TEACHING NOTE 97-06:
PRICING AND VALUATION OF INTEREST RATE AND CURRENCY SWAPS

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Pricing a swap means to determine the terms of the swap at the start of the transaction. In the case of a plain vanilla (fixed-for-floating) interest rate swap, pricing means to determine the fixed rate that will be exchanged for the floating rate. The fixed rate is determined as the rate that establishes the present value of the fixed payments as equal to the present value of the floating payments. In the case of a currency swap, in which either or both sides can pay fixed or floating, pricing the swap establishes the notional principals in the two currencies. In all cases, pricing a swap is accomplished by identifying the terms that make the value of the swap zero to both parties at the start.

This teaching note uses an example to illustrate how interest rate and currency swap pricing is accomplished. We assume that the term structure of spot interest rates is given and we let the notional principal for the dollar interest rate swap be $1.[1] There is no need for a complete pricing model, such as Cox-Ingersoll-Ross or Heath-Jarrow-Morton, as swap pricing is somewhat analogous to futures pricing. Such models are, of course, necessary in pricing interest rate options.

Another procedure related to swap pricing is valuation. Normally one thinks of pricing and valuation as the same process, at least in an efficient market. In the case of swaps and forward contracts, however, pricing means to determine the terms of the transaction at the start. Valuation means to determine the market value of the transaction at a time later during its life. Pricing is, therefore, simply valuation at the start of the contract when the value is forced to zero, and the terms consistent with zero value are then inferred. The last section of this note covers the valuation of a swap.

Let us begin with notation. In order to handle currency swaps, we shall price interest rate swaps in two countries, which we choose to be the U. S. and Switzerland. Let $r_{US}(0,1)$, $r_{US}(0,2)$,

[1]Swap payoffs are linearly homogeneous with respect to the notional principal so the results for any notional principal can be obtained from these results by multiplying by the appropriate notional principal.

...$r_{US}(0,n)$ be the term structure of interest rates in the U. S. Let $r_{SW}(0,1)$, $r_{SW}(0,2)$, ..., $r_{SW}(0,n)$ be the term structure of interest rates in Switzerland. These are LIBOR-style rates, meaning that when interest is paid it is added on to the principal and adjusted by multiplying by days/360 where days is the actual day count over the period or assuming 30 days in a month, whichever method the counterparties to the swap decide on. For example, if $1 is put in a LIBOR time deposit for d days at the rate r, the deposit grows to a value of $1[1 + r(d/360)].

We can simplify the notation for swap pricing considerably by working with present value factors. By definition, a present value factor is the price of a zero coupon bond with $1 face value. If that bond pays interest in the LIBOR manner, its price is found as $B_{US}(0,i) = 1/[1 + r_{US}(0,i)i/360]$. A corresponding specification holds for Switzerland. By definition the forward rates are given as $r_{US}(0,a,b) = \{[1 + r_{US}(0,b)b/360]/[1 + r_{US}(0,a)a/360] - 1\}(360/(b-a)) - 1$ for the U. S. and a corresponding formula for Switzerland, where the 0,a,b arguments indicate that the rate is observed at time 0 for the period beginning at time a and ending at time b. If a = 0 then the rate r(0,0,b) is expressed as r(0,b), which is a spot rate. The Appendix contains numerical examples that will be used in illustrating the results demonstrated herein.

In the material that follows, we shall use these discount factors/zero coupon bond prices to obtain present values. The swap payments themselves are determined by multiplying the notional principal times the rate times days/360; however, for purposes of simplifying the notation, we shall assume a $1 (or SF1) notional principal and leave off the days/360 adjustment. For the numerical examples in the appendix, we shall make the days/360 adjustment.

Pricing Plain Vanilla Interest Rate Swaps

Consider a swap of maturity n, which means that there are n specified payments. One party pays a floating rate and the other pays a fixed rate, which we designate as f. Pricing the swap means to find the fixed rate such that the present value of the fixed payments equals the present value of the floating payments. Clearly the challenge is to find the present value of the floating payments. At first glance this may appear to be a formidable task but in comparison to the difficulty of pricing a stock, which has unknown floating payments, a swap is a remarkably simple instrument whose pricing is aided by the existence of a term structure of interest rates. For the first floating payment, we use the spot rate r(0,1), which will be the first payment, and for the remaining floating payments we use the one-period forward rates. Thus, the second payment is currently set at r(0,1,2), the third at r(0,2,3). *This procedure in no way invokes the expectations theory. These forward rates are not expectations of future spot rates.* Rather, these rates are arbitrage-free values. A party making floating payments can hedge the risk by substituting forward contracts that lock in the commitment to pay the forward rates currently embedded in the term structure.

At time 0 the present value of the floating payments is

$$V_{FL}(0) = \sum_{i=1}^{n} r(0,i-1,i)B(0,i).$$

Thus, the first payment is valued at r(0,1)B(0,1), the second at r(0,1,2)B(0,2), etc. We define the present value of the fixed payments as

$$V_{FX}(0) = \sum_{i=1}^{n} fB(0,i) = f\sum_{i=1}^{n} B(0,i).$$

Since f is the only unknown, the solution is easily found by setting $V_{FL}(0) = V_{FX}(0)$ and solving for f. Let us, however, obtain the value in a different manner. The floating payments in a swap can be viewed as a floating rate bond, which will naturally involve a principal repayment at time n. Likewise, the fixed payments in a swap can be viewed as a fixed rate bond, which also involves a principal repayment at time n. The principal repayments offset so the combination of issuing (buying) a fixed-rate bond and buying (issuing) a floating-rate bond replicates a pay-fixed (-floating), receive-floating (-fixed) swap. Adding the principals, however, facilitates pricing. The value of the floating-rate bond is, therefore,

$$V_{FLRB}(0) = \sum_{i=1}^{n} r(0,i-1,i)B(0,i) + B(0,n).$$

The value of the fixed-rate bond is

$$V_{FXRB}(0) = f\sum_{i=1}^{n} B(0,i) + B(0,n).$$

The value of a floating rate bond when initially offered equals par of 1.0, a result of the fact that the first coupon is set to equal the one-period market discount rate and of the assumption that the floating payment at each reset date will equal the then-current one-period discount rate.[2] Since the present value of the fixed payments must equal the present value of the floating payments, the swap fixed rate can be found as the solution, f, to

$$1.0 = f\sum_{i=1}^{n} B(0,i) + B(0,n),$$

which is an easily solvable problem whose solution is

$$f = \frac{1.0 - B(0,n)}{\sum_{i=1}^{n} B(0,i)},$$

and is equivalent to finding the fixed rate on a par value fixed rate bond. See the Appendix for calculations of the swap fixed rate for U. S. and Swiss markets.

Pricing Currency Swaps

Now let us price the four possible currency swaps. We can have fixed payments in dollars vs. fixed payments in Swiss francs, floating payments in dollars vs. floating payments in Swiss francs, and fixed payments in either and floating payments in the other. Let \$S(0) be the spot exchange rate, stated as dollars per Swiss franc. We assume a \$1 notional principal for the dollar payments. Nearly all currency swaps involve the exchange of notional principals at the beginning and at the end of the swap but we shall also evaluate currency swaps without the exchange of principals.

First consider the fixed-fixed swap assuming paying dollars and receiving Swiss francs. We know that by using the U.S. fixed rate, f_{US}, the present value of the fixed payments is \$1. The present value of the Swiss fixed payments is SF1. To equate these present value streams, we need only adjust the notional principal of the Swiss payments to \$1/\$S(0), a value we denote as N_{SW}. Thus,

[2]It is a simple matter to prove this point. Simply substitute the definition of the forward rate, in terms of the corresponding spot rates, for the appropriate floating payment. Cancellation of appropriate terms leaves a value of 1.0.

the initial exchange of cash in the two currencies has equivalent value at the start. See the Appendix for calculations.

For the floating-floating swap the present value of the dollar payments is, again, \$1. The present value of the Swiss franc payments is, again, SF1. Again, by adjusting the Swiss notional principal to N_{SW} = \$1/\$S(0), the present value of the Swiss franc payments in dollars is \$1. See the Appendix for sample calculations.

Based on these results, it is easy to see that with either party paying fixed and the other paying floating, the notional principals will be \$1 and N_{SW} = \$1/\$S(0).

The logic behind the initial and final exchange of principals in a currency swap is that the transaction is a simplified representation of the process of issuing a (fixed- or floating-rate) bond in one currency and using the proceeds to purchase a (floating- or fixed-rate) bond in another currency. The initial exchange of principals has equivalent value but the final exchange does not have equivalent value, assuming an exchange rate change. Some currency swaps will omit the initial and final exchange of principals. As we show here, this affects the pricing. Consider the receive fixed dollars vs. pay fixed Swiss francs swap. The present value of the dollar payments is

$$V_{\$}(0) \; = \; f_{US}\sum_{i=1}^{n} B_{US}(0,i)$$

and the present value of the Swiss franc payments is

$$V_{SF}(0) \; = \; f_{SF}\sum_{i=1}^{n} B_{SF}(0,i).$$

To equate these two, we need only multiply V_{SF} by the exchange rate \$S(0) times the Swiss notional principal, N_{SW}. In other words, we have

$$N_{SW}\$S(0)V_{SF}(0) \; = \; V_{\$}(0),$$

and the solution is found by solving for the one unknown N_{SW}.

A similar approach is found in the floating dollars-floating Swiss francs swap. The answer, N_{SW}, is the same as the fixed-fixed case. Similarly the answer is the same for the floating dollars-fixed Swiss francs and fixed dollars-floating Swiss francs. Solutions are provided in the Appendix.

Pricing Interest Rate Swaps as Forward Contracts

It is well known that interest rate swaps can be modeled as combinations of forward rate agreements or FRAs. An FRA is a forward contract on an interest rate and is essentially a swap with one payment. The fixed rate on an FRA is set at the appropriate forward rate. In practice an FRA settles at the point of expiration. For example, a 3x6 FRA (pronounced "3 by 6") on LIBOR is a forward contract calling for one party to pay the 90 day LIBOR that prevails three months from now, the other party paying a fixed rate, with payments occurring in three months. In a swap, and almost all other interest rate derivatives, the payment is deferred. For example a one year swap on 90 day LIBOR with settlement each quarter will involve the payment of 90 day LIBOR vs. a fixed rate where the LIBOR is set at the beginning of the quarter and both payments are made at the end of the quarter.

To replicate a swap by FRAs, one would combine a series of FRAs with expirations corresponding to the swap settlement dates but where the FRA payments are made at the end of the settlement period. In a swap, however, all of the fixed payments are the same, while in an FRA each fixed payment would differ, given a non-flat term structure. Thus, a swap is actually a series of incorrectly priced FRAs, sometimes called *off-market FRAs*. Also, note that since the first floating payment in a swap is known when the swap is initiated, the first payment is simply an exchange of known but different cash flows and not technically an FRA.

Consider a pay-fixed, receive-floating swap of n payments. Decomposing the swap into a series of off-market FRAs, we can express the value of the swap as

$$V_{SWAP}(0) = \sum_{i=1}^{n} V_{FRA}(0,i,f),$$

where $V_{FRA}(0,i,f)$ is the value at time 0 of an FRA expiring at time i-1, with payment at time i where the fixed payment is f and the floating payment is r(0,i-1,i). As of the initiation date of the swap, the appropriate floating payments are the forward rates r(0,i-1,i). Thus, $V_{FRA}(0,i,f) = [r(0,i-1,i) - f]B(0,i)$.

As noted above, since the first floating payment is known, it is technically not an FRA, but the notation [r(0,1) - f]B(0,1) still correctly specifies the value of that payment. See the Appendix for computations using the U.S. and Swiss term structures.

Pricing Currency Swaps as Forward Contracts

Likewise a currency swap can be priced as a combination of currency forward contracts, with, however, the first contract being simply an exchange of known cash flows at the current exchange rate. To value the swap as a combination of currency forwards, we require the forward exchange rates. Recalling that \$S(0) is the spot exchange rate, let \$S(0,i) be the forward exchange rate observed at time 0 for time i. When i = 0, then \$S(0,0) = \$S(0).

Now consider the fixed dollars-fixed Swiss francs swap. In terms of dollars, its value can be expressed as

$$V_{FXFXCS}(0) = \sum_{i=1}^{n} V_{CF}(0,i,f_{SW},f_{US})$$

where $V_{CF}(0,i,f_{SW},f_{US})$ is the value of a currency forward expiring at time i involving the payment of a fixed rate in dollars and the receipt of a fixed rate in Swiss francs. It is given as $V_{CF}(0,i,f_{SW},f_{US})$ = $[f_{SW}N_{SW}\$S(0) - f_{US}]B_{US}(0,i)$. For i = n, $V_{CF}(0,n,f_{SW},f_{US})$ = $[(1+f_{SW})N_{SW}\$S(0) - (1+f_{US})]B_{US}(0,n)$, reflecting the repayment of both principals. Also, at the start there is an exchange of $N_{SW}\$S(0)$ against \$1, but of course there is no net value associated with this initial exchange of principals.

For the floating dollars-floating Swiss francs, the value can be expressed as

$$V_{FLFLCS}(0) = \sum_{i=1}^{n} V_{CF}(0,i,r_{SW}(0,i-1,i),r_{US}(0,i-1,i)),$$

which indicates that the rate $r_{US}(0,i-1,i)$ will be paid and the rate $r_{SW}(0,i-1,i)$ will be received. Here the component currency forward involves a floating payment on both sides and its value can be expressed as $V_{CF}(0,i,r_{SW}(0,i-1,i),r_{US}(0,i-1,i))$ = $[r_{SW}(0,i-1,i)N_{SW}\$S(0) - r_{US}(0,i-1,i)]B_{US}(0,i)$. Naturally there is a valueless initial exchange of principals, and the forward contract at time n would need to include the exchange of N_{SW} Swiss francs for \$1.

Likewise, the remaining currency swaps (dollar fixed-Swiss franc floating and dollar floating-Swiss franc fixed) can be valued in a similar manner, using either the appropriate floating rate r(0,i-1,i) or fixed rate for the U.S. and Switzerland. For currency swaps with no notional principal exchange, slight adjustments are required to remove the notional principals at the beginning and at the end. See the appendix for numerical examples.

Swaps can also be priced as combinations of call and put options. A LIBOR pay-fixed, receive-floating swap can be priced as a combination of long calls and short puts on LIBOR with equivalent exercise rates and where the exercise rate equals the fixed rate on the corresponding swap. Likewise, a currency swap can be viewed as a combination of currency options. Naturally pricing these options would require an arbitrage-free term structure model.

Naturally in practice, payments would be adjusted to reflect the length of the settlement period. In other words, most rates are quoted as annual rates but settlement periods are typically less than a year. Thus, given a rate r, spanning six months, the payment would be adjusted to r/2 or r(days/360 or 365), depending on whatever the parties agree. Discounting would also have to be adjusted accordingly.

Valuation of Swaps

Now that we have identified the terms that permit a swap to have zero value at the start, we can proceed to understand how the swap value changes during the life of the swap. This process is called *valuation*. The value of a swap is simply the amount it is worth. To value a swap is nothing more than to determine the present value of the promised inflows net of the present value of the promised outflows. It is sometimes referred to as *replacement value*, mostly in the context of the cost to the holder of a swap that has positive value in the event that the counterparty defaults.

Valuation of a swap is quite easy. After the swap is in place, the terms, except for any promised floating payments, are fixed. For a plain vanilla swap one simply discounts the fixed payments using the new interest rates. One then discounts the floating payments using one of two procedures, corresponding to those we identified in the pricing section. We can either add the notional principal to the floating payments and the fixed payments or simply solve for the new forward rates and use those as the unknown future floating payments.

Suppose we are at time t which is after time 0 but before time 1. We have a new term structure of interest rates r(t,1), r(t,2), etc. and a new set of discount bond prices B(t,1), B(t,2), etc. Note, however, that a rate like r(t,1) is a rate spanning a period of less than 1 since $0 < t < 1$. The present value at t of the fixed payments, previously set at the rate f, is obviously,

$$V_{FX}(t) = f\sum_{i=1}^{n} B(t,i).$$

Finding the present value of the floating payments is just slightly harder. The upcoming floating payment was fixed at the last reset date, time 0 here, at r(0,1). All remaining floating payments are yet to be determined. In the section on pricing we demonstrated that a swap can be shown to be a series of FRAs. Consequently we can use the new forward rates, obtained using the new spot rates, as our floating payments. Thus,

$$V_{FL}(t) = \sum_{i=1}^{n} r(t,i-1,i)B(t,i).$$

For i = 1, the upcoming payment, we have r(t,0,1)B(t,1). For i = 2, the second floating payment, which is currently unknown, we use r(t,1,2), which is the forward rate from period 1 to period 2 observed at time t. Valuation of the swap is then easily found by subtracting the present value of one stream of payments from that of the other.

A simpler way to value the swap is, as we did when pricing the swap, to add the notional principal to both sides. Treating the stream of payments as a fixed rate bond, its value is

$$V_{FXRB}(t) = f\sum_{i=1}^{n} B(t,i) + B(t,n).$$

This trick will greatly facilitate finding the present value of the floating payments. As previously noted, the present value of the floating payments plus a final principal will always be par when the next payment is reset to the current market interest rate for that upcoming period. Consequently, at the next payment date we would receive the next known floating payment plus a claim on the remaining unknown floating payments and the final principal, which at that time must have a value of par. Thus, treating the stream of payments as a floating rate bond, its value is

$$V_{FLRB}(t) = (1 + r(t,0,1))B(t,1).$$

To value a currency swap, we follow the same procedure, determining the value in the respective currencies of each set of payments. We then convert the foreign currency to the domestic currency using the new current spot exchange rate. Alternatively, we could value the currency swap as a series of currency forward contracts and determine their overall net value.

Calculations in the appendix demonstrate how swaps are valued.

Appendix

This appendix illustrates the results presented herein with numerical examples involving a three-period swap. Let the following information about the U. S. and Swiss term structures be given. (US and SW subscripts are omitted here but used later where necessary).

U.S. Term Structure	*Switzerland Term Structure*
r(0,1) = .08, r(0,2) = .09, r(0,3) = .10	r(0,1) = .088, r(0,2) = .093, r(0,3) = .105
B(0,1) = 1/[1 + .08(360/360)] = 0.9259	B(0,1) = 1/[1 + .088(360/360)] = 0.9191
B(0,2) = 1/[1 + .09(720/360)] = 0.8475	B(0,2) = 1/[1 + .093(720/360)] = 0.8432
B(0,3) = 1/[1 + .10(1080/360)] = 0.7692	B(0,3) = 1/[1 + .105(1080/360)] = 0.7605

The implied forward rates for one-period transactions are

(in the U.S.)

r(0,1,2) = {([1 + .09(720/360)]/[1 + .08(360/360)]) - 1}(360/360) = 0.0926

r(0,2,3) = {([1 + .10(1080/360)]/[1 + .09(720/360)]) - 1}(360/360) = 0.1017

(in Switzerland)

r(0,1,2) = {([1 + .093(720/360)]/[1 + .088(360/360)]) - 1}(360/360) = 0.0901

r(0,2,3) = {([1 + .105(1080/360)]/[1 + .093(720/360)]) - 1}(360/360) = 0.1088

Let the spot exchange rate be \$S(0) = \$0.70. The forward exchange rates can be easily derived as

\$S(0,1) = \$0.70[1 + .08(360/360)]/[1 + .088(360/360)] = \$0.6949

\$S(0,2) = \$0.70[1 + .09(720/360)]/[1 + .093(720/360)] = \$0.6965

$$\$S(0,3) = \$0.70[1 + .10(1080/360)]/[1 + .105(1080/360)] = \$0.6920,$$

which, of course, obtain from applying the interest rate parity rule.

Pricing Plain Vanilla Swaps

Now let us price the plain vanilla swap in both countries. For the U. S., the present value of the floating payments is

$$V_{FL}(0) = 0.08(0.9259) + 0.0926(0.8475) + 0.1017(0.7692) = 0.2308,$$

which is set equal to the present value of the fixed payments:

$$V_{FX}(0) = f_{US}(0.9259 + 0.8475 + 0.7692) = 0.2308.$$

Solving for the fixed payment gives $f_{US} = .0908$. As an alternative, we could simply note that the present value of the floating payments plus a final principal is 0.2308 + 1(0.7605) = 1.0000, as it should be. Then equating the present value of the fixed payments plus a final principal, we have $f_{US}(0.9259 + 0.8475 + 0.7692) + \$1(0.7692) = 1.0000$. Solving for f_{US} again gives .0908. Of course, this alternative method avoids having to compute the forward rates.

For Switzerland, the present value of the floating payments is

$$V_{FL}(0) = 0.088(0.9191) + 0.0901(0.8432) + 0.1088(0.7605) = 0.2396,$$

and equating to the present value of the fixed payments, we have

$$V_{FX}(0) = f_{SW}(0.9191 + 0.8432 + 0.7605) = 0.2396.$$

Solving gives $f_{SW} = .0950$. Likewise, adding a principal repayment gives a present value of the floating payments as 0.2395 + 0.7605 = 1.0000 and a present value of the fixed payments set at $f_{SW}(0.9191 + 0.8432 + 0.7605) + 0.7605 = 1.0000$. Again, $f_{SW} = .0950$.

Pricing Currency Swaps

Now let us price the currency swap involving payment of U.S. dollars and receipt of Swiss francs with an initial and final exchange of notional principals. First we price the fixed-fixed currency swap. With a $1 US notional principal, the Swiss franc notional principal should be set at $1/$0.70 = SF1.4286. This is easily verified by finding the present values of both streams of payments and converting to a common currency, here the dollar. The present value of the dollar fixed payments we already know is $1. The present value of the Swiss franc fixed payments on SF1.4286 notional principal is 1.4286[.0950(0.9191) + .0950(0.8432) + .0950(0.7605) + 1(0.7605)] = 1.4286. At a $0.70 exchange rate, this is equivalent to $1 at the start.

For the floating-floating currency swap, the results are the same. Given that we found the plain vanilla fixed rates in each country by equating the present value of the floating payments to the present value of the fixed payments, we know that a notional principal of \$1 and SF1.4286 will have the equivalent value, whether the payments are fixed or floating. Likewise, the currency swaps where one party will pay fixed and the other floating will have the same notional principals, \$1 and SF1.4286.

For currency swaps with no initial or final exchange of principal, the pricing will be different. For the fixed-fixed swap, the present value of the dollar fixed payments is .0908(0.9259 + 0.8475 + 0.7692) = 0.2308. The present value of the Swiss franc fixed payments for an unspecified notional principal of N_{SW} is N_{SW}[.0950(0.9191 + 0.8432 + 0.7605)](\$0.70) = N_{SW}(0.1678). Equating this value to 0.2308 and solving for N_{SW} gives a Swiss franc notional principal of SF1.3754. The same result would be obtained for the swap with floating-floating or one fixed, the other floating.

Plain Vanilla Swaps as Combinations of Forward Rate Agreements (FRAs)

Now let us value these swaps as combinations of forward contracts. For the plain vanilla swaps, we can value the US swap as the following combination of FRAs:

$$V_{US}(0) = (0.08 - 0.0908)(0.9259) + (0.0926 - 0.0908)(0.8475) + (0.1017 - 0.0908)(0.7692)$$
$$= -0.0100 + 0.0015 + 0.0084 \approx 0.0000,$$

and for the Swiss plain vanilla swap, we have

$$V_{SW}(0) = (0.088 - 0.0950)(0.9191) + (0.0901 - 0.0950)(0.8432) + (0.1088 - 0.0950)(0.7605)$$
$$= -0.0064 + -0.0041 + 0.0105 \approx 0.0000.$$

We see that in both cases, the component FRAs are not individually valued at zero, as they would be if taken out separately. That is, each FRA, would ordinarily be priced at the appropriate forward rate to make its value be zero. Hence, we call these off market FRAs. Naturally the first payment is actually not an FRA but rather a spot transaction. Collectively these transactions have zero value for each party and in each respective country.

Currency Swaps as Combinations of Currency Forwards

Now let us look at the currency swaps as combinations of currency forwards. For the fixed-fixed currency swap, involving payment of dollars and receipt of Swiss francs, the value is found as

$$V_{FXFXCS} = [0.0950(1.4286)(0.6949) - 0.0908]0.9259$$
$$+ [0.0950(1.4286)(0.6965) - 0.0908]0.8475$$
$$+ [(0.0950 + 1)(1.4286)(0.6920) - (0.0908 + 1)].7692$$
$$= 0.0032 + 0.0032 - 0.0064 \approx 0.0000$$

Given our equivalence between the fixed and floating payments in each country, it should be easy to see that the remaining swap values can be similarly decomposed into currency forwards. The component forward values will not match up individually with these three values but the overall value of the combination of contracts will be zero.

Valuation of Swaps

Let us now assume that we are six months into the life of the swap. The new term structures are

U. S. Term Structure	*Switzerland Term Structure*
r(0.5,1) = .082, r(0.5, 2) = .094, r(0.5,3) = .105	r(0.5,1) = .09, r(0.5, 2) = .096, r(0.5,3) = .108
B(0.5,1) = 1/[1 + .082(180/360)] = 0.9606	B(0.5,1) = 1/[1 + .09(180/360)] = 0.9569
B(0.5,2) = 1/[1 + .094(540/360)] = 0.8764	B(0.5,2) = 1/[1 + .096(540/360)] = 0.8741
B(0.5,3) = 1/[1 + .105(900/360)] = 0.7921	B(0.5,3) = 1/[1 + .108(900/360)] = 0.7874

The new implied forward rates are

(in the U.S.)

r(0.5,1,2) = {([1 + .094(540/360)]/[1 + .082(180/360)]) - 1}(360/360) = .0961

r(0.5,2,3) = {([1 + .105(900/360)]/[1 + .094(540/360)]) - 1}(360/360) = .1065

(in Switzerland)

r(0.5,1,2) = {([1 + .096(540/360)]/[1 + .09(180/360)]) - 1}(360/360) = .0947

r(0.5,2,3) = {([1 + .108(900/360)]/[1 + .096(540/360)]) - 1}(360/360) = .1101.

Let the new spot exchange rate be $S(0) = $0.725. Recall that the original fixed rates were .0908 in the U. S. and .0950 in Switzerland. The original notional principals were $1 in the U. S. and SF1.4286.

First let us look at valuing domestic plain vanilla swaps. We take the position of a party paying fixed and receiving floating. Let us initially add the notional principals. For a U. S. swap, the fixed payments can be treated like a fixed-rate bond, which will have a present value of

$$V_{FXRB}(t) = .0908(0.9606) + .0908(0.8764) + 1.0908(0.7921) = 1.0308.$$

Remember that the present value of the floating payments, with notional principal included, is found as the present value of the next floating payment, which is known to be the time 0 one-period spot rate of .08, plus the present value of par value of 1.0 at time 1. Remember that 1.0 at time 1 represents the value at time 1 of all remaining floating payments plus par value paid at time n. Thus, treating the floating payments like a floating-rate bond, we have

$$V_{FLRB}(t) = 1.08(0.9606) = 1.0374.$$

Hence, a swap paying fixed and receiving floating is worth 1.0374 - 1.0308 = 0.0066. In Switzerland a SF1 notional principal plain vanilla swap would be found as

$$V_{FXRB}(t) = 0.0950(0.9569) + 0.0950(0.8741) + 1.0950(0.7874) = 1.0361,$$

$$V_{FLRB}(t) = 1.088(0.9569) = 1.0411,$$

and, thus, the swap value would be 1.0411 - 1.0361 = 0.0050 per SF1 notional principal.

Alternatively, we could value the swap as a combination of forward contracts, using the new forward rates to reflect the unknown future floating payments. Again, we repeat, *this does not mean that the new forward rates are our expectations of the future floating rates*. They simply reflect the substitutability of forward rates for future spot rates that is permissable when forward contracts can be perfectly hedged.

In the U. S. market, the present value of the fixed payments is

$$V_{FX}(t) = 0.0908(0.9606 + 0.8764 + 0.7921) = 0.2387,$$

and the present value of the floating payments is,

$$V_{FL}(t) = 0.08(0.9606) + 0.0961(0.8764) + 0.1065(0.7921) = 0.2454,$$

which uses the new forward rates for the second and third floating payments. The net difference is 0.0067, which is off by one basis point, a result of round-off error.

In the Swiss market, the present value of the fixed payments is

$$V_{FX}(t) = 0.0950(0.9569 + 0.8741 + 0.7874) = 0.2487,$$

and the present value of the floating payments is,

$$V_{FL}(t) = 0.088(0.9569) + 0.0947(0.8741) + 0.1101(0.7874) = 0.2537.$$

The net difference is 0.0050, which is exactly what we previously obtained.

Valuation of the currency swap is then simple. Consider a currency swap with no notional principal involving the payment of U. S. dollars and receipt of Swiss francs. We previously determined that a $1 notional principal is equivalent to a SF1.3754 notional principal. The new spot exchange rate is $0.725. For the fixed-fixed swap, we pay dollars at the fixed rate of .0908 and receive Swiss francs at the fixed rate of .0950. The value of the swap is, therefore,

$$V_{FXFX}(t) = 1.3754(0.2487)(0.725) - 0.2387 = 0.0093,$$

which uses the fact that we have previously determined the present value of the Swiss francs fixed payments per SF1 notional principal to be 0.2487 and the present value of the dollar fixed payments to be 0.2387. If we were paying dollars floating and receiving Swiss francs fixed, the swap value would be

$$V_{FXFL}(t) = 1.3754(0.2487)(0.725) - 0.2454 = 0.0026,$$

which uses the fact that we have already determined the present value of the Swiss franc fixed payments per SF1 to be 0.2487 and the present value of the dollar floating payments to be 0.2454. If we were paying dollars fixed and receiving Swiss francs floating, the swap value would be

$$V_{FLFX}(t) = 1.3754(0.2537)(0.725) - 0.2387 = 0.0143,$$

which uses the fact that we have already determined the present value of the Swiss franc fixed payments per SF1 to be 0.2537 and the present value of the dollar fixed payments to be 0.2387. If we were paying dollars floating and receiving Swiss francs floating, the swap value would be

$$V_{FLFL}(t) = 1.3754(0.2537)(0.725) - 0.2454 = 0.0076,$$

which uses the fact that we have already determined the present value of the Swiss franc floating payments to be 0.2537 per SF1 and the present value of the dollar floating payments to be 0.2454. These swap values can also be determined by valuing the component payments as currency forward contracts. The notional principals are added if relevant.

References

A variety of sources cover swap pricing.

Arditti, F. D. *Derivatives: A Comprehensive Resource for Options, Futures, Interest Rate Swaps, and Mortgage Securities* (Boston: Harvard Business School Press, 1996), Chs.14, 15, 16.

Chance, D. M. *An Introduction to Derivatives and Risk Management, 5th ed.* (Fort Worth: Harcourt, Inc., 2001), Chs. 13, 14.

Hull, John C. *Options, Futures, and Other Derivative Securities*, 4th ed. (Upper Saddle River, New Jersey: Prentice-Hall, 2000), Ch. 5.

Marshall, J. F. and K. R. Kapner. *The Swaps Market* (Miami: Kolb Publishing, 1993).

Sundaresan, S. *Fixed Income Markets and Their Derivatives* (Cincinnati: South-Western Publishing, 1997).

An interesting article showing the parity relations among interest rate and currency swaps and options is

Yaksick, R. "Swaps, Caps, and Floors: Some Parity and Price Identities." *Journal of Financial Engineering* 1 (1992), 105-115.

An alternative view of pricing swaps is provided in

Jarrow, R. A. and S. M. Turnbull. *Derivative Securities*, 2nd ed. (Cincinnati: South-Western, 2000), Chapter 14.

Jarrow and Turnbull argue that you cannot add the principals to both sides of a plain vanilla swap because the two streams of payments must be discounted at different rates, the floating being discounted at LIBOR and the fixed being discounted at the Treasury bill rate. The majority view is consistent with the approach taken in this teaching note and not with the Jarrow-Turnbull position. JT's view seems to confound market and credit risk, which is the primary factor that distinguishes t-bill from LIBOR rates. JT argue that the risks of the streams are different but if that is the case, then JT themselves discount improperly. On the floating side, they find the present value of the floating payments by subtracting the present value of the principal from the value of a floating rate security by discounting the principal at LIBOR. Yet, if they treat fixed payments as appropriately discounted at the t-bill rate, then the present value of the fixed principal repayment should be discounted at the t-bill rate.