Global Financial Management

Debt Policy, Capital Structure, and Capital Budgeting

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7.0 Capital Structure: An Introduction

In this lecture we analyze the firm's capital structure and its impact on the value and returns of a company. Capital structure is defined as the composition of all the securities the firm issues in order to finance its operations. These securities can be different in many respects:

- Leverage: the amount of debt financing relative to equity financing
- Maturity: we can have long-term and short-term debt.
- Fixed versus variable payments: some bonds and loan agreements have interest rates fixed until maturity, whereas others are reset in certain intervals of time.
- Seniority: We can have different priorities associated with debt, where senior creditors have always to be paid before junior creditors.
- Currency: Bonds and bank loans can be denominated in different currencies, and some firms have even issued hybrid "dual currency" bonds that have interest payments in one and principal payments denominated in another currency.
- Control: Securities confer control rights to the owner, e.g., owners of common stock can vote at annual general meetings, whereas creditors often have the right to stop the firm from paying dividends or selling assets that serve as collateral.
- Contingencies: Some securities have repayments that are contingent on future events. Convertible bonds are bonds that can be converted into stock at the discretion of the bondholder, trading an entitlement to future coupons and principal payments into an entitlement to future dividend payments and capital gains.

This list is not complete, and in this note we will only attempt a first approach on this problem.

Many of the considerations regarding the choice between these classes of securities are firm specific whereas others follow more general principles. In this lecture we will focus on the choice between debt and equity as an example of the complexities involved in capital structure decisions. However, many of the arguments here extend in a straightforward way to the other dimensions of capital structure choice.

7.1 Objectives

After completing this class you should be able to:

- Show why in a world with perfect capital markets, capital structure is irrelevant.
- Compute the firm’s Weighted Average Cost of Capital (WACC).
- Explain why in a world with perfect capital markets the required return on a project is independent of the firm’s capital structure.
- Compute the required return demanded by equity-holders.
- Evaluate investment decisions where the project is financed in a different proportion of debt and equity than is the existing firm and has a different risk.
- Lever and unlever *beta*s to reflect differences in capital structures.
- Compute appropriate required returns for projects in industries different from the existing assets of the firm.

7.2 Does Leverage Increase Risk?

Our first approach is somewhat indirect: We first analyze a benchmark case that has the property that leverage does not matter to the firm at all and develop the famous propositions by Modigliani and Miller. These propositions rest on assumptions, some of which are unrealistic. However, it turns out that this benchmark case is helpful in guiding us to sensible capital structure choices, since we have to identify exactly which assumption is violated in order to understand in which dimension and how to choose an optimal capital structure.

In this section we investigate the relationship between leverage and risk. Consider a firm that is entirely financed with equity and has assets worth 200 that generate risky payoffs. In the good scenario, which materializes with probability of 50%, the return is 25% on the assets. In a more favorable "excellent" scenario the return is 40% with probability 25%. The firm can also lose money in which case returns are only -10% with probability 25%. Payoffs and returns are given in the following table:

Table 1

Scenario	Probability	Payoff	Return
Excellent	0.25	180	-10%
Good	0.50	250	25%
Bad	0.25	280	40%
Average	-	240	20%
Volatility	-	36.7	18%

The balance sheet of this firm is simple: the assets are worth 200, and the equity is worth 200. What happens if this firm decides to issue riskless debt with a face value of 55 if the required rate of return on riskless assets it 10%? And how would this scenario change again if the debt issued has face value of 110 or even 204? Consider first the impact on cash flows:

Table 2

		Face value = 55		Face value = 110		Face value = 204	
Scenario	**Assets**	**Debt**	**Equity**	**Debt**	**Equity**	**Debt**	**Equity**
Bad	180	55	125	110	70	180	0
Good	250	55	195	110	140	204	46
Excellent	280	55	225	110	170	204	76
Average	240	55	185	110	130	198	42
Volatility	36.74	0.0	36.74	0.0	36.74	5.77%	27.17

Depending on the face value of debt, we observe that the risk inherent in the company's cash flow is allocated between debtholders and equityholders. However, debtholders are always served first, whereas equityholders are paid a dividend only if debtholders are repaid in full. Hence, if the debt value is not "too high" (here: if the face value is below 180), then debtholders are repaid in all states of the world and the risk is entirely borne by equityholders. However, if we increase the debt-level (beyond a face value of 180), then debt becomes risky too, and the company is in default on its debt with probability 0.25. In order to see how increasing the debt level affects equity, it is instructive to look at these numbers in terms of returns:

Table 3

		Leverage = 25%		Leverage = 50%		Leverage=90%	
Scenario	**Asset return**	**Equity Payoff**	**Equity Return**	**Equity Payoff**	**Equity Return**	**Equity Payoff**	**Equity Return**
Bad	-10%	125	-16.7%	70	-30%	0	-100.0%
Good	25%	195	30.00%	140	40%	46	130%
Excellent	40%	225	50.00%	170	70%	76	280%
Average	20%	185	23.33%	130	30%	42	110%
Volatility	18%	36.7	24%	36.7	37%	38	136%
Total debt	0	50		100		180	
Total equity	0	150		100		20	

In order to understand how the elements of the table are constructed, consider the case of a debt level of 55. This leads to a value of debt of 50 (the risk free rate is 10%, and 55/1.1=50), hence the value of equity must be 150. (We demonstrate this in more detail below). Then, in the good scenario, an equity payoff of 195 corresponds to a return of 195/150-1=0.3 or 30%. Leverage here is defined as $\frac{Debt}{Debt + Equity}$, or as a percentage of debt in the total value of the firm, which is 200. Observe how increasing the debt level increases the risk of equity: the volatility of returns increases from a moderate 34% to 191% if the value of debt is 90% of the value of the firm. However, the riskiness of the firm as a whole is unaffected! This observation is fundamental: increasing leverage increases the risk of equity, and also increases the risk of debt at a certain point, where repayment of the face value of the debt cannot be guaranteed (here 180). However, this changes only the way risk is *allocated*, but it does not change the riskiness of the firm as a whole.

In order to emphasize the last observation, suppose the firm engages in a financing strategy where it issues bundles of debt and equity so that each investor always holds the securities of the firm in exactly the same proportion as the firm issues them. So, if the firm has leverage of 25%, then an investor who invests $100 in the firm holds $25 worth of debt and 75$ worth of equity. Then this investor realizes returns that depend only on the value of the firm, but not on the amount of leverage: her position is always the same as if the firm had no leverage at all, and she had just invested $100 in the equity of the unlevered firm. This principle is important and we will explore it in greater detail below. For this section we conclude:

Increasing leverage increases the riskiness of debt and the riskiness of equity, but the riskiness of the cash flows of the firm remains unchanged.

We should keep in mind some assumptions we needed to make for this analysis:

- Cash flows are given. We have assumed that a higher leverage does not change the behavior of managers or workers, so the cash flows of the firm do not depend on leverage as such.

- No taxes; in the US and some other countries, interest payments are deducted before corporation tax is calculated, so that an increase in the debt level reduces the tax liability. Then our analysis is correct only if we include the government as a third claimant on the cash flows of the firm.

- No bankruptcy costs. Suppose that every time the firm is in default a certain amount needs to be paid to lawyers and investment bankers for a financial restructuring. This would imply that the net cash flows to investors are less in bankruptcy then we assumed above

- No differences in expectations. We have analyzed the example by assuming that all investors share their views of the probabilities and payoffs of the firm.

Obviously, none of these assumptions is correct. However, it is instructive to make these assumptions in order to develop a benchmark case. We continue to make these four assumptions in the subsequent analysis, and then discuss what the implications of changing any single one of them are.

7.3 Does Leverage affect the value of the firm

In the previous section we assumed that the firm value was given as 200. We should investigate a little more why this was a sensible assumption to make. Suppose that there are two versions of the same firm, U (for "unlevered") and L (for "levered"). These firms are replicas of each other. In their seminal article, Modigliani and Miller introduced the notion of risk classes, where firms in the same risk class have the same business risk. From investors' point of view, firms in the same risk class are perfect substitutes. Reconsider again the firm without leverage (U), and compare it with a levered firm (L) with a leverage of 75%. Suppose the assets of both firms are identical and given by Table 1 above. We use the following notation:

E_L	Value of the total equity of firm L
D_L	Value of the total debt of firm L
E_U	Value of the total equity of firm U
V_L	Total value of firm L = $E_L + D_L$
V_U	Total value of firm U = E_U

Consider the case of an investor who wishes to own exactly 1% of the equity of firm U, and compare this portfolio strategy with that of buying 1% of firm L's debt and 1% of firm L's equity. The value of the first portfolio is $0.01E_U$, and the value of the second portfolio is $0.01E_L+0.01D_L$. Then we have the payoffs to both investors as:

State	Invest in firm U's equity	Invest in firm L's equity	Invest in firm L's debt	Total Payoff from investing in firm L
Bad	**1.8**	0.15	1.65	**1.8**
Good	**2.5**	0.85	1.65	**2.5**
Excellent	**2.8**	1.15	1.65	**2.8**

Evidently, both strategies lead to exactly the same results. Hence the investor is not willing to pay more for the portfolio consisting of 1% of the equity and 1% of the debt of firm L, then she is willing to pay for the portfolio consisting of 1% of the equity of firm U. This gives us:

$$E_L + D_L = E_U \tag{1}$$

or, equivalently,

$$V_U = V_L \tag{2}$$

Now, consider an investor who starts out with an investment in 1% of firm L's equity. What would be the maximum price for shares in firm U's equity she is willing to pay? We derive an upper bound by using a strategy that involves buying firm U's equity "on margin", i. E., buy 1% of firm U's equity and borrow an amount equivalent to 1% of firm L's debt. The outlay of the second portfolio is $0.01E_U$-$0.01D_L$. At this point we introduce the additional assumption that the investor can borrow at the same interest rate as firm U. Hence, the interest obligations of the investor are exactly 1% of the interest obligations of firm L. Then we have:

Table 4

State	Invest in 1% of firm L's equity	Invest in 1% of firm U's equity	Borrow equivalent of 1% of firm L's debt	Total of leveraged portfolio
Bad	0.15	1.8	-1.65	0.15
Good	0.85	2.5	-1.65	0.85
Excellent	1.15	2.8	-1.65	1.15

Again, both strategies lead to the same result, hence the maximum the investor is willing to pay for buying 1% of firm U's equity on margin is $0.01E_L$ and we have:

$$E_U - D_L = E_L$$

which repeats (1). In words: the value of a firm is independent of leverage. This result is known as Modigliani and Miller's Proposition I:

In a world without taxes, bankruptcy costs and differences in investor's assessments of the payoffs of the firm, leverage has no influence on the value of the firm.

Before we move on to analyze the implications of this result, we retrace our steps in this section to understand the intuition behind this argument. We started by assuming that leverage does not affect the investment policy of the firm, hence the operating income of the firm is unaffected if we change the capital structure. However, if operating income is unaffected, then all changes of capital structure can do is dividing operating income between debtholders (who receive it in the form of interest and principal) and equityholders (who receive it in the form of dividends and capital gains). Clearly, we must have that:

Operating Income=Payments to debtholders + payments to equityholders

Since we have learned from previous lectures that:

Value of debt=Present value of payments to debtholders (interest/principal)

Value of equity=Present value of payments to equityholders(ultimately all dividends)

Value of firm=value of debt + value of equity

we must have:

Value of firm=Present value of operating income

The following table shows these relationships:

Value (Firm)	=	PV(Operating Income)
=		=
Debt	=	PV(Interest/Principal)
+		+
Equity	=	PV(Dividends)

Hence, if two firms have the same operating income and risk characteristics, then they must have the same value, even if this value is split differently between debt and equity.

7.4 Arbitrage and Leverage

We now argue that the equality in equation (2) has also a practical value, because whenever (2) is violated, we can form an arbitrage portfolio and make a riskless profit. Suppose, for example, that investors discount the dividends of the levered firm too much (risk adjustment too high), so that we have $E_L + D_L < E_U$. We show that we can form an arbitrage portfolio as follows: buy low, that is, invest in the undervalued securities of firm L, and sell high, by (short) selling the equity of firm U. We buy and sell 1% of each security as before. Hence, the payoff of this portfolio is:

$$0.01(E_U - D_L - E_L) > 0$$

Hence, this portfolio gives us a strictly positive payoff. We have to establish that it is also riskless:

State	Sell 1% of equity of L	Sell 1% of debt of L	Buy 1% of equity of U	Total payoff of portfolio
Bad	-0.15	-1.65	1.8	0
Good	-0.85	-1.65	2.5	0
Excellent	-1.15	-1.65	2.8	0

Hence, the portfolio has no payoffs in the future and the arbitrage profit is therefore riskless. It is easy to see that in case $E_L + D_L > E_U$, we can simply reverse our strategy.

7.5 Modigliani and Miller's Proposition I: A general proof

In this section we demonstrate the argument of the previous section more generally. We need the following additional notation:

Div	Dividend payment
CF	Cash flow
IP	Payment of interest and principal
S	State of the world

We proceed in exactly the same steps. Consider an investor who bought a fraction α of the equity of firm U. Then he could also invest in a portfolio consisting of a fraction α of firm L's equity and α of firm L's debt. The payoffs are:

Transaction	Investment (today)	Payoff in state s (future)
Buy levered equity	$-\alpha E_L$	$\alpha Div_L(s)=\alpha(CF_L(s)-IP_L(s))$
Buy debt	$-\alpha D_L$	$\alpha IP_L(s)$
Total (debt + equity of L)	$-\alpha(E_L+D_L)=-\alpha V_L$	$\alpha CF_L(s)$
Buy unlevered equity	$-\alpha E_U=-\alpha V_U$	$\alpha CF_U(s)$

Hence, since $CF_L(s)=CF_U(s)$, and since α is arbitrary, we conclude that equation buying unlevered equity, and buying levered equity and debt in equal proportions give the same payoff, hence the cost to both strategies must be the same. Similarly, an investor who buys α of firm L's equity could also buy α of firm U's equity on margin by borrowing αD_L:

Transaction	Investment (today)	Payoff in state s (future)
Buy levered equity	$-\alpha E_L$	$\alpha Div_L(s)=\alpha(CF_L(s)-IP_L(s))$
Buy unlevered equity	$-\alpha E_U$	$\alpha Div_U=\alpha CF_U(s)$
Borrow	αD_L	$-\alpha IP_L(s)$
Total	$-\alpha(E_U-D_L)$	$\alpha(CF_U(s)-IP_L(s))$

Again, since $CF_L(s)=CF_U(s)$, and since α is arbitrary, we conclude that both strategies lead to the same payoffs, hence the initial outlays must also be the same. This proves Modigliani and Miller's argument more generally. The last argument is better known as "home made leverage" and has important implications. Note that the argument above not invalidated by the fact that not all investors can borrow on the same terms as the firm. It is sufficient if intermediaries (banks, fund management groups) can do this. Hence, even though a small, individual investor cannot borrow on margin at the current interest rate, she could invest in shares of a hedge fund, and the hedge fund pursues a strategy of buying securities on margin.

The arbitrage argument can also be generalized in a straightforward way. From equation (1) we can see that the optimal strategy is:

$E_L + D_L < E_U$	Firm L is undervalued relative to firm U	Buy firm L, sell firm U
$E_L + D_L > E_U$	Firm U is undervalued relative to firm L	Buy firm U, sell firm L

We demonstrate the second case:

Transaction	Investment (today)	Payoff in state s (future)
Sell levered equity	αE_L	$-\alpha Div_L(s)=-\alpha(CF(s)-IP_L(s))$
Buy unlevered equity	$-\alpha E_U$	$\alpha Div_U=\alpha CF(s)$
Borrow	αD_L	$-\alpha IP_L(s)$
Total	$\alpha(E_U-(E_L+D_L))>0$	$\alpha(CF(s)-IP_L(s))$ $+\alpha\ CF(s)-\alpha IP_L(s)=0$

Hence, the payoff of this portfolio is positive today, and zero in all future states.

7.6 The argument with taxes

Obviously, some of the Modigliani-Miller-assumptions are unrealistic. In this section we investigate what happens if we introduce taxes. Companies have to pay corporation tax on their income, but they can deduct interest expenses as a cost, so tax is payable only on net income (operating income minus interest). There is therefore a difference in the tax treatment of debt and equity: the income of debtholders (interest) is tax-free, whereas the income of equityholders is taxable. We adapt the argument of the previous section slightly, and assume that we have an unlevered firm that pays off a constant cash flow CF every period, and that the tax rate is t. Then the equityholders of this firm receive (1-t)*CF, whereas the government receives the remaining t*CF. How would the value of this company change if it where to take on some debt? Assume the debt is perpetual.[1] Then the firm has to pay interest every period, which we denote by INT. This interest payment is deducted from the cash flow before determining the tax liability of the company, hence taxes are:

$$Tax=t*(CF-INT)$$

Debtholders receive INT, and equityholders receive the remainder as a dividend:

$$Div=(1-t)*(CF-INT)$$

We have the following relationship between payoffs and transactions for the unlevered firm:

Transaction	**Outlay**	**Payoff per period**
Buy unlevered equity	E_U	(1-t)*CF
Total	$V_U=E_U$	(1-t)*CF

And for the levered firm:

Transaction	**Outlay**	**Payoff per period**
Buy levered equity	E_L	(1-t)*(CF – INT)
Buy debt	D_L	INT
Total	$V_L=E_L + D_L$	(1-t)*CF+t*INT

Since the value of receiving a perpetual stream of payments equal to INT is D_L, we can now write:

$$V_L = E_L + D_L = E_U + t * D_L \tag{3}$$

[1] This is not a necessary assumption. There is no difference for this example between assuming that debt is perpetual and that debt is always rolled over, i. e., every repayment of principal is financed through new

In other words: the value of the levered firm is strictly *greater* than the value of the unlevered firm. The difference comes from the fact that the cash flows are now split between three parties: equityholders, bondholders, and the government. The value of the firm is the value to equityholders and debtholders combined, and this is increased whenever the share of the government is reduced. Since interest expenses are tax deductible, increasing leverage increases the value of the firm. The amount of the increase is equal to t^*D_L and is called the *tax shield* from borrowing. Hence, if a firm borrows $1,000 and the tax rate is 30%, then the value of the company to shareholders and debtholders combined increases by $300.

Reconsider the example of the firm we studied above, and suppose the corporate tax rate is 30%. Clearly, in this case the value of the firm is only 0.7*200=140. The following table gives the relevant calculations:

Scenario	Before tax	Return	After tax
Bad	180	-10%	126
Good	250	25%	175
Excellent	280	40%	196
Average	240	20%	168
Value			140.00

Now, if leverage is 50%, our calculation changes to:

	Debt	Equity	
Scenario	Payment	Before tax	After tax
Bad	110	70	49
Good	110	140	98
Excellent	110	170	119
Average	110	130	91
Value	100.00		70.00

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borrowing.

The value of the firm has increased by 30 to 170, which is exactly equivalent to the value of debt (=100) multiplied by the tax rate (=30%).

Hence, we can see that the Modigliani-Miller irrelevance proposition has now changed substantially: the value of the firm increases with the debt level, and higher levels of debt are clearly preferable. This leads to the conclusion that companies could easily increase their value by borrowing as much as possible. While there may be some merit to this argument, there are two important qualifications:

1. Companies pay corporation tax, but investors also pay personal taxes. They value a company most if it minimizes the *total* tax burden from corporate and personal taxes combined. Since personal taxes on debt income are higher than on equity income, the tax advantage from debt is inflated if we only look at corporate taxes.

2. The argument above still assumes that there are no bankruptcy costs and no implications on the firm's operating policy. We shall see below that these assumptions are not likely to be valid for high debt levels.

These arguments lead us to the conclusion that the optimal policy is not to borrow somewhat less than the maximum. Unfortunately, these considerations give no clear quantitative guideline as what the optimal amount of borrowing actually is, but the amount of borrowing of many US companies is arguably lower than what would be optimal given the above considerations.

7.7 Another perspective: What is the role of the corporate treasurer?

The ideas presented above sometimes look unrealistic, especially the construction of twin levered/unlevered companies. However, these concepts have some important implications, especially for the role financial strategy can play for increasing corporate value.

As an example, suppose investors for some reason wish to invest in volatile, highly levered securities. Then the Modigliani-Miller argument says that the firm cannot hope to extract a premium from these investors by offering new types of securities, simply because investors can "make" these securities, i.e., replicate their characteristics by combining the equity of the firm with borrowing as described in the previous sections.

Consider the position of the treasurer of the firm we described in tables 1-3 above. Suppose the firm is currently 75% equity financed (current face value of debt=55, see tables 2,3), but the treasurer reckons that investors would prefer to hold more risky, volatile securities, and would be willing to pay a higher price for them. The treasurer reckons that leverage of 90% would be optimal (face value=204, see tables 2,3). We can show that investors can "make" the securities they desire themselves. Evidently, we could run through another version of the argument presented in section 7.3 above and show how investors can form a portfolio of the firm's equity and borrowing on their own account to accomplish their goal. However, the more likely scenario is that an intermediary would step in and create the desired securities. In this case, a bank would set up an investment fund with the following balance sheet:

Assets		Liabilities	
Equity of Firm	50	Investment fund certificates	20
Debt of Firm (Face value = 55)	150	Bonds or bank loan (Face value - 204)	180
Total assets	200	Total liabilities	200

Hence, the fund would invest by buying all of the firm's equity and debt and issue fund certificates (effectively a form of equity) for 10% of the value of the firm, and fund the remaining 90% with borrowing.[2] Clearly, in each state of the world the fund is entitled to the entire operating income of the firm, hence the fund's income is 180, 250 and 280 in the bad, good and excellent state, respectively. Then the payoffs to the bondholders who invest in the fund is 180/204/204, and the owners of the fund certificates earn nothing in the bad state, 46 in the good, and 76 in the excellent state of the world. (These numbers follow directly from Table 3 above). Hence, by way of setting up an intermediary, investors obtain *exactly* the same kind of investments as they would obtain if the firm itself had taken on 90% debt.

This argument has important implications. Firstly, it is correct to observe that most investors cannot borrow on the same terms as IBM. However, it is incorrect to conclude from this observation that the Modigliani-Miller-assumption that investors can borrow on the same terms as the firm is invalid. Investors can always invest in funds offered by intermediaries that offer the securities they desire, and it is sufficient to assume that intermediaries can borrow on the same terms as the corporation, a much more realistic assumption. The Modigliani-Miller conclusion about "home-made leverage" can therefore hold in a world where individual investors are unable to borrow. A second observation is equally important. If our treasurer wishes to offer investors

[2] Mutual funds would not be allowed to engage in this kind of borrowing, so the example would be closer to the operations of a hedge fund.

more desirable securities, then he faces competition from any intermediary that can set up a fund with the above description. The firm does not have a comparative advantage over outside investors and banks to market its own cash flows. Hence, one immediate conclusion from the Modigliani-Miller theorem is that the opportunities of the treasurer to improve the firm's profits are small. Now we can use the assumptions of the proposition to be more precise. The treasurer can only hope to contribute to increasing the company's value if:

- changing the capital structure can save taxes (all taxes, corporate taxes and personal taxes paid by individual investors combined);

- changing the capital structure changes the company's operating policy, for example by creating incentives to cut unnecessary expenditures;

- changing the capital structure changes the risk profile of the firm and reduces the incidence of bankruptcy costs.

This list is not exhaustive. However, the assumptions of the Modigliani-Miller proposition give clear guidance: the treasurer can only increase the value of the company through changes in the capital structure if one of the Modigliani-Miller-assumptions is violated *and* if the change in the capital structure cannot be replicated by individual investors or intermediaries.

7.8 Does leverage affect the required return on the firm?

One of our primary goals is to establish how the cost of capital, i.e., the discount rates we should be using for project appraisal and company valuation, depends on leverage. We change our notation slightly and use subscripts now to refer to different points in time, t=0 for "today" and t=1 for some date in the future, depending on the interval over which we measure returns, which can range from a day to a number of years.

Table 5

D_t	Value of debt at time t
E_t	Value of equity at time t
$V_t=E_t+D_t$	Value of firm as a whole at time t
r_D	Return on debt
r_E	Return on equity
r_A	Return on firm as a whole

Evidently, these concepts are related:

$$r_D = \frac{D_1 - D_0}{D_0} \qquad r_E = \frac{E_1 - E_0}{E_0} \qquad r_A = \frac{V_1 - V_0}{V_0} \tag{4}$$

However, using the fact that $V_1 = E_1 + D_1$ and $V_0 = E_0 + D_0$ we find:

$$\begin{aligned} r_A &= \frac{V_1 - V_0}{V_0} = \frac{E_1 - E_0 + D_1 - D_0}{V_0} = \frac{E_1 - E_0}{E_0}\frac{E_0}{V_0} + \frac{D_1 - D_0}{D_0}\frac{D_0}{V_0} \\ &= r_E \frac{E_0}{V_0} + r_D \frac{D_0}{V_0} \end{aligned} \tag{5}$$

Hence, the return on the assets of the firm as a whole is a weighted average of the return on equity and the return on debt, where each return is weighted with the market value proportions of debt and equity respectively. Note that all we have used in the transformations in (5) are the definitions of returns in (4). Hence, being able to write the return on the firm as a weighted average is a consistency requirement that follows directly from the definition of returns. However, we need to go one step further in order to understand how to interpret equation (5).

More specifically, does causality run from the left hand side to the right hand side from the equation (returns on the firm determine the returns to debt and equity) or the other way round (returns to debt and equity determine the returns on the firm)? It is instructive to return to our example above:

Table 6

		Leverage = 25%		**Leverage = 90%**	
Scenario	**Asset return**	**Equity return**	**Debt return**	**Equity return**	**Debt return**
Bad	-10%	-16.67%	10%	-100%	0.0%
Good	25%	30.00%	10%	130%	13.33%
Excellent	40%	50.00%	10%	280%	13.33%
Average	20%	23.33%	10%	110%	10%
Volatility	26%	34%	10%	191%	8%

Observe that the return patterns of debt and equity both change as leverage increases from a moderate 25% to a significantly higher 90%. Using formula (5) we now compute (using superscripts to distinguish between different degrees of leverage):

$$\begin{aligned} r_A^{25\%} &= \frac{V_1^{25\%} - V_0}{V_0} = \frac{E_0^{25\%}}{V_0} r_E^{25\%} + \frac{D_0^{25\%}}{V_0} r_D^{25\%} = \frac{150}{200} 23.33\% + \frac{50}{200} 10\% = 20\% \\ r_A^{90\%} &= \frac{V_1^{90\%} - V_0}{V_0} = \frac{E_0^{90\%}}{V_0} r_E^{90\%} + \frac{D_0^{90\%}}{V_0} r_D^{90\%} = \frac{20}{200} 110\% + \frac{180}{200} 10.00\% = 20\% \end{aligned} \tag{6}$$

Hence, we find that for both cases, the return on the firm is 20%, even though leverage and the riskiness of the firm’s securities (both debt and equity!) has increased dramatically when raising leverage from 25% to 90%. It turns out that this is a general property. In order to see this, note that the return of the firm depends only on the value of the firm today and the value of the firm tomorrow. However, we have already seen that the value of the firm as a whole *does not depend* *on leverage*. In our example we have $V_1^{90\%} = V_1^{25\%} = V_1$, independently of the degree of leverage. From this it follows immediately that $r_A^{90\%} = r_A^{25\%} = r_A$, hence, the return of the firm does not depend on leverage. We state this as the main result of this section, which is known as Modigliani and Miller’s Proposition II:

Under the assumptions of Modigliani and Miller’s Proposition I, the expected return on the firm as whole and the firm’s cost of capital are independent of leverage.

Obviously, r_A is the rate at which the market discounts future expected payoffs of the firm as a whole, so in our example we have that the expected payoff of the firm is:

$$\frac{0.25 * 180 + 0.5 * 250 + 0.25 * 280}{1.2} = \frac{240}{1.2} = 200$$

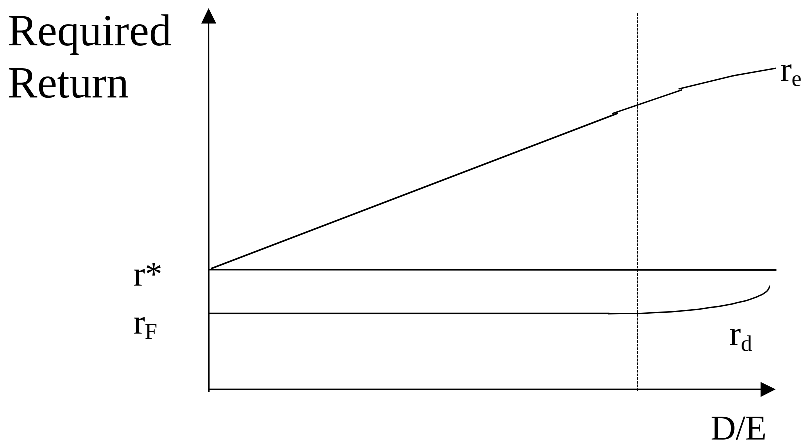
Hence, we refer to r_A synonymously as the "expected return of the firm", the "cost of capital of the firm" and the "required rate of return".

Now we are in a position to put all the insights of our discussion so far together. We observe that leverage has an impact on the riskiness of the firm’s securities, and, generally, both debt and equity become more risky as leverage increases. As a result, the average return of equity and debt also goes up. (At least it does not go down; in our example it stays the same, but this is not always the case.) Hence, the required rate of return on the firm’s securities always goes up as leverage increases. On the other hand, increasing leverage has a countervailing effect, because we use more of the cheaper source of finance, in our case debt with a lower required rate of return.

The insight behind the Modigliani Miller theorems is that the two effects cancel exactly. Why? Let us return to the discussion in the first section of this note. There we observed that leverage does not change any fundamental properties of the firm given our assumptions. If the tax system does not discriminate between payments from debt and equity securities, there are no bankruptcy costs, and the investment policy of the firm is given, then the returns of the firm are unaffected by leverage. The only impact of increasing leverage is then to change the *allocation* of risk *between* debtholders and equityholders, but not the total *amount* of risk borne by debtholders and equityholders collectively. As a result, leverage changes the riskiness of each security, but not the riskiness of all securities taken together. Hence, equation (5) must be interpreted as saying that the return r_A on the left-hand side of the equation is given, and that the returns to the securities change as leverage changes. Solving (5) for r_E gives:

$$r_E = r_A + \frac{D_0}{E_0}(r_A - r_D) \tag{7}$$

The following figure displays the relationships implied by equations (5) and (7). The required rate of return on the firm (the cost of capital) is constant and independent of leverage. However, for higher leverage the required return on debt increases. For very high leverage, default risk on debt increases until debt becomes effectively like equity and the firm is technically always bankrupt; in this case debt holders become residual claimants of the cash flows. In our example this would happen if we increased the face value of debt to 280 or more. Then equityholders would never receive anything, and there is no difference between owning 100% of the debt or 100% of the equity of an unlevered firm. As a result, the required rate of return on debt approaches the cost of capital. The cost of equity is given from equation (5). If leverage is zero, then the cost of equity is simply the cost of capital. When the debt is risk-free, r_D is equal to the risk free rate and the cost of equity increase in proportion to D/E by $r_A - r_F$ until leverage becomes so high that debt becomes risky and the cost of debt increases above r_F as indicated by the curved section of the lower line.



If we want to extend the formula above to a world with taxes, we need to take into account that interest is tax deductible. Then we obtain:

$$r_A = r_E \frac{E_0}{V_0} + r_D (1-t) \frac{D_0}{V_0} \qquad (5\text{ a})$$

where t is again the tax rate. This revises formula (5) above and recognizes the fact that the government practically "subsidizes" debt financing by making interest deductible as an expense. The weight of the cost of debt is therefore lower. We can now turn to implications of the results we have discovered so far.

7.9 Applications: Computing the required rate of return

Equation (5) has one powerful application: we can use it in order to determine the cost of capital we use for company valuation and project appraisal purposes as a weighted average of the cost of equity and the cost of debt. This is known as the ***weighted average cost of capital (WACC)***. In general, r_A is a number that is not directly related to publicly traded securities, hence we have to use a more indirect approach. The approach following from equation (5) is simple to implement:

1. Determine the cost of debt and the cost of equity, using an asset pricing model like the CAPM, the APT, or the Fama and French 3-factor model. This gives us estimates of r_D and r_E.

2. Determine the proportions of debt and equity using the *market* value of debt and the *market* value of equity, to determine E_0, D_0 and therefore V_0.

3. Use equation (5) to determine the weighted average cost of capital (WACC).

One problem that typically arises in the above procedure is the determination of the costs of debt r_D. A common mistake is to use the interest rate quoted on bank loans or the coupon rate or yield to maturity of the bonds of the firm. However, such a procedure is entirely inappropriate. Reconsider our example above. When leverage is 90% we have that the firm issues debt worth 180 with a face value of 204. Hence, the interest rate stated by the bank, which in this case is equal to the yield to maturity of a zero-coupon bond, is:

$$\frac{204}{180} = 1.133$$

or 13.33%. However, using the probabilities of the states and computing the expected return of debt gives us r_D=10%. It is the second number we require, and we used in our calculations all along. Evidently, since there is a probability (25% in our example) that the firm defaults on its obligations to debtholders, debtholders will charge a premium for their default risk (3.33% in our example). Suppose they did not and naively charged the firm an interest rate of 10%, so that the face value would be only 198. Then the expected payoff would be 0.25*180+0.75*198=193.5, or a return of 13.5/180=7.5%, much less than the 10% they require to break even! This sometimes causes confusion. The easiest way to remember the difference between the cost of debt r_D we have introduced here, and the interest rate on a bank loan is to recall that r_D is an *expected* value, which is sometimes higher and sometimes lower than the realized return which ranges from 0% to 13.33%. The interest rate charged by a bank is always the *highest* return the bank is ever going to realize since no company is going to pay *more* interest than initially agreed. It is therefore always higher than the *expected* repayment, which falls short of the contractual payment in case of bankruptcy. Hence, the interest rate that is stated on the loan contract is a biased estimator of r_D since it is always higher. A shortcut that is appropriate in all cases where leverage is not too high is to use the risk free rate of interest as an estimate for the expected return on debt. Since government bonds are not subject to repayment risk, the yield to maturity of a government bond is a good estimate of the expected return on this bond, and if leverage is low the error made in our calculations is only moderate.

It is important to recognize that causality and computation run in opposite directions. Effectively, **causality** runs from the asset side to the liability side of the balance sheet of the firm: the risk of operations (assets) determines the riskiness of debt and equity. Debt and equity (or any other liability of the firm) is nothing but a claim to the operating cash flows, and the structure of these claims (priority and seniority rights) determines how the risks from cash flows are *allocated* among different claimants of the firm. Hence, causality runs from the left hand side to the right hand side of equation (5). However, we do not have direct access to data on the asset side of the firm, since the assets of a firm are typically not traded. Hence, we infer the riskiness of the assets by analyzing the riskiness of the liabilities of the firm. The process of **computation** of the cost of capital runs therefore from the liability side to the asset side to the balance sheet. Computing the weighted average cost of capital runs therefore from the right hand side to the left-hand side of (5). This is sometimes confusing, since the logic of the weighted average cost of capital formula (equation (5) above) seems to suggest that the cost of equity and the cost of debt **determine** the cost of capital, because this is the way in which they are computed. However, we have seen that changing leverage changes only the way in which the operating cash flows of the firm are distributed, not the total size of these cash flows. Hence, the cost of capital *determines* the cost of debt and equity, even though WACC computations run exactly the other way.

7.10 Another method to calculate the cost of capital: unlevering betas

We have noted above that we can use the weighted average cost of capital formula by using an asset pricing model like the CAPM to estimate r_D and r_E. This suggest an alternative approach that follows directly from the following transformation of equation (5):

$$
\begin{aligned}
r_A &= \frac{E}{V} r_E + \frac{D}{V} r_D \\
&= \frac{E}{V}\left(r_F + \beta_E\left(r_M - r_F\right)\right) + \frac{D}{V}\left(r_F + \beta_D\left(r_M - r_F\right)\right) \\
&= r_F + \left(\frac{E}{V}\beta_E + \frac{D}{V}\beta_D\right)\left(r_M - r_F\right)
\end{aligned} \tag{8}
$$

The second line of equation (8) uses the CAPM in order to estimate the required returns of debt and equity by using the market betas of debt and equity respectively. The last line of equation (8) follows from just regrouping terms. Observe that the last line is simply the CAPM where we use a weighed average of the debt and equity betas. This leads us to define the ***asset beta*** of the firm as follows:

$$\beta_A \equiv \frac{E}{V}\beta_E + \frac{D}{V}\beta_D \tag{9}$$

We will frequently refer to the asset beta of the firm also as the **unlevered beta**. Hence, we have found an alternative formula for the cost of capital of the firm, substituting (9) into (8):

$$r_A = r_F + \beta_A\left(r_M - r_F\right) \tag{10}$$

This suggests an alternative method to compute the cost of capital:

1. Determine the market value proportions of debt and equity in the capital structure, E/V and D/V, respectively.

2. Use the CAPM to determine the equity and debt betas.

3. Compute the asset beta of the company or project using equation (9).

4. Compute the cost of capital using equation (10).

The problem we had with using the WACC has of course not disappeared in this method. When we unlever betas, we need an estimate of the debt beta as well as the equity beta in order to compute the asset beta. Whereas equity betas are easy to compute, debt is frequently not publicly traded, and we do not have a figure readily available. The same argument as in the previous section applies, and for moderate debt levels we can use a debt beta of zero so that (9) simplifies to $\beta_A = (E/V) * \beta_E$.

7.11 The costs and benefits of leverage: Implications of the Modigliani Miller propositions

The argument in the previous sections shows that the choice of leverage does not in any way affect the value or the cost of capital of the firm in a world that conforms to the assumptions made above. This is obviously only a benchmark case, and we can easily see that several of these assumptions are violated. However, the analysis above can still guide us in order to identify how a corporate management can create value using the capital structure. Specifically, we can conclude that:

- If changing the capital structure only changes the *allocation* of risk among security holders without affecting the operating decisions of the firm, then it is unlikely that value can be created.

- Issuing securities that have lower required return does *not* reduce the cost of capital, since the expected return on all other securities will increase.

- Offering new securities that suit particular investor's preferences (regarding currency, risk, leverage, etc.) does not reduce the cost of capital if investors (or their intermediaries) can replicate these securities.

These conclusions are important, since they show that some common assumptions about what determines the cost of capital are misguided. These principles extend easily:

- If interest rates on bonds denominated in Yen are lower than those denominated in dollars, then issuing Yen-bonds does not reduce the cost of capital.

- Issuing convertible bonds with a lower coupon than straight bonds does not reduce the cost of capital.

- Issuing subordinated loans at a higher interest rate is not more expensive than issuing senior, secured loans.

Hence, the Modigliani Miller propositions help us to distinguish between real and spurious opportunities to create value through capital structure choices.

- Taxes: If taxes discriminate in favor of payments from some securities (generally: debt, since interest is deductible before income tax is computed), then issuing these securities can generate value for the firm. This is an important but complicated topic, since taxes are paid at the firm level (corporation tax) and by individual investors (income tax, capital gains tax) and different countries/states and investors are subject to different tax statutes. Note that if taxes are high but all securities are taxed in the same way, the conclusions of Modigliani and Miller still hold.

- Bankruptcy costs: Bankruptcy is a costly process, hence managers will generally try to avoid this scenario by using only limited amounts of debt. Thus, issuing foreign currency bonds may be a form of hedging currency risk that can help reduce the likelihood of bankruptcy, and this creates value for the company.

- Homogeneous investors. Investors differ in various respects, and some groups may be willing to pay a premium for a security that suits their circumstances. Especially institutional investors (pension funds, mutual funds) are subject to different regulations that also vary across countries. Designing securities that target these investors and suit their requirements may allow the firm to raise cheaper capital *if* other companies or banks cannot offer these securities. This is a delicate argument, since the firm must have sufficient "monopoly power" and be able to offer securities that cannot easily be replicated. Again, if investors prefer dollar-bonds, issuing sterling-bonds is not more expensive, since investors can swap sterling for dollar liabilities. (This extends the "home made leverage" argument through investing in hedge funds in section 5 above).

- Independence of financing choices and the returns from assets. This assumption requires that capital budgeting choices are independent of leverage or other capital structure choices. This assumption is correct if managers accept all positive NPV projects and reject all negative

NPV projects. The subsequent section shows when this assumption fails and discusses this particular assumption in more detail.

This discussion exemplifies the dilemma of capital structure theory: in the ideal world of Modigliani and Miller we have a clear set of propositions and a couple of hard and fast rules that depend on a set of rather stringent assumptions. Once we relax these assumptions these conclusions give way to a general "it all depends" that is best explored by studying the capital structure choices of companies using case studies. We need to know the specific issues of the company at hand, and the details of the tax law, investor clientele, sources of risk etc. that give rise to deviations from Modigliani and Miller's world to determine an optimal capital structure.

7.12 The costs of leverage: the dependence of capital budgeting decisions on financing

This section shows how excessive leverage can have adverse implications for the value of the firm even if a firm is not in financial distress. Modigliani and Miller assumed that the investment policy of the firm is independent of leverage. However, suppose that managers try to maximize the value of the *equity* of the firm. This will be the case if managers try to maximize “shareholder value”, for example because they are compensated with stock or stock options. We show in the following two examples that excessive leverage induces managers to make capital budgeting decisions that increase the value of the equity at the expense of the firm’s creditors, and that this may reduce the value of the firm. Hence, leverage may have implicit costs because it induces a conflict of interest between shareholders and debt holders.

Suppose the firm has assets in place that pay off 100 in a boom (the “good state”) and 25 in a recession (the “bad state”). Suppose booms occur with probability 20% and recessions with probability 80%, and the firm has debt outstanding with a face value of 50. Then payoffs to debt holders and equity holders are:

	Firm	Equity	Debt
Prob=20%	100	50	50
Prob=80%	25	0	25
Total value (Expectation)	40	10	30

In the good state, debt holders are paid the face value of debt, and shareholders receive the difference. In the bad state, the firm is bankrupt, debt holders receive all the cash flows, and shareholders receive nothing. In market value terms, the leverage of the firm is 75% (debt/(debt+equity)), and the debt/equity- ratio is 300%. Now consider two investment projects managers are currently considering.

7.12.1 Risk shifting: When managers gamble

The first project has the following characteristics:

- Outlay today: 10
- Payoff in good state: 40
- Payoff in bad state: 0

First note that the project has a negative NPV since 0.2*40+0.8*0-10=-2. Hence, accepting this project reduces the value of the firm by 2. However, how does accepting this project affect the value of debt and equity? Note that taking the project changes the payoff in the boom from 100 to 130 (=100+40-10), and in the recession to 15 (=25+0-10). Hence, the table becomes:

	Firm	Equity	Debt
Prob=20%	130	80	50
Prob=80%	15	0	15
Total value (Expectation)	38	16	22

Hence, the value of the firm has dropped by 2 from 40 to 38. However, creditors and shareholders have been affected very differently: shareholder value has increased from 10 to 16. Who has paid the price if equity holders win and the firm as a whole has not? The value of creditors' claims has dropped from 30 to 22 by 8: of these 8, 6 are a straight transfer to shareholders, and the other 2 are simply the loss in firm value.

Hence, if managers try to maximize shareholder value, they will accept the project. They will effectively gamble and bet on the occurrence of the high state, where all gains are passed on to shareholders. The costs of the project are mainly borne by creditors whose claims get diluted. This phenomenon is referred to in the literature as *risk shifting.* A famous historical example of this behavior were thrifts who invested in high-risk assets like real estate and junk bonds, which increased equity value, whereas the costs of non-performing investments were borne by others (in this case the FDIC). Other examples include highly leveraged airlines that offered excessively generous airmiles rewards that reduced the value of the company, but increased the probability of not going bankrupt.

7.12.2 Risk shifting: The general case[3]

The economics of this argument are much more general than the numerical example above. Debt with face value F, equity is a like call option on the assets of the firm with strike price F: the payoff to shareholders is Max(V-F,0). We have also seen that the value of an option increases in the volatility of the underlying asset. Hence, for equity holders it is generally advantageous to increase the riskiness of the firm's payoffs, other things being equal. Who has written this call option? Creditors claim can be written as:

$$\left.\begin{array}{lll} F & if & V > F \\ & & \\ V & if & V < F \end{array}\right\} = Min(V,F) = F - Max(V - F,0) \qquad (15)$$

Hence, creditors have a fixed claim against the firm and a short position in the call option: the value of their claims *falls* if the riskiness of the operations increases. Hence, increasing risk shifts wealth from creditors to shareholders. In practice, creditors try to protect themselves against situations like this by introducing covenants into the debt contract: these can for example stipulate that asset sales must be used to pay down debt, or that dividends cannot be paid if leverage is too high.

7.12.3 The Problem of Debt Overhang

Now consider the second project that has the following characteristics:

- Outlay today: 10

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[3] Full appreciation of this argument requires some understanding of options. You may wish to skip this and return to it after the option lecture.

- Payoff in good state: 5
- Payoff in bad state: 15

This project has a positive NPV of +3.

	Firm	Equity	Debt
Prob=20%	95	45	50
Prob=80%	25	0	30
Total value (Expectation)	43	9	34

Now the value of the firm has increased by the positive NPV of the project to 43. However, for equity holders this project is unattractive: the value of their claims has dropped by 1 from 10 to 9. Where has the value gone? This project is the opposite of the one before: it pays off a lot in recessions and the payoffs of the investment are *negatively* correlated with the assets in place, hence it reduces the risk of the firm. The increase in value accrues to bondholders, but equity holders bear a disproportionate amount of the costs.

The situation may be even worse. Suppose managers had an incentive scheme so that they maximized the value of the firm, but they needed external finance in order to start this project. Suppose debt has a covenant that stipulates that no claims senior to the existing debt can be issued. Than managers need to raise equity (or some form of junior debt). Before taking the project, the value of equity is 10, hence they maximum value of a debt claim which dilutes equity completely would be 10. However, since the value generated with this project would be transferred to creditors, they can only raise 9, too little to be able to finance the project. This problem is referred to in the literature as the *debt overhang problem.* A firm cannot finance a positive NPV project, because the payoffs of the project would benefit existing debt holders, so that new creditors or equity holders brought into the firm would not be able to get the money back on their investment. Of course, in situations like this managers can try to find a solution, for example by renegotiating the debt contracts or repurchasing debt to the point where equity holders could internalize the benefits of the project. Sometimes it is also possible to incorporate a project as a separate legal entity (project financing). This helps, because old claims against the existing assets would not have priority for the cash flows of the new project, and the new project could be financed through issuing new securities whenever (and only if) it had a positive NPV.

Appendix: Important formula

The weighted average cost of capital formula:

$$r_A = r_E \frac{E}{V} + r_D \frac{D}{V}$$

Unlevering betas formula:

$$\beta_A \equiv \frac{E}{V} \beta_E + \frac{D}{V} \beta_D$$