QUANTITATIVE METHODS

1. Time Series Analysis and Forecasting

- Economic Time Series Data can be broken down into **4** Components
    - **Secular Trend**: the smooth, long-tem direction of time series
    - **Cyclical Variation**: SIGNIFICANT fluctuations above and below the secular trend that occur over an extended period of time (+1 year)
    - **Seasonal Variation**: Fluctuations in data that REGULARLY occur at about the same time each year
    - **Irregular Variation:** variations that are neither cyclical nor seasonal, 2 types
        - ***Episodic Variations***: identified with random occurring events (war, pestilence, famine)
        - ***Residual Variations***: small RANDOM fluctuations that are unpredictable, and associated with neither specific events nor cyclical variations

a. *Measuring Linear Secular Trends*

- If the Secular trend is LINEAR, there are 2 methods that are commonly used to measure the Secular Trend Linear Regression Analysis and Moving Average

i. Linear Regression Analysis

- Fits the Time Series data to a straight line equation of the following form:

$$\mathbf{Y_t = a + b(t)}$$

Where:

Y_t = the value of the time series (Y_t) at time, t

a = the Intercept: the value of Y_t when t equals zero (the BASE from which time is measured

b = the REGRESSION COEFFICIENT, the amount by which the value of the time series (Y_t) increases per unit increase in time (t):

$$b = (\Delta Y_t / \Delta t)$$

- The Method of LEAST SQUARES regression is normally used to determine the values of the INTERCEPT (a) and the REGRESSION COEFFICIENT (b).

$$b = (n\Sigma tY - (\Sigma Y)(\Sigma t)) / (n\Sigma t^2 - (\Sigma t)^2)$$

$$a = \bar{Y} - b\bar{t}$$

- *For Example*; Using Linear Regression Analysis, construct an equation for and draw a graph of the secular trend of the following data:

Year	Sales
19X0	100
19X1	120
19X2	90
19X3	140
19X4	150

Answer: Construct a Table of the Summations needed to Calculate the Regression Coefficients

t	**Y**	**tY**	**t^2**
0	100	0	0
1	120	120	1
2	90	180	4
3	140	420	9
4	150	600	16
10	**600**	**1320**	**30**

$b = (n\Sigma tY - (\Sigma Y)(\Sigma t)) / (n\Sigma t^2 - (\Sigma t)^2)$

$= [5(1320) - (600)(10)] / [(5)(30) - 10^2] = 12$

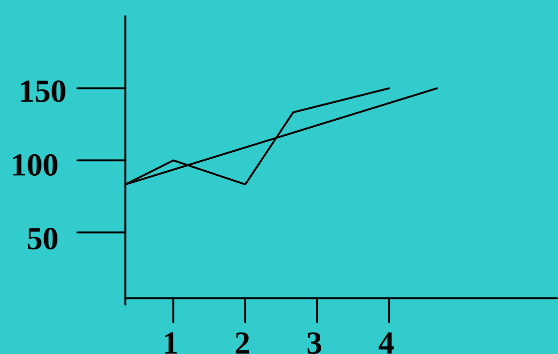
$a = \bar{Y} - b\bar{t}$

$= (600)(5) - 12(10/5) = 96$

$Y_t = a + bt$

$= 96 + 12t$

Plotting the results of this equation against the actual data:



The Trendline (straight) measures the LINEAR SECULAR TREND, while the variations about the Trend measure the CYCLICAL and IRREGULAR VARIATIONS (Seasonal variations are not shown when only Annual Data is given)

ii. Moving Average

- If the SECULAR trend is Approximately linear and the Variations around the trend are Approximately PERIODIC, the Cyclical, Seasonal, and Irregular variations around the trend can be mostly eliminated by using a centered MOVING Average of the Time Series Data whose Length is Equal to the Frequency of the Cyclical Variation

For Example: Calculate and Plot the 5-year, centered Moving Average of the following Data:

YEAR	SALES
1	100
2	130
3	140
4	140
5	120
6	150
7	180
8	190
9	170
10	160
11	190
12	210
13	200
14	180
15	180
16	240
17	250
18	250
19	240
20	220

Answer: To calculate the 5-year moving average, compute the average of every 5 successive data points in the time series. Plot the resulting average at the mid point of each of the five values used to compute the average, as indicated below (intentional error with year numbers)

First: Get the 5 Year Moving Total (Uncentered); sum yrs 1-5, 2-6, etc.

Second: Center the 5 Year Moving Total (by plotting Year 20 in 18, 19 in 17 and going back until 5 is in 3)

Third: Get 5 Year Moving Average (centered) by dividing that centered 5 year moving total by 5

iii. Loglinear Regression Models

- Not all regression models are Linear. Loglinear and Power function models are often encountered when working with economic and financial data. However, these functions can be transformed into linear forms by using logarithms. Consequently, after making such a transformation, it is possible to apply the linear regression methods to these types of NONLINEAR relationships.
- <u>*Loglinear Transformation*</u>

  A **Loglinear** Model is one that has the Mathematical form:

  $$Y = ab^x$$

  Rewriting this equation in logarithmic form produces:

  $$\text{Log } Y = \log(a) + \log(b)x$$

  Because a & b are constants, their logarithms are also constants; thus, in logarithmic form, this is a linear model:

  $$U = m + nx$$

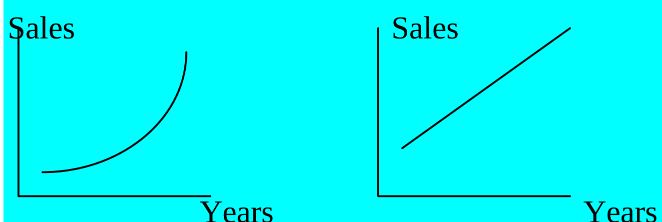
  Where $U = \log Y$
  $m = \log a$
  $n = \log b$

  Loglinear functions are commonly used in time series analysis, where a variable grows at some CONSTANT AVERAGE GROWTH RATE over time

For Example: suppose sales are $1,000 in 19X0 and they are growing at a rate of 5% per year. The mathematical formulation of this relationship is:

$$S_t = S_0\,(1+g)^t = 1{,}000(1.05)^t$$

S_t = the sales at any time, t
t = the time measured in years from 19X0 when t = 0



Arithmetic Graph Paper   Semilogarithmic Graph Paper

Also, there is a relationship between the slope of this straight line on the semilogarithmic graph and the average annual growth rate of sales, as depicted by the curved relationship on the arithmetic graph. Note:

$S_t = S_0\,(1+g)^t = 1000(1.05)^t$
$\text{Ln}S_t = \ln S_0 + [\ln(1+g)]*t = 6.9078 + .0488t$
$S_0 = e^{6.9078} = 1000$
$G = e^{.0488} - 1 = .05$

- *Power Function Transformation*

  A Power function is an equation that appears as follows

  $Y = aX^b$

  Rewriting this into a logarithmic form produces the following

  $LogY = \log(a) + (b)\log(X)$

  This is a linear model in the form of

  $U = m + bV$

  Where $U = \log(Y)$

  $M = \log(a)$

  $V = \log(X)$

  Power functions are often used to represent supply and demand functions in economics, where b is the elasticity of the supply or demand function

b. *Measuring Nonlinear Secular Trends*

- The Secular trends of MOST economic and financial time series are NOT Linear: rather, they are EXPONENTIAL trends that grow at an average annual growth rate

For Example: Describe the Secular Trend and Average Annual Growth Rate of the Following Time Series of Earnings.

YEAR	t	EARNINGS(E)
X1	1	1.20
X2	2	0.90
X3	3	1.40
X4	4	1.50
X5	5	1.30

TWO Approaches could be used to ESTIMATE the Secular Growth Rate of these data.

FIRST, use the CONVENTIONAL Compound Interest Formulation

$E = E_0(1+g)^t$

$FV = PV(1+g)^t$

Using only the FIRST and LAST Data Points, this method implies an Average Growth Rate of 2.02%

FV = 1.30 , PV = 1.20; t = 5-1=4

$FV/PV = (1+g)^t$ → 1.30/1.20 = 1.0833 → $(1+g)^4 = (1.0833)^{.25}$ →g= .0202

This first method, while easy to compute, relies on only 2 data points. It ignores the rest of the data and consequently, this method computes the ACTUAL (HISTORICAL) Average ANNUAL Growth Rate for the period between the first and last point in a time series. But it is NOT an accurate measure of the secular growth rate that should be used to make long-term earnings projections. (preferable only to use historical method for measuring the past ACTUAL growth rates).

Instead, when projecting secular trends, the SECOND METHOD to use is a LOGLINEAR Regression Model

$E = E_0 (1+g)^t$

$Ln(E) = \ln(E_0) + [\ln(1+g)]*(t)$

Since $U = a + b(t)$

And, $\ln(E) = U$, $\ln(E_0) = a$, and $\ln(1+g) = b$

Using the data from above, a Simple Linear Regression can be performed when you transfer the data into logarithmic form

T	U=ln(E)	t(U)	t^2
1	.182322	.182322	1
2	-.105361	-.210722	4
3	.336472	1.009416	9
4	.405465	1.621860	16
5	.262364	1.311820	25
15	**1.081262**	**3.914696**	**55**

U = a + bt

b = ln(1+g)

b = [(t)(Σ(t)(lnE)) – (Σt)(Σ(lnE))] / [(t)(Σt^2) –(t)(Σt^2)]

[(5)(3.914696) - (15)(1.081262)] / [(5)(55) – 225)] = .067091

1+g = $e^{.067091}$ = 1.0694

g = 6.94%

a = ln(E_0)

a = [(Σln(E)/t) – (b)(t-2)]

a = [(1.081262/5) – (.067091)(3)] = .01479

E_0 = $e^{.014979}$ = $1.015

E = $1.015(1.0694)^t

This is the BEST Estimate of the TRENDLINE SECULAR Growth rate that can be used to project the LONG-TERM Future Earnings Trent (starting with a value of $1.015 in the base year) is 6.94% per year.

c. *Seasonal Indexes*

- The ratio of every data point in a time series to a 1 YEAR moving average of the Data can be used to determine whether or not there is a STRONG Seasonal component to the time series

For Example: consider the following 5 years of quarterly sales data

YEAR	Q1	Q2	Q3	Q4
1	500	550	520	510
2	540	570	550	530
3	580	600	570	560
4	630	660	620	610
5	680	700	670	650

In order to construct SEASONAL INDEXES for these data

1. COMPUTE a CENTERED 1 year (4 quarter) MOVING Average of the Data
2. AVERAGE successive centered moving averages so that the result corresponds to the same period as the quarterly raw data
3. COMPUTE the Ratio of each data point in the time series (for which a moving average exists to its corresponding average moving average value. These Ratios are called SPECIFIC SEASONAL INDEXES

YEAR	QUARTER	SALES	4Q Moving Average(CTR)	Avg of Successive Moving Averages	Specific Seasonal Index
1	I	500			
	II	550	520		
	III	520	530	525	.9905
	IV	510	535	532.5	.9577
2	I	540	542.5	538.75	1.0023
	II	570	547.5	545	1.0459
	III	550	557.5	552.5	.9955
	IV	530	565	561.25	.9443
3	I	580	570	567.5	1.0220
	II	600	577.5	573.75	1.0458
	III	570	590	583.75	.9764
	IV	560	605	597.5	.9372
4	I	630	617.5	611.25	1.0307

	II	660	630	623.75	1.0581
	III	620	642.5	636.25	.9745
	IV	610	652.5	647.5	.9421
5	I	680	665	658.75	1.0323
	II	700	675	670	1.0448
	III	670			
	IV	650			

4. AVERAGE the Specific Seasonal Indexes for the 1st, 2nd 3rd and 4th quarters in order to obtain a first approximation of the seasonal index for each quarter

QUARTERLY SPECIFIC SEASONAL INDEXES

	Q1	**Q2**	**Q3**	**Q4**
	1.0023	1.0459	.9905	.9577
	1.0220	1.0458	.9955	.9443
	1.0307	1.0581	.9764	.9372
	1.0323	1.0448	.9745	.9421
Σ	**4.0873**	**4.1946**	**3.9369**	**3.7813**
Average	**1.0218**	**1.0487**	**.9842**	**.9453**

5. In order to force the sum of the three average seasonal indexes to equal 4.00, perform the following correction:

   Correction Factor $= n / (Avg.n_1 + Avg.n_2 + Avg.n_3 + Avg.n_4)$

   Here (4.00) / (1.0218 + 1.0487 + .9842 + .9453) = 1.00

   Usually the CORRECTION FACTOR is Slightly above or below 1.00

6. The Seasonal Indexes for the 4 quarters are the average of the specific seasonal indexes, multiplied by the CORRECTION FACTOR

Quarter	**Avg. Specific Index ***	**Correction Factor =**	**Seasonal Index**
1	1.0218	1.00	1.0218
2	1.0487	1.00	1.0487
3	.9842	1.00	.9842
4	.9453	1.00	.9453

Note: It is rare to have to do this much number crunching on the exam

i. *Deseasoninalizing Data*

- Once a set of SEASONAL Indexes have been Constructed for a TIME SERIES, UNSEASONALIZED Data can be DESEASONALIZED by DIVIDING the RAW Data by the SEASONAL INDEX

For Example: The Raw quarterly sales data given above can be deseasonalized as follows

YEAR	**Quarter**	**Unadjusted /**	**Seasonal Index =**	**Deseasonalized Sales**
1	I	500	1.0218	489.33
	II	550	1.0487	524.46
	III	520	.9842	528.35
	IV	510	.9453	539.51
2	I	540	1.0218	528.48
	II	570	1.0487	543.53
	III	550	.9842	558.83
	IV	530	.9453	560.67
3	I	580	1.0218	567.63
	II	600	1.0487	572.14
	III	570	.9842	579.15
	IV	560	.9453	592.40
4	I	630	1.0218	616.56
	II	660	1.0487	629.35
	III	620	.9842	629.95
	IV	610	.9453	645.30
5	I	680	1.0218	665.49
	II	700	1.0487	667.49
	III	670	.9842	680.76
	IV	650	.9453	687.61

The Deseasonalized DATA contain the SECULAR trend, Cyclical Variation and Irregular Variation components

d. Time Series Projections

- If it is Desired to PROJECT a Time Series trend and Make actual forecasts from it, do the following procedure.

1. Start with Historical DESEASONALIZED Data

2. Perform a LINEAR or LOGLINEAR Regression on the Deseasonalized Time Series to Project the SECULAR Trend.

- *For Example*: Using the Deseasonalized data from the preceding table and fit the Data to a Linear Regression line, will have the following Results

YEAR	**Quarter**	**(Y) Sales**	**(t)**	**tY**	**t^2**
1	I	489.33	1	489.33	1
	II	524.46	2	1048.92	4
	III	528.35	3	1585.05	9
	IV	539.51	4	2158.04	16
2	I	528.48	5	2642.40	25
	II	543.53	6	3261.18	36
	III	558.83	7	3911.81	49
	IV	560.67	8	4485.36	64
3	I	567.63	9	5108.67	81
	II	572.14	10	5721.40	100
	III	579.15	11	6370.65	121
	IV	592.40	12	7108.80	144
4	I	616.56	13	8015.28	169
	II	629.35	14	8810.90	196
	III	629.95	15	9449.25	225
	IV	645.30	16	10324.80	256
5	I	665.49	17	11313.33	289
	II	667.49	18	12014.82	324
	III	680.76	19	12934.44	361
	IV	687.61	20	13752.20	400
		11806.99	210	130506.63	2870

$b = [n\Sigma tY - (\Sigma t)(\Sigma Y)] / [n\Sigma t^2 - (\Sigma t)^2]$

$= [20(130506.63) - (210)(11806.99)] / [(20)(2870) - (210)^2]$

$= 9.8244$

$a = \bar{Y} - b\bar{X}$

$= (11806.99)/(20) - (9.8244)(210/20)$

$= 487.1933$

3. The Regression equation can then be used to project the secular trend for the next **4** quarters, which is the DESEASONALIZED Sales projections for Quarters 21, 22, 23, and 24:

Projection of Secular Trend

YEAR	**QUARTER**	**t**	**Deseasonalized Trend ($Y_t = a + bt$)**
6	I	21	693.51
	II	22	703.33
	III	23	713.15
	IV	24	722.98

4. These deseasonalized quarterly projections are then MULTIPLIED by their RESPECTIVE SEASONAL INDEXES to Obtain the actual, SEASONALIZED estimates for the Next 4 Quarters

YEAR	**Quarter**	**Deseasonalized Sales**	**X Seasonal Index =**	**Sales Estimate**
6	I	693.51	1.0218	708.63
	II	703.33	1.0487	737.58
	III	713.15	.9842	701.88
	IV	722.98	.9453	683.43