DERIVATIVES – OPTIONS

A. INTRODUCTION

There are 2 Types of Options

- **Calls**: give the holder the RIGHT, at his discretion, to BUY a Specified number of a Specified Asset at a Specified Price on, or until, a specified Expiration Date
- **Puts**: give the holder the RIGHT, at his discretion, to SELL a Specified number of a Specified Asset at a Specified Price on, or until, a specified Expiration Date
- **American**: they can be exercised at any time until they expire
- **European**: can be exercised ONLY on the Expiration Date

Mechanics of the Options Market

- Either OTC or on an Exchange
- Stock Option Contracts: 100 Shares of an Underlying Stock
- Bond Option Contracts: 100 Bonds ($100,000 Par Value). Usually quoted in 64^{th}, i.e., a Quote of 2-29 means the dollar cost of acquiring the option is $2,453.125 (2 29/64 percent of $100,000)
- Stock Index Option Contracts: Settle in Cash (not delivery). Usually a Dollar Multiplier of 100.
- Stock Index Futures Option Contracts: Also uses a Dollar Multiplier. Usually $500.
- Foreign Currency Option Contracts: DM is DM125,000 and various.

B. VALUATION OF OPTION CONTRACTS

There are 2 Parts to an Option's Value

- **Intrinsic Value**: depends only upon the price of the underlying asset to the exercise price of the option.
  $C = S - X$, as long as positive, else $C = 0$
  $P = X - S$, as long as positive, else $P = 0$
- **Time Premium**: function of the probability that the option could change in value by the time it expires.

Factors Determining the Value of Options

1. *Exercise (Strike) Price of the Option*
2. *Price of the Underlying Asset*
3. *Volatility of the Underlying Asset*
4. *Time Until Expiration*
5. *Level of Interest Rates*
6. *Dividends on Stocks*
7. *Type (American or European) of Option*

Valuing Call Options Using the Single-Period Binomial Model (1-2-3) Forecast 1 period ahead

1. *Determine the Hedge Ratio*
   HR_{option} is the PLUG in the Following Equation →
   $\text{Price Stock}_{\text{Ending Low}} + (\text{Price Option}_{\text{Ending Low}} * \text{Plug}) = \text{Price Stock}_{\text{Ending High}} + (\text{Price Option}_{\text{Ending High}} * \text{Plug})$
2. *Discount the Hedged Ending Wealth to its Present Value using r_f as the Discount Rate*
   $\text{Ending Wealth}_{PV} = [\text{Stock Price}_{End} - (\text{Option Price}_{End})(HR_{Option})] / (1 + r_f)^t$
   Note, t is fraction of the year that the option is alive
3. *Determine the Value of the Call Option based upon the Current Price of the Stock*
   $\text{Ending Wealth}_{PV} = \text{Stock Price}_{\text{Current}} - (HR_{Option} * C)$

Valuing Put Options with Put-Call Parity

$S + P - C = [X/(1+r_f)^t] + [D_P/(1+r_f)^t]$

FACTORS Impacting the Valuation of Options

1. *Price of the Underlying Stock*
   DELTAs are the Sensitivities of the Put & Call Prices to Changes in the Price of the Underlying Security
   $\Delta_{Call} = (\Delta \text{ Call} / \Delta \text{ Stock})$ $\Delta_{Put} = (\Delta\text{Put} / \Delta \text{ Stock})$ $\Delta_{Call} - \Delta_{Put} = 1$
   GAMMA is the Rate at which the delta of an option changes as the price of the underlying security changes
   $\text{Gamma}_{Call} = (\Delta\Delta_{call}/\Delta\text{Stock})$ $\text{Gamma}_{Put} = (\Delta\Delta_{put}/\Delta\text{Stock})$
2. *Volatility of the Underlying Stock*
   VEGA is the Sensitivity of the Price of the Put & Call to Changes in the Volatility of the Underlying Stock
   $\text{Vega}_{Call} = (\Delta\text{Call} / \Delta\sigma_{Stock})$ $\text{Vega}_{Put} = (\Delta\text{Put} / \Delta\sigma_{Stock})$
3. *Level of the Risk-free Rate*
   RHO is the sensitivity of the Price of the Put & Call to Changes in the Volatility of the Risk Free Rate
   $\text{RHO}_{Call} = (\Delta\text{Call} / \Delta r_f)$ $\text{RHO}_{Put} = (\Delta\text{Put} / \Delta r_f)$
4. *Time 'til Expiration*
   THETA is the Sensitivity of the Price of the Put & Call to changes in the Time 'til Expiration
   $\text{Theta}_{Call} = (-\Delta\text{Call} / \Delta\text{Time})$ $\text{Theta}_{Put} = (-\Delta\text{Put} / \Delta\text{Time})$
5. *Dividends*
   No big deal here.

Valuing a Call Option using the Multi-period Binomial Model

- Kind of Difficult. No need to Memorize anything

Valuing a Call Option using the Black-Scholes Model

Assumptions

- Options are European
- r_f and σ_{Stock} are CONSTANT over the life of the option
- No Dividend paid by the Underlying Stock
- No Transaction Costs

Model

$C = S * N(d_1) - Xe^{-rt}N(d_2)$

→ N(d) is area under the Normal Curve from extreme left tail to (d)

→ $d_1 = [(\ln (S/X) + (r + 0.5\ \sigma_{stock}^{2})t] / \sigma_{stock}(t)^{1/2}$

→ $d_2 = d_1 - \sigma_{stock}(t)^{1/2}$

Drawbacks

- Performs well only near 'at-the-money' options
- Tends to undervalue American Options (since Assumes European)
- Tends to Overvalue Call Options while undervaluing put options on stock that pay dividends (since assumes no dividends)
- Hard to find an appropriate σ_{Stock} (try using either Historical, Scenario or Implied Volatility Approach)
- Must use Put-Call Parity to determine Price for Puts $S + P - C = Xe^{-rt}$
- Merton Model: Adjusts the Black-Scholes Model to account for Dividends (assumes continuous payments of dividends to simplify the mathematics) Doesn't work well for stock options, but GREAT for FOREIGN Currency Options which continuously compound interest

C. STRATEGIES EMPLOYING OPTIONS

Reasons for Employing Options

1. For Speculation (low priced & volatile leading to great speculating opportunities for profit)
2. Alter the Risk/Return Characteristics of a Portfolio (produce non-linear payoff patterns)
3. Lowers transactions costs for Short-term investment horizons
4. Used to execute some tax strategies
5. To Avoid Stock Restrictions

Must Analyze Strategies using:

- Maximum Loss that the Strategy can produce
- Maximum Gain that the Strategy can produce
- Break-even Point of the Strategy
- USE a PAY-OFF ANALYSIS TABLE

Common Strategies

1. Speculative Strategies

a. Buy a Naked Call

S **$C_{intrinsic}$ = Max (S-X;0)** **Value (100*C)** **Profit (100 *($C_{intrinsic}$-c)**
Max Loss = 100*C
Max Gain = Unlimited
Breakeven → S_{BE} = K+C
Use when Expect: BULL

b. Write a Naked Call

S **$C_{intrinsic}$ = Max (S-X;0)** **Value (-100*C)** **Profit (100 *(-$C_{intrinsic}$+c)**
Max Loss = Unlimited
Max Gain = 100*C
Breakeven → S_{BE} = K+C
Use when Expect: BEAR

c. Buy a Naked Put

S **$P_{intrinsic}$ = Max (X-S;0)** **Value (100*P)** **Profit (100 *($P_{intrinsic}$-p)**
Max Loss = 100*P
Max Gain = 100(X-P)
Breakeven → S_{BE} = X – P
Use when Expect: BEAR

d. Write a Naked Put

S **$P_{intrinsic}$ = Max (X-S;0)** **Value (-100*P)** **Profit (100 *(-$P_{intrinsic}$+p)**
Max Loss = - (X –P)*100
Max Gain = 100*P
Breakeven → S_{BE} = (X – P)
Use when Expect: BULL

2. Writing Covered Calls

Compare with Owning the Stock Straight-up
S **$C_{intrinsic}$ = Max (S-X;0)** **Value = 100(S-c)** **Profit=100(S-c-S_0+c_0)**
Lowers the Breakeven Point, limits upside potential and can produce a large downside loss
Use when Expect: STABILITY

3. Protective Put Strategy

Buy the Stock & Buy a Put on that Stock
S **$P_{Intrinsic}$ = Max (X-S;0)** **Value = 100(S+p)** **Profit = 100(S+$P_{Intrinsic}$ – S_0 –P_o)**
Limits Downside with Unlimited Upside Potential. But strategy RAISES the Breakeven point. It is a Form of PORTFOLIO insurance.

4. *Bull & Bear Spreads*

a.) Bull Spreads using Call Options

Buy Call low Strike & Sell Call with High Strike → Profit when S rises

S **$Call_{Low}$ =Max(S-C_{low};0)** **$Call_{High}$=Max(S-C_{High};0)** **$Value_{Portfolio}$(100(C_{low}-C_{high})** **Profit = 100(C_{low}-C_{High} – $C_{0\ Low}$ + $C_{0\ High}$)**

Max Gain: calculate

Max Los: Calculate

Break-even: Calculate

Use when expect: BULL

b.) Bear Spreads using Call Options

Buy Call with High Strike & Sell Call with Low Strike → Profit when S falls

S **$Call_{High}$ =Max(S-C_{High};0)** **$Call_{Low}$=Max(S-C_{Low};0)** **$Value_{Portfolio}$(-100(C_{High}- C_{Low})** **Profit = 100(C_{High} – C_{Low} – $C_{0\ High}$ + $C_{0\ Low}$)**

Use When Expect: BEAR

c.) Bull Spreads using Put Options

Buy Put LOW Strike and Sell Put with HIGH Strike

Calculate

d.) Bear Spreads using Put Options

Buy Put with HIGH Strike & Sell Put with LOW Strike

5. *Box Spreads*

Combination of BULL Spread with CALLS + BEAR Spread with PUTS

Arbitrage Strategy (same result at expiration regardless of price)

$P_{Box} = [X_{High} – X_{Low}] / [1 + (r_f t_m/360)]$

6. *Straddles*

Own a Put & Call with the Same Parameters. Works if Stock Soars or Plummets. Bad if Flat

7. *Strangles*

LONG Straddle: Own a Put with X below underlying Asset & Own a Call with X above underlying Asset

SHORT Straddle: Own a Put with X Above underlying Asset & Own a Call with X below underlying Asset

8. *Butterfly Spreads*

LONG: Buy 1 Call with High X, Sell 2 Calls with Med. X, Buy 1 Call with Low X

SHORT: Sell 1 Call with High X, Buy 2 Calls with Med. X, Sell 1 Call with Low X

LONG: Buy 1 Put with Low X, Sell 2 Puts with Med. X, Buy 1 Put with High X

SHORT: Sell 1 Put with Low X, Buy 2 Puts with Med. X, Sell 1 Put with High X

9. *Condor Spreads*

LONG: Buy 1 Call with Low X, Sell 1 Call with High X, Sell 1 Call with Higher X, Buy 1 Call with Higher X

SHORT: Sell 1 Call with Low X, Buy 1 Call with Higher X, Buy 1 Call with Higher X, Sell 1 Call with Higher X

LONG: Buy 1 Put with Low X, Sell 1 Put with Higher X, Sell 1 Put with Higher X, Buy 1 Put with Higher X

SHORT: Sell 1 Put with Low X, Buy 1 Put with Higher X, Buy 1 Put with Higher X, Sell 1 Put with Higher X

10. *Ratio Spreads*

Employs 2 or more related options that are traded in a specified proportion. Can be infinite.

11. *Calendar Spreads*

USING OPTION STRATEGIES

Strategy	Expectation of Asset Price
Buy Naked Call	⇑
Short a Naked Call	⇓
Buy a Naked Put	⇓
Short a Naked Put	⇑
Long Straddle	**Volatile**
Short Straddle	**Stable**
Long Strangle	**Very Volatile**
Short Strangle	**Stable**
Bull Spread	⇑
Bear Spread	⇓
Box Spread	**Arbitrage Mispricing**
Long Butterfly Spread	**Stable**
Short Butterfly Spread	**Volatile**
Long Condor Spread	**Stable**
Short Condor Spread	**Volatile**
Ratio Spreads	**Analyze Separately**

D. *MIMICKING & SYNTHESIZING PORTFOLIO CHARACTERISTICS USING PUT-CALL PARITY*

Owning a Stock & a Put, then borrowing X dollars at risk-free rate creates a portfolio that will replicate the behavior of a call option on the stock with a strike price = dollars borrowed and an exercise date = exercise date on the put.

$C = S + P – [X/(1+r_f)^t]$

Same for a Synthetic Put

$P = [X/(1+r_f)^t] + C - S$

Same for Synthetic Stock

$S = [X/1+r_f)^t] – P + C$

Put-Call-Forward Parity

$C – P = [F_0 – X] / [(1+r_f)^t]$

E. *USING STOCK OPTIONS TO HEDGE STOCK HOLDINGS*

N_o = - (Hedge Ratio/Contract Size) * (Quantity of Shares Being Hedged)

The Hedge Ratio is the Reciprocal of the DELTAs of the Options: $HR = 1/\Delta_{option}$

Implications

1. Selling Calls Short against a stock generates revenue for the hedger. IN addition, this is writing a covered call. This works best if the price of the stock does not change much either way; works badly when price of stock changes significantly.
2. Buying Puts incurs a higher cost for the hedger. This is essentially a Protective Put. Works Best if the Price of the Stock rises or falls significantly, bad if price remains flat.
3. As the price of the Stock drops below the strike on a Call, the Hedge ratio moves towards ∞; thus calls can’t be used for long, thus puts are required.
4. As the price of stock rises above the strike on a put, the hedge ratio moves towards ∞. Ditto from above.
5. Also, Option Deltas are not as predictable in reality as in theory.
6. Options are only available for the short term; can’t hedge in the long term
7. Cost of hedge varies with the cost of the option

F. VALUATION OF WARRANTS

$W = C / [1 + (N_W/N_S)]$

N_w is the number of shares that the Warrant Converts into

N_s is the number of shares outstanding (excluding the potential dilution from the exercise of the warrants)

- Warrants are basically Long-term Call options issued by a Corporation (rather than an independent option writer). When Exercised, the firm issues additional shares of stock in exchange for cash.