NCEA 3.4 – Complex Numbers and Solving Equations
Revision Sheet No 1

1. Solve the following equations:

a. $3^x = 2^{4-x}$

$\Rightarrow \log 3^x = \log 2^{4-x}$

$\Rightarrow x \log 3 = (4-x)\log 2$

$\Rightarrow x \log 3 = 4\log 2 - x\log 2$

$\Rightarrow x\log 3 - x\log 2 = 4\log 2$

$\Rightarrow x(\log 3 - \log 2) = 4\log 2$

$\Rightarrow x = \frac{4\log 2}{(\log 3 - \log 2)} = 6.838$

b. $\sqrt{2x+2} = 2x - 10$ ①

$\Rightarrow 2x+2 = (2x-10)^2$

$\Rightarrow 2x+2 = 4x^2 - 40x + 100$

$\Rightarrow 4x^2 - 42x + 98 = 0$

$\Rightarrow 2(2x^2 - 21x + 49) = 0$

$\Rightarrow (x-7)(2x-7) = 0$

$\Rightarrow x = 7 \text{ or } 7/2$

sub 7 into ①

$4 = 2(7) - 10$ ✓

7 is a solution

sub 7/2 into ①

$3 \neq 2(7/2) - 10$

7/2 is NOT a solution

c. $\log_5(2x+3) = 2.048$

$2x + 3 = 5^{2.048}$

$\Rightarrow x = \frac{5^{2.048} - 3}{2}$

$\Rightarrow x = 12.004$

d. $x^3 - 3x^2 - 40x + 84$

using factor theorem

$f(2) = (2)^3 - 3(2)^2 - 40(2) + 84$

$= 8 - 12 \quad 80 + 84$

$= 0 \Rightarrow (x-2)$ is a factor.

$$\begin{array}{r} x^2 - x \mp 42 \\ x-2 \overline{)\, x^3 - 3x^2 - 40x + 84} \\ \underline{x^3 - 2x^2} \\ -x^2 - 40x \\ \underline{-x^2 + 2x} \\ -42x + 84 \end{array}$$

hmmm!

$\Rightarrow x^2 - x + 42$

$= (x+6)(x-7)$

solutions $x = 2, 7, -6$

e. $z^2 - 3z + 9 = 0$

$a = 1 \quad b = -3 \quad c = 9$

$x = \frac{3 \pm \sqrt{9 - 36}}{2}$

$\Rightarrow x = \frac{3 \pm \sqrt{27}}{2}$

$x = \frac{3}{2} \pm \frac{5.1}{2} i$

2. Evaluate the following if:

$a = 3 + 2i \qquad b = 5 - 7i \qquad c = 3 + 2\sqrt{5} \qquad d = 6 - \sqrt{5}$

a. $2a + \bar{b} = 2(3+2i) + (5+7i)$

$= 6 + 4i + 5 + 7i$

$= 11 + 11i$

b. $\bar{a}b = (3-2i)(5-7i)$

$= 15 - 21i - 10i + 14i^2$

$= 1 - 31i$

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c. $cd = (3+2\sqrt{5})(6-\sqrt{5})$

$= 18 - 3\sqrt{5} + 12\sqrt{5} - 10$

$= 8 + 9\sqrt{5}$

d. $\frac{c}{d} = \frac{3+2\sqrt{5}}{6-\sqrt{5}}$

$= \frac{(3+2\sqrt{5})}{(6-\sqrt{5})} \times \frac{(6+\sqrt{5})}{(6+\sqrt{5})}$

$\Rightarrow \frac{c}{d} = \frac{18+3\sqrt{5}+12\sqrt{5}+10}{31}$

$= \frac{28+15\sqrt{5}}{31}$

3. Convert $^{-}3+5i$ to polar coordinates.

$r = \sqrt{9+25}$

$= \sqrt{34}$

θ:

$\theta = \sin^{-1}\frac{5}{\sqrt{34}}$

$= 59^\circ$ (2sf)

$\therefore {}^{-}3+5i = \sqrt{34}\,\text{cis}\,59^\circ$

4. Convert $1.732cis\left(-\frac{2\pi}{3}\right)$ to rectangular coordinates

$x = r\cos\theta$

$= 1.732\cos\left(-\frac{2\pi}{3}\right)$

$= -0.866$

$y = r\sin\theta$

$= 1.732\sin\left(-\frac{2\pi}{3}\right)$

$= -1.5$

$1.732\,\text{cis}\left(-\frac{2\pi}{3}\right) = {}^{-}0.866 - 1.55i$

5. Evaluate z_1z_2 and $\frac{z_1}{z_2}$ when:

$z_1 = 3cis\left(-\frac{\pi}{6}\right)$ and $z_2 = 6cis\left(\frac{\pi}{4}\right)$

$z_1z_2 = 3\,\text{cis}\left(-\frac{\pi}{6}\right) \times 6\,\text{cis}\left(\frac{\pi}{4}\right)$

$= 18\,\text{cis}\left(\frac{3\pi-2\pi}{12}\right)$

$= 18\,\text{cis}\left(\frac{\pi}{12}\right)$

$\frac{z_1}{z_2} = \frac{3\,\text{cis}\left(-\frac{\pi}{6}\right)}{6\,\text{cis}\left(\frac{\pi}{4}\right)}$

$= \frac{1}{2}\,\text{cis}\left(\frac{-2\pi-3\pi}{12}\right)$

$= \frac{1}{2}\,\text{cis}\left(\frac{-5\pi}{12}\right)$

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6\. Evaluate z^5 when $z = 2cis\left(\frac{\pi}{3}\right)$

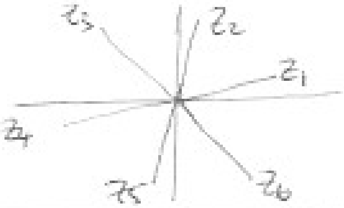
$$z^5 = \left[2cis\left(\frac{\pi}{3}\right)\right]^5$$
$$= 2^5 cis\left(5 \times \frac{\pi}{3}\right)$$
$$= 32\ cis\left(\frac{5\pi}{3}\right.$$
$$= 32\ cis\left(-\frac{\pi}{3}\right)$$

7\. Evaluate $z^{1/6}$ when $z = cis\left(\frac{\pi}{2}\right)$

$$z^{1/6} = \left[cis\left(\frac{\pi}{2}\right)\right]^{1/6}$$
$$z_1 = cis\left(\frac{\pi}{12}\right)$$

solution are symmetric and $\frac{2\pi}{6} = \frac{\pi}{3}$ apart

$$z_2 = cis\left(\frac{\pi}{12} + \frac{\pi}{3}\right)$$
$$= cis\left(\frac{5\pi}{12}\right)$$
$$z_3 = cis\left(\frac{5\pi}{12} + \frac{\pi}{3}\right)$$
$$= cis\left(3\pi/4\right)$$
$$z_4 = cis\left(3\pi/4 + \pi/3\right)$$
$$= cis\left(13\pi/12\right)$$
$$= cis\left(-11\pi/12\right)$$
$$z_5 = cis\left(-11\pi/12 + \pi/3\right)$$
$$= cis\left(-7\pi/12\right)$$
$$z_6 = cis\left(-7\pi/12 + \pi/3\right)$$
$$= cis\left(-3\pi/12\right) = cis\left(-\pi/4\right)$$



8\. Solve $z^4 = 8\sqrt{3} - 8i$ leaving all solutions in polar form.

$$z^4 = \sqrt{64 \times 3 + 64}\ cis\ 30^\circ$$
$$= 16\ cis\ 30^\circ$$
$$\therefore z = \left(16\ cis\ 30^\circ\right)^{1/4}$$
$$= 2\ cis\left(\frac{30^\circ + k360^\circ}{4}\right)$$
$$k = 0 \Rightarrow z_1 = 2cis\ 7\tfrac{1}{2}^\circ$$
$$k = +1 \Rightarrow z_2 = 2cis\left(390/4^\circ\right)$$
$$= 2cis\ 97\tfrac{1}{2}^\circ$$
$$k = -1 \Rightarrow z_3 = 2cis\left(-330/4^\circ\right)$$
$$= 2cis\left(-82\tfrac{1}{2}^\circ\right)$$
$$k = -2 \Rightarrow z_4 = 2cis\left(-690/4^\circ\right)$$
$$= 2cis\left(-172\tfrac{1}{2}^\circ\right)$$

solution are:

$z = 2cis\ 7\tfrac{1}{2}^\circ$

$2cis\ 97\tfrac{1}{2}^\circ$

$2cis\ -82\tfrac{1}{2}^\circ$

$2cis\ -172\tfrac{1}{2}^\circ$