Global Financial Management

Asset Pricing Models

Latest Revision: December 9, 1997

5.0 Overview

In this class we derive the Capital Asset Pricing Model (CAPM). This model is widely used in capital budgeting exercises in practice and is one of the cornerstones of modern finance. One of the main uses of the CAPM is in determining the appropriate discount for capital budgeting. This module highlights the difference between systematic risk (which is priced or rewarded by investors) and diversifiable risk (which is not awarded).

5.1 Objectives

- Show that, in large diversified portfolios, an individual asset's contribution to the risk of the portfolio is its covariance with the returns of the existing portfolio and that individual variances are irrelevant.
- Explain why, when a riskless asset is introduced, all investors will hold the market portfolio and the riskless asset in some proportion.
- Understand why the return/risk tradeoff has to be the same for all assets in equilibrium.
- Understand and use the Capital Market Line where appropriate.
- Understand and use the Security Market Line or CAPM where appropriate.
- Understand the difference between systematic and diversifiable risk.
- Use the CAPM in a capital budgeting exercise.

5.2 The Capital Market Line

In the previous lecture on portfolio theory we introduced the Capital Market Line (CML) (see end of section 5.12). Specifically, the CML described all possible mean-variance efficient portfolios that were a combination of the risk-free asset and the tangency portfolio of risky assets

M. We now proceed to derive mathematically the CML using the mean-variance mathematics we have learnt so far.

The expected return for any portfolio *p* on the CML is a linear combination of the risk-free rate and the expected return on the tangency portfolio M,

$$E(r_p) = w \cdot E(r_M) + (1-w) \cdot r_f \tag{1}$$

Similarly, the variance and standard deviation of this portfolio are,

$$Var(r_p) = \sigma_p^2 = w^2 \cdot \sigma_M^2 \Rightarrow \sigma_p = w \cdot \sigma_M \tag{2}$$

We can solve for *w* in the standard deviation equation (2) and substitute it into the return equation (1). This gives,

$$\begin{aligned} E(r_p) &= r_f + w \cdot \left(E(r_M) - r_f\right) \\ &= r_f + \frac{\sigma_p}{\sigma_M} \cdot \left(E(r_M) - r_f\right) \end{aligned} \tag{3}$$

This equation describes **all** portfolios *p* on the CML, which are combinations of the riskless asset and portfolio *M* where r_f is the intercept and $(E(r_M)-r_f)/\sigma_M$ is the slope of the line. As a consequence, all such portfolios *p* that contain some proportion of portfolio *M* and the risk free asset will be perfectly correlated with it.

But what about portfolios and assets that do not lie on the CML? Can we relate the expected return on such assets to the expected return of portfolio *M* as well? Consider the following example:

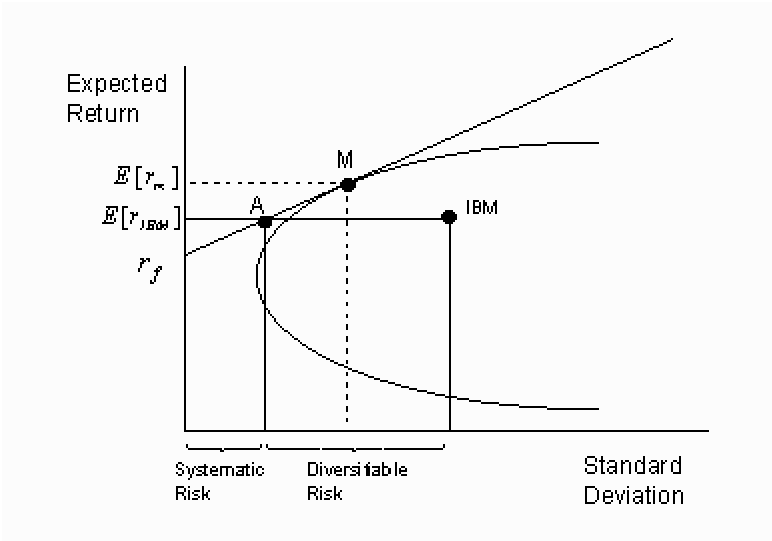


Figure 1

Figure 1 shows why assets that are not perfectly correlated with *M* do not fall on the CML. We shall use an investment in IBM stock as an example. The figure shows that there are two ways to receive an expected return of $E[r_{IBM}]$: simply buy shares in IBM, or buy portfolio A. For a risk averse investor (as we all are) portfolio A is preferred to an investment solely in IBM since it produces the same return with less risk. It is impossible to earn an expected return of $E[r_{IBM}]$ incurring less risk than that of portfolio A. The total risk of IBM can therefore be decomposed into ***systematic risk***, the minimum risk required to earn that expected return and ***diversifiable risk***, that portion of the risk that can be eliminated, without sacrificing any expected return, simply by diversifying. Investors are rewarded for bearing this systematic risk, but they are not rewarded for bearing diversifiable risk, because it can easily be eliminated at no cost.

In the next section we will derive the expected return that an investor will earn if she was to invest in an asset such as IBM.

5.3 Finding the Market Portfolio and Measuring Systematic Risk- The CAPM

Two questions remain:

(1) **What is portfolio *M*?** and

(2) **How do we quantify the systematic risk for assets that do not lie on the CML?**

Each of these questions is explored below.

From section 5.2 we know that investors will hold portfolios that combine the riskfree asset and the risky portfolio *M*. Hence, the first important observation is that only two funds are held by every investor: a fund comprised of the risk-free instrument and a fund which is the market portfolio. We call this the 2-fund theorem. This does not mean that the only investment portfolio that is held is *M*. We know that people have differing degrees of risk aversion. What is implied that there is only one portfolio of risky assets held in conjunction with the risk-free asset. Investors with different degrees of risk aversion will hold different proportions of portfolio *M* and the risk free asset, but they will not hold risky assets in different proportions.

Our second observation is that all investors will want to hold the same portfolio of risky assets (*M*, the tangency portfolio). As a result, the market portfolio represents the total invested wealth in risky assets. Hence, it is a portfolio with weights defined to be the total value of the asset divided by the total value of all risky assets. These weights are referred to as *value weights*. This is the key implication of the Capital Asset Pricing Model (CAPM).

It is important to see that no investor can do any better than holding the market portfolio. Specifically, let w_j be the weight of asset j in the market portfolio and write the expected return and variance of the market portfolio:

$$E(r_M) = \sum_{j=1}^{N} w_j \left(E(r_j) - r_f\right) + r_f$$
$$Var(r_M) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i \cdot w_j Cov(r_i, r_j) \tag{4}$$

Now, assume that there existed an asset j that offered a higher expected return relative to its risk. Our first step would be to buy this asset and add it to our portfolio of risky assets. Since our original portfolio was the market portfolio, increasing the weight of asset j results in an increase of expected return in proportion to $E(r_j)$-$r_{f.}$ Mathematically, we have:

$$\frac{\partial E(r_M)}{\partial w_j} = E(r_j) - r_f \tag{5a}$$

The symbol $\frac{\partial E(r_M)}{\partial w_j}$ means in words: "the increase in $E(r_M)$ if we increase the weight of asset j in the portfolio by a small amount." In addition, the riskiness of the portfolio increases by

$$\frac{\partial Var(r_M)}{\partial w_j} = \sum_{i=1}^{N} w_i \cdot Cov(r_j, r_i) = Cov(r_j, r_M) \tag{5b}$$

Hence, if we purchase more of asset j, the return/risk gain is:

$$\frac{E(r_j) - r_f}{Cov(r_j, r_M)} \tag{6}$$

Equation (6) results from dividing (5a) by (5b). It gives the increase in expected return of our portfolio relative to the increase in risk of our portfolio if we deviate from the market portfolio by increasing our investment in asset j by a small amount. The key observation is that - in equilibrium - it must be the case that all assets provide the same return/risk tradeoff. If this were not the case, then we would purchase the assets with high return/risk and sell those with the low ratio. Given this logic, asset j's return/risk tradeoff must be equal to all other assets in the market and hence to the return/risk tradeoff of the market itself. That is:

$$\frac{E(r_j)-r_f}{Cov(r_j,r_M)} = return/risk\ \ tradeoff\ \ \ for\ \ all\ \ assets = \frac{E(r_M)-r_f}{Var(r_M)} \qquad (7)$$

We are in a position now to answer the second question that we posed in the beginning of this section namely: quantify the expected return that investors demand for holding any risky asset.

Using the equation (7) we can rewrite it as follows:

$$\begin{aligned} E(r_j) &= r_f + \frac{Cov(r_j,r_M)}{Var(r_M)}\cdot\left(E(r_M)-r_f\right) \\ &= r_f + \beta_j\left(E(r_M)-r_f\right) \\ & where \\ \beta_j &= \frac{Cov(r_j,r_M)}{Var(r_M)} \end{aligned} \qquad (8)$$

This equation gives the relationship between the risk and expected return for individual stocks and portfolios. It is the central implication of the CAPM (also called the Sharpe-Lintner CAPM) and is called the **Security Market line**. Compare equation (3) with equation (8) and observe that (8) holds for any asset or portfolio of assets, not only for portfolios on the CML.

To gain additional intuition into the Security Market Line (SML) note that the we measure the riskiness of each asset using its β with respect to the market portfolio *M*. The reward for bearing high risk comes in the form of $\beta\cdot(E(r_M)-r_f)$. Thus high beta assets earn in equilibrium higher average return.

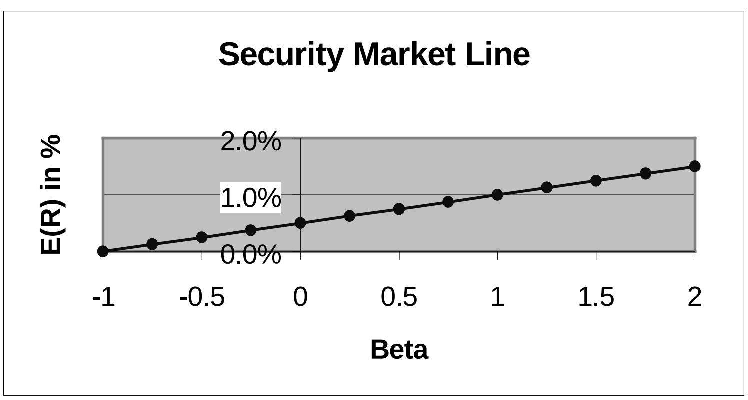
Finally, note that the beta of the market portfolio is one:

$$\beta_M = \frac{Cov(r_M, r_M)}{Var(r_M)} = \frac{Var(r_M)}{Var(r_M)} = 1 \qquad (9)$$

This provides a reference point against which the risk of other assets can be measured. The average risk (or beta) of all assets is the beta of the market, which is one. Assets or portfolios that have a beta greater than one have above average risk, tending to move more than the market. For example, if the riskless rate of interest (T-bill rate) is 5% per year and the market rises by 10%, assets with a beta of 2 will tend to increase by 15%. If however, the market falls by 10%, assets with a beta of 2 will tend to fall by 25% on average. Conversely, assets with betas less than one are of below average risk and tend to move less than the market portfolio. Assets that have betas less than zero tend to move in the opposite direction to the market. These assets are known as **hedge assets**.

The next graph plots the SML:

Figure 2



5.4 Black's CAPM

The above derivation of the CAPM measure assumes that all individuals hold the same portfolio and that this portfolio must be the market portfolio. The most general version of the CAPM requires only that individuals hold mean variance efficient portfolios. In this version, each individual can hold a different portfolio of risky assets. The market portfolio, which is just a weighted sum of the individual portfolios, will itself be on the efficient frontier and hence will be an efficient portfolio. This more general version of the CAPM also relaxes the assumption that individuals can borrow and lend at the riskless rate. In fact, it treats all assets as risky. Rather than relying on the existence of a riskless asset, all that is required is the existence of an asset whose returns are uncorrelated with those of the market portfolio (a zero-beta portfolio). The final equation for this model is:

$$E(r_i) = E(r_z) + \beta_i \cdot E(r_M - r_z) \qquad (10)$$

where E(r_z) is the expected return of the zero-beta portfolio and the other variables are as previously defined. This version of the CAPM is know as the Black CAPM and was derived by Fischer Black.

5.5 Implementing the CAPM

In the previous section, we derived a relation between expected excess returns on an individual security and the beta of the security. We can write this as a regression equation. This is a special regression where the intercept is equal to zero.

$$r_i - r_f = \beta_i \left(r_M - r_f\right) + \varepsilon_i \qquad (11)$$

This holds for all *i*. The beta, which was defined above, is the covariance between the security *i*'s return and the market return divided by the variance of the market return.

So the CAPM delivers an expected value for security *i*'s excess return that is linear in the beta which is security specific. We will interpret the beta as the individual security's contribution to the variance of the entire portfolio. When we talk about the security's risk, we will be referring to its contribution to the variance of the portfolio's return -- not to the individual security's variance.

This relation holds for all securities and portfolios.

Example 1

If we are given a portfolio's beta and the expected excess return on the market, we can calculate its expected return. Finally, we have a tool that we can help us evaluate the advertisement presented in the Diversification lecture. The ad that appeared in the Wall Street Journal provided data on Franklin Income Fund and some other popular portfolios. The returns over the past 15 years were:

The Franklin Income Fund	516%
Dow Jones Industrial Average	384%
Salomon's High Grade Bond Index	273%

First, let's convert these returns into average annual returns:

The Franklin Income Fund	12.9%
Dow Jones Industrial Average	11.1%
Salomon's High Grade Bond Index	9.2%

Note that the average annual returns are not nearly as impressive as the total return over 15 years. This is due to the compounding of the returns.

In order to use the CAPM, we need some extra data. We need the expected return on the market portfolio, the security or portfolio betas and the riskfree rate. Suppose that the average return on the market portfolio is 13% and the riskfree return is 7%. Furthermore, suppose the betas of the portfolios are:

The Franklin Income Fund	1.000
Dow Jones Industrial Average	0.683
Salomon's High Grade Bond Index	0.367

These are reasonable beta estimates. The Dow is composed of 30 blue chip securities that are generally less risky than the market. Remember that the beta of the market is 1.00. Any security that has a beta greater than 1.00 is said to have extra market risk (extra-market covariance). The long-term bond portfolio has a very low market risk. If we had a short-term bond portfolio, it would have even lower market risk (beta would probably be 0.10). The assumption that Franklin's beta is equal to the market's beta is conservative. In reality, The Franklin Growth Fund probably has a beta that is much higher because growth stocks are usually small and have higher market risk. Income stocks are usually larger and have market risk about equal to the market or lower.

Now let's calculate the expected excess returns on each of these portfolios using the CAPM.

$$E(r_i) = r_f + \beta\left(E\left(r_M\right) - r_f\right)$$

The Franklin Income Fund	13.0% = 7% + 1.000 x (13% - 7%)
Dow Jones Industrial Average	11.1% = 7% + 0.683 x (13% - 7%)
Salomon's High Grade Bond Index	9.2% = 7% + 0.367 x (13% - 7%)

Note that the expected returns for the Dow and the Salomon Bond Index were exactly what the actual average returns were. Note also that the expected return on the Franklin Income Fund was higher than what was realized. The market expected 13% performance and the Fund delivered 12.9%. The difference between the expected performance and the actual is called the abnormal return. The abnormal return is often used in performance evaluation.

So now we have a powerful tool with which to calculate expected returns for securities and portfolios. We can go beyond examination of historical returns and determine what the risk adjusted expected return for the security is.

5.6 Characteristics of Betas

To get a deeper insight into risk, consider the estimation of the beta coefficient from an ordinary least squares regression:

$$r_{it} - r_{ft} = \alpha_i + \beta_i (r_{Mt} - r_{ft}) + \varepsilon_{it} \tag{12}$$

In this regression, the beta is the ratio of the covariance to the variance of the market return. The alpha is the intercept in the regression. *This is not the CAPM equation*. This is a regression that allows us to estimate the stock's beta coefficient. The CAPM equation suggests that the higher the beta, the higher the expected return. Note that this is the only type of risk that is rewarded in the CAPM. The beta risk is referred to in some textbooks as systematic or non-diversifiable or market risk. This risk is rewarded with expected return.

The other type of risk which we encountered earlier is called non-systematic, diversifiable, non-market or idiosyncratic risk. This type of risk is the residual term in the above time-series regression.

$$r_{it} - r_{ft} = \alpha_i + \underbrace{\beta_i (r_{Mt} - r_{ft})}_{\text{Nondiversifiable or Systematic Risk}} + \underbrace{\varepsilon_{it}}_{\text{Diversifiable, Nonsystematic or Idiosyncratic Risk}} \tag{13}$$

The asset's characteristic line is the line of the best fit for the scatter plot that represents simultaneous excess returns on the asset and on the market.

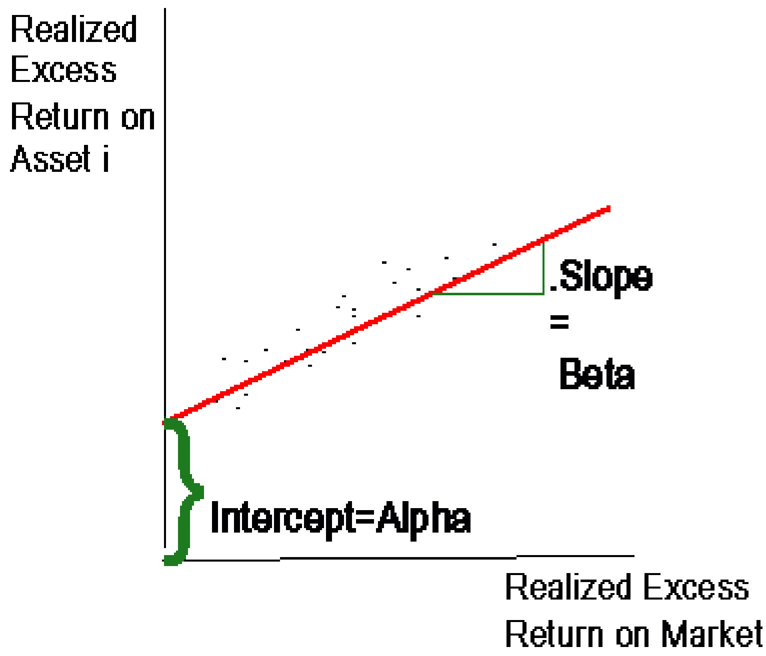


Figure 3

This is just the fitted values from a regression line. As mentioned above, the beta will be the regression slope and the alpha will be the intercept. The error in the regression, epsilon, is the distance from the line (predicted) to each point on the graph (actual).

The CAPM implies that the alpha is zero. So we can interpret, in the context of the CAPM, the alpha as the difference between the expected excess return on the security and the actual return. The alpha for Franklin would have been -.10 whereas the alpha for both the Dow and the Salomon Bonds were zero.

5.7 IBM's Alpha and Beta

Any security's alpha and beta can be estimated with an ordinary least squares regression. We have provided some results for IBM from 1926-1994. The returns data comes from the *Center for Research in Security Prices* (CRSP) and the riskfree return is the return on the one month Treasury Bill from *Ibbotson Associates*. This type of regression is usually estimated over 5-year sub-periods if the data is monthly. The market index used is the CRSP value weighted NYSE stock index. Value weighting means that stock *i* is given a weight equal to the market value of the stock of *i* divided by the market value of all securities on the NYSE.

Usually, this type of regression is estimated over 5-year sub-periods. We have provided estimates over the entire time period and some shorter subperiods. The results are summarized below.

Time	**Alpha**	**t-stat**	**Beta**	**t-stat**	R^2
1926-95	0.0076	4.3	0.79	25.5	0.48
1926-35	0.0148	2.8	0.79	14.4	0.64
1936-45	0.0058	1.6	0.49	8.4	0.37
1946-55	0.0080	1.9	0.83	7.5	0.33
1956-65	0.0091	2.2	1.39	11.6	0.53
1966-75	0.0040	0.9	0.89	10.2	0.47
1976-85	0.0017	0.4	0.82	9.1	0.41
1986-95	-0.0011	0.5	0.93	8.1	0.39
1971-75	-0.0019	-0.3	0.88	7.2	0.47
1976-80	-0.0043	-0.9	0.87	8.0	0.52
1981-85	0.0075	1.3	0.77	5.3	0.33
1986-90	0.0035	0.9	0.89	4.3	0.37
1991-95	-0.0045	0.8	0.97	6.3	0.43

The results indicate that the beta of IBM varied between .5 to 1.4 over the period examined. In recent years (from 1971), the beta has been around 0.9. Notice that in recent years the alpha is indistinguishable from zero. This indicates that there has been no abnormal return from investing in IBM.

5.8 Portfolio Beta

The beta of an individual asset is:

$$\beta_i = \frac{Cov(r_i, r_M)}{Var(r_M)} \tag{14}$$

Now consider a portfolio with weights w_p. The portfolio beta is:

$$\beta_p = \frac{\text{cov}(r_p, r_M)}{Var(r_M)} = \frac{Cov\left(\left(\sum_{i=1}^{N} w_i \cdot r_i\right), r_M\right)}{Var(r_M)}$$
$$= \frac{\sum_{i=1}^{N} w_i \cdot Cov(r_i, r_M)}{Var(r_M)} = \sum_{i=1}^{N} w_i \cdot \beta_i \tag{15}$$

The beta of the portfolio is the weighted average of the individual asset betas where the weights are the portfolio weights. So we can think of constructing a portfolio with whatever beta we want. All the information that we need is the betas of the underlying asset. For example, if we wanted to construct a portfolio with zero market (or systematic) risk, then we should choose an appropriate combination of securities and weights that delivers a portfolio beta of zero.

As an example of some portfolio betas, the next table includes some average beta values for industry portfolios. These betas are ranked by size. The industry with the highest beta was Air Transport and the lowest beta industry was Gold Mining.

Industry	Beta
Air transport	1.80
Real Property	1.70
Travel, outdoor rec.	1.66

Electronics	1.60
Misc. Finance	1.60
Nondurables, entertain	1.47
Consumer durables	1.44
Business machines	1.43
Retail, general	1.43
Media	1.39
Insurance	1.34
Trucking, freight	1.31
Producer goods	1.30
Aerospace	1.30
Business services	1.28
Apparel	1.27
Construction	1.27
Motor vehicles	1.27
Photographic, optical	1.24
Chemicals	1.22
Energy, raw materials	1.22
Tires, rubber goods	1.21
Railroads, shipping	1.19
Forest products, paper	1.16
Miscellaneous, conglom	1.14
Drugs Medicine	1.14
Domestic oil	1.12
Soaps, cosmetics	1.09
Steel	1.02

Containers	1.01
Nonferrous metals	0.99
Agriculture	0.99
Liquor	0.89
International oil	0.85
Banks	0.81
Tobacco	0.80
Telephone	0.75
Energy, utilities	0.60
Gold	0.36

5.9 Investment Decisions Under Uncertainty

Recall that the net present value of a project (**NPV**) is

$$NPV = \sum_{t=1}^{N} \frac{\Delta X_t}{(1+r_p)^t} - I \qquad (16)$$

or

NPV = PV[Future Incremental Cash Flows] - Initial Investment Cost

where ΔX_t is the expected incremental net cash flow from the project in year *t*; *n* is the number of cash flows generated by the investment; *I* is the initial investment r_p is the required return for this particular project.

In a world of uncertainty, r_p is the return required for the risk involved and is given by the CAPM. The CAPM states that the expected return on any risky asset equals the risk-free rate of interest plus a risk premium equal to the market risk premium times the project's beta.

Consider the marginal cost of capital for an all-equity firm ($r_{project}$) and assume that the investment that the firm is considering is in the same risk class as the firm. Hence, the marginal cost of capital for the investment project can be computed by estimating the stock's beta, the market risk premium, and the riskless rate of interest, and substituting into the CAPM equation above.

Example 2

Consider the following investment opportunity available to a firm.

Year	**0**	**1**	**2**	**. . .**	**7**
E[Cash Flow]	-$60,000	$15,000	$15,000	. . .	$15,000

This investment opportunity is in the same risk class as the other investments of the firm. The expected return on the market is 12%, the risk free rate is 8% and the firm's stock has a beta = 1.5. Should the firm undertake this investment (assume that the firm is all equity)?

First, we acknowledge that this is a risky investment and calculate the required return for the risk involved. This required return is given by the CAPM as follows:

$$E(r_{project}) = r_f+\beta_{firm}(E(r_M-r_f)) = 8+1.5(12-8) = 14\%$$

Thus the required rate of return, r_p is 14%. Finally, we can calculate the NPV in the usual way using r_p = 14%.

$$NPV = \$15,000[1-(1+r_{project})^{-n}/r_{project}] - I = \$15,000[1-(1.14)^{-7}/0.14]-\$60,000=\$4,324.57$$

Since NPV>0 we accept this project.

For additional examples see the appendix to this lecture.

5.10 Tests of the CAPM

Let's review what we have learned so far. There is a statistical model that describes realized excess returns through time:

$$r_{it} - r_{ft} = \alpha_i + \beta_i (r_{Mt} - r_{ft}) + \varepsilon_{it} \qquad (17)$$

This type of model can be estimated with ordinary least squares regression. We assume that the expected value of the error is zero and that it is uncorrelated with the independent variable. If we were to take expected values of each side of this model we would get:

$$E(r_{it} - r_{ft}) = \alpha_i + \beta_i E(r_{Mt} - r_{ft}) \qquad (18)$$

which looks like the CAPM. But the asset-pricing model that we developed imposes the following constraint on expected returns for all securities i*:*

$$\begin{aligned} E(r_{it} - r_{ft}) &= \beta_i E(r_{Mt} - r_{ft}) \\ &\mathit{so} \\ \alpha_i &= 0 \end{aligned} \qquad (19)$$

The security's expected excess return is linear in the security's beta. The beta represents the risk of security i in the market portfolio -- or the contribution of security i to the variance of the market portfolio. The beta risk is the only type of risk that is rewarded or priced in equilibrium. *What makes the CAPM different from the statistical model is that the CAPM imposes the constraint that the intercept or alpha is zero*. Below we describe empirical tests that examine this restriction.

5.11 Time Series Tests of the CAPM

One test of the CAPM is to test whether the alpha of any security or portfolio is statistically different from zero. The regression would be run with available stock return data. The null hypothesis is (the CAPM holds) that the intercept is equal to zero. Under the alternative hypothesis, the intercept or alpha is not equal to zero. The standard test is a t-test on the intercept of the regression. If the intercept is more than 2 standard errors from zero (or having a t-statistic greater than 2), then there is evidence against the null hypothesis (the CAPM).

5.12 Cross-Sectional Tests of the CAPM

We have learnt that the Capital Asset Pricing Model implies that each security's expected return is linear in its beta. A possible strategy for testing the model is to collect securities' betas at a particular point in time and to see if these betas can explain the cross-sectional differences in average returns. Consider the cross-sectional regression:

$$R - r_f = \gamma_0 + \gamma_1 \beta + \varepsilon \qquad (20)$$

In this regression, R represents the returns of many securities at a particular cross-section of time and beta represents the betas on many firms.

According to the CAPM, γ_0 should be equal to zero and γ_1 should equal the expected excess return on the market portfolio. We can test this.

The first tests of the theory were carried out by Black, Jensen and Scholes (1972) and Fama and MacBeth (1973). Both of these tests were cross sectional tests. We will examine the Black, Jensen and Scholes (1972). Portfolios of stocks are created ranging from high beta portfolios to low beta portfolios. A cross sectional regression was run to see if the betas were able to explain the differences in the returns across securities.

$$E - r_f = \gamma_0 + \gamma_1 \cdot \beta + \varepsilon \qquad (21)$$

The results are:

$$E - r_f = 0.0036 + 0.0108 \cdot \beta$$
$$[6.53] \quad [20.77]$$

The t-statistics are in parentheses. The CAPM suggests that $\gamma_0 = 0$. The regression evidence provides evidence against that hypothesis. The CAPM theory also suggests that $\gamma_1 > 0$ and is equal to the expected return on the market less the risk free rate. Over the period 1931--1965, the average return on the market less the risk free rate was 0.0142. The regression evidence suggested a coefficient of 0.0108. The evidence suggests that there is a positive trade off between risk and return -- but the γ_1 coefficient is lower than expected.

There are many possible explanations as to why the data does not exactly support the CAPM. The CAPM is still used as a benchmark in risk analysis.

5.13 Multi-risk formulations

The goal of most economic models is to simplify reality so that we can gain a greater understanding of how the world works. The CAPM is an example of a simplification. A very complicated process (how prices are set in equilibrium) is reduced to a single firm-specific parameter - the beta. The beta is multiplied by the risk premium for beta (the expected excess return on the market portfolio) to obtain the expected excess return on the security.

Obviously, the CAPM is a very simple model. As for all models, with enough data we can "statistically" reject the model. However, this does not mean that the model is not useful. On the contrary, a rejection of the model leads us to refinement and expansion of the generality of the model. One of the most obvious sources of generalization is to add additional risk factors.

Asset pricing theory has been generalized to multiple sources of risk in important papers by Ross (1976, *Journal of Economic Theory*), Sharpe (1982), Merton (1973, *Econometrica*) and Long (*Journal of Financial Economics*, 1974). The intuition of these models is that assets have exposures to various types of risk: inflation risk, business-cycle risk, interest rate risk, exchange rate risk, and default risk. It is difficult to capture all of these risk measures with the beta of the CAPM. The multirisk models have multiple betas. Instead of running a regression of the asset return at time t on the market return at time t, we run a regression of the asset return at time t on various "factors" at time t, like the change in the interest rate. The betas from this augmented regression are sometimes called factor loadings, risk sensitivities or risk exposures. The basic idea of the CAPM is maintained. The higher the exposure the greater the expected return on the asset.

Multifactor asset pricing formulations which tried to explain average returns with average risk loadings in Roll and Ross (1980, *Journal of Finance*) and Chen, Roll and Ross (1986, *Journal of Business*). However, these studies assume that risk is constant, risk premiums are constant and expected returns are constant.

Another popular formulation is the 3-factor model of Fama and French (1993). In this model, there are three betas. The stock returns are regressed on (1) the market; (2) a High book value to market value portfolio Minus Low book value to market value portfolio (HML); and (3) a Small capitalization portfolio Minus a Big capitalization portfolio (SMB). The first beta is the standard beta from the CAPM. The second beta measures distress (high book to market ratios are indicative of firms in distress). The third beta captures the size of the firm which is often correlated with risk.

While substantial advances have been made in asset pricing theory, there is still much work to be done. In addition, standard asset pricing models need to be modified to be applied to international settings. For example, Bekaert and Harvey (1995) ‘‘Time-Varying World Market Integration,” *Journal of Finance,* detail the impact of capital market integration on asset pricing. Two markets are integrated when the same risk project commands the same expected returns in both markets. Regulations that prevent foreigners from transacting in the domestic market or that prevent domestic investors from diversifying their portfolios internationally lead to market segmentation.

APPENDIX

Example 3

Assume now that the investment opportunity available to the firm is in a different industry to that of the firm's normal operations. This industry is 40% riskier than the firm's industry. If the cash flows are as above and the expected return on the market is 12%, the riskfree rate is 8% and the firm's beta = 1.5, should the firm undertake the investment?

Again, we recognize the fact that the firm is confronted by a risky investment and calculate the return required for the risk involved. Since this investment is 40% riskier than the normal investments of the firm, the beta of the investment is given by beta = 1.4(1.5) = 2.1. Hence, the required return is given by:

$$E(r_{project}) = r_f + \beta_{firm}(E(r_M - r_f)) = 8 + 2.1(1.2-8) = 16.4\%$$

Thus the required rate of return, r_p, is 16.4%. Again, we calculate the NPV in the usual way using r_p = 16.4%.

$$NPV = \$15,000[1-(1+r_p)^{-n}/r_p] - I = \$15,000[1-(1.164)^{-n}/0.164] - \$60,000 = -\$128.54$$

Since NPV<0, we reject this proposal.

Example 4

The *Marlet Fishing Boat Company* currently has a market value of $4 million. Its required rate of return is 18%. The company is evaluating an $88,000 investment project which is expected to generate after-tax cash flows of $176,000 a year indefinitely. The project is 40% riskier than the firm's average operations. If the riskless rate of interest is 8% should the project be accepted? Assume that the market risk premium ($E[r_m] - r_f$) = 7.5%.

First, we calculate the required return for the risk involved. We are not told the beta of the normal investments, but we do know that:

$$E(r_i)=r_f+\beta_i(E(r_M)-r_f)$$

Or

$$18\%=8+\beta_i(E(r_M)-r_f)=8+\beta_i(7.5)$$

so β_i=1.33 for a normal project. Since this project is 40% riskier, we know that

$$\beta_p=1.4\cdot\beta_i=1.4(1.33)=1.866$$

So the beta of the project is 1.866.

Plugging this back into the CAPM yields:

$$r_p=r_f+\beta_p(E(r_M)-r_f)=8+1.866(7.5)=22\%$$

Hence, the required rate of return, r_p = 22% and the NPV is given by

$$NPV = (\$176{,}000 / 0.22) - \$800{,}000 = 0$$

Hence we are indifferent between undertaking the project or rejecting it.

Acknowledgement

Some of the material for this lecture is drawn from Douglas Breeden, "*Capital Asset Pricing Model: Tests and Extensions*".