Global Financial Management

Forward and Future Contracts

Latest Revision: December 10, 1997

9.0 Overview:

This class provides an overview of **forward** and **futures** contracts. Forwards and futures belong to the class of securities known as **derivatives** since their value is derived from the value of some other security. The price of a foreign exchange forward contract, for example, depends on the price of the underlying currency and the price of a pork belly futures contract depends on the price of pork bellies. Derivatives trade both on exchanges (where contracts are standardized) and over-the-counter (where the contract specification can be customized). The focus of this class is on

1) definitions and contract specifications of the major exchange-traded derivatives,
2) the mechanics of buying, selling, exercising, and settling forward and futures contracts,
3) derivative trading strategies including hedging, and
4) the relationships between derivatives, the underlying security and risk-less bonds. In particular, it is possible to form combinations of derivatives and the underlying security that are risk-less, providing a means of valuing derivatives.

9.1 Objectives

After completing this class, you should be able to:

- Determine the payoffs of forward and futures contracts.

- Determine the possible payoffs of portfolios of futures, forwards, and the underlying asset.
- Understand the mechanics of buying, selling, exercising, and settling forward and futures contracts.
- Explain the use of margin accounts and the procedure of "marking to market".
- Examine market prices to find arbitrage opportunities in futures and forward markets.
- Use standard valuation techniques to determine the price of forward and futures contracts.
- Construct optimal hedges for hedging exchange rate risk, commodity price risk, stock market risk and interest rate risk using forwards and futures contracts.
- Explain the concept of basis risk.

9.2 Introduction

Despite the recent adverse press they have received, **derivative securities** provide a number of useful functions in the areas of risk management and investments. In fact, derivatives were originally designed to enable market participants to eliminate risk. A wheat farmer, for example, can fix a price for his crop even before it is planted, eliminating price risk. An exporter can fix a foreign exchange rate even before beginning to manufacture the product, eliminating foreign exchange risk. If misused, however, derivative securities are also capable of dramatically increasing risk.

This module focuses on the mechanics of **forward** and **futures** contracts. There is a particular emphasis on the interrelationship between the various contracts and the **spot price** of the underlying asset. The spot price is the price of an asset now (spot market) i.e. the sale transaction and settlement is to occur immediately.

9.3 Forward Contracts

The Mechanics of Forward Contracts

A **Forward Contract** is a contract made today for delivery of an asset at a prespecified time in the future at a price agreed upon today. The buyer of a forward contract agrees to take delivery of an underlying asset at a future time, T, at a price agreed upon today. No money changes hands until time T. The seller agrees to deliver the underlying asset at a future time, T, at a price agreed upon today. Again, no money changes hands until time T.

A forward contract, therefore, simply amounts to setting a price today for a trade that will occur in the future. Example 1 illustrates the mechanics of a forward contract. Since forward contracts are traded over-the-counter rather than on exchanges, the example illustrates a contract between a user and a producer of the underlying commodity.

Example 1: Forward Contract Mechanics

> A wheat farmer has just planted a crop that is expected to yield 5000 bushels. To eliminate the risk of a decline in the price of wheat before the harvest, the farmer can sell the 5000 bushels of wheat forward. A miller may be willing to take the other side of the contract. The two parties agree today on a forward price of 550 cents per bushel, for delivery five months from now when the crop is harvested. No money changes hands now. In five months, the farmer delivers the 5000 bushels to the miller in exchange for $27,500. Note that this price is *fixed* and does not depend upon the spot price of wheat at the time of delivery and payment. As a result, the farmer does not have to worry about the price at which he is going to sell the wheat, and the miller does not have to worry about the price at which he is going to buy wheat. They have hedged the price of their output and input respectively.

9.4 Valuation of Forward Contracts

Forward contracts are the simplest type of derivative securities, and we analyze the principles of their valuation in greater detail. The common characteristic of almost all approaches to valuing derivatives is realizing that they can be *replicated* using other securities. We have already encountered the concept of replication in the context of coupon bonds. There we argued that coupon bonds can be replicated using a portfolio of zero-coupon bonds. Correspondingly, we will find a portfolio of that replicates a forward contract and calculate its price. As a result this portfolio's price must be the forward contract's price.

Consider the simplest case of a stock trading at $45 today that pays no dividends during the next 3 months. Assume that the risk free rate of interest is 8%, and that large investors in the market can borrow and lend at this rate. Now compare the following two strategies:

(A) Buy one share for $45 and sell the share forward in three months for the forward price *F*.

(B) Invest $45 at the risk free rate of interest

The payoffs from strategy (A) are:

	Today	**3 Months from now**
Buy share	-$45	+SP
Sell forward	0	F-SP
Total Portfolio A	-$45	F
Portfolio B	-$45	$45.90

Hence, the payoff of the portfolio is certain and equal to F, even though we do not know the stock price. Compare this to strategy B, where we invest $45 today, and receive $45*(1.02)=$45.90 in three months time, which is also riskless. By the principle of no-arbitrage, portfolio A and portfolio B must have the same payoff, hence the forward price can be determined as F=$45.90.

Let's investigate the principle of no-arbitrage a little bit further. This principle says that prices must always adjust so that no market participant can make a riskless profit. Suppose that the forward contract is overpriced and F=$47.00. How could anybody profit from this? If the forward contract is overpriced, then we can (1) buy portfolio A and (2) sell portfolio B (i. e., borrow) and obtain:

	Today	**3 Months**
Buy Stock	-$45	+SP
Sell forward	0	+$47.00-SP
Borrow $45	$45	-$45.90
Total	0	$1.10

Evidently, with this portfolio strategy, we earn $1.10 without making any investment today. The profit of $1.10 is independent of the future stock price; hence it is risk free. It is easy to see that in case the forward contract is underpriced (F=$43, say), the opposite strategy (sell the stock, buy it forward and invest the proceeds from the sale in the risk free asset) leads to an arbitrage profit (If F=$43, it is $2.90).

When we price forwards – or any other asset for that matter – by the principle of no-arbitrage, we assume that risk-less profit opportunities without investment as in the case above cannot exist. We therefore conclude that the only price that is consistent with arbitrage free market is F=$45.90.

We can now generalize this argument and include the fact that stocks may actually pay dividends sometime during the contract. This is important, because the owner of the stock receives all dividends paid out to stockholders while holding the stock, whereas the buyer of a forward contract does not own the stock yet, hence receives no dividends. We use the following notation:

S_0	The current stock price, which is known
S_T	The stock price at maturity of the contract, which is not known when the contract is entered into
r_T	The interest rate for the period from now (time '0') to maturity (time 'T')
F	The forward price, which is known and part of the contract
D_T	The cumulative value of all dividends paid between now and maturity, expressed in period T dollars
d	The dividend yield of the stock (annual percentage rate)
q	cost of carry of commodities (annual percentage rate, see below)

Following our strategy above, we form exactly the same portfolios A and B as before.

	Now (time 0)	**Maturity (time T)**
Buy stock	$-S_0$	$+S_T(+D_T$ if dividend)
Sell the stock forward	0	$F-S_T$
Total portfolio A	$-S_0$	$F(+D_T$ if dividend)
Portfolio B (Invest S_0 at the risk free rate)	$-S_0$	$+S_0(1+r_T)$

Then equating the payoffs from both portfolios gives us:

$$\begin{aligned} F &= S_0(1+r_T) \quad \text{if the stock pays no dividends} \\ F &= S_0(1+r_T)-D_T \quad \text{if the stock pays dividends} \end{aligned} \tag{2}$$

Suppose the stock pays a dividend with a total cumulative dollar value of D_T and a present value of D_0. We can easily relate the two by writing $D_T = (1+r_T)D_0$, the standard formula for future values. Then we obtain:

$$F = (S_0 - D_0)(1+r_T) \tag{3}$$

Formula (2) suggests another form of building an arbitrage portfolio:

Portfolio A: Invest in $1-\frac{D_0}{S_0}$ (1 minus dividend yield) units of the stock, and reinvest all dividends by buying another $\frac{D_0}{S_0}$ units of the stock; own 1 unit of stock at maturity.

Portfolio B: Borrow S_0-D_0.

Then the previous table changes to:

	Now (time 0)	**Maturity (time T)**
Buy $1-\frac{D_0}{S_0}$ units of stock and reinvest all dividends in buying more stock	$-S_0+D_0$	$+S_T$
Sell the stock forward	0	$F-S_T$
Total portfolio A	$-S_0+D_0$	F
Portfolio B (Invest S_0-D_0 at the risk free rate)	$-S_0+D_0$	$(S_0 - D_0)(1 + r_T)$

Equating the payoffs from both portfolios results in equation 3 above.

Another aspect about formula (2) that is inconvenient for applications is the use of discrete compounding. It is generally preferable to use continuous compounding to obtain:

$$\begin{aligned} F &= S_0 e^{rT} \quad \text{if the stock pays no dividends} \\ F &= S_0 e^{(r-d)T} \quad \text{if the stock pays dividends} \end{aligned} \tag{4}$$

Formula (4) expresses the fact that with a dividend paying stock, we only need to buy e^{-dT} (This is a number smaller than 1) units of the stock. Here the dividend yield is expressed as an annual percentage rate, similar to an interest rate. Conceptually, there is no difference between the discrete compounding case and the continuous compounding case. When we obtain dividends while holding the stock, reinvesting the dividends enables us to purchase another $1-e^{-dT}$ units of the stock and implies that at maturity we own exactly one unit of the stock. The arbitrage relationships constructed above can then be expressed as:

	Now (time 0)	**At maturity (time T)**
Buy e^{-dT} units of the stock; reinvest dividends	$-e^{-dT}S_0$	S_T
Sell one unit of the stock forward	0	$F-S_T$
Borrow $S_0 e^{-dT}$	$S_0 e^{-dT}$	$-\left(S_0 e^{-dT}\right)e^{rT}$
Total	0	$F-\left(S_0 e^{-dT}\right)e^{rT} = F - S_0 e^{(r-d)T}$

Arbitrage free markets then require that the total payoff of this portfolio is zero at maturity, which is equivalent to (4).

Example 2:

Suppose you wish to value a six-month forward contract on a stock that is currently trading at $95 and has a dividend yield of 2.0%. Assume the risk free rate is 7%. We wish to show that the 6-month forward should be priced at $97.40. Suppose you buy $e^{-0.5(0.02)} = 0.99$ units of the stock, i. e., you invest $$94.05. You reinvest all dividends, so in six months you own 1 unit of the stock. You sell this unit forward, so the return on your portfolio is riskless. You could also invest your $94.05 at the risk free rate, and obtain a safe payoff equal to $\$94.05 * e^{0.5*0.07}$ or $97.40 (this is of course equal to $\$95.00 * e^{0.5(0.07-0.02)} = \$95.00 * 1.02532$). If the forward price would be less than $97.40, we could make an arbitrage profit by selling stock and buying it back forward, investing the proceeds in bonds. If the forward price would be more than $97.40, we could make an arbitrage profit by buying stock and selling it forward, where we would borrow the money for purchasing the stock.

It may at first not be obvious, but the same logic applies to commodity futures. With commodity futures we can either buy commodities outright, or we can buy them forward. Again, as in the case of stock, there is a difference between owning commodities and buying them forward. The owner of commodities has to maintain their value. This requires storage (wheat, gold), feeding (live hogs), or security (gold). These cost are called **cost of carry** and also expressed as an annual percentage rate q. The treatment of these cost is simply as a negative dividend, and we therefore obtain a valuation formula for commodity futures as:

$$F = S_0 e^{(r+q)T} \tag{5}$$

Effectively, (5) is not a new formula, we have simply used (4) and replaced –d with q.

Example 3: Forward arbitrage.

Suppose the spot price of wheat is 550 cents per bushel, the six-month forward price is 600, the riskless rate of interest is 5% p.a., and the cost of carry is 6% p.a. To execute an arbitrage, you borrow money, buy a bushel of wheat, pay to store it and sell it forward. The cash flows are:

Position	**Initial Cash Flow**	**Terminal Cash Flow**
Buy one unit of commodity	-550	S_T
Pay Cost of Carry	$-550\ (e^{0.06x0.5} - 1)$	0
Borrow	$550\ e^{0.06x0.5}$	$-550\ e^{(0.06+0.05)T}$
Enter 6-month forward sale	0	$600 - S_T$
Net Portfolio Value	0	$600-550\ e^{(0.06+0.05)0.5} = 18.90$

That is, it is possible to lock in a sure profit that requires no initial cash outlay.

9.5 Hedging With Forward Contracts

The primary motivation for the use of forward contracts is risk management. The wheat farmer in example 1 was able to eliminate price risk by selling his crop forward. Example 4 contains a more comprehensive example concerning foreign exchange risk management.

Example 4: Forward contacts and risk management.

XYZ is a multinational corporation based in the US. Its manufacturing facilities are located in Pittsburgh and hence its labor and manufacturing costs are incurred in US dollars (USD). A large fraction of its sales, however, are made to German customers who pay for the goods in Deutschemarks (DM).

There is a six-month lead time between the placement of a customer order and delivery of the product. XYZ's cost of production is 80% of the sale price. Suppose XYZ receives a $1MM DM order and that the current USD/DM exchange rate is 0.60 (i.e. 1 DM = 0.60 USD). The cost of production of this order is $480,000 (0.60 x $1MM x 0.80). The exchange rate six months from now is, of course, uncertain in which case XYZ is exposed to exchange rate risk.

If the exchange rate stays at 0.60, then XYZ will convert the 1MM DM to $600,000 and earn a 25% profit on the $480,000 cost of production. If, however, the exchange rate falls to 0.40 six months from now, XYZ will convert the 1MM DM to only $400,000, registering a loss on the sale.

Conversely, if the exchange rate rises to 0.80 six months from now, XYZ will convert the 1MM DM to $800,000, registering a very large profit on the sale. Whereas XYZ are very good at manufacturing and marketing their product, they have no expertise in forecasting exchange rate movements. Therefore, they want to avoid the exchange rate risk inherent in this transaction (i.e., the risk that they do everything right and then lose money on the sale, solely because exchange rates move against them). They can do this by **selling forward** 1MM DM. This involves entering a contract today with, say, an investment bank under which XYZ agrees to deliver 1MM DM six months from now in exchange for a fixed number of US dollars. This rate of exchange is the **six-month forward rate**. Suppose the six-month forward rate is 0.62 (which is set according to market expectations and relative interest rates as described below). Then, when XYZ receives 1m DM from its customer, they deliver it to the investment bank in exchange for $620,000 (locking in a profit) **regardless of whether the exchange rate happens to be 0.40 or 0.80 at that time**.

9.6 Futures Contracts

The Mechanics of Futures Contracts

A **futures contract** is similar to a forward contract except for two important differences. First, intermediate gains or losses are posted each day during the life of the futures contract. This feature is known as **marking to market**. The intermediate gains or losses are given by the difference between today's futures price and yesterday's futures price. Second, futures contracts are traded on organized exchanges with standardized terms whereas forward contracts are traded **over-the-counter** (customized transactions between a buyer and a seller).

Example 5 illustrates the marking to market mechanics of the All Ordinaries Share Price Index (SPI) futures contract on the Sydney Futures Exchange. The SPI contract is similar to the Chicago Mercantile Exchange (CME) S&P 500 contract and the London International Financial Futures Exchange (LIFFE) FTSE 100 contract. The mechanics are the same for all of these contracts.

Example 5: Marking to market.

Suppose an Australian futures speculator buys one **SPI futures contract** on the **Sydney Futures Exchange** (SFE) at 11:00am on June 6. At that time, the futures price is 2300. At the close of trading on June 6, the futures price has fallen to 2290 (what causes futures prices to move is discussed below).

Underlying one futures contract is $25 x Index, so the buyer's position has changed by $25(2290-2300)=-$250. Since the buyer has bought the futures contract and the price has gone down, he has lost money on the day and his broker will immediately take $250 out of his account. This immediate reflection of the gain or loss is known as **marking to market**.

Where does the $250 go? On the opposite side of the buyer's buy order, there was a seller, who has made a gain of $250 (note that futures trading is a zero-sum game - whatever one party loses, the counterparty gains). The $250 is credited to the seller's account. Suppose that at the close of trading the following day, the futures price is 2310. Since the buyer has bought the futures and the price has gone up, he makes money. In particular, *$25(2310-2290)=+$500* is credited to his account. This money, of course, comes from the seller's account.

This concept of *marking to market* is standard across all major futures contracts. Contracts are marked to market at the close of trading each day until the contract expires. At expiration, there are two different mechanisms for settlement. Most financial futures (such as stock index, foreign exchange, and interest rate futures) are cash settled, whereas most physical futures (agricultural, metal and energy futures are settled by delivery of the physical commodity. Example 6 illustrates cash settlement.

Example 6: Cash settlement.

> Suppose the SPI futures contact price was 2350 at the close of trading on the day before expiration and 2360 at the close of trading on the expiration day. Settlement simply involves a payment of $25(2360-2350) = $250 from the seller's account to the buyer's account. The expiration day is treated just like any other day in terms of standard marking to market.

An alternative to cash settlement is physical delivery. Consider the SFE wool futures contract which requires delivery of 2500 kg of wool when the contract matures. Of course, there are different grades of wool, so a set of rules governing **deliverable quality** is required. These are detailed rules that govern the standard quality of the underlying commodity and a schedule of discounts and premiums for delivery of lower and higher quality respectively.

Example 7 illustrates the process of physical delivery for the SFE greasy wool futures contract. The process is similar for most commodity futures contracts.

Example 7: Physical delivery.

> Suppose the greasy wool futures contact price was 700 cents at the close of trading on the expiration day. Settlement involves physical delivery from the seller of the futures contract to the buyer of the underlying quantity of wool (2,500 kilograms) on the business day following the expiration day. Delivery, therefore, involves the seller delivering 2,500 kg of wool to the buyer, in return for a payment of $17,500.
>
> If the wool is of better quality than is specified in the contract, a premium must be paid. Conversely wool of lower quality involves a discount. It is the seller of the

futures who must make delivery of the wool and he has the option to choose what quality he will deliver, subject to the schedule of discounts and premiums.

9.7 Margin

Although futures contracts require no initial investment, futures exchanges require both the buyer and seller to post a security deposit known as **margin**. Margin is typically set at an amount that is larger than a usual one-day moves in the futures price. This is done to ensure that both parties will have sufficient funds available to mark to market. Residual credit risk exists only to the extent that (1) futures prices move so dramatically that the amount required to mark to market is larger than the balance of an individual's margin account, and (2) the individual defaults on payment of the balance. In this case, the exchange bears the loss so that participants in futures markets bear essentially zero credit risk. Margin rules are stated in terms of **initial margin** (which must be posted when entering the contract) and **maintenance margin** (which is the minimum acceptable balance in the margin account). If the balance of the account falls below the maintenance level, the exchange makes a **margin call** upon the individual, who must then restore the account to the level of initial margin before the start of trading the following day. Example 8 illustrates the margining procedure.

Example 8: Margin

Suppose a contract requires initial margin of $7,000 and maintenance margin of $5,000. The following table illustrates the margining procedure and the cash flows required for the buyer of a futures contract.

Time	Value of Futures Contract	Margin Balance before Calls	Margin Call	Margin Balance after Calls
0	25,000	0	7,000	7,000
1	24,000	6,000	0	6,000
2	22,000	4,000	3,000	7,000
3	24,500	7,000	0	7,000

Note that when the margin balance falls below the maintenance margin, it must be restored to the initial level. Note also that when the futures moves favorably (as at time 3) the marking to market cash inflow can be immediately withdrawn - it need not remain in the margin account.

9.8 Some common futures

Futures are traded on exchanges, like the Chicago Mercantile Exchange (CME), the Chicago Board of Trade (CBOT), and others. Prices on futures are recorded and published regularly in the financial press. In this section we introduce examples of the most common types of futures: (1) commodity futures, (2) stock index futures, (3) currency futures and (4) interest rate futures. Table 1 gives the price and trading data for wheat futures as published in the Wall Street Journal:

Table 1

Wheat (CBT), 5,000 bu.; cents per bu.						
	Open	**High**	**Low**	**Settle**	**Change**	**Open Interest**
Dec	360 ½	364 ½	358 ½	363 1/4	+2 3/4	52,809
Mr98	374	379	373 1/2	377 3/4	+3 3/4	27,345
May	381 3/4	386	381	385 1/2	+3 3/4	6,423
July	384 1/2	388	383 1/2	387 3/4	+3 1/4	14,941
Sept	391 1/2	392 1/2	391 1/2	391 1/2	+2	141
Dec	400	403 1/2	398 3/4	402 1/2	+4 1/4	2,948
Est vol 14,000; vol Fri 10,573; open int 104,635						

The table tells us that Wheat futures are traded on the Chicago Board of Trade, and that contracts are per 5,000 bushels. All prices are given in cents per bushel; hence the opening price was $3.605/bu for the contract maturing in December, or $18,025 for the whole contract. Trading volume was 14,000 contracts.

The dominant **stock market index futures** contract is the S&P 500 futures contract. This contract trades on the Chicago Mercantile Exchange and has delivery months March, June, September, and December. The underlying quantity is $250 times the level of the S&P 500 index. The minimum price move is 0.05 index points, which is $12.5 per contract. Example 9 illustrates the settlement mechanics for the S&P 500 contract.

Example 9: Settlement of the S&P 500 Futures Contract.

> It is currently November 15 and the S&P 500 index is at 382.62. The December S&P 500 futures price is 383.50. If you buy 1 December S&P 500 futures contract, how much will you gain if the futures price at expiration is $393.50? The gain on your futures position is $\$250(F_t\text{-}F_0) = \$250(393.50\text{-}383.50)=\$2{,}500$. That is, to settle the contract, your counterparty will give you $2,500.

Table 2 gives the trading data on S&P500 stock index futures. You see that, for example, the settlement price for the S&P500 futures maturing in December was 947.50 index points, hence the price of the underlying asset for one futures contract was $236,875.

Table 2

S&P 500 INDEX (CME) $250 times index						
	Open	**High**	**Low**	**Settle**	**Change**	**Open Interest**
Dec	924.00	946.00	921.10	945.70	+21.70	192,508
Mr98	944.00	955.60	942.00	955.40	+21.90	6,503
June	953.00	966.50	953.00	965.80	+22.50	1,607
Sept	968.00	976.00	966.40	976.00	+23.30	251
Dec		….	….	986.30	+23.60	323
Est vol 72,447; vol Fri 62,765; open int 201,315						

A number of **foreign currency futures** contracts trade on the International Monetary Market division of the Chicago Mercantile Exchange. The currencies on which contracts are based, and the underlying notional amounts are listed in table 3. Delivery months for all contracts are March, June, September, and December. Prices are quoted as US dollars per unit of foreign currency. For example, if one Swiss franc buys 69.15 US cents, the price will be quoted as 0.6915.

Table 3

Denomination of Foreign Currency Futures Contracts

Currency	**Underlying Amounts**
British Pound	62,500 L
Canadian Dollar	100,000 C$
German Mark	125,000 DM
Japanese Yen	12,500,000 Y
Swiss Franc	125,000 SF
French Franc	250,000 FF
Australian Dollar	125,000 A$

Table 4 gives the data for futures on the Canadian dollar. These contracts are denominated in units of foreign currency, here C$100,000. Hence, if we buy C$1million for December, then the settlement price would oblige us to pay USD 711,700. You can see that the December contract is more heavily used than the later ones.

Table 4

Canadian Dollar (CME)- 100,000 dlrs.; $per Can $						
	Open	**High**	**Low**	**Settle**	**Change**	**Open Interest**
Dec	.7117	.7148	.7116	.7123	+.0013	68,920
Mar98	.7165	.7176	.7148	.7154	+.0012	3,571
June	.7194	.7200	.7175	.7178	+.0012	661
Sept	.7211	.7215	.7200	.7198	+.0012	351
Est vol 6,478; vol Fri 5,507; open int 73,595						

There are two primary **interest rate futures contracts** that trade on US exchanges. The Eurodollar Futures Contract trades on the Chicago Mercantile Exchange and the US T-Bill Futures Contract trades on the Chicago Board of Trade.

The **Eurodollar contract** is the more successful and heavily traded contract. At any point in time, the notional loan amount underlying outstanding Eurodollar futures contracts is in excess of $4 trillion. This contract is based on **LIBOR** (London Interbank Offer Rate), which is an interest rate payable on **Eurodollar Time Deposits**. This rate is the benchmark for many US borrowers and lenders. For example, a corporate borrower may be quoted a rate of LIBOR+200 basis points on a short-term loan. Eurodollar time deposits are non-negotiable, fixed rate US dollar deposits in banks that are not subject to US banking regulations. These banks may be located in Europe, the Caribbean, Asia, or South America. US banks can take deposits on an unregulated basis through their international banking facilities. LIBOR is the rate at which major money center banks are willing to place Eurodollar time deposits at other major money center banks. Corporations usually borrow at a spread above LIBOR since a corporation's credit risk is greater than that of a major money center bank. By convention, LIBOR is quoted as an annualized rate based on an actual/360-day year (i.e., interest is paid for each day at the annual rate/360). Example 10 demonstrates how interest is calculated on a LIBOR loan according to the conventions.

Example 10: LIBOR Conventions.

If 3-month (90 actual days) LIBOR is quoted as 8%, the interest payable on a $1 million loan at the end of the 3-month borrowing period is

$$(.08)\left(90/360\right)\$1{,}000{,}000 = \left((.08)/4\right)\$1{,}000{,}000 = \$20{,}000$$

The Eurodollar futures contract is based on a 3-month $1 million Eurodollar time deposit. It is cash settled, so no actual delivery of the time deposit occurs when the contract expires. Delivery months are March, June, September, and December. The minimum price move is $25 per contract which is equivalent to 1 basis point: *(.0001/4)1,000,000=25*. With interest rate futures we have a similar problem to the one we faced on stock market indices: we have to convert interest rates (denominated in percentage points or basis points) into units of currency. This is conventionally done by determining the price of an interest rate futures as

Price=100-interest rate

Hence, we can work out the implied interest rate as the difference between 100 and the price. E.g., if the price is 94.25 we find that the settlement price for the Eurodollar contract implies an interest rate of 100-94.25=5.75%. Note that this implies that the contract price increases when interest rates decrease, and decreases when interest rates increase. The Eurodollar futures price at expiration (time *T*) is determined as $F_T = 100$-LIBOR. Prior to expiration, the futures price implies the interest rate that can be effectively locked in for a 3-month loan that begins on the day the contract matures. Settlement of the Eurodollar futures contract is illustrated in Example 11:

Example 11: Settlement of Eurodollar Futures Contract.

Suppose you purchased 1 December Eurodollar futures contract on November 15 when the price was 94.86. If interest rates fall 100 basis points between November 15 and expiration of the futures contract in December, what is your total gain or loss on the contract at settlement?

First note that no money changes hands at the time you buy the contract. This is the nature of all futures contracts. The November 15 price of 94.86 implies that the LIBOR rate of interest was 100-94.86=5.14% at that time. If LIBOR falls 100 basis points by the time the December contract expires, LIBOR will then be 4.14%. Therefore, the expiration futures price will be 100-4.14=95.86. The total gain is therefore:

$$0.25(1{,}000{,}000)(F_t - F_0) = 0.25(1{,}000{,}000)(0.9586 - 0.9486) = \$2{,}500$$

Table 5 gives the data for interest rate futures. You can see from the table below that the term structure implied by the futures contract has interest rates rising from 5.75% (settlement, Nov 97) to 6.56% (settlement, Dec 2003).

Table 5

Eurodollar (CME)- $1 million; pts of 100%						
	Open	**High**	**Low**	**Settle**	**Change**	**Open Interest**
Nov	94.24	94.25	94.24	94.25	-.02	21,549
Dec	94.30	94.30	94.22	94.24	-.03	545,769
Mr98	94.24	94.24	94.18	94.19	-.05	432,549
June	94.16	94.16	94.11	94.14	-.05	344,826
Sept	94.08	94.08	94.04	94.07	-.05	256,704
Dec	94.06	93.98	93.93	93.96	-.05	220,405
Mr99	93.95	93.95	93.92	93.94	-.05	153,169
June	93.92	93.92	93.88	93.90	-.05	135,629
Sept	93.88	93.88	93.84	93.87	-.05	104,780
Dec	93.81	93.81	93.78	93.81	-.05	88,750
Mr00	93.82	93.82	93.78	93.81	-.05	69,722
June	93.79	93.79	93.75	93.78	-.05	57,977
Sept	93.76	93.76	93.73	93.75	-.05	51,417
Dec	93.70	93.70	93.67	93.69	-.05	45,752
Mr01	93.69	93.69	93.68	93.69	-.05	43,113
June	93.66	93.66	93.65	93.66	-.05	37,354
Sept	93.63	93.63	93.62	93.63	-.05	37,061
Dec	93.57	93.57	93.55	93.57	-.05	23,647
Mr02	93.57	93.57	93.55	93.57	-.05	21,714
June	93.54	93.54	93.52	93.54	-.05	18,331
Sept	93.51	93.51	93.49	93.51	-.05	15,008
Dec	93.45	93.45	93.44	93.44	-.05	7,820
Mr03	93.45	93.45	93.44	93.44	-.05	6,634
Est vol 283,821; vol Fri 453,085; open int 2,802,570						

4.9 Valuation of Futures Contracts

Whereas the valuation of forward contracts is relatively straightforward, the marking to market feature complicates the valuation of futures contracts. The cash flows associated with forward and futures contracts are illustrated in the following table.

Cash Flows of Forward and Futures Contracts

Time	**0**	**1**	**2**	**...**	**T**
Forward Cash Flow	**0**	**0**	**0**	**...**	S_T-FO_0
Futures Cash Flow	**0**	FU_1-FU_0	FU_2-FU_1	**...**	FU_T - FU_{T-1}

For both contracts, no money changes hands at the time the contract is initiated (time *0*). For the forward contract, no money changes hand until the contract matures (time *T*). For the futures contract, money changes hands daily depending upon movements in the futures price. Hence, futures contracts come with a sequence of cash flows associated with them, and in principle we have to value the whole sequence of cash flows to value the contract.

In some circumstances, however, a futures contract is perfectly equivalent to a forward contract in which case the two contracts must have the same value. Since forward contracts are relatively easy to value using a no-arbitrage argument, this provides a convenient way of valuing a futures contract. In particular, if interest rates are constant (at a continuously compounded annual rate of r) over the life of the contract then the prices of the futures contract and the forward contract are identical: in this case the present value of the intermediate cash flows is zero.

4.10 Arbitrage Relationships

For the remainder of this module, we assume that interest rates are indeed constant over the period of the contract and hence the futures price equals the forward price. That is, we can consider the price and payoffs of a futures contract to be identical to those of a forward contract. This simplifies things because a forward contract has only a single payoff at maturity.

Consider, for example, the valuation of a futures contract on the *S&P 500* stock index. This contract, which trades on the Chicago Mercantile Exchange (CME) entitles the buyer to receive the cash value of the *S&P 500* stock index at the end of the contract period. There are always four contracts in effect at any one time expiring in March, June, September, and December. It is easy to see that we can treat the stock price index just as we did dividend paying stocks above (see equation (4) and example 2). To see why this relationship must hold, consider the strategy of (1) borrowing $\mathbf{e^{-dT}S_0}$ through time *T*, (2) using this to purchase $\mathbf{e^{-dT}}$ units of the index and reinvesting all dividends back into the index, and (3) selling a futures contract that matures at time *T*. In particular, the (equivalent) cash flows associated with this strategy are tabulated in the following table. Note that reinvestment of the dividends has resulted in the initial investment of $\mathbf{e^{-dT}}$ units of the index growing at a rate of *d* to amount to one unit by maturity.

Arbitrage Relationship between Spot and Futures Contract

Position	**Time 0**	**Time T**
Borrow	$e^{-dT}S_0$	$e^{rT}e^{-dT}S_0$
Buy e^{-dT} units of index	$-e^{-dT}S_0$	S_T
Sell one Futures Contract	0	$F - S_T$
Net Position	0	$F - S_0e^{(r-dT)}$

Once again, since this strategy requires no initial cash outlay, the cash flow at maturity must also be zero or an arbitrage opportunity exists. In particular, if $F > S_0$ $e^{(r-dT)}$ the strategy of buying the index and selling the futures generates an arbitrage profit. Conversely, if $F < S_0$ $e^{(r-dT)}$ the strategy of selling the index and buying the futures generates an arbitrage profit. Examples 12 and 13 illustrate how to execute a riskless arbitrage if equality does not hold.

Example 12: Futures arbitrage: Buy index - Sell futures

Suppose the *S&P 500* stock index is at *$295* and the six-month futures contract on that index is at *$300*. If the prevailing T-Bill rate is *7%* and the dividend rate is *5%*, an arbitrage opportunity exists because $F=300 > S\ e^{(r-d)T} = 297.96$. The arbitrage can be executed by buying low and selling high. In this case, the futures contract is relatively overvalued, so we sell the futures and buy the index.

In particular, the strategy is to

- Borrow $e^{-dT}S_0 = \$287.72$ at *7%* repayable in 6 months.
- Use this *$287.72* to buy $e^{-dT} = 0.975$ units of the *S&P* index, and reinvest all dividends in the index.
- Sell a futures contract for delivery of the index in six months.

This generates the following cash flows:

Position	**Time 0**	**Time T**
Borrow	287.72	-297.96
Buy e^{-dT} units of index	-287.72	S_T
Sell one futures contract	0	300 - S_T
Net Position	**0**	**2.04**

Hence this strategy generates an arbitrage profit of *$2.04* six months from now.

Example 13: Futures arbitrage: Sell index - Buy futures

Suppose the *S&P 500* stock index is at *$300* and the six-month futures contract on that index is at *$300*. If the prevailing T-Bill rate is *7%* and the dividend rate is *5%*, an arbitrage opportunity exists because $F = 300 < S\ e^{(r-d)T} = 303.02$. The arbitrage can be executed by buying low and selling high. In this case, the futures contract is relatively undervalued, so we buy the futures and sell the index.

In particular, the strategy is to

- Short sell e^{-dT} units of the *S&P* index generating $Se^{-dT} = 292.59$.
- Lend the *$292.59* proceeds of the short sale at *7%* repayable in 6 months.

- Buy a futures contract for delivery of the index in six months.

This generates the following cash flows:

Position	**Time 0**	**Time T**
Sell e^{-dT} units of index	292.59	$-S_T$
Lend	-292.59	303.02
Buy one futures contract	0	$S_T - 300$
Net Position	**0**	**3.02**

Hence this strategy generates an arbitrage profit of *$3.02* six months from now.

9.11 Hedging with Futures

In this section, we examine how three common business risks - interest rate risk, stock market risk, and foreign exchange risk - can be hedged in a practical setting. In each case, we describe the nature of the risk and illustrate, through a series of practical examples, how the risk can be managed.

9.11.1 Hedging Interest Rate Risk

Interest rate risk arises whenever you expect to borrow or invest cash in the future at terms that are yet unknown. E.g., if expect to borrow a certain amount today because a loan matures that you need to refinance, then the interest rate on the new loan may be higher than you expect it to be. Conversely, if you expect to receive cash flows from a sale or a securities issue, you may not wish to use them straight away and need to invest them and some uncertain future interest rate. We have already talked about the conventions and procedures relating to Eurodollar contracts. Example 13 explains the use of this contract for interest rate hedging.

Example 14: Hedging with the Eurodollar Futures Contract.

It is currently November 15 and your company is aware that it needs to borrow $1 million on December 16 to pay a liability which falls due on that day. The loan can be repaid on March 16 when an account receivable will be collected. Your company's borrowing rate is LIBOR plus 200 basis points. The current LIBOR rate is 5.14%. At this rate, you would borrow at 7.14%. Your company is concerned that interest rates will rise between now and December 16, in which case you will pay a higher rate of interest on your loan. How can your company lock in the current rate of 5.14%?

Your company stands to lose if interest rates increase. Therefore, you want to enter a futures position that increases in value if interest rates rise. Then, if interest rates rise, your company loses by paying higher interest charges on the loan, but your company gains by profiting on the futures position. Conversely, if interest rates fall, your company gains by paying lower interest charges on the loan, but your company loses on the futures position. Ideally, the loss and the gain would exactly cancel, whether interest rates rise or fall.

From the construction of the Eurodollar futures contract, we know that if the interest rate rises, the futures price will fall. Therefore, you will sell 1 December Eurodollar futures contract at 94.86. Underlying this contract is a notional 3-month $1 million dollar loan to be entered into on December 16 (the day the contract expires).

If we could lock in the rate of 5.14%, the total interest on the loan would be *0.0714($1 million)/4 = $17,850*.

First, suppose that on December 16 LIBOR is 6.14%. Interest on the loan will be 0.0814($1 million)/4 = $20,350, and the gain on the futures position will be -10000(93.86-94.86)/4 = $2,500. This yields a net cash outflow of -

$20,350+$2,500 = -$17,850, which is the same as 3-month's interest on $1 million at 7.14%.

Now suppose that on December 16 LIBOR is 4.14%. Interest on the loan will be 0.0614($1 million)/4 = $15,350, and the gain on the futures position will be -10000(95.86-94.86)/4 = -$2,500. This yields a net cash outflow of -$15,350-$2,500 = -$17,850, which is the same as 3-month's interest on $1 million at 7.14%. Hence, your interest rate exposure is hedged.

9.11.2 Hedging Market Risk

Another source of risk that an individual or organization may wish to hedge is stock market risk. For example, a person nearing retirement may wish to hedge the value of the equities component of his retirement fund against a stock market crash before he retires. A fund manager, who believes he can pick winners among individual stocks, may wish to hedge market-wide movements. Example 16 contains a detailed illustration of how the S&P 500 futures contract can be used to hedge stock market risk.

Example 15: Hedging with the S&P 500 Futures Contract.

A portfolio manager holds a portfolio that mimics the S&P 500 index. The S&P 500 index started the year at 306.8 and is currently at 382.62. The December S&P 500 futures price is currently 383.50. The manager's fund was valued at $76.7 million at the beginning of the year. Since the fund has already generated a handsome return for the year, the manager wishes to lock in its current value. That is, he is willing to give up potential increases in order to ensure that the value of the fund does not decrease. How does he lock in the current value of the fund?

First note that at the December futures price of 383.50, the return on the index, since the beginning of the year, is 383.5/306.80-1 = 25%. If the manager is able to lock in this return on his fund, the value of the fund will be *1.25($76.7 million) =*

$95.875 million. Since the notional amount underlying an S&P 500 futures contract is *500(383.50) = $191,750*, the manager can lock in the 25% return by selling 95,875,000/191,750=500 contracts. To illustrate that this position does indeed form a perfect hedge, we examine the net value of the hedged position under two scenarios.

First, suppose the value of the S&P 500 index is 303.50 at the end of December. In this case, the value of the fund will be *(303.50/383.50)95.875 million = 75.875 million*. The gain on the futures position will be *-500(500)(303.50-383.50) = 20 million.* Hence the total value of the hedged position is 75.875+ 20 = 95.875 million, locking in a 25% return for the year.

Now suppose that the value of the S&P 500 index is 403.50 at the end of December. In this case, the value of the fund will be *403.50/383.50(95.875 million) = 100.875 million.* The gain on the futures position will be *-$500(500)(403.50-383.50) = -5 million.* Hence the total value of the hedged position is 100.875-5=95.875 million, again locking in a 25% return for the year.

9.11.3 Hedging Foreign Exchange Risk

Another source of risk that an individual or organization may wish to hedge is foreign exchange risk. For example, a person who will be travelling overseas in the coming months may wish to hedge the value of the amount of money he intends to spend abroad against a devaluation of his domestic currency relative to the foreign currency. An exporter who sells goods overseas on credit may wish to hedge against a devaluation of the foreign currency in which payment occurs. Example 17 contains a detailed illustration of hedging exchange risk.

Example 16: Hedging with the Swiss Franc Futures Contract.

Your company sells 10 machines to a Swiss company. The sale price is 100,000 Swiss Francs each and payment is to be made at the end of the calendar year. The December futures price for Swiss Francs is 0.6915. You are worried that the Swiss Franc will depreciate against the US Dollar between now and the end of the year. How can you hedge this exchange rate risk?

Note that since (1) the total exposure is one million Swiss Francs and (2) each futures contract is for 125, 000 Francs, eight contracts are required to hedge the exposure. Further, since (1) the company stands to lose if the Swiss Franc depreciates (each Swiss Franc can be converted back into a smaller number of Dollars) and (2) the futures contracts decrease in value if the Swiss Franc depreciates (since the basis of the contract is Swiss Francs per Dollar), the contracts should be **sold**.

To illustrate that selling eight futures contracts provides an adequate hedge, first suppose that the value of the Swiss Franc is 0.30 at the end of December. In this case, the US Dollar value of the payment for the machines will be 0.30(10)(100,000) = $300,000. The gain on the futures position will be -8(125,000)(0.30-0.6915) = $391,500. Hence the total income is $691,500, which equals the unhedged income in dollars if the exchange rate does not fluctuate.

9.12 Basis Risk

There is no such thing as a perfect hedge. You can never completely eliminate a cash position's risk. So far we have assumed that there is always a contract that matches the source of risk exactly, so we can construct perfect hedges. However, there may be instances where a contract that matches our needs (in terms of currency, maturity, denomination, etc.) does not exist, and we have to use a contract that is close to the one we ideally desire. Then we incur a residual risk which is called "basis risk." There are many reasons for basis risk:

- First, the good or instrument being hedged may be different from the good or instrument for which there is a futures contract. This would be the case if a corporate bond offering is hedged with Treasury bond futures; basis risk arises due to the uncertainty of the yield differential between corporates and treasuries at the time the hedge is lifted.
- Second, in commodity futures, there is basis risk due to locational differentials. For example, a cattle farmer in Texas who hedges with a cattle futures contract that calls for delivery in Omaha has the uncertainty of the closeout differential between the Texas steer price and the Omaha steer price. This is called *locational basis risk*. This is usually an important factor in agricultural contracts. The risk is compounded by the fact that the seller usually has the option of where delivery is made.
- The third type of basis risk arises because the seller of the futures contract often has the option to choose the *quality* of the goods or financial instrument delivered. For example, the Treasury bond futures market calls for delivery of any U.S. Treasury bond that is not callable within 15 years. Since there are many instruments that are candidates for delivery, the hedge has the risk of fluctuations in the yield spread between the instrument hedged and the instrument ultimately delivered.
- Fourthly, with most futures contracts, the seller has the choice of the date of delivery within the delivery month. This choice is an uncertain value and thus contributes to basis risk.
- Finally, the mark to market aspect of futures results in hedging risk. The uncertainty is about the amount of interest earned or forfeited due to the daily transfers of profits and losses. In fact, the equations for net revenue are not exactly right due to the omission of interest earned (lost) on futures profits (losses).

9.13 The Volatility of Futures

A common mistake made is to assume that futures are much more volatile than stocks. Percentage changes of futures prices are generally less volatile than the percentage changes of a typical stock. Annualized standard deviations for most futures contracts are in the 15-20% range whereas a typical stock's is about 30%.

There is no reason that the futures should be played in a high risk manner by a large investor. Of course, if the futures investor does not have enough capital (5-8 times margin), then he is required to play with considerable leverage or not at all. Before taking great leverage, the small investor should consider looking at a smaller contract (grain on CBT is 5,000 bushels whereas Mid-America contract is 1,000 bushels).

The effect of leverage is to increase volatility. Borrowing to meet the margin requirements will increase gains but also increase losses. Setting aside larger amounts of capital which are invested in a safe asset will decrease the volatility.

9.14 Risk in the Futures Markets

As we have already seen, one the most important applications of the futures is for hedging. Futures contracts were initially introduced to help farmers that did not want to bear the risk of price fluctuations. The farmer could *short hedge* in March (agree to sell his crop) for a September delivery. This effectively locks in the price that the farmer receives. On the other side, a cereal company may want to guarantee in March the price that it will pay for grain in September. The cereal company will enter into a *long hedge*.

There are a number of important insights that should be reviewed. The first is that we should be careful about what we consider the *investment* in a futures contract. It is unlikely that the margin is the investment for most traders. It is rare that somebody plays the futures with a total equity equal to the margin. It is more common to invest some of your capital in a money market fund and draw money out of that account as you need it for margin and add to that account as you gain on the futures contract. It is also uncommon to put the full value of the underlying contract in the money market fund. It is more likely that the futures investor will put a portion of the value of the futures contract into a money market fund. The ratio of the value of the underlying contract to the equity invested in the money market fund is known as the leverage. The leverage is a key determinant of both the return on investment and on the volatility of the investment. The higher the leverage -- the more volatile are the returns on your portfolio of money market funds and futures. The most extreme leverage is to include no money in the money market fund -- only commit your margin.

The second important insight had to do with hedging with futures contracts. The concept of *basis risk* was introduced. It is extremely unlikely that you can create a perfect hedge. With a perfect hedge the loss on your cash position is exactly offset by the gain in the futures position. We suggested some reasons why it is unlikely that we can construct a perfect hedge.

- The most obvious case is when you are trying to hedge a cash position with futures positions in different instruments. This is the case that we introduced in one of the first lectures when we hold the Ginnie Mae security and want to hedge this security with a combination of T-Bonds and Euros. It is unlikely, however, that at the expiration of the futures contract, the cash price of the T-Bond and Euros will equal the Ginnie Mae. This is the basis risk.
- A second type of basis risk arises out of the *quality option*. If you are a farmer and want to lock in the price for your crop of wheat, you may use a futures contract that may call for delivery of a number of different types of wheat. Similarly, in the T-

Bond and T-Note contracts, there is a whole range of instruments that are available for delivery. This difference will induce basis risk.

- Third, there is a timing option. The futures contract is different from an options contract. Most futures call for delivery within the contract month. It is unclear when the short will deliver the goods. This uncertainty leads to basis risk.
- The fourth type of risk is locational basis risk. This is mainly applicable to agricultural commodities. There could be a difference the cash price of the good that you are selling (cattle) and the futures price at a different location.
- The last type of uncertainty is linked to the uncertain interest rate flows from the money you make in excess of the margin.

7.15 Potential sources of confusion

There is some common confusion about how the different arguments regarding futures are related. It is important to understand that there are three aspects: 1. Valuation by arbitrage, 2. Construction of arbitrage portfolios and 3. Hedging. These are all related, but they are also quite distinct aspects of securities:

- Valuation by arbitrage. Here we use (1) the fact that a derivative security can be replicated by a portfolio of other securities, the price of which we know and (2) the assumption that there are no arbitrage opportunities. The objective is to find the correct price.
- Construction of arbitrage portfolios. Here we use (1) the fact that we can replicate a derivative with a portfolio of securities and (2) compare the correct price we determined in 1. with the actual market price, but we do *not* assume that no arbitrage opportunities exist. If the correct price differs from the market price, an arbitrage opportunity exists. Then the objective is to construct an arbitrage portfolio that allows us to benefit from mispricing. This portfolio consists of (A) a long position in the derivative and a short position in the replicating portfolio if the derivative is undervalued (correct price larger than market price), (B) a short position in the

derivative and a long position in the replicating portfolio if the derivative is overvalued.

- Hedging. Hedging is the main use of derivative securities. For hedging we assume that derivatives are correctly priced and no arbitrage opportunities exist. The purpose is to find a balancing position against risky positions you already have: e.g., if you receive foreign currency in the future, you sell it forward, if you have to pay in foreign currency, you buy it forward. Since the derivatives used for hedging can be replicated using the portfolios mentioned under 1. and 2. above, there is always an alternative strategy to hedge by using the replicating portfolio rather than the derivative itself.

For all three aspects of derivatives it is important to understand the concept of a replicating portfolio, but the status of this portfolio is different in each case. For valuation, you use the market price of the replicating portfolio in order to infer the correct price of the derivative. If you wish to construct arbitrage portfolios you need to know the replicating portfolio, and for hedging it may be useful to know the replicating portfolio in order to construct hedges without using derivatives.

Summary of Important Formulas

$$F = S_0 e^{(r+q)T} \quad (2)$$

The price of a forward contract when there is a cost of carry *q*. When interest rates are constant, the same relationship holds for a futures contract.

$$F = S_0 e^{(r-d)T} \quad (3)$$

The price of a forward contract when there is a dividend benefit *d*. When interest rates are constant, the same relationship holds for a futures contract.