TEACHING NOTE 97-11:

AN OVERVIEW OF OPTION TRADING STRATEGIES: PART II

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This teaching note provides an overview of several advanced option trading strategies, covering bull spreads, bear spreads, collars, butterfly spreads, calendar spreads and straddles. It builds on Teaching Note 97-10, which covers the basic option strategies.

Let us re-state the terms: A(0) = price of underlying asset today, X = exercise price of option, T = expiration date of option. We assume no dividends are paid or costs incurred on holding the underlying asset. Consequently, A(T) = asset price at option expiration and T - 0 = T = time to expiration. We shall work with European options. At any time t, the call price is c(A(t),X,T-t) and the put price is p(A(t),X,T-t). Given values for the risk-free rate (r) and volatility (σ), the option prices are generally provided by the Black-Scholes formula. At expiration, c(A(T),X,0) = Max(0,A(T) - X) and p(A(T),X,0) = Max(0,X - A(T)). With one exception, in what follows we shall examine the values of various option strategies at the end of two holding periods, the first where we close the option position prior to expiration and the second where we hold the position all the way to the option's expiration.

First let us consider what is meant by a spread strategy. A spread is an option transaction in which the investor is long one option and short another. The two options are alike in all respects except one, either the exercise price or the time to expiration. The former are called *money spreads* or *strike spreads.*

In analyzing money spreads we must introduce more than one exercise price. We do so by denoting the exercise prices as X_1, X_2 and for butterfly spreads, we add X_3. The subscript is a reminder of the relationship between the exercise prices: $X_1 < X_2 < X_3$. Consequently, the prices of these calls are denoted as c(A(t),X_i,T-t) for i = 1, 2, or 3 and a similar construction is used for puts.

We shall derive formulas for the value of the position and illustrate the results graphically for a range of possible asset prices at the end of the holding period. For the numerical examples, we assume the following values: A(0) = 100, r = 5.5 %, and σ = .35. We let the option have 90 days to expiration at the start. When we close the position early, we do so 20 days later, that is, with the option having 70 days remaining until expiration. There is an exception for the case of calendar spreads. For the graphs we analyze the position value for asset values when the position is closed ranging from 75 to 125.

Bull Spread

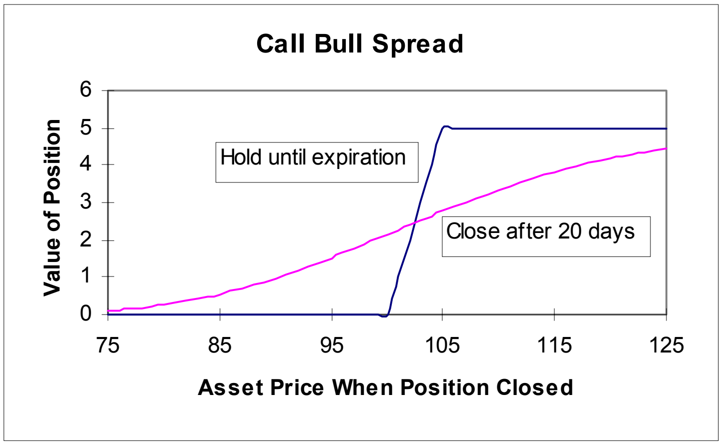
This transaction involves a long position in the call with exercise price X_1 and a short position in the call with exercise price X_2. Its value today is $c(A(0),X_1,T) - c(A(0),X_2,T)$. We know that of two calls differing only by exercise price, the one with the lower exercise price will have the higher price, with the exception of when we are at the expiration and both options are out-of-the-money. Consequently, $c(A(0),X_1,T) - c(A(0),X_2,T) > 0$, meaning simply that establishment of the position requires a net investment of funds, as opposed to a net inflow of funds, which would be the case if the position were reversed. A bull spread is done in anticipation of a rising market, hence the name *bull spread*. It has less risk than just a long position in the X_1 call option or just a short position in the X_2 call option because the other position provides some protection.

The value of the position when the transaction is terminated at time $t \leq T$ is given as

$$V(t) = c(A(t), X_1, T-t) - c(A(t), X_2, T-t) \quad \text{(bull spread with calls)}$$

where these values are provided by the Black-Scholes formula for any chosen value of A(t). Note also that the remaining time to expiration is T-t. When t = T, we still have $V(T) = c(A(T),X_1,0) - c(A(T),X_2,0)$. This value is, therefore, $\text{Max}(0,A(T) - X_1) - \text{Max}(0,A(T) - X_2)$. Consequently, for $A(T) < X_1$, $V(T) = 0 - 0 = 0$, for $X_1 \leq A(T) < X_2$, $V(T) = A(T) - X_1$, and for $A(T) \geq X_2$, $V(T) = A(T) - X_1 - (A(T) - X_2) = X_2 - X_1$. In the first case, both options end up out-of-the-money, in the second case, the long option ends up in-the-money and the short option ends up out-of-the-money and in the third case, both options end up in-the-money.

The figure below illustrates this result for the example chosen with $X_1 = 100$ and $X_2 = 105$.



For a given asset price below the exercise price, the value is lower at expiration and for a given asset price above the exercise price, the value is higher at expiration. To see this, consider this point. An option's time value is greater, the closer the asset price is to the exercise price. Prior to expiration with the asset price below (above) X_1 = 100 (X_2 = 105), the time value is greater on the long (short) call. Over the remaining life of the options, each will lose all of its time value and end up worth its exercise value. If the asset value remains below (above) the exercise price of 100 (105), the long (short) call will have more time value to lose. This hurts (benefits) the call bull spread holder by causing the long (short) position to lose more time value than the short (long) position.

If one knew that the asset price would not change, it would be best to close the position immediately if the asset price is below the lower exercise price and hold the position if the asset price is above the upper exercise price, but of course one does not know that the asset price will not move. The longer the position is held the more time there is for a large asset price move.

The call with exercise price of 100 would cost \$7.57 when purchased and the call with exercise of 105 would produce \$5.40 when sold. Thus, the net cost of the bull spread would be \$2.17. It would be unprofitable if the spread value when closed were less than this amount. As is apparent, the minimum value is zero while the maximum value is the difference - *the spread* - between the exercise prices, \$5.

If a person reversed the transaction, it would be called a bear spread. Such a position would be done in anticipation of a down market. Bear spreads, however, are more often done with puts.

Bear Spread

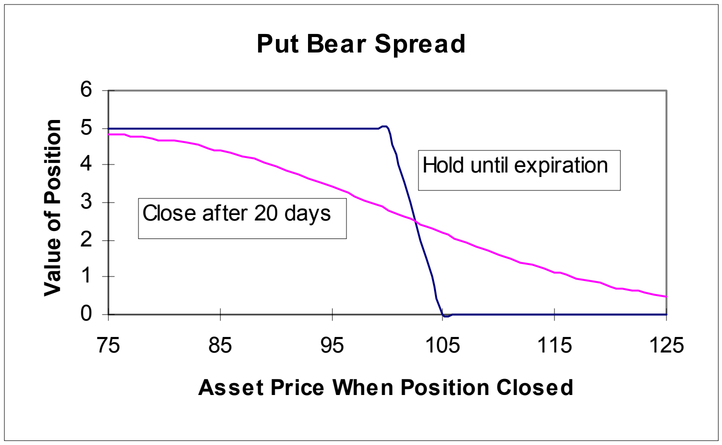
This transaction involves a long position in the put with exercise price X_2 and a short position in the put with exercise price X_1. Its value today is $p(A(0),X_2,T) - p(A(0),X_1,T)$. We know that of two puts differing only by exercise price, the one with the higher exercise price will have the higher price, with the exception of when we are at the expiration and both options are out-of-the-money. Consequently, $p(A(0),X_2,T) - p(A(0),X_1,T) > 0$, meaning simply that establishment of the position requires a net investment of funds, as opposed to a net inflow of funds, which would be the case if the position were reversed. A bear spread is done in anticipation of a falling market, hence the name *bear spread*. It has less risk than just a long position in the X_2 put option or just a short position in the X_1 put option because the other position provides some protection.

The value of the position when the transaction is terminated at time $t \leq T$ is given as

$$V(t) = p(A(t), X_2, T - t) - p(A(t), X_1, T - t) \quad \text{(bear spread with puts)}$$

where these values are provided by the Black-Scholes formula for any chosen value of A(t). Note also that the remaining time to expiration is T-t. When t = T, we still have V(T) = $p(A(T),X_2,0) - p(A(T),X_1,0)$. This value is, therefore, $Max(0,X_2 - A(T)) - Max(0,X_1 - A(T))$. Consequently, for $A(T) < X_1$, $V(T) = X_2 - A(T) - (X_1 - A(T)) = X_2 - X_1$, for $X_1 \leq A(T) < X_2$, $V(T) = X_2 - A(T)$, and for $A(T) \geq X_2$, $V(T) = 0 - 0 = 0$. In the first case, both options end up in-the-money, in the second case, the long option ends up in-the-money and the short option ends up out-of-the-money and in the third case, both options end up out-of-the-money.

The figure below illustrates this result for the example chosen with $X_1 = 100$ and $X_2 = 105$.



For a given asset price below the exercise price, the value is higher at expiration and for a given asset price above the exercise price, the value is lower at expiration. Recall the point that an option's time value is greater the closer the asset price is to the exercise price. Prior to expiration with the asset price below (above) X_1 (X_2), the time value is greater on the short (long) put. Over the remaining life of the options each option will lose all of its time value and end up worth its exercise value. If the asset value remains below (above) the exercise price of 100 (105), the short (long) put will have more time value to lose. This helps (hurts) the put bear spread holder by causing the short (long) position to lose more time value than the long (short) position.

If one knew that the asset price would not change, it would be best to close the position immediately if the asset price is above the lower exercise price and hold the position if the asset price is below the upper exercise price, but of course one does not know that the asset price will not move. The longer the position is held the more time there is for a large asset price move.

The put with exercise price of 105 would cost $8.99 when purchased and the put with exercise of 100 would produce $6.23 when sold. Thus, the net cost of the bear spread would be $2.76. It would be unprofitable if the spread value when closed were less than this amount. As is apparent, the minimum value is zero while the maximum value is the spread between the exercise prices, $5.

If a person reversed the transaction, it would be called a bull spread. Such a position would be done in anticipation of an up market. Bull spreads, however, are more often done with calls.

Collar

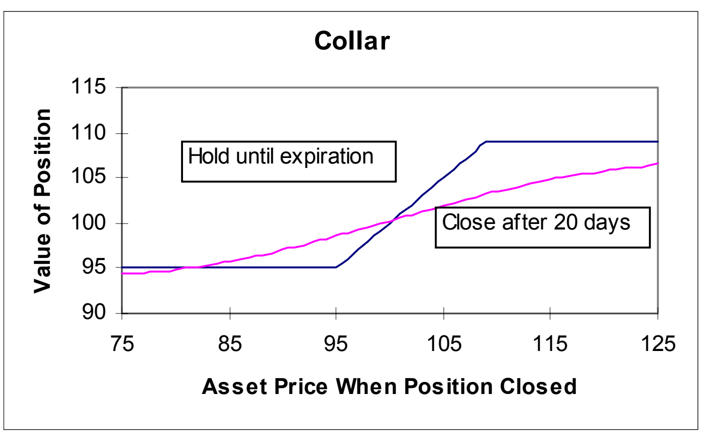
This transaction involves long positions in the underlying asset and the put with exercise price $X_1 < A(0)$ and a short position in the call with exercise price $X_2 > A(0)$. A collar is similar to a protective put in that the underlying asset as well as a put option are purchased; however, a call is also sold to help compensate for the cost of the put. Even though we can choose any call with an exercise price higher than that of the underlying asset's value, we usually create a zero–cost collar where the premium for the call offsets the premium for the put. The collar's value today is $A(0) + p(A(0),X_1,T) – c(A(0),X_2,T)$.

. The value of the position when the transaction is terminates at time $t \leq T$ is given as

$$V(t) = A(t) + p(A(t),X_1,T - t) – c(A(t),X_2,T - t) \text{ (collar)}$$

where these values are provided by the Black–Scholes formula for any chosen value of A(t). Note also that the remaining time to expiration is T–t. When t = T, we have $V(T) = A(T) + p(A(T),X_1,0) – c(A(T),X_2,0)$. This value is, therefore, $A(T) + Max(0, X_1 – A(T)) – Max (0, A(T) – X_2)$. Consequently, for $A(T) < X_1$, $V(T) = A(T) + X_1 – A(T) – 0 = X_1$, for $X_1 < A(T) < X_2$, $A(T) + 0 – 0 = A(T)$, and for $A(T) \geq X_2 = A(T) + 0 – (A(T) – X_2) = X_2$. In the first case, the call is out–of–the–money and the put is in–the–money. In the second case, both options are out-of-the- money and in the third case the call is in–the–money but the put is out–of–the–money.

The figure below illustrates this result for the example chosen with X_1 = 95 and X_2 = 109.03.



For a given asset price below the exercise price, the value of the collar is usually lower at expiration, and for a given asset price above the exercise price, the value is higher at expiration. Also remember that an option's time value is greater the closer the asset price is to the exercise price.[1]

If one knew that the asset price would not change, it would be best to close the position immediately if the asset price is below the lower exercise price and hold the position if the asset price is above the upper exercise price, but of course one does not know that the asset price will not move.[2] The longer the position is held the more time there is for a large asset price move.

The put with exercise price of 95 would cost \$4.03 when purchased, the underlying asset would cost \$100 when purchased and the call with exercise price \$109.03 would produce \$4.03 when sold. Thus the net cost of the collar would be \$100. It would be unprofitable if the spread value when closed were less than this amount.

Looking back to the bull spread, one should immediately note the similarities between the two strategies by applying put–call parity: a long put plus the underlying asset equals a long call and risk free bond paying the exercise price at expiration. When comparing the two strategies one would find that the collar is equivalent to the bull spread plus a risk–free bond paying X_1 at expiration.

Butterfly Spread

This transaction involves long positions in one call with exercise price X_1 and one call with exercise price X_3 and two short positions in the call with exercise price X_2. Its value today is $c(A(0),X_1,T) - 2c(A(0),X_2,T) + c(A(0),X_3,T)$. We know that of two calls differing only by exercise price, the one with the lower exercise price will have the higher price, with the exception of when we are at the expiration and both options are out-of-the-money. Consequently, $c(A(0),X_1,T) - c(A(0),X_2,T) > 0$, and $-c(A(0),X_2,T) + c(A(0),X_3,T) < 0$. Note that the former combination, long the X_1 call and short the X_2 call, is a bull spread and the latter combination, short the X_2 call and long the X_3 call is a bear spread. The former difference, $c(A(0),X_1,T)\ \ -c(A(0),X_2,T)$, is greater in an absolute sense than the latter, $-c(A(0),X_2,T)$ +

[1] At certain low exercise prices, however, a European put rises in value as expiration approaches, and consequently, we see a slight tendency for the spread value to increase as time passes. For most of the range of asset prices below X_1, the collar decreases in value as expiration approaches.

[2] An exception would be at the lowest asset prices, as explained in the previous footnote.

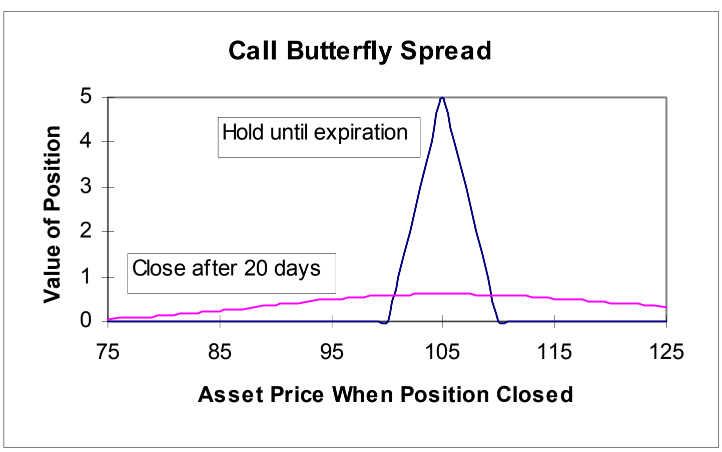
$c(A(0),X_3,T)$, because the advantage of the lower exercise price call over the higher exercise price call is reduced, the higher are the exercise prices. This is because in such a case, the probability of both calls expiring out-of-the-money is greater. Consequently, $c(A(0),X_1,T) - 2c(A(0),X_2,T) + c(A(0),X_3,T) > 0$, meaning that the establishment of the position requires a net investment of funds, as opposed to a net inflow of funds, which would be the case if the position were reversed.

The value of the position when the transaction is terminated at time $t \leq T$ is given as

$$V(t) = c(A(t), X_1, T - t) - 2c(A(t), X_2, T - t) + c(A(t), X_3, T - t) \quad \text{(butterfly spread with calls)}$$

where these values are provided by the Black-Scholes formula for any chosen value of A(t). Note also that the remaining time to expiration is T-t. When t = T, we have $V(T) = c(A(T),X_1,0) - 2c(A(T),X_2,0) + c(A(T),X_3,0)$. This value is, therefore, $\text{Max}(0,A(T) - X_1) - 2\text{Max}(0,A(T) - X_2) + \text{Max}(0,A(T) - X_3)$. Consequently, for $A(T) < X_1$, $V(T) = 0 - 2(0) - 0 = 0$, for $X_1 \leq A(T) < X_2$, $V(T) = A(T) - X_1$, for $X_2 \leq A(T) < X_3$, $V(T) = A(T) - X_1 - 2(A(T) - X_2) = -A(T) + X_2 - X_1$, and for $A(T) \geq X_3$, $V(T) = A(T) - X_1 - 2(A(T) - X_2) + A(T) - X_3 = -X_3 + 2X_2 - X_1$. If the exercise prices are equally spaced, as they usually are, the latter result is simply zero. In the first outcome, both options end up out-of-the-money, in the second case, one long option ends up in-the-money, in the third case, one long option and two short options end up in-the-money, and in the fourth case, all options end up in-the-money.

The figure below illustrates this result for the example chosen with $X_1 = 100$, $X_2 = 105$, and $X_3 = 110$.



Using the same reasoning we previously employed regarding time value decay, we see that gains are greatest if the asset prices stays near the exercise price of the two short options. If the asset price moves beyond the upper (lower) exercise price and stays there, the long options have greater time value to lose.

If one knew that the asset price would not change, it would be best to close the position immediately if the asset price is near the upper or lower exercise price and hold the position if the asset price is around the middle exercise price, but of course one does not know that the asset price will not move. The longer the position is held the more time there is for a large asset price move.

The call with exercise price of 100 would cost \$7.57 when purchased and the call with exercise of 105 would produce \$5.40 when sold. The call with exercise price of 110 would cost \$3.74 when purchased. Thus, the butterfly spread would cost \$7.57 - 2(\$5.40) + \$3.74 = \$0.51. It would be unprofitable if the spread value when closed were less than this amount. As is apparent, the minimum value is zero while the maximum value is the difference between either pair of exercise prices, \$5.

A person could reverse the transaction, which would invert the graph. Also, it could be done with puts with outcomes nearly identical.

Calendar Spreads

Calendar spreads, also called *time spreads*, are constructed by taking a long position in an option with one expiration and a short position in an otherwise identical option with a different expiration. Using a common exercise price, X, let us denote the expiration dates as T_1 and T_2 where $T_2 > T_1$. Analysis of a calendar spread must proceed in a slightly different manner than for a money spread. Because the options expire at different times, it is impossible to speak in terms of holding a position until the options expire. If one holds the position past the expiration of the shorter-maturity option, the position turns into either a standard long or short position in a single option. We typically analyze the calendar spread by examining its value before the first option expires, and then at the expiration of the first option.

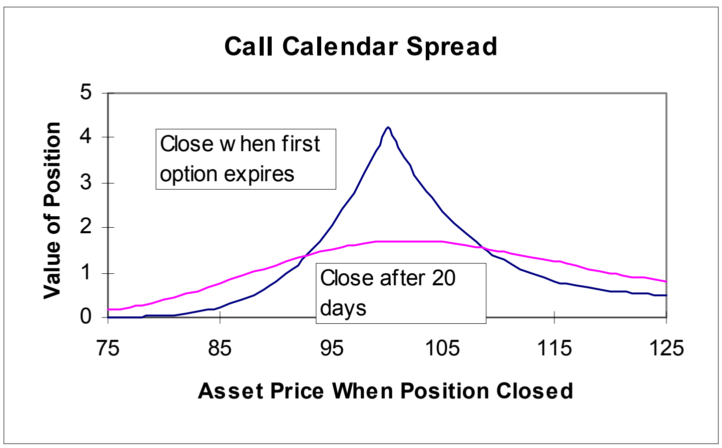
Let us define our calendar spread as being long the option with the longer expiration and short the option with the shorter expiration. The calendar spread's value today is $c(A(0),X,T_2) - c(A(0),X,T_1)$. We know that of two calls differing only by expiration, the one with the longer time to expiration will have the higher price. Consequently, $c(A(0),X,T_2) - c(A(0),X,T_1) > 0$, meaning that the position requires the outlay of funds.

The value of the position when the transaction is terminated at time $t \leq T$ is given as

$$V(t) = c(A(t), X, T_2 - t) - c(A(t), X, T_1 - t) \quad \text{(calendar spread with calls)}$$

where these values are provided by the Black-Scholes formula for any chosen value of A(t). Note also that the remaining time to expiration is T_2-t for the long option and T_1 - t for the short option. When $t = T_1$, we have $V(T_1) = c(A(T_1),X,T_2-T_1) - c(A(T_1),X,0)$. This value is, therefore, $c(A(T_1),X,T_2-T_1) - Max(0,A(T_1) - X)$. Again, the former term must be obtained using the Black-Scholes model.

Let us illustrate the calendar spread with two call options with an exercise price of \$100 but where the longer term option has an expiration of 90 days and the shorter term option has an expiration of 60 days. The figure below illustrates this result for the example chosen.



Both options have the same exercise value, so the performance of the strategy is strictly determined by the different rates of time value decay on the two options. Both option are losing time value as one moves forward in time but the option expiring earlier must lose it faster. Consequently, when the asset price stays around the exercise price, the shorter-term option, which you are short, loses its time value faster than the longer term option, which you are long. The opposite occurs when the asset price makes a large move up or down.

If one knew that the asset price would not change, it would be best to close the position immediately if the asset price is away from the exercise price and hold the position if the asset price is around the exercise price. The longer the position is held the more time there is for a large asset price move.

The call with 90 days to go would cost \$7.57 when purchased and the call with 60 days to go would produce \$6.09 when sold. Thus, the calendar spread would cost \$7.57 - \$6.09 = \$1.48. It would be unprofitable if the spread value when closed were less than this amount. As is apparent, the minimum value is zero. The maximum could be computed from the Black-Scholes model, where it would be the value of the longer-term call with 30 days to go with the asset price equal to the exercise price, which in this case is \$4.22.

Clearly the calendar spread where you are long the longer-term option and short the shorter-term option is done in anticipation of lower than expected volatility. A person could reverse the transaction, which would invert the graph and would be done in anticipation of higher than expected volatility. Also, the calendar spread could be done with puts with similar implications.

Straddle

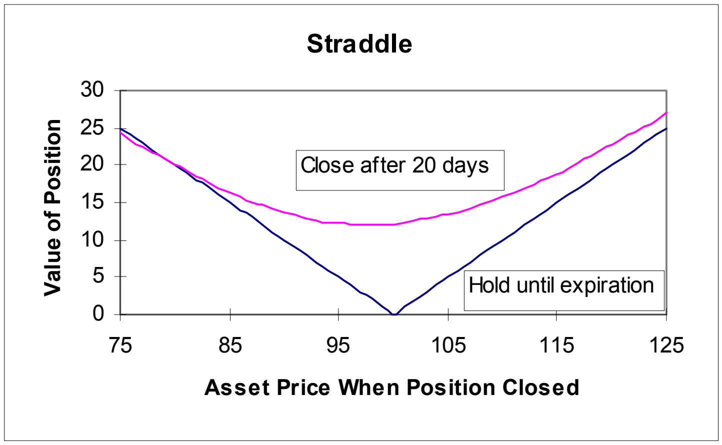
This transaction involves a long position in the call with exercise price X and a long position in the put with the same exercise price and expiration. Its value today is c(A(0),X,T) + p(A(0),X,T).

The value of the position when the transaction is terminated at time $t \leq T$ is given as

$$V(t) = c(A(t), X, T - t) + p(A(t), X, T - t) \quad \text{(straddle)}$$

where these values are provided by the Black-Scholes formula for any chosen value of A(t). Note also that the remaining time to expiration is T-t. When t = T, we have V(T) = c(A(T),X,0) + p(A(T),X,0). This value is, therefore, Max(0,A(T) - X) + Max(0,X - A(T). Consequently, for A(T) < X, V(T) = X - A(T), for A(T) ≥ X, V(T) = A(T) - X. In the first case, the put ends up in-the-money and the call out-of-the-money and in the second case, the call ends up in-the-money and the put out-of-the-money.

The figure below illustrates this result for the example chosen.



For a given asset price, being long both a call and a put, the value declines as you move through time. This is due to the time value decay on both the call and the put. If an investor knew that the asset price would not move, he should close the position immediately. Naturally, however, the longer the position is held, the greater the chance of a large price move.[3]

The call would cost $7.57 when purchased and the put would cost $6.23 when purchased. Thus, the net cost of the straddle would be $13.80. It would be unprofitable if the spread value when closed were less than this amount. As is apparent, the minimum value is zero while the maximum value is unlimited.

If a person reversed the transaction, it would be called a short straddle and would have a limited gain, occurring if the asset price were near the exercise price, and an unlimited loss, due to the unlimited loss potential on the short call. Straddles are sometimes modified by adding a single call or put, the former strategy called a *strap* and the latter called a *strip*. A strap increases the gains if the market goes up, but of course costs the premium on the additional call. A strip increases the gains if the market goes down, but costs the premium on the additional put.

References

The following books devote extensive material to option trading strategies:

Chance, D. M. *An Introduction to Derivatives and Risk Management*, 5th ed. Fort Worth: Harcourt (2001), Chs. 6, 7.

[3] As noted in an earlier footnote, for a European put, the value can increase as time passes for extremely low asset prices.

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