{o1})(r_{o3}+r_{o2})}{r_{o1}r_{o3}\left[1+(g_{m2}-g_{m3})r_{o2}\right]}\right]=g_{m1}\cdot V_{in}$$ 3–12

$$\frac{V_{out}}{V_{in}}=-\frac{g_{m1}r_{o1}r_{o3}\left[1+(g_{m2}-g_{m3})r_{o2}\right]}{r_{o1}\left[1+(g_{m2}-g_{m3})r_{o2}\right]+(1+g_{m3}\cdot r_{o1})(r_{o3}+r_{o2})}$$



(g)

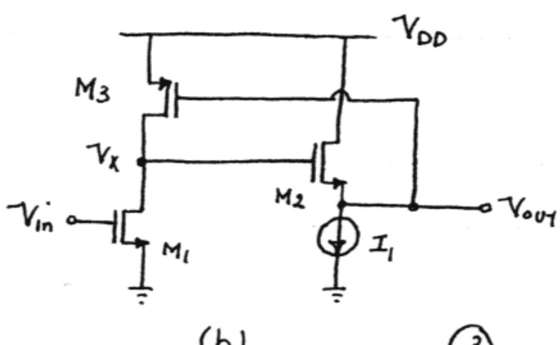
$$V_X=\frac{\frac{1}{r_{o2}}+g_{m2}-g_{m3}}{\frac{1}{r_{o2}}+\frac{1}{r_{o3}}}\cdot V_{out}$$

$$-\frac{V_X}{r_{o3}}-g_{m3}V_{out}=\frac{V_{out}}{r_{o1}}+g_{m1}\cdot V_{in}$$

$$-\frac{\frac{1}{r_{o2}}+g_{m2}-g_{m3}}{\frac{r_{o3}}{r_{o2}}+1}V_{out}-g_{m3}V_{out}=\frac{V_{out}}{r_{o1}}+g_{m1}V_{in}$$

$$-V_{out}\left[\frac{1+(g_{m2}-g_{m3})r_{o2}}{r_{o3}+r_{o2}}+g_{m3}+\frac{1}{r_{o1}}\right]=g_{m1}V_{in}$$

$$\frac{V_{out}}{V_{in}}=-\frac{g_{m1}r_{o1}(r_{o2}+r_{o3})}{r_{o1}\left[1+(g_{m2}-g_{m3})r_{o2}\right]+(r_{o2}+r_{o3})(1+g_{m3}\cdot r_{o1})}$$



(h)

① ②

$$-\left(\frac{V_X}{r_{o3}}+g_{m3}V_{out}\right)=g_{m1}V_{in}+\frac{V_X}{r_{o1}}$$

③

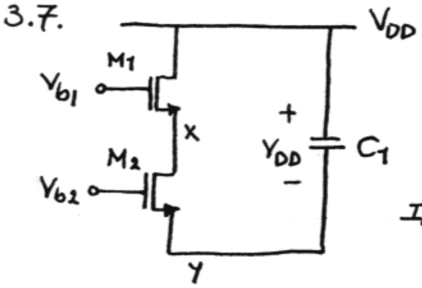
$$-\frac{V_{out}}{r_{o2}}+g_{m2}(V_X-V_{out})=0$$ @ output node

③ → $$V_X=\frac{\frac{1}{r_{o2}}+g_{m2}}{g_{m2}}\cdot V_{out}=\frac{1+g_{m2}r_{o2}}{g_{m2}r_{o2}}V_{out}$$

①, ② → $$-\left[\left(\frac{1}{r_{o3}}+\frac{1}{r_{o1}}\right)\frac{1+g_{m2}\cdot r_{o2}}{g_{m2}\cdot r_{o2}}+g_{m3}\right]V_{out}=g_{m1}\cdot V_{in}$$

$$\frac{V_{out}}{V_{in}}=-\frac{g_{m1}\cdot g_{m2}r_{o1}r_{o2}r_{o3}}{(r_{o1}+r_{o3})(1+g_{m2}\cdot r_{o2})+g_{m2}g_{m3}r_{o1}r_{o2}r_{o3}}$$

3.7.



(a)

$$V_Y(t=0) = -V_{C1} + V_{DD} = -V_{DD} + V_{DD} = 0$$

$$I_{D1} = I_{D2} \rightarrow \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[V_{b1} - V_X(t=0) - V_{TH1}\right]^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2$$

$$V_X(t=0) = V_{b1} - V_{TH1} - \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\,(V_{b2} - V_{TH2})$$

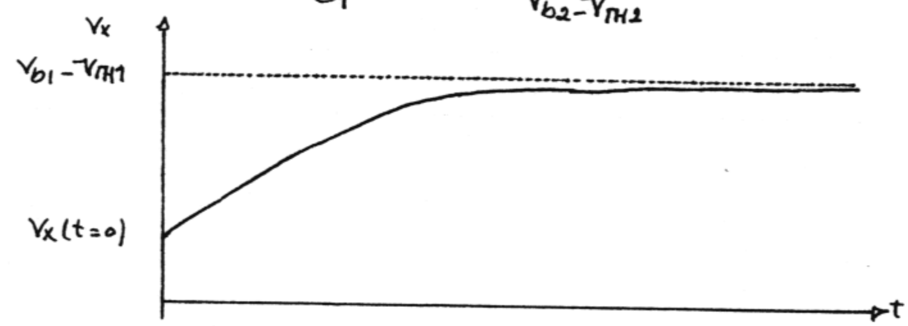
We assume that $V_X(t=0) > V_{b2} - V_{TH2}$, therefore, M_2 is always saturated.

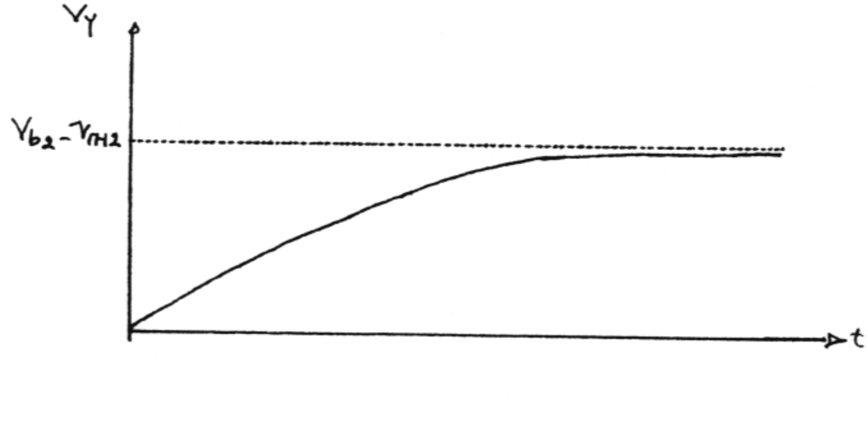
$$\underbrace{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_X - V_{TH1})^2}_{①} = \underbrace{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_Y - V_{TH2})^2}_{②} = \underbrace{C_1\frac{dV_Y}{dt}}_{③}$$

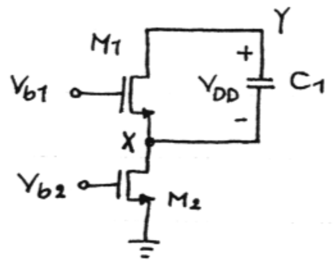
$$②, ③ \rightarrow \frac{dV_Y}{(V_{b2} - V_Y - V_{TH2})^2} = \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 \cdot dt$$

$$\frac{1}{V_{b2} - V_Y - V_{TH2}} = \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 t + K, \quad K = \frac{1}{V_{b2} - V_{TH2}} \quad \text{because } V_Y(t=0) = 0$$

$$V_Y = V_{b2} - V_{TH2} - \frac{1}{\frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 t + \frac{1}{V_{b2} - V_{TH2}}}, \quad V_X = V_{b1} - V_{TH1} - (V_{b2} - V_Y - V_{TH2})\sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}} \leftarrow ②, ①$$







(b)

3-14

The drain current of M_2 is zero, therefore, M_2 operates in deep triode region, pulling down V_X to zero potential.

$V_X = 0$ for $0 < t < \infty$

$V_Y(t=0) = V_{DD} \rightarrow M_1$ starts in saturation.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2 = -C_1\frac{dV_{C1}}{dt} = -C_1\frac{dV_Y}{dt}$$

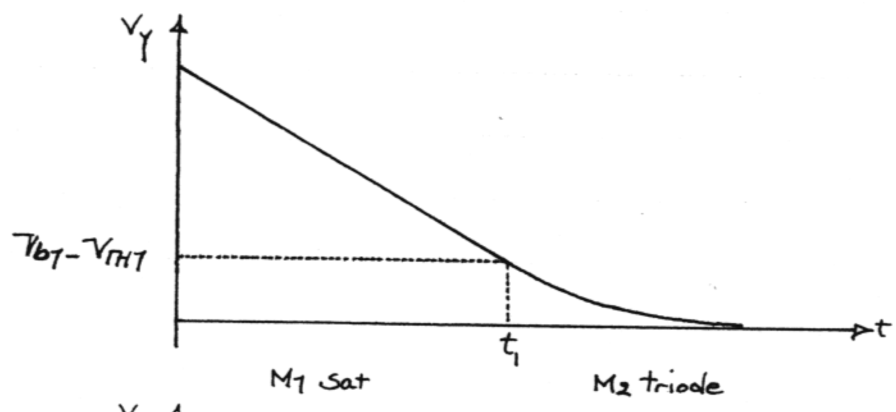
$$① \; V_Y = V_{C1} = V_{DD} - \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2 t$$

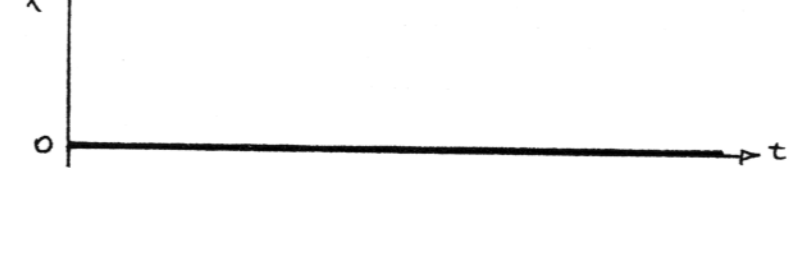
When $V_Y = V_{b1} - V_{TH1}$ ②, M_1 enters triode region.

Substituting ② in ①, we calculate the time when M_1 is at the edge of triode region.

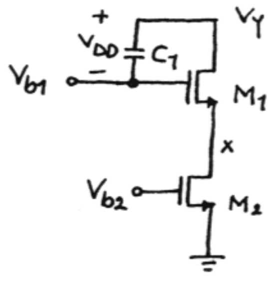
$$t_1 = \frac{V_{DD} - V_{b1} + V_{TH1}}{\frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2}$$

for $t > t_1$: $\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{b1} - V_{TH1})V_Y - \frac{V_Y^2}{2}\right] = -C_1\frac{dV_Y}{dt} \rightarrow V_Y = \ldots$





3-15



(c)

$V_Y(t=0) = V_{DD} + V_{b1}$, both transistors are saturated.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_X - V_{TH1})^2$$

$$V_X = V_{b1} - V_{TH1} - (V_{b2} - V_{TH2})\sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}$$

$$C_1 \frac{dV_{C1}}{dt} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 \rightarrow V_{C1} = V_{DD} - \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t$$

$$V_Y = V_{C1} + V_{b1} = V_{DD} + V_{b1} - \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t$$

@ $t = t_1$, we have $V_Y = V_{b1} - V_{TH1}$, polarity of voltage across C_1 has already changed.

$$V_{DD} + V_{b1} - \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t_1 = V_{b1} - V_{TH1}$$

$$t_1 = \frac{2(V_{DD} + V_{TH1})C_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2}$$

for $t > t_1$, M_1 enters triode region. We assume that still M_2 is saturated.

$$V_Y = V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2} \cdot t \quad \text{where } I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2$$

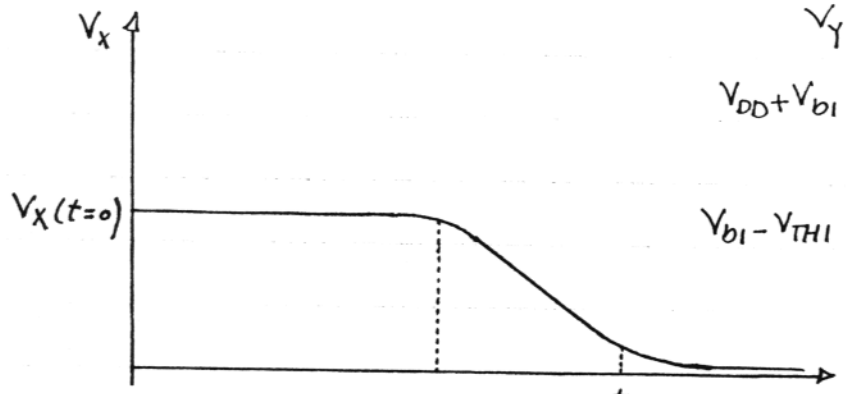
and $I_{D2} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{b1} - V_X)\left(V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2} \cdot t - V_X\right) - \frac{\left(V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2} \cdot t - V_X\right)^2}{2}\right]$

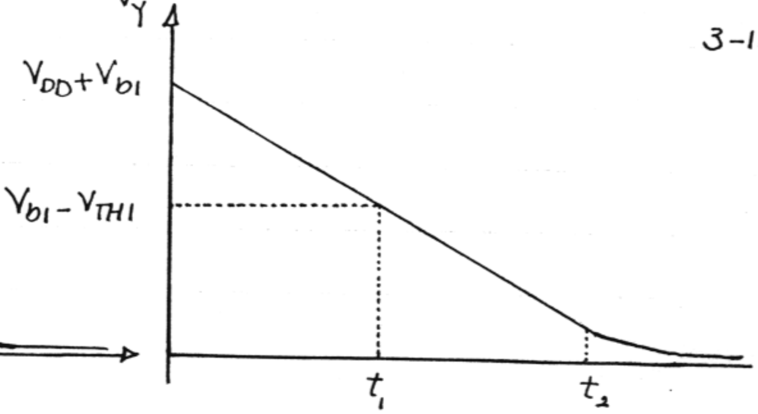
$\rightarrow V_X$ is obtained

When $V_X = V_{b2} - V_{TH2}$, M_2 enters the triode region, too.

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_2 \left[(V_{b2} - V_{TH2})V_X - \frac{V_X^2}{2}\right] = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{b1} - V_X - V_{TH1})(V_Y - V_X) - \frac{(V_Y - V_X)^2}{2}\right] = -C_1 \frac{dV_Y}{dt}$$

V_X and V_Y are obtained. This regime continues until V_X and V_Y drop to zero, and C_1 Charges up to $-V_{b1}$.



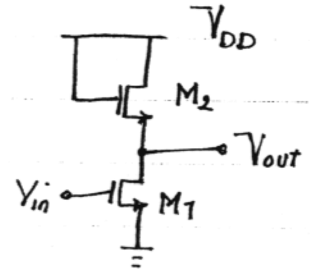


3-16

$$t_1 = \frac{2(V_{DD}+V_{TH1})C_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2}-V_{TH2})^2}$$

For $0<t<t_1$ M_1 Sat, M_2 Sat, for $t_1<t<t_2$ M_1 Triode, M_2 Sat.
for $t_2<t$ M_1 Triode, M_2 triode

3.8



$\left(\frac{W}{L}\right)_1 = \frac{50}{0.5}$, $\left(\frac{W}{L}\right)_2 = \frac{10}{0.5}$, $I_{D1} = I_{D2} = 0.5\,mA$

$$\mu_n C_{ox} = 350\,\frac{Cm^2}{V.S} \times \frac{8.85\times10^{-14}\times 3.9\ Farad/Cm}{9\times10^{-7}\,Cm} =$$

$$1.34225\times10^{-4}\ A/V^2$$

$$\mu_p C_{ox} = \frac{100\ Cm^2}{V.S} \times \frac{8.85\times10^{-14}\times3.9\ Farad/Cm}{9\times10^{-7}\,Cm} =$$

$$3.835\times10^{-5}\ A/V^2$$

$r_{o1} = r_{o2} = \frac{1}{\lambda_N I_1} = 20K$, $I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS2}-V_{TH2})^2(1+\lambda_N V_{DS2})$,

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\times20\left[3-V_o-0.7-0.45\left(\sqrt{0.9+V_o}-\sqrt{0.9}\right)\right]^2\left[1+0.1(3-V_o)\right]$$

$$2.3-0.45\left(\sqrt{0.9+V_o}-\sqrt{0.9}\right)-\sqrt{\frac{1}{2.6845\,(1.3-0.1V_o)}} = V_o \rightarrow V_o = 1.466\ V$$

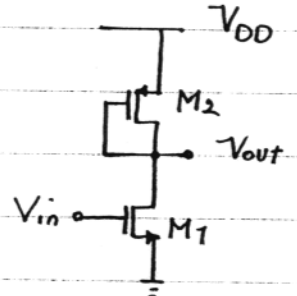
$$g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times100\times0.5\times10^{-3}} = 3.66\times10^{-3}\ A/V$$

$$g_{m2} = \sqrt{2\times1.34225\times10^{-4}\times20\times0.5\times10^{-3}} = 1.63\times10^{-3}\ A/V$$

$$g_{mb2} = \frac{\gamma.\,g_{m2}}{2\sqrt{2\phi_F+V_{SB}}} = \frac{0.45}{2\sqrt{0.9+1.466}}\times1.63\times10^{-3} = 2.3843\times10^{-4}\ A/V$$

$$R_{out} = \frac{1}{g_{m2} + g_{mb2} + r_{o2}^{-1}} \parallel r_{o1} = \frac{1}{1.63\times10^{-3} + 2.3843\times10^{-4} + (20\times10^{3})^{-1}} \parallel 20\times10^{3}$$ 3.17

$R_{out} = 508\ \Omega$ $\quad A_V = -g_{m1}\cdot R_{out} = -3.66\times10^{-3} \times 508 = -1.85$



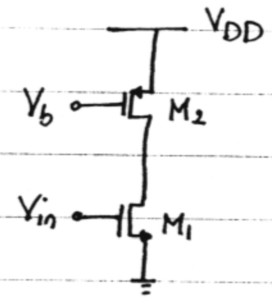
$$g_{m2} = \sqrt{2\times 3.835\times10^{-5} \times 20 \times 0.5\times10^{-3}} = 8.7578\times10^{-4}$$

$$r_{o2} = \frac{1}{\lambda_p I} = \frac{1}{0.2\times0.5\times10^{-3}} = 10K$$

$$R_{out} = \frac{1}{g_{m2} + r_{o2}^{-1}} \parallel r_{o1} = 974.8628\ \Omega$$

$$A_V = -g_{m1}\cdot R_{out} = -0.8537$$

3.9.



$\left(\frac{W}{L}\right)_1 = 50/0.5,\quad \left(\frac{W}{L}\right)_2 = 50/2,\quad I_{D1} = I_{D2} = 0.5\ mA$

$$r_{o1} = \frac{1}{\lambda_N I} = \frac{1}{0.1\times0.5\times10^{-3}} = 20K,\quad r_{o2} = \frac{1}{\lambda_p I} = \frac{1}{0.2\times0.5\times10^{-3}} = 40K$$

$$g_{m1} = \sqrt{2\times 1.34225\times10^{-4} \times 100 \times 0.5\times10^{-3}} = 3.6636\times10^{-3}$$

$$A_V = -g_{m1}(r_{o1} \parallel r_{o2}) = -48.84$$

If we assume that M_1 is in the edge of the triode region, then, we have:

$V_{GS} - V_{TH1} = V_{DS1} = V_{out},\quad I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2 (1 + \lambda_N V_{DS})$

$0.5\times10^{-3} = \frac{1}{2}\times 1.34225\times10^{-4} \times 100\, V_{DS}^2 (1 + 0.1 V_{DS}) \rightarrow \sqrt{\frac{1}{13.4225(1+0.1V_{DS})}} = V_{DS}$

$V_{DSmin} = V_{o\,min} = 0.2693$

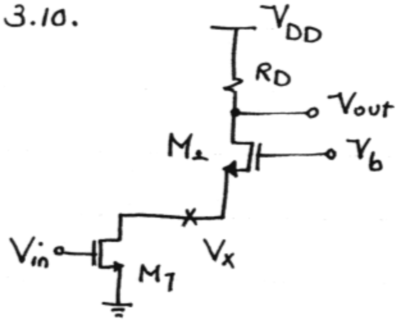
If we assume that M_2 is in the edge of the triode region, then, we have:

$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{SG} - |V_{TH2}|)^2 (1 + \lambda_p V_{SD}),\quad 0.5\times10^{-3} = \frac{1}{2}\times 3.835\times10^{-5} \times 25\, V_{SD}^2 (1 + \lambda_p V_{SD})$

$\sqrt{\frac{1}{0.95875(1+0.05V_{SD})}} = V_{SD} \rightarrow V_{SDmin} = 0.9967V,\quad V_{o\,max} = V_{DD} - V_{SDmin},$

$V_{o\,max} = 2V$

3.10.



$\left(\frac{W}{L}\right)_1 = 50/0.5, \quad \left(\frac{W}{L}\right)_2 = 10/0.5 \quad I_{D1} = I_{D2} = 0.5\,mA$ 3–18

$R_D = 1K\Omega$

$V_{DS,sat1} = V_{GS1} - V_{TH1} = \left(\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\right)^{1/2} = \left(\frac{2\times 0.5\times 10^{-3}}{1.34225\times 10^{-4}\times 100}\right)^{1/2}$

$V_{DS,sat1} = 0.2729\,V$

$V_{X,Bias} = 0.2729 + 50\times 10^{-3} = 0.3229\,V$

$V_{TH2} = V_{TH0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right) = 0.7 + 0.45\left(\sqrt{0.9 + 0.3229} - \sqrt{0.9}\right)$

$V_{TH2} = 0.77073\,V$

$V_{GS2} = V_{TH2} + \left(\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}\right)^{1/2} = 0.77073 + \left(\frac{2\times 0.5\times 10^{-3}}{1.34225\times 10^{-4}\times 20}\right)^{1/2} = 1.38107\,V,$

$V_b = V_{GS2} + V_X$

$V_b = 1.38107 + 0.3229 = 1.7\,V, \quad g_{m1} = \sqrt{2\times 1.34225\times 10^{-4}\times 100\times 0.5\times 10^{-3}} = 3.6636\times 10^{-3}\,A/V$

$g_{m2} = \sqrt{2\times 1.34225\times 10^{-4}\times 20\times 0.5\times 10^{-3}} = 1.6384\times 10^{-3}\,A/V$

$g_{mb2} = \frac{0.45}{2\sqrt{0.9 + 0.3229}}\,1.6384\times 10^{-3} = 3.3336\times 10^{-4}, \quad r_{o1} = r_{o2} = \frac{1}{\lambda_N I} = \frac{1}{0.1\times 0.5\times 10^{-3}} = 20K$

$R_{out} = R_D \,\|\, \left\{\left[1 + (g_{m2} + g_{mb2})r_{o2}\right]r_{o1} + r_{o2}\right\} = 10^3 \,\|\, \left\{\left[1 + (1.6384\times 10^{-3} + 3.3336\times 10^{-4})\,20\times 10^3\right]20\times 10^3 + 20\times 10^3\right\}$

$R_{out} = 998.7947\,\Omega, \quad G_m = \frac{g_{m1}\cdot r_{o1}\left[r_{o2}(g_{m2} + g_{mb2}) + 1\right]}{r_{o1}\cdot r_{o2}(g_{m2} + g_{mb2}) + r_{o1} + r_{o2}}$

$G_m = \frac{3.6636\times 10^{-3}\times 20\times 10^3\left[20\times 10^3(1.6384\times 10^{-3} + 3.3336\times 10^{-4}) + 1\right]}{(20\times 10^3)^2(1.6384\times 10^{-3} + 3.3336\times 10^{-4}) + 2\times 20\times 10^3} = 3.5751\times 10^{-3}\,A/V$

$A_v = -G_m R_{out} = -3.57$

We obtain the small signal voltage gain from input to node x.

$R_{out@X} = r_{o1} \,\|\, \frac{R_D + r_{o2}}{1 + (g_{m2} + g_{mb2})r_{o2}} = 20\times 10^3 \,\|\, \frac{10^3 + 20\times 10^3}{1 + (1.6384\times 10^{-3} + 3.3336\times 10^{-4})\,20\times 10^3}$

$R_{out@X} = 506.2$

$A_{VX} = -g_{m1}\cdot R_{out@X} = -1.8545$

If $V_X = V_{X\,min} = V_{DS,sat1}$, $\Delta V_X = -50\,mV \rightarrow \Delta V_{in} = \frac{-50\times 10^{-3}}{-1.8545} = 26.96\times 10^{-3}$

$\Delta V_{out} = 26.96 \times 10^{-3} \times (-3.57) = -96.25 \times 10^{-3}$

$V_{out\,min} = V_{DD} - R_D I_D + \Delta V_o = 3 - 1 \times 0.5 - 96.25 \times 10^{-3} = 2.4V$

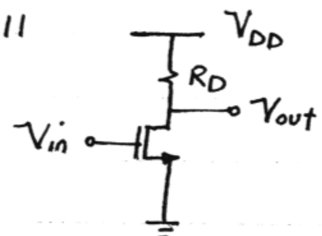
$V_{out,max} = 3V, \quad \Delta V_o = 3 - 2.5 = 0.5V, \quad \Delta V_{in} = \dfrac{0.5}{-3.57} = -0.14V$

$\Delta V_X = -1.8545(-0.14) = 0.2597$

$V_{X,max} = V_{X,Bias} + 0.2597 = 0.3229 + 0.2597 = 0.5826V$

If we take $V_{out,min} = V_b - V_{TH2} = 1.7 - 0.77073 = 0.92924V$, $\Delta V_o = -1.57$ which translates into a huge negative swing at x that makes the final voltage at node x negative. Therefore, M_1 limits the negative going output swing because the voltage gain from input to node x is quite large.

3.11



$\left(\frac{W}{L}\right)_1 = 50/0.5, \; R_D = 2K\Omega, \; \lambda = \emptyset$

$r_{o1} = \dfrac{1}{\lambda_N I_D} = \dfrac{1}{0.1 \times 10^{-3}} = 10K$

$R_{out} = r_{o1} \parallel R_D = 10K \parallel 2K = \dfrac{5000}{3}\,\Omega$

$g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 10^{-3}} = 5.1812 \times 10^{-3}$

$A_V = -g_{m1} \cdot R_{out} = -5.1812 \times 10^{-3} \times \dfrac{5000}{3} = -8.6353$

At the edge of the triode region: $V_{out} = V_{GS} - V_{TH} = V_{GS} - 0.7$, $I_{D1} = \dfrac{V_{DD} - V_{out}}{R_D} = \dfrac{3 - V_{GS} + 0.7}{2 \times 10^3} = \dfrac{3.7 - V_{GS}}{2 \times 10^3}$, $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2$

$$\frac{3.7 - V_{GS}}{2 \times 10^3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 100\,(V_{GS} - 0.7)^2$$

$13.4225\,V_{GS}^2 - 17.7915\,V_{GS} - 10.277025 = 0 \rightarrow V_{GS} = 1.137V$

I_D @ the edge of the triode $= \frac{1}{2} \times 1.34225 \times 10^{-4} \times 100\,(1.137 - 0.7)^2 = 1.28151 \times 10^{-3}$

g_{m1} @ the edge of the triode $= \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 1.28151 \times 10^{-3}} = 5.8653 \times 10^{-3}$

$r_{o1} = \dfrac{1}{0.1 \times 1.28151\; 10^{-3}} = 7.8 \times 10^3$

A_V @ the edge of the triode $= -g_{m1}(r_{o1} \parallel R_D) = -5.8653 \times 10^{-3}(7.8 \times 10^3 \parallel 2 \times 10^3)$

$A_V = -9.3374$

V_o @ the edge of the triode $= V_{DD} - R_D \times I_D = 3 - 2 \times 1.2815 \times 10^{-3} = 0.4369\,V$ 3-20

$$V_{DS} = V_{DS,sat1} - 50\times10^{-3} = 0.4369 - 50\times10^{-3} = 0.3869\,V$$

$$I_D = \frac{V_{DD} - V_{DS}}{R_D} = \frac{3 - 0.3869}{2\times10^3} = 1.3065\times10^{-3}$$

$$I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{GS} - V_{TH1})V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$1.3065\times10^{-3} = 1.34225\times10^{-4} \times 100 \left[(V_{GS} - 0.7)\,0.3869 - \frac{(0.3869)^2}{2}\right] \Rightarrow V_{GS} = 1.145$$

$$g_{m1} = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot V_{DS}$$

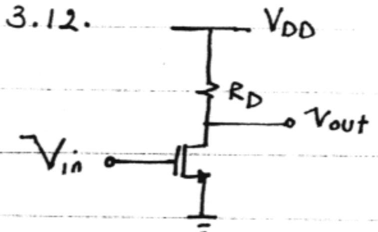
g_{m1} @ the point where 50 mV into the triode $= 1.34225\times10^{-4} \times 100 \times 0.3869 = 5.1942\times10^{-3}$

$$R_o^{-1} = \frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1} - V_{DS}) \rightarrow R_o = \frac{1}{1.34225\times10^{-4} \times 100(1.145 - 0.7 - 0.38}$$

$$R_o = 1.2835\times10^3\ \Omega$$

A_V @ 50 mV into the triode region $= -5.1942\times10^{-3}(1.2835\times10^3 \,\|\, 2\times10^3) = -4$

3.12.



$$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad R_D = 2K, \quad \lambda = 0$$

$$I_{D1}@V_{out}=1V = \frac{V_{DD} - V_o}{R_D} = \frac{3-1}{2\times10^3} = 10^{-3}\,A$$

$$V_{in} = V_{TH1} + \left(\frac{2 I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\right)^{1/2} = 0.7 + \left(\frac{2\times10^{-3}}{1.34225\times10^{-4}\times100}\right)^{1/2} \rightarrow V_{in}@V_{out}=1V = 1.086\,V$$

$$I_{D1}@V_{out}=2.5V = \frac{3-2.5}{2\times10^3} = 2.5\times10^{-4},\ V_{in}@V_{out}=2.5V = 0.7 + \left(\frac{2\times2.5\times10^{-4}}{1.34225\times10^{-4}\times100}\right)^{1/2} = 0.893\,V$$

$$g_{m1}@V_{out}=1V = \sqrt{2\times1.34225\times10^{-4}\times100\times10^{-3}} = 5.1812\times10^{-3}$$

$$g_{m1}@V_{out}=2.5V = \sqrt{2\times1.34225\times10^{-4}\times100\times2.5\times10^{-4}} = 2.59\times10^{-3}$$

$r_{o1} @ V_{out} = 7V = \frac{1}{0.1 \times 10^{-3}} = 10K$, $R_{out} = r_{o1} \| R_D = 10000 \| 2000 = \frac{5000}{3} \Omega$ 3-21

$A_V @ V_{out}=7V = -g_{m1} \cdot R_{out} = -5.1812 \times 10^{-3} \times \frac{5000}{3} = -8.6353$

$r_{o1} @ V_{out}=2.5V = \frac{1}{0.1 \times 2.5 \times 10^{-4}} = 40K$, $R_{out} = r_{o1} \| R_D = 40000 \| 2000 = 1.9 \times 10^{3}$

$A_V @ V_{out}=2.5V = -g_{m1} \cdot R_{out} = -2.59 \times 10^{-3} \times 1.9 \times 10^{3} = -4.9221$

3.13. $\left(\frac{W}{L}\right) = 50/0.5$ $|I_D| = 0.5mA$
& $100/1$

For NMOS device with $\left(\frac{W}{L}\right) = 50/0.5$, $r_o = \frac{1}{\lambda_N I_D} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 20K$

$g_m = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 0.5 \times 10^{-3}} = 3.6636 \times 10^{-3}$

$g_m r_o = 73.27$

For PMOS device with $\left(\frac{W}{L}\right) = 50/0.5$, $r_o = \frac{1}{\lambda_P I_D} = \frac{1}{0.2 \times 0.5 \times 10^{-3}} = 10K$

$g_m = \sqrt{2 \times 3.835 \times 10^{-5} \times 100 \times 0.5 \times 10^{-3}} = 1.9583 \times 10^{-3}$

$g_m r_o = 19.5831$

For NMOS device with $\left(\frac{W}{L}\right) = 100/1$, $r_o = \frac{1}{\frac{0.1}{2} \times 0.5 \times 10^{-3}} = 40K$

$g_m = 3.6636 \times 10^{-3}$, $g_m r_o = 146.5469$

For PMOS device with $\left(\frac{W}{L}\right) = 100/1$, $r_o = \frac{1}{\frac{0.2}{2} \times 0.5 \times 10^{-3}} = 20K$

$g_m = 1.9583 \times 10^{-3}$, $g_m r_o = 39.1663$

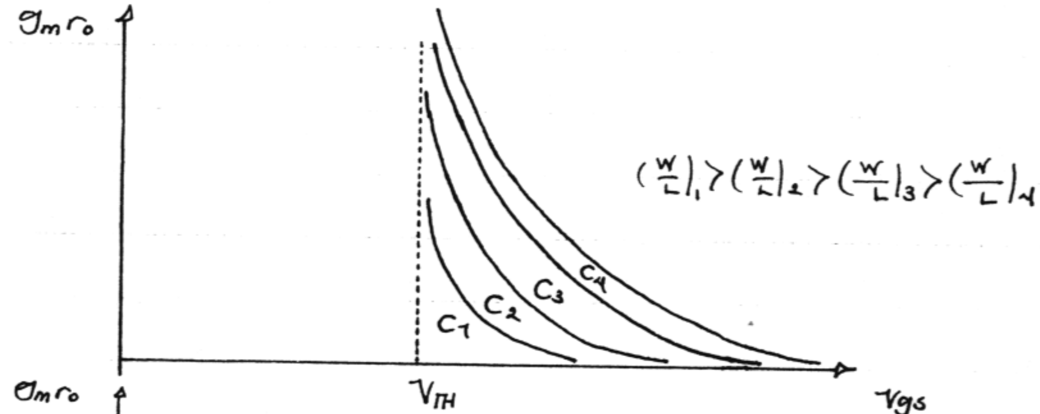
3.14. $I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$ ①

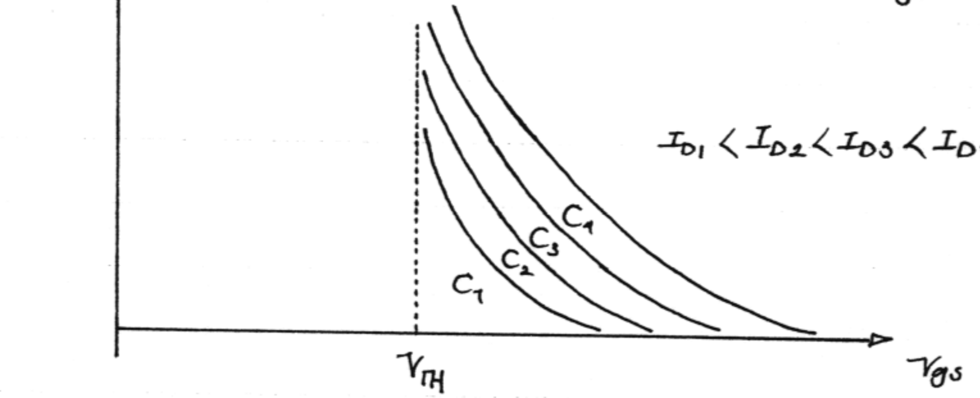
$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH}) (1 + \lambda V_{DS})$ ②

Substituting $(1+\lambda V_{DS})$ from ① in ②, we have, 3-22

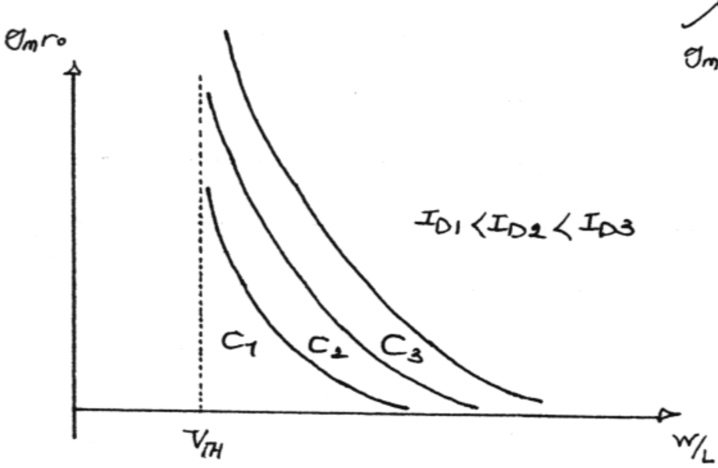
$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})\frac{I_D}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2} = \frac{2I_D}{V_{GS}-V_{TH}}$$

$$g_m r_o = \frac{2I_D}{V_{GS}-V_{TH}}\,\frac{1+\lambda V_{DS}}{\lambda I_D} = \frac{2(1+\lambda V_{DS})}{\lambda(V_{GS}-V_{TH})} = \frac{4I_D}{\mu_n C_{ox}(V_{GS}-V_{TH})^3\lambda\left(\frac{W}{L}\right)}$$

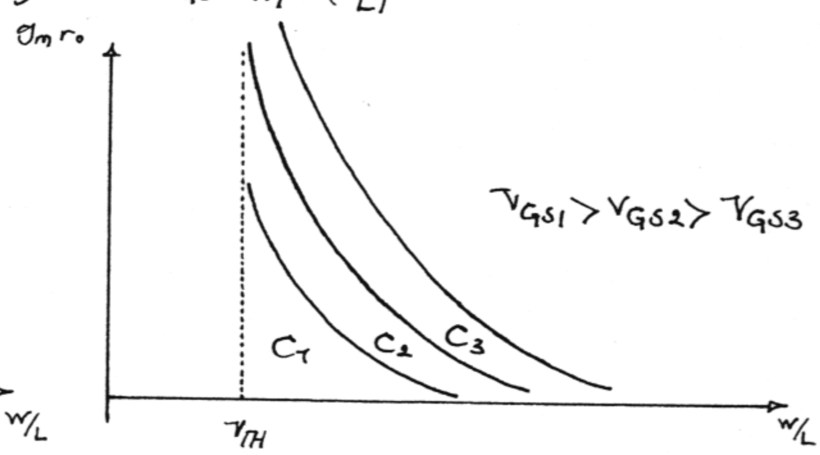




3.15. From 3.14. we have: $g_m r_o = \dfrac{4I_D}{\mu_n C_{ox}(V_{GS}-V_{TH})^3\lambda\left(\frac{W}{L}\right)}$



(a)



(b)

3.16. $\frac{W}{L} = 50/0.6 \quad V_G = +1.2V \quad V_S = 0 \quad 0 < V_D < 3 \quad V_{bulk} = 0$

$V_{Dsat} = V_{GS} - V_{TH} = 1.2 - 0.7 = 0.5V$, for a saturated device $g_m r_o = \dfrac{2(1+\lambda V_{DS})}{\lambda(V_{GS}-V_{TH})}$

@ the edge of the triode region $g_m r_o = \dfrac{2(1+0.5\times 0.1)}{0.1\,(1.2-0.7)} = 42$

We cannot neglect the channel-length modulation in the triode region, because it would lead to a discontinuity at the transition point between the saturation and the triode region.

@ triode region

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\left(\frac{W}{L}\right) V_{DS}\,(1+\lambda V_{DS})$$

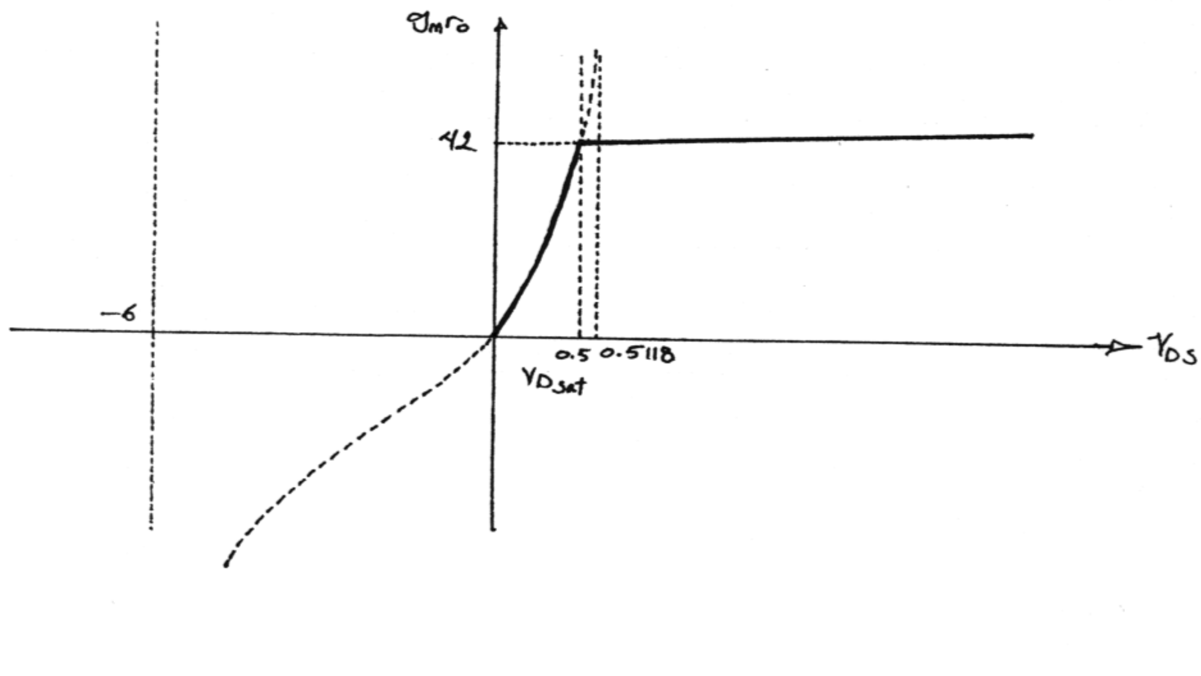
$$g_o = \frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)\left\{(V_{GS}-V_{TH}-V_{DS})(1+\lambda V_{DS}) + \lambda\left[(V_{GS}-V_{TH})V_{DS} - \frac{V_{DS}^2}{2}\right]\right\}$$

in the triode region $g_m r_o = \dfrac{(1+\lambda V_{DS})V_{DS}}{(V_{GS}-V_{TH}-V_{DS})(1+\lambda V_{DS}) + \lambda\left[(V_{GS}-V_{TH})V_{DS} - \frac{V_{DS}^2}{2}\right]}$

In Saturation $\quad g_m r_o = \dfrac{2(1+0.1V_{DS})}{0.1(1.2-0.7)} = 40 + 4V_{DS} \qquad V_{DS} > 0.5V$

In triode $\quad g_m r_o = \dfrac{(1+0.1V_{DS})V_{DS}}{(0.5-V_{DS})(1+0.1V_{DS}) + 0.1\times 0.5V_{DS}(1-V_{DS})}$

$$g_m r_o = \frac{0.1V_{DS}^2 + V_{DS}}{-0.15V_{DS}^2 - 0.9V_{DS} + 0.5}$$



3-24

$V_{bulk} = -1V$, $V_{SB} = +1V$

$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right) = 0.7 + 0.45\left(\sqrt{0.9+1} - \sqrt{0.9}\right) = 0.8933\ V$

In Saturation $g_m r_o = \dfrac{2(1+0.1V_{DS})}{0.1(1.2-0.8933)} = 65.2262 + 6.5226\, V_{DS}$

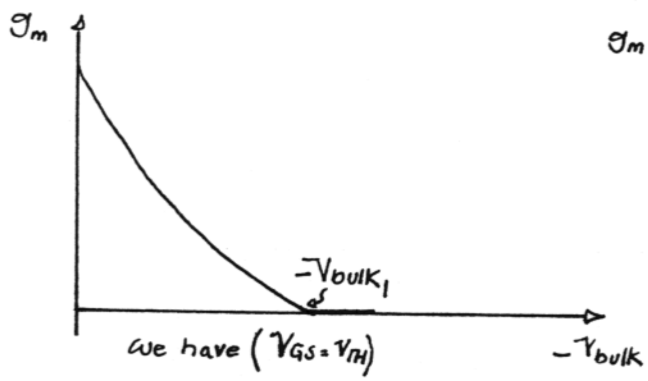
$V_{DSsat} = V_{GS} - V_{TH} = 1.2 - 0.8933 = 0.3066\ V$, @the edge of the triode $g_m r_o = 67.2262$

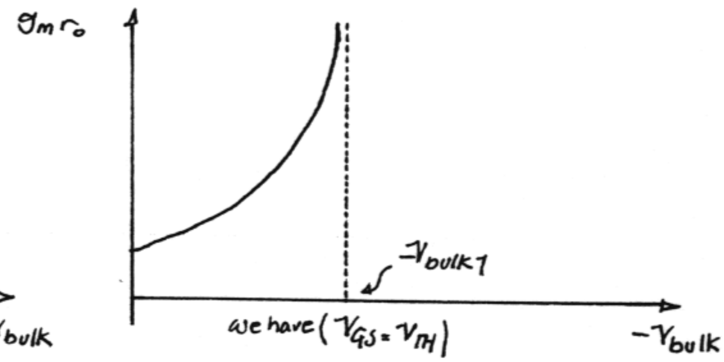
$$g_m r_o = \frac{(1+0.1V_{DS})V_{DS}}{(1.2-0.8933-V_{DS})(1+0.1V_{DS}) + 0.1\left[(1.2-0.8933)V_{DS} - 0.5V_{DS}^2\right]}$$

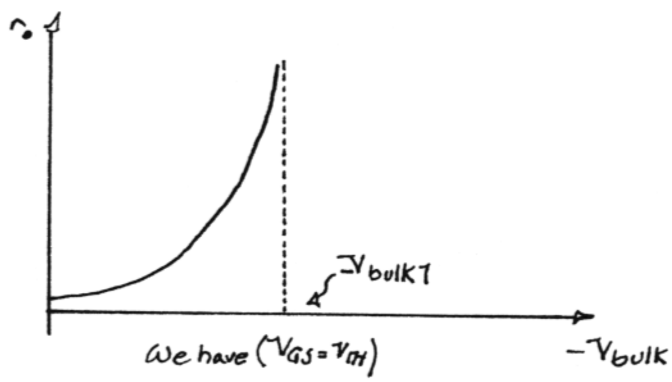
$$g_m r_o = \frac{(1+0.1V_{DS})V_{DS}}{-0.15V_{DS}^2 - 0.9386V_{DS} + 0.3066}$$

3.17. $g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[V_{GS} - V_{TH0} - \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right)\right](1+\lambda V_{DS})$

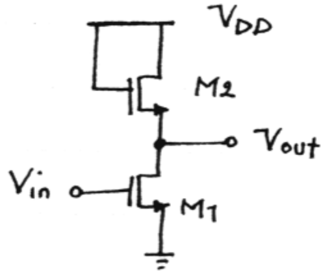
$$r_o = \frac{1}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2\lambda}, \quad g_m r_o = \frac{2(1+\lambda V_{DS})}{\lambda(V_{GS}-V_{TH})}$$







3.18.



$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad \left(\frac{W}{L}\right)_2 = 10/0.5 , \quad \lambda = \gamma = 0$

M_1 at the edge of the triode region, $\rightarrow V_{out} = V_{in} - V_{TH1}$

$$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} + V_{TH1} - V_{TH2})^2$$

$$\left(\frac{W}{L}\right)_1^{1/2} (V_{in} - V_{TH1}) = \left(\frac{W}{L}\right)_2^{1/2} (V_{DD} - V_{in}) \rightarrow (V_{in} - V_{TH1}) = \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\,(V_{DD} - V_{in})$$

$$V_{in} = \left(\sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\, V_{DD} + V_{TH1}\right) \Big/ \left(1 + \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\right) = \left[\left(\frac{10}{50}\right)^{1/2} \times 3 + 0.7\right] \Big/ \left[1 + \left(\frac{10}{50}\right)^{1/2}\right] = 1.41V$$

$$A_V = -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}} = -\sqrt{\frac{50}{10}} = -2.236 .$$

At the edge of the triode region $V_{out} = 1.41 - 0.7 = 0.71\,V$

50 mV into the triode region $V_{out} = 0.71 - 50\times10^{-3} = 0.66V$

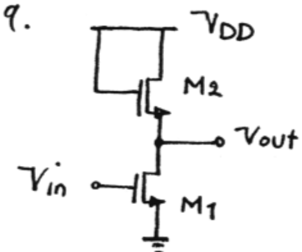
$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})^2 = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{TH1}) V_{out} - \frac{V_{out}^2}{2}\right]$$

$$V_{in} = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} \frac{(V_{DD} - V_{out} - V_{TH2})^2}{V_{out}} + \frac{V_{out}}{2} + V_{TH1} = \frac{10}{50}\,\frac{(3 - 0.66 - 0.7)^2}{0.66} + \frac{0.66}{2} + 0.7$$

$V_{in} = 1.8437$, $\quad I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{TH1}) V_{out} - \frac{V_{out}^2}{2}\right]$, $\quad \frac{\partial I_D}{\partial V_{in}} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot V_{out}$

$$A_V = -\frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot V_{out}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})} = -\frac{\frac{50}{0.5} \times 0.66}{\frac{10}{0.5} \times (3 - 0.66 - 0.7)} = -2.015$$

3.19.



3-26

$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad \left(\frac{W}{L}\right)_2 = 10/0.5 \quad \lambda = 0$

$V_{out} = V_{in} - V_{TH1}. \quad V_{TH2} = V_{TH2,0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right)$

$I_{D1} = I_{D2} \Rightarrow \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} + V_{TH1} - V_{TH2,0} - 0.45\left(\sqrt{0.9 + V_{out}} - \sqrt{0.9}\right)\right)^2$

$\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \left(\frac{W}{L}\right)_2 \left[V_{DD} - V_{in} - 0.45\left(\sqrt{0.9 + V_{in} - 0.7} - \sqrt{0.9}\right)\right]^2$

$V_{in} = \sqrt{\frac{1}{5}}\left[3 - V_{in} - 0.45\left(\sqrt{0.2 + V_{in}} - \sqrt{0.9}\right)\right] + 0.7 \rightarrow$ After enough iterations $\rightarrow$

$V_{in} = 1.3685, \quad V_{out} = 0.6685, \quad \eta_2 = \frac{\gamma}{2(2|\phi_F| + V_{SB})^{1/2}} = \frac{0.45}{2(0.9 + 0.6685)^{1/2}} = 0.1796$

$A_V = -\frac{g_{m1}}{g_{m2}(1+\eta_2)} = -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}}\frac{1}{1+\eta_2} = -\sqrt{\frac{50}{10}}\frac{1}{1+0.1796} = -1.8955$

$V_{out} = 0.6685 - 50\times10^{-3} = 0.6185$

$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})^2 = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{TH1})V_{out} - \frac{V_{out}^2}{2}\right]$

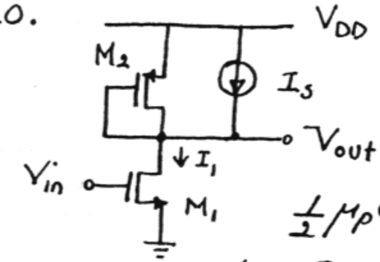
$V_{TH2} = V_{TH2,0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right) = 0.7 + 0.45\left(\sqrt{0.9 + 0.6185} - \sqrt{0.9}\right) = 0.8276$

$24.1453 = \frac{50}{0.5}\left[(V_{in} - 0.7)\,0.6185 - \frac{0.6185^2}{2}\right] \rightarrow V_{in} = 1.3996$

$\eta_2 = \frac{0.45}{2(0.9 + 0.6185)^{1/2}} = 0.1825 \qquad A_V = -\frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot V_{out}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TH2})(1+\eta_2)} =$

$\frac{\left(\frac{W}{L}\right)_1 V_{out}}{\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})(1+\eta_2)} = \frac{50\times0.6185}{10(3 - 0.6185 - 0.8276)(1 + 0.1825)} = -1.6829$

3.20.



$\left(\frac{W}{L}\right)_1 = 20/0.5,\ I_1 = 1\,mA,\ I_S = 0.75\,mA,\ \lambda = 0$ 3.27

M_1 at the edge of the triode region $V_{out} = V_{in} - V_{TH1}$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - |V_{TH2}|)^2 + I_S = \frac{1}{2}\mu_n C_{ox}(V_{in} - V_{TH1})^2 = 10^{-3}$$

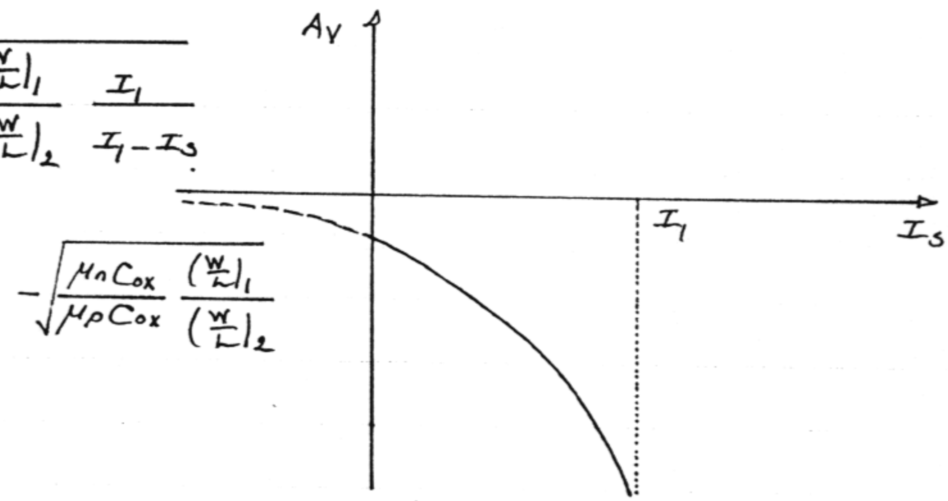
$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} + V_{TH1} - |V_{TH2}|)^2 + I_S = \frac{1}{2}\mu_n C_{ox}(V_{in} - V_{TH1})^2 = 10^{-3}$$

$$(V_{in} - V_{TH1})^2 = \frac{2I_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1} \rightarrow V_{in} = \sqrt{\frac{2I_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} + V_{TH1} = 0.7 + \sqrt{\frac{2\times10^{-3}}{1.34225\times10^{-4}\times 20/0.5}} = 1.31$$

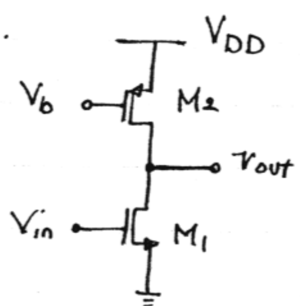
$$\frac{1}{2}\times 3.835\times10^{-5}\left(\frac{W}{L}\right)_2 (3 - 1.31 + 0.7 - 0.8)^2 + 0.75\times10^{-3} = 10^{-3},\quad \left(\frac{W}{L}\right)_2 = 5.159$$

$$A_V = -\frac{g_{m1}}{g_{m2}} = -\sqrt{\frac{\mu_n C_{ox}}{\mu_p C_{ox}} \times \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2} \times \frac{I_1}{I_2}} = -\sqrt{\frac{1.34225\times10^{-4}}{3.835\times10^{-5}} \times \frac{20/0.5}{5.159} \times \frac{10^{-3}}{2.5\times10^{-4}}} = -10.418$$

3.21. $$A_V = -\sqrt{\frac{\mu_n C_{ox}}{\mu_p C_{ox}}\ \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}\ \frac{I_1}{I_1 - I_S}}$$



3.22.



output voltage Swing = 2.2

$I_{D1} = I_{D2} = 1\ mA$

$A_V = 100$

$$V_{out,min} = \left(\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\right)^{1/2},\quad V_{out,max} = V_{DD} - \left(\frac{2I_{D2}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}\right)^{1/2}$$

3_28

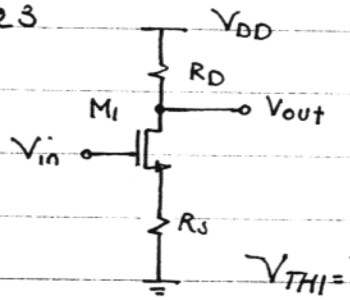
$$V_{DD} - \left(\frac{2I_D}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2}\right)^{1/2} - \left(\frac{2I_D}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}\right)^{1/2} = 2.2, \quad r_{o1} = \frac{1}{\lambda_1 I_D} = \frac{1}{0.1\times10^{-3}} = 10K$$

$$r_{o2} = \frac{1}{\lambda_2 I_D} = \frac{1}{0.2\times10^{-3}} = 5K, \quad r_{o1} \| r_{o2} = \frac{10^4}{3}, \quad g_{m1}(r_{o1}\|r_{o2}) = 100 \rightarrow g_{m1} = \frac{100\times3}{10^4} = 0.03$$

$$2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \times 10^{-3} = 9\times10^{-4} \rightarrow \left(\frac{W}{L}\right)_1 = \frac{9\times10^{-4}}{2\times1.34225\times10^{-4}\times10^{-3}} = 3352.5796$$

$$3 - \left(\frac{2\times10^{-3}}{1.34225\times10^{-4}\times3352.5796}\right)^{1/2} - 2.2 = \left(\frac{2\times10^{-3}}{3.835\times10^{-5}\left(\frac{W}{L}\right)_2}\right)^{1/2} \rightarrow \left(\frac{W}{L}\right)_2 = 96.97$$

3.23



$$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad R_D = 2K \quad R_S = 200\Omega$$

$$r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{0.1\times0.5\times10^{-3}} = 20K, \quad V_S = R_S I_D = 200\times0.5\times10^{-3} = 0.1$$

$$V_{TH1} = V_{TH1,0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right) = 0.7 + 0.45\left(\sqrt{0.9+0.1} - \sqrt{0.9}\right)$$

$$V_{TH1} = 0.723, \quad V_{out} = V_{DD} - R_D \cdot I_D = 3 - 2\times10^3\times0.5\times10^{-3} = 2$$

$$V_{DS} = 2 - 0.1 = 1.9$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})(1 + \lambda V_{DS}) = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (1+\lambda V_{DS}) I_D}$$

$$g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times\frac{50}{0.5}(1+0.1\times0.9)\times0.5\times10^{-3}} = 3.8249\times10^{-3}$$

$$\eta_1 = \frac{0.45}{2(0.1+0.9)^{1/2}} = 0.225 \quad G_m = \frac{g_{m1}\cdot r_{o1}}{R_S + \left[1 + (1+\eta_1) g_{m1}\cdot R_S\right] r_{o1}} =$$

$$G_m = \frac{3.8249\times10^{-3}\times20\times10^3}{200 + \left[1 + (1+0.225)\,3.8249\times10^{-3}\times200\right]20\times10^3} = 1.7644\times10^{-3}$$

$$R_{out} = \left[1 + (g_m + g_{mb}) r_o\right] R_S + r_o$$

Seen looking down at the drain of M1

$R_{out} = \left[1 + (1+0.225)\, 3.8249 \times 10^{-3} \right] 200 + 20 \times 10^{3} = 20.2 \times 10^{3}$ 3-29

$R_{out,total} = R_{out} \,//\, R_D = 1819.8274$, $A_V = -G_m \cdot R_{out,total} = -1.96 \times 10^{-3} \times 1819.8$

$A_V = -3.57$

$V_{out} = V_{in} - V_{TH1}$ @ the edge of the triode region

$V_{in} = V_{GS1} + R_S I_D$

$V_{DD} - R_D I_D = V_{out}$, $V_{DD} - R_D I_D = V_{GS1} + R_S I_D - V_{TH1}$, $V_{DD} - (R_S + R_D) I_D = V_{GS1} - V_{TH1}$

$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[V_{DD} - (R_S + R_D) I_D \right]^2 =$

$\frac{1}{2} \times 1.34225 \times 10^{-4} \times \frac{50}{0.5} \left[3 - (2000 + 200) I_D \right]^2$

$I_D = 6.71125 \times 10^{-3} (3 - 2200 I_D)^2 \rightarrow 32482.45 I_D^2 - 89.5885 I_D + 60.40125 \times 10^{-3} = 0$

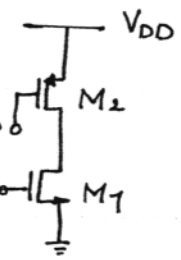
$I_{D1} = 1.5844 \times 10^{-3}$ (not acceptable), $I_{D2} = 1.17355 \times 10^{-3}$ (acceptable!)

$V_{in} = V_{DD} - R_D I_D + V_{TH1} = 3 - 2000 \times 1.17355 \times 10^{-3} + 0.7 = 1.35288\,V$

$g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times \frac{50}{0.5} \times 1.17355 \times 10^{-3}} = 5.6128 \times 10^{-3}$

$G_m = \frac{g_{m1}}{1 + g_{m1} \cdot R_S} = 2.6443 \times 10^{-3}$ $A_V = -G_m R_D = -2.6443 \times 10^{-3} \times 2000 = -5.2887$

3.24



$A_V = -5$, $\left(\frac{W}{L}\right)_1 = 20/0.5$, $I_{D1} = 0.5\,mA$, $V_b = 0$

$g_{m1} = 2 \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (1 + \lambda V_{DS}) I_D$

$g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times \frac{20}{0.5} (1 + 0.1 V_{DS}) \times 0.5 \times 10^{-3}}$

$r_{o1} = \frac{1}{\lambda_N I} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 20K$, $r_{o2} = \frac{1}{\lambda_P I} = \frac{1}{0.2 \times 0.5 \times 10^{-3}} = 10K$

The key point here is that the channel length modulation effect in M_1 cannot be neglected because its drain-source voltage is quite large. We take this effect into account with a few iterations.

First we let $V_{DS1}=0$, then we have, $g_{m1}=2.31711\times10^{-3}$ (as $A_v=-5$) 3-31

$R_{out,total}=2157.86\Omega$

$$r_{o2}=\frac{1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{SG}-|V_{TH2}|-V_{SD})}=2418.8356$$

$0.5\times10^{-3}=\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[(V_{SG}-|V_{TH2}|)V_{SD}-\frac{V_{SD}^2}{2}\right]$, by dividing these two relations together.

$$1.2094=\frac{(3-0.8)V_{SD}-0.5V_{SD}^2}{3-0.8-V_{SD}}=\frac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}},\quad V_{SD}^2-6.8188V_{SD}+5.3214=0$$

$V_{SD}=0.8987$, now second iteration starts, with the aid of the value we obtain for V_{SD} (or V_{DS}) from the first iteration, we have:

$g_{m1}=2.5489\times10^{-3}$, $R_{out}=1961.6020\Omega$

$$r_{o2}=2174.9182,\quad 1.087459=\frac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}},\quad V_{SD}^2-6.5749V_{SD}+4.7848=0$$

$V_{SD}=0.8336V$

Third iteration starts now:

By substituting the value of V_{SD} from the second iteration in the relation for g_{m1}, we get:

$g_{m1}=2.5558\times10^{-3}$, $R_{out}=1956.3119$, $r_{o2}=2168.4169\Omega$

$$1.0842=\frac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}},\quad V_{SD}^2-6.5684V_{SD}+4.77051=0$$

$V_{SD}=0.8315$

By doing the forth iteration:

$g_{m1}=2.5560\times10^{-3}$

$$R_{out}=1956.1662,\quad r_{o2}=2168.2379,\quad 1.0841189=\frac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}}$$

$V_{SD}^2-6.5682V_{SD}+4.77012=0$

$$\boxed{V_{SD}=0.8315}$$

3.31

$I = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 \left[(V_{SG} - |V_{TH2}|) V_{SD} - \frac{V_{SD}^2}{2} \right] . \left(\frac{W}{L}\right)_2 = \frac{0.5 \times 10^{-3}}{3.835 \times 10^{-5} \left[(3-0.8) 0.8315 - \frac{0.8315^2}{2} \right]}$

$\left(\frac{W}{L}\right)_2 = 8.7878$

If M_1 is at the edge of the triode region: $V_{out} = V_{in} - V_{TH1} = V_{in} - 0.7$

$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1}) = I_{D2} = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 \left[(V_{DD} - |V_{TH2}|)(V_{DD} - V_o) - \frac{(V_{DD} - V_o)^2}{2} \right] \times \left[1 + 0.2(V_{DD} - V_o)\right]$

$V_{out} = \sqrt{\frac{2 \times 3.835 \times 10^{-5}}{1.34225 \times 10^{-4}} \frac{8.7878}{40} \left[2.2(3 - V_o) - \frac{(3 - V_o)^2}{2} \right] (1.6 - 0.2 V_o)}$

$V_o = 0.6663$, $V_{in} = 1.3663$, $g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1}) = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{out} =$
$1.34225 \times 10^{-4} \times \frac{20}{0.5} \times 0.6663 = 3.5773 \times 10^{-3}$

However, M_2 is no longer in triode region because $V_o = 0.66 < V_b + |V_{TH2}| = 0.8$
Therefore, we should recalculate V_o with the assumption that M_2 is saturated

$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{out}^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_b - |V_{TH2}|)^2 \left[1 + \lambda_p (V_{DD} - V_o)\right]$

$536.9 V_o^2 + 32.623 V_o - 260.9845 = 0$, $V_{out} = 0.6674$, $V_{in} = 1.3674$

$g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH}) = 3.5837 \times 10^{-3}$, $I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{out}^2 = 1.196 \times 10^{-3}$

$r_{out} = r_{o1} \parallel r_{o2} = \frac{1}{(\lambda_p + \lambda_N) I} = 2786.962\,\Omega$

$A_v = -g_{m1} \cdot r_{out} = -9.9877$

$V_{out} = 0.8$, $\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2 \left[1 + \lambda_p (V_{DD} - V_o)\right]$

$1.34225 \times 10^{-4} \times 40 \times (V_{GS} - 0.7)^2 = 3.835 \times 10^{-5} \times 8.7878 (3 - 0.8)^2 \left[1 + 0.2(3 - 0.8)\right]$

$V_{in} = 1.3614$

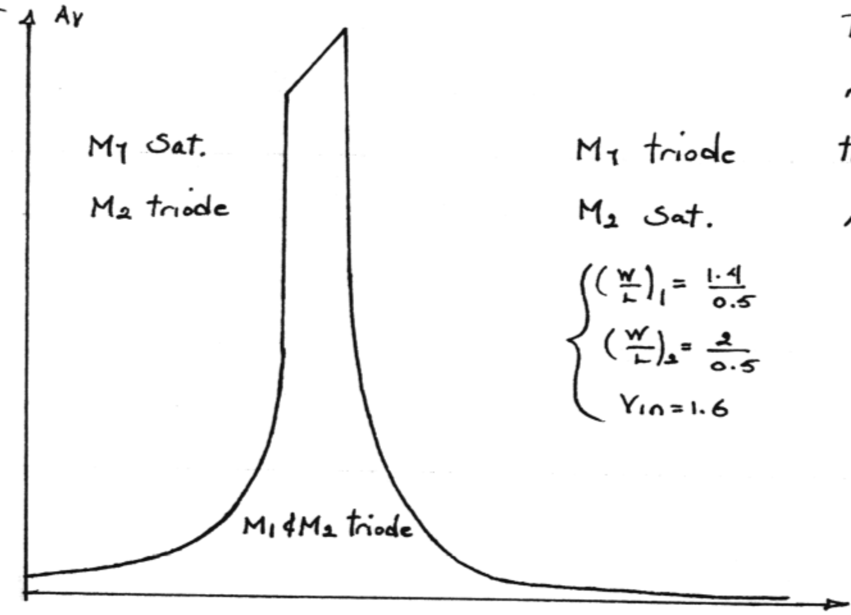
$$g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1}) = 3.5512 \times 10^{-3}$$

$$I = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2 = 1.1744 \times 10^{-3}$$

$$r_{out} = \frac{1}{(\lambda_P + \lambda_N) I} = 2838.2553$$

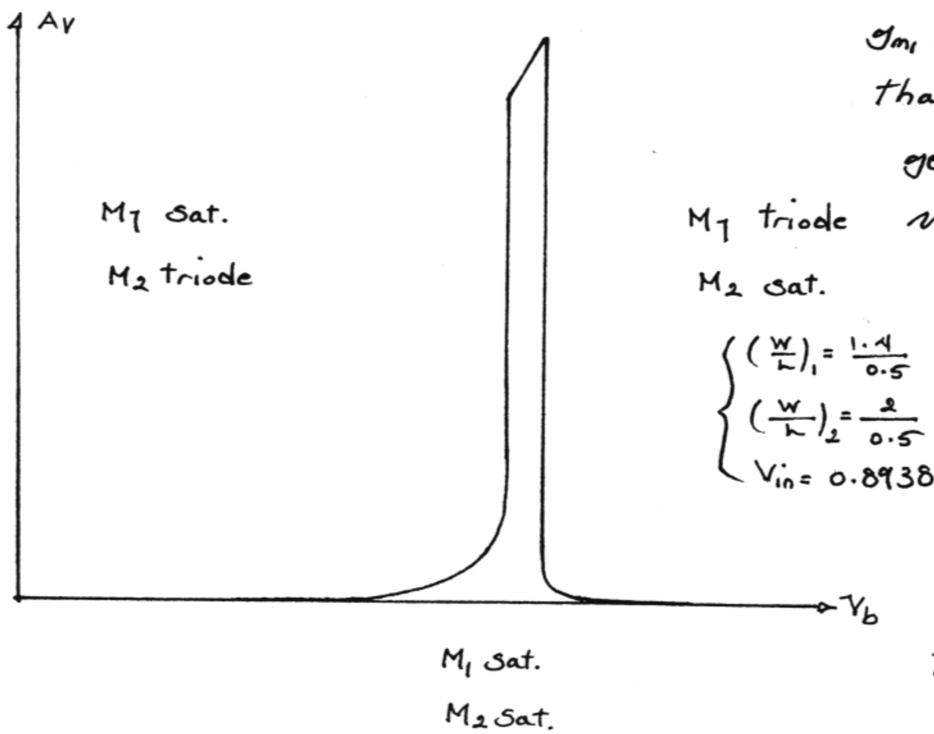
$$A_V = -g_{m1} \cdot r_{out} = -10.08$$

3.25

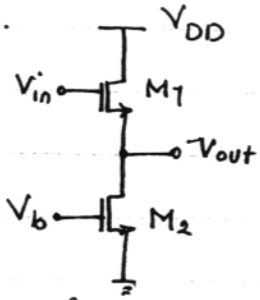


For M_1 to enter the triode region before M_2 is saturated, the overdrive voltage of M_1 must be increased.

Comparing the two curves, we observe that at $V_b = 0$ small signal voltage gain in (a) is higher than that in (b). That is because g_{m1} in (a) is higher than that in (b). However, generally, small signal voltage gain in (a) is less than that in (b), because when v_b sweeps all the way from 0 to V_{DD}, nowhere are both devices simultaneously in the saturation region.



3.26.



$V_{in} - V_{out} = 1V, \ I_{D1} = I_{D2} = 0.5\,mA, \ V_{GS2} - V_{GS1} = 0.5$

$\lambda = \gamma = 0$

$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{out} - V_{TH1})^2 =$
$\frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_b - V_{TH2})^2$

$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \left(\frac{W}{L}\right)_1 (1 - 0.7)^2$

$\left(\frac{W}{L}\right)_1 = 82.77 \qquad V_{GS2} = 0.5 + 1 = 1.5$

$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \left(\frac{W}{L}\right)_2 (1.5 - 0.7)^2 \rightarrow \left(\frac{W}{L}\right)_2 = 11.64$

$\gamma = 0.45\,V^{-1}, \ V_{in} = 2.5V, \ V_{in} - V_{out} = 1, \ I_{D1} = I_{D2} = 0.5\,mA, \ V_{GS2} - V_{GS1} = 0.5$

$V_{out} = V_{in} - 1 = 2.5 - 1 = 1.5, \ V_{GS2} = 0.5 + (2.5 - 1.5) = 1.5V$

$V_{TH1} = V_{TH0} + \gamma\left(\sqrt{2|\phi_f| + V_o} - \sqrt{2|\phi_f|}\right) = 0.7 + 0.45\left(\sqrt{0.9 + 1.5} - \sqrt{0.9}\right) = 0.97022$

$I_{D1} = I_{D2} = 0.5 \times 10^{-3} = \frac{1}{2}\mu_n C_{ox} S_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox} S_2 (V_{GS2} - V_{TH2})^2$

$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} S_1 (1 - 0.97)^2 = \frac{1}{2} \times 1.34225 \times 10^{-4} S_2 (1.5 - 0.7)^2$

$S_1 = \left(\frac{W}{L}\right)_1 = 8278$

$S_2 = \left(\frac{W}{L}\right)_2 = 11.64$

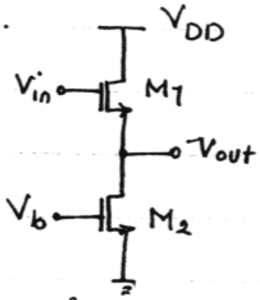
$V_{out} = V_b - V_{TH2} = 1.5 - 0.7 = 0.8$

$V_{TH1} = 0.7 + 0.45\left(\sqrt{0.9 + 0.8} - \sqrt{0.9}\right) = 0.8598$

$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 8278 (V_{in} - 0.8 - 0.8598)^2$

$V_{in} = 1.6897$

3.26.



$V_{in} - V_{out} = 1V, \quad I_{D1} = I_{D2} = 0.5\ mA, \quad V_{GS2} - V_{GS1} = 0.5$

$\lambda = \gamma = 0$

$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{out} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_b - V_{TH2})^2$

$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\left(\frac{W}{L}\right)_1 (1-0.7)^2$

$\left(\frac{W}{L}\right)_1 = 82.77 \qquad V_{GS2} = 0.5+1 = 1.5$

$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\left(\frac{W}{L}\right)_2 (1.5-0.7)^2 \rightarrow \left(\frac{W}{L}\right)_2 = 11.64$

$\gamma = 0.45\ V^{-1}, \quad V_{in} = 2.5V, \quad V_{in} - V_{out} = 1, \quad I_{D1} = I_{D2} = 0.5\ mA, \quad V_{GS2} - V_{GS1} = 0.5$

$V_{out} = V_{in} - 1 = 2.5 - 1 = 1.5, \quad V_{GS2} = 0.5 + (2.5-1.5) = 1.5V$

$V_{TH1} = V_{TH0} + \gamma\left(\sqrt{2|\phi_F| + V_0} - \sqrt{2|\phi_F|}\right) = 0.7 + 0.45\left(\sqrt{0.9+1.5} - \sqrt{0.9}\right) = 0.97022$

$I_{D1} = I_{D2} = 0.5\times10^{-3} = \frac{1}{2}\mu_n C_{ox} S_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox} S_2 (V_{GS2} - V_{TH2})^2$

$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4} S_1 (1-0.97)^2 = \frac{1}{2}\times1.34225\times10^{-4} S_2 (1.5-0.7)^2$

$S_1 = \left(\frac{W}{L}\right)_1 = 8278$

$S_2 = \left(\frac{W}{L}\right)_2 = 11.64$

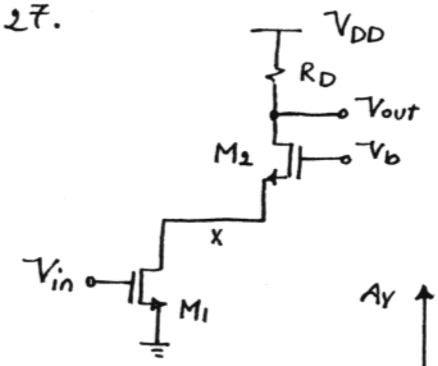
$V_{out} = V_b - V_{TH2} = 1.5 - 0.7 = 0.8$

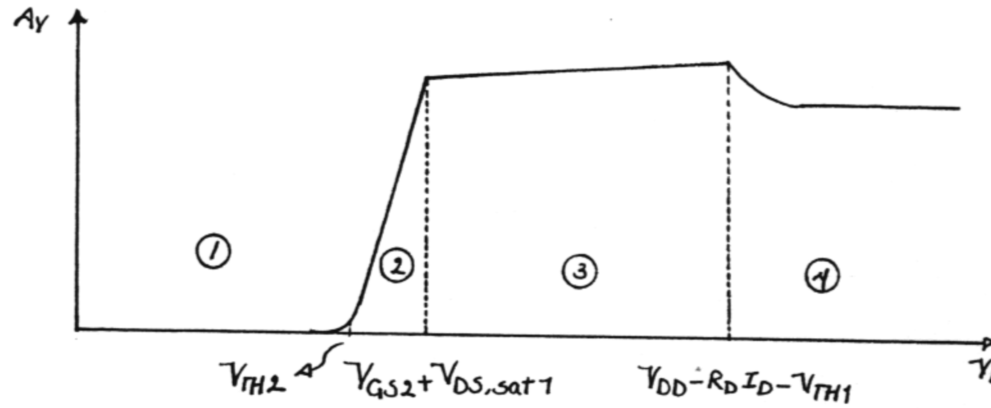
$V_{TH1} = 0.7 + 0.45\left(\sqrt{0.9+0.8} - \sqrt{0.9}\right) = 0.8598$

$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\times8278\,(V_{in} - 0.8 - 0.8598)^2$

$V_{in} = 1.6897$

3.27.





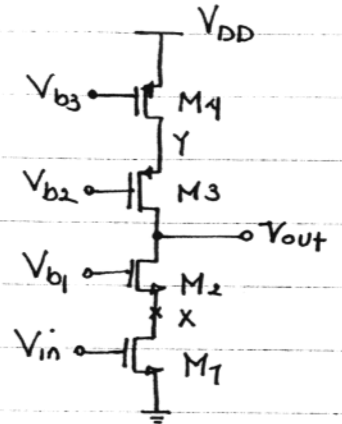
① In this region V_b is less than V_{TH2}, so M_1 and M_2 are off. It is worth mentioning that M_2 is saturated-off and M_1 is off in triode region.

② V_b is increasing above V_{TH2}, as a result, a current establishes in circuit. M_1 operates in triode region and M_2 does in saturation. The higher V_b, the higher the drain-source voltage of M_1, increasing the output impedance of M_1 which, in turn, causes the small signal voltage gain of the circuit increases.

③ Both devices are in saturation region and the maximum gain is attainable in this region. The slight increase in A_v is because of increasing the transconductance of M_1 with increasing V_x (or V_b).

④ M_2 enters the triode region, as a result, the total output impedance decreases down to the limit of $r_{o1} \| R_D$. Consequently, the small signal voltage gain experiences a similar change.

3.28



3.35

output swing = 1.9 V

$I_{bias} = 0.5\,mA$

$\gamma = 0 \quad \left(\frac{W}{L}\right)_{1-4} = \left(\frac{W}{L}\right)$

$V_{b1} - V_{TH1} < V_{out} < V_{b2} + V_{TH3}$

$V_{b2} + V_{TH3} - (V_{b1} - V_{TH1}) = 1.9$

$V_{b2} + 0.8 - V_{b1} + 0.7 = 1.9\ , \quad V_{b2} - V_{b1} = 0.4$

$$0.5 \times 10^{-3} = \frac{1}{2}\mu_n C_{ox} S (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox} S (V_{b1} - V_X - V_{TH2})^2 =$$

$$\frac{1}{2}\mu_p C_{ox} S (V_Y - V_{b2} - |V_{TH3}|)^2 = \frac{1}{2}\mu_p C_{ox} S (V_{DD} - V_{b3} - |V_{TH4}|)^2$$

$$V_{DD} - V_{SD\,min,4} - V_{SD\,min,3} - V_{DS\,min,1} - V_{DS\,min,2} = 1.9$$

$$1.1 = \left(\frac{2I_D}{\mu_p C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_p C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2}$$

$$1.1 = 2\sqrt{2I_D}\left(\frac{1}{\sqrt{\mu C_{ox}}} + \frac{1}{\sqrt{\mu_n C_{ox}}}\right)\frac{1}{\sqrt{S}} \rightarrow S = \frac{8I_D\left(\frac{1}{\sqrt{\mu_p C_{ox}}} + \frac{1}{\sqrt{\mu_n C_{ox}}}\right)^2}{1.1^2}$$

$$S = \frac{8 \times 0.5 \times 10^{-3}\left(\frac{1}{\sqrt{1.34225 \times 10^{-4}}} + \frac{1}{\sqrt{3.835 \times 10^{-5}}}\right)^2}{1.1^2} = 202.98 \rightarrow S = 203$$

$$V_{DS\,min,1} = \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2} = \left(\frac{2 \times 0.5 \times 10^{-3}}{1.34225 \times 10^{-4} \times 203}\right)^{1/2} = 0.1915$$

$$V_{SD\,min,4} = \left(\frac{2 \times 0.5 \times 10^{-3}}{3.835 \times 10^{-5} \times 203}\right)^{1/2} = 0.3584$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 203\,(V_{b1} - V_X - 0.7)^2$$

$V_{b1} - V_X = 0.8915$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 3.835 \times 10^{-5} \times 203\,(V_Y - V_{b2} - 0.8)^2$$

$V_Y - V_{b2} = 1.1584$

$V_{b2} - V_{b1} = 0.4$

If $V_X = 0.1915 \rightarrow V_{b1} = 1.083$, $V_{b2} = 1.483$, $V_Y = 2.6414$

$V_{SD4} = V_{DD} - V_Y = 0.3586$, as a result, M_7 and M_2 are at the edg of the triode region.

$$g_{m1} = \sqrt{2\mu_n C_{ox} S(1+\lambda V_{DS}) I_D} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 203 \times 0.5 \times 10^{-3}}$$

$$g_{m1} = g_{m2} = 5.2199 \times 10^{-3}$$

$$r_{o1} = r_{o2} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 20K \qquad r_{o3} = r_{o4} = \frac{1}{0.2 \times 0.5 \times 10^{-3}} = 10K$$

$$G_m = \frac{g_{m1} \cdot r_{o1} \cdot (1 + g_{m2} \cdot r_{o2})}{r_{o1} \cdot r_{o2}\, g_{m2} + r_{o1} + r_{o2}} = \frac{5.2199 \times 10^{-3} \times 20 \times 10^{3} (20 \times 10^{3} \times 5.2199 \times 10^{-3} + 1)}{(20 \times 10^{3})^2 \times 5.2199 \times 10^{-3} + 2 \times 20 \times 10^{3}}$$

$G_m = 5.17 \times 10^{-3}$, neglecting the body effect.

$$R_{out} = \left[(1 + g_{m2} r_{o2}) r_{o1} + r_{o2} \right] \| \left[(1 + g_{m3} r_{o3}) r_{o1} + r_{o3} \right]$$

$$R_{out} = \left[(1 + 5.2199 \times 10^{-3} \times 20 \times 10^{3}) 20 \times 10^{3} + 20 \times 10^{3} \right] \| \left[(1 + 2.79 \times 10^{-3} \times 10 \times 10^{3}) 10 \times 10^{3} + 10 \times 10^{3} \right]$$

$R_{out} = 262.1766 \times 10^{3}$, $A_V = -G_m R_{out} = -5.17 \times 10^{-3} \times 262.1766 \times 10^{3}$

$A_V = -1355.45$ $\qquad g_{m3} = g_{m4} = \sqrt{2 \times 3.835 \times 10^{-5} \times 203 \times 0.5 \times 10^{-3}} = 2.79 \times 10^{-3}$

Chapter 4: Differential Amplifiers

4.1

(a) $A_V \cong -\dfrac{g_{mN}}{g_{mP}} = -\sqrt{\dfrac{\mu_n (W/L)_N}{\mu_P (W/L)_P}}$ (4.52)

$$A_V = -\sqrt{\frac{350}{100} \times \frac{50/0.5}{50/1}} = -\sqrt{7} = \underline{-2.65}$$

(b) $A_V = -g_{mN}(r_{ON} \| r_{OP})$ (4.53)

$I_D = \dfrac{I_{SS}}{2} = 0.5^{mA}$ $\quad \mu_n C_{ox} = 350 \times \dfrac{8.85 \times 10^{-14} \times 3.9}{9 \times 10^{-7}} = 0.134 \ mA/V^2$

$$g_{mN} = \sqrt{2 I_D \mu_n C_{ox} \frac{W}{L}} = \sqrt{2 \times 0.5^m \times 0.134^m \times 100} = 3.66 \ m\Omega^{-1}$$

$L_N = 0.5^{\mu} \Rightarrow \lambda_n = 0.1 \rightarrow r_{ON} = \dfrac{1}{\lambda_n I_D} = \dfrac{1}{0.1 \times 0.5^m} = 20^{K\Omega}$

$L_P = 1^{\mu}$; $\lambda_P = 0.2$ for $L = 0.5^{\mu}$; $\lambda \propto \dfrac{1}{L} \rightarrow \lambda_P = 0.1$

$r_{OP} = \dfrac{1}{\lambda_P I_D} = \dfrac{1}{0.1 \times 0.5^m} = 20^{K\Omega}$

$$A_V = -g_{mN}(r_{ON} \| r_{OP}) = -3.66^m (20^k \| 20^k) = \underline{-36.6}$$

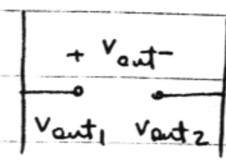
$(V_{in,cm})_{min} = 0.4 + V_{GS1}$ for both circuits

$$V_{GS1} = V_{TH} + \sqrt{\frac{2 I_D}{\mu_n C_{ox} (W/L)_N}} = 0.7 + \sqrt{\frac{2 \times 0.5^m}{0.134^m \times 100}} = 0.7 + 0.27 = 0.97^V$$

$\rightarrow (V_{in,CM})_{min} = 0.4 + 0.97 = 1.37^V$

max output voltage swing:

(a) $(V_{out1,2})_{max} = V_{DD} - |V_{TH,P}| = 3 - 0.8 = 2.2^V$



There are two constraints for $(V_{out1,2})_{min}$:

1) M_1 enters triode: $(V_{out1,2})_{min} = 0.4 + V_{GS1} - V_{TH,n}$

$$= 0.4 + 0.97 - 0.7 = 0.67^V$$

2) all of I_{SS} goes through M_3:

$$(V_{out1,2})_{min} = V_{DD} - |V_{GS3}|_{I_D = I_{SS}}| = V_{DD} - |V_{TH,P}| + \sqrt{\frac{2 I_{SS}}{\mu_p C_{ox} (\frac{W}{L})_3}}$$

$$= 3 - 0.8 - \sqrt{\frac{2 \times 1^m}{38.3^\mu \times 50}} = 3 - 0.8 - 1.02 = 1.18^V$$

$$\mu_p C_{ox} = 100 \times \frac{8.85 \times 10^{-14} \times 3.9}{9 \times 10^{-7}} = 38.3 \ \mu A/V^2 \qquad \Rightarrow (V_{out1,2})_{min} = 1.18^V$$

Max swing of $V_{out1,2} = 2.2 - 1.18 = 1.02^V$

Max swing of $V_{out} = 2 \times 1.02 = 2.04^V$

(b) $(V_{out1,2})_{max} = V_{DD} - |V_{GS3} - V_{TH,P}| = 3 - 0.72 = 2.28^V$

$(V_{out1,2})_{min} = 0.4 + V_{GS1} - V_{TH,n} = 0.67^V$

Max swing of $V_{out} = 2(2.28 - 0.67) = 3.22^V$

4.2 $I_{SS} = 1^{mA}$

(a)

$$A_v = -g_{m1}\left(\frac{1}{g_{m3}} \| r_{o1} \| r_{o3} \| r_{o5}\right) \approx -\frac{g_{m1}}{g_{m3}} = \sqrt{\frac{\mu_n}{\mu_p} \times \frac{I_{D1}}{I_{D3}}}$$

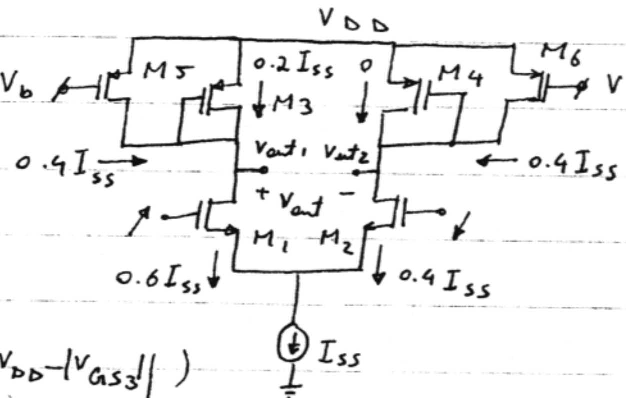
$$= \sqrt{\frac{350}{100} \times \frac{\frac{1}{2} I_{SS}}{0.2\,\frac{I_{SS}}{2}}} = -4.18$$

(b) $I_{D5} = I_{D6} = 0.8\left(\frac{I_{SS}}{2}\right) = 0.4^{mA}$

$$|V_{GS5}| = V_{DD} - V_b \Rightarrow V_b = V_{DD} - |V_{GS5}| = V_{DD} - |V_{TH,P}| - \sqrt{\frac{2 I_{D5}}{\mu_p C_{ox} \frac{W}{L}}}$$

$$V_b = 3 - 0.8 - \sqrt{\frac{2 \times 0.4^m}{38.3^{\mu} \times 100}} = 1.74$$

(c)



$$(V_{out1,2})_{max} = \min(V_b + |V_{TH,P}|, V_{DD} - |V_{TH,P}|)$$

$$= (1.74 + 0.8, 3 - 0.8) = 2.2^V$$

$$(V_{out1,2})_{min} = \max\left(V_{I_{SS},min} + V_{GS1}\Big|_{I_D = 0.6 I_{SS}} - V_{TH,n},\; V_{DD} - |V_{GS3}|\Big|_{I_D = 0.2 I_{SS}}\right)$$

$$V_{GS1}\Big|_{I_D = 0.6 I_{SS}} = V_{TH,n} + \sqrt{\frac{2 \times 0.6 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = V_{TH,n} + 0.299^V$$

$$|V_{GS3}|\Big|_{I_D = 0.2 I_{SS}} = |V_{TH,P}| + \sqrt{\frac{2 \times 0.2 I_{SS}}{\mu_p C_{ox} \frac{W}{L}}} = 0.8 + 0.323^V = 1.12^V$$

$$(V_{out1,2})_{min} = \max(0.4 + 0.299, 3 - 1.12) = 1.88^V$$

Max swing of $V_{out} = 2(2.2 - 1.88) = 0.64^V$

4.2 $I_{SS} = 1^{mA}$

(a)

$$A_v = -g_{m1}\left(\frac{1}{g_{m3}} \| r_{o1} \| r_{o3} \| r_{o5}\right) \approx -\frac{g_{m1}}{g_{m3}} = \sqrt{\frac{\mu_n}{\mu_p} \times \frac{I_{D1}}{I_{D3}}}$$

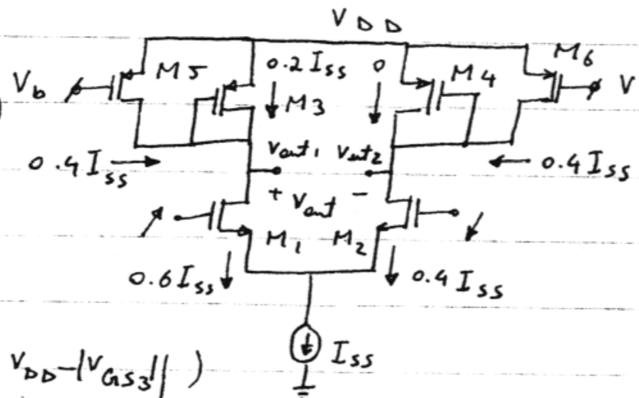
$$= \sqrt{\frac{350}{100} \times \frac{\frac{1}{2} I_{SS}}{0.2 \frac{I_{SS}}{2}}} = -4.18$$

(b) $I_{D5} = I_{D6} = 0.8\left(\frac{I_{SS}}{2}\right) = 0.4^{mA}$

$$|V_{GS5}| = V_{DD} - V_b \Rightarrow V_b = V_{DD} - |V_{GS5}| = V_{DD} - |V_{TH,P}| - \sqrt{\frac{2 I_{D5}}{\mu_p C_{ox} \frac{W}{L}}}$$

$$V_b = 3 - 0.8 - \sqrt{\frac{2 \times 0.4^m}{38.3^{\mu} \times 100}} = 1.74$$

(c)



$$(V_{out1,2})_{max} = \min(V_b + |V_{TH,p}|, V_{DD} - |V_{TH,p}|)$$

$$= (1.74 + 0.8, 3 - 0.8) = 2.2^V$$

$$(V_{out1,2})_{min} = \max\left(V_{I_{SS},min} + V_{GS1}\Big|_{I_D = 0.6 I_{SS}} - V_{TH,n},\ V_{DD} - |V_{GS3}|\Big|_{I_D = 0.2 I_{SS}}\right)$$

$$V_{GS1}\Big|_{I_D = 0.6 I_{SS}} = V_{TH,n} + \sqrt{\frac{2 \times 0.6 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = V_{TH,n} + 0.299^V$$

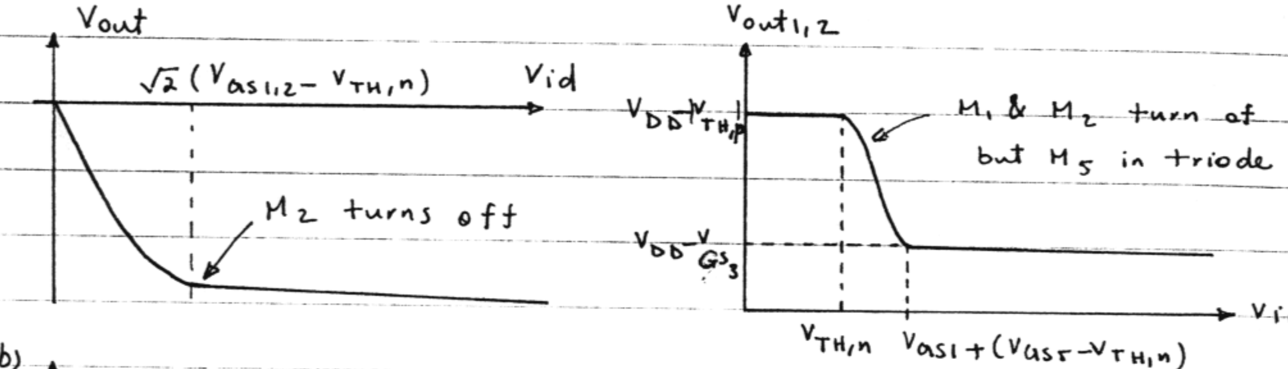
$$|V_{GS3}|\Big|_{I_D = 0.2 I_{SS}} = |V_{TH,P}| + \sqrt{\frac{2 \times 0.2 I_{SS}}{\mu_p C_{ox} \frac{W}{L}}} = 0.8 + 0.323^V = 1.12^V$$

$$(V_{out1,2})_{min} = \max(0.4 + 0.299, 3 - 1.12) = 1.88^V$$

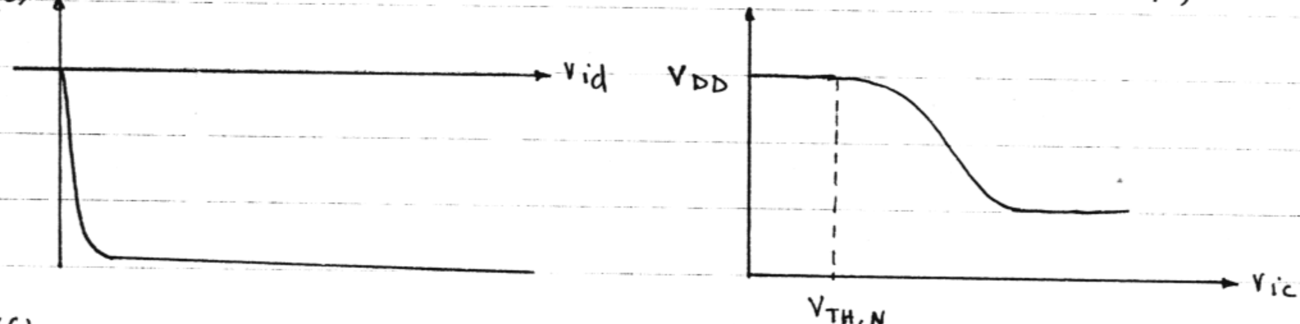
Max swing of $V_{out} = 2(2.2 - 1.88) = 0.64^V$

4.3

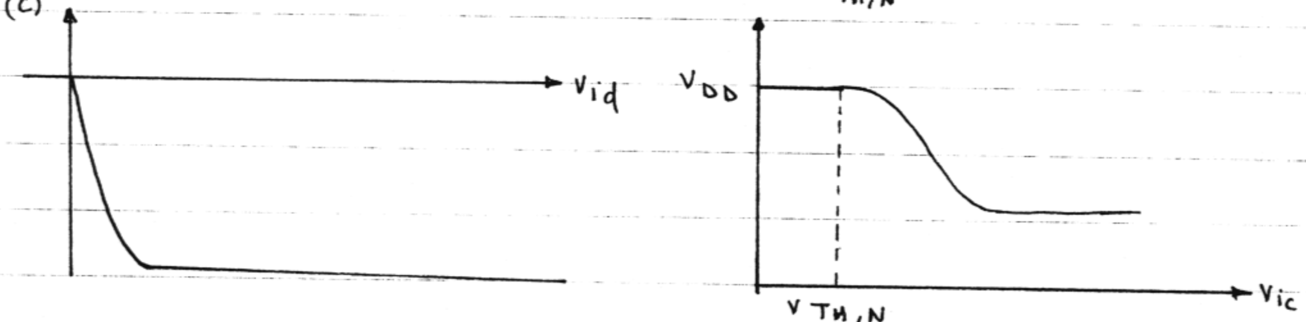
(a)


Vout
√2 (VGS1,2 − VTH,n)
Vid
M2 turns off
Vout1,2
VDD−|VTH,p|
M1 & M2 turn of
but M5 in triode
VDD−VGS3
Vic
VTH,n
VGS1 + (VGS5 − VTH,n)

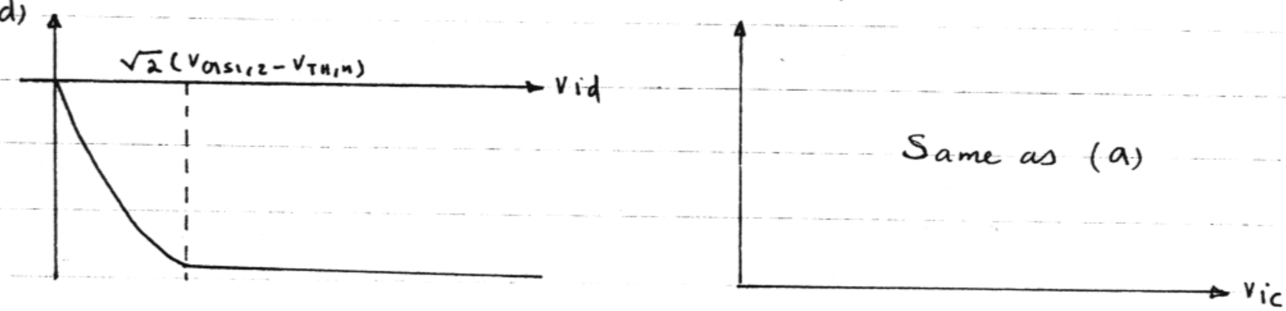
(b)


Vid
VDD
Vic
VTH,N

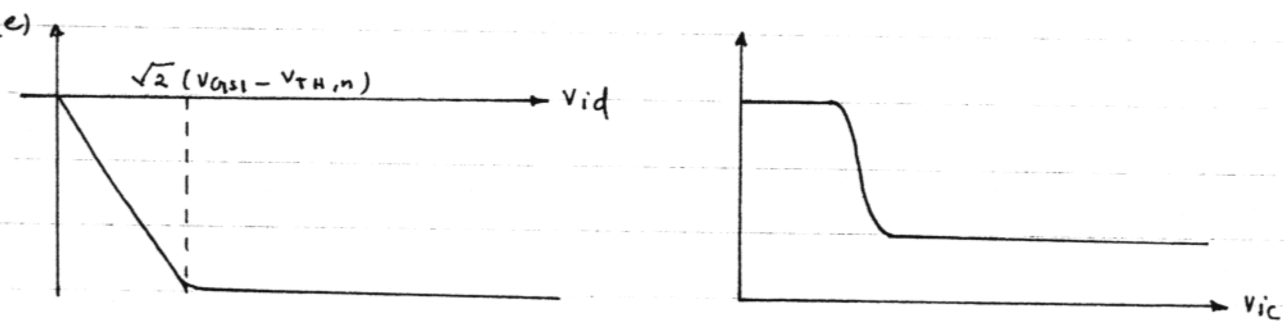
(c)


Vid
VDD
Vic
VTH,N

(d)


√2 (VGS1,2 − VTH,n)
Vid
Same as (a)
Vic

(e)


√2 (VGS1 − VTH,n)
Vid
Vic