TEACHING NOTE 97-07:

VALUE-AT-RISK (VaR)

Value at Risk or VaR is a concept developed in the field of risk management that is defined as *the minimum amount of money that one could expect to lose with a given probability over a specific period of time.* The concept has taken hold as a way to manage the position of a derivatives dealer or end user. While VaR is widely used, it is, nonetheless, a controversial concept, one that has generated some intensely heated debates, primarily due to the diversity of methods for obtaining VaR, the widely divergent values so obtained and for a fear that management will rely too heavily on VaR with little regard for other information about a firm's risk.

The VaR concept embodies three factors. The first is a given time horizon. A risk manager might be concerned about possible losses over one day, one week, one month, etc. Second, VaR is associated with a probability. The stated VaR represents the minimum possible loss over a given period of time with a given probability. Third, there is the actual dollar amount itself. Consider for example a dealer with a $20 million position. He might find that his VaR for a one day period, with a one percent probability is $500,000. This means that the dealer can expect to lose at least $500,000 in any given day about one percent of the time, or in other words, 2.5 times in a year (assuming 250 trading days). Of course, the user can specify any probability or holding period and there are no hard and fast rules to help one decide on the appropriate probability and holding period.

The concept of VaR is a very appealing one. It can be developed for any kind of portfolio and can be aggregated across portfolios of different kinds of instruments. For example, a bank might have a portfolio of interest rate swaps, a portfolio of currencies, positions in some commodities and a portfolio of common stocks, as well as its regular loan and bond portfolios. The VaR for each separate portfolio can be calculated and aggregated across all portfolios. This does not imply that estimating VaR for a portfolio is a simple process; the correlations across asset classes must be accounted for, a point we shall cover later. VaR does, however, provide a consistent measure across

portfolios. Thus, with appropriate consideration for all correlations, VaR can provide the bank with an overall measure of exposure.

An attractive feature of VaR is that it is stated in terms of dollars, or whatever currency is appropriate. Senior management and anyone without much technical knowledge of derivatives can still easily understand the concept.

Now let us examine how VaR is computed. First we begin with a simple example that takes the required inputs as given. Later we shall look at how this information is obtained. Suppose that a firm has compiled the following information about the change in the value of a portfolio over a one-week period.

Change in Value	Probability
≤ -$1,000,000	.01
-$500,000 to -$999,999	.04
-$250,000 to -$499,999	.15
$0 to -$249,999	.30
$1 to $249,999	.30
$250,000 to $499,999	.15
$500,000 to $999,999	.04
≥ $1,000,000	.01

From this information we can say that VaR at .01 is -$1,000,000. This means that the firm would expect to lose at least $1 million one percent of the time. A common misconception about VaR is that it is the maximum expected loss. It should be apparent that this is not what VaR is. We do not say that the most the firm could lose 1 percent of the time is $1 million. Obviously the firm could lose more than $1 million.

From the above table we can also easily define VaR at 5 percent, which is -$500,000. There is a 4 percent chance of a loss of between $500,000 and $999,999 plus a one percent chance of a loss of at least $1 million. Thus, the firm can expect to lose at least $500,000 five percent of the time.

The probability distribution given above is a simple discrete distribution. Sometimes a firm will prefer to use a continuous distribution, the normal being a common one. When that is the case, then the user must know the expected change in value, $E(\Delta V)$, and the standard deviation, $\sigma(\Delta V)$. From there one can compute VaR as

$$VaR(1 \%) = E(\Delta V) - 2.33\sigma(\Delta V)$$

$$VaR(5\%) = E(\Delta V) - 1.65\sigma(\Delta V).$$

These numbers, 2.33 and 1.65, represent the number of standard deviations to the left of the expected value that leave 1 percent and 5 percent, respectively, in the tail of the distribution.

For example, suppose a firm estimates that the change in its portfolio value is $1,000,000 with a standard deviation of $1.5 million. Thus, VaR(1 %) = $1,000,000 - 2.33($1,500,000) = -$2,495,000 and VaR(5 %) = $1,000,000- 1.65($1,500,000) = -$1,475,000. Thus, the firm could expect to lose at least $2,495,000 one percent of the time and lose at least $1,475,000 five percent of the time.

A discrete distribution is, in many cases, merely a simplification of a continuous distribution. In finance virtually all risks are associated with continuous distributions. The more critical problem, however, is not whether to use a discrete or continuous distribution but how to obtain the inputs. There are several approaches, which can be broadly classified into two groups: the **historical method** and the **analytical method**.

The historical method requires that the user obtain historical information on the performance of all financial instruments that are in the current portfolio. This means a time series of the prices or returns on these assets. From this historical data, the user can calculate means, variances and covariances and, thereby, estimate the parameters, $E(\Delta V)$ and $\sigma(\Delta V)$ of the portfolio currently in place. Obviously this method relies heavily on the assumption that the probability distribution from the past continues to hold for the future.

Alternatively, the user might do a Monte Carlo simulation. Using the parameters estimated from past data, the user can then generate random numbers to represent market prices, interest rates, etc. and for each simulation run, the value of the portfolio is determined. The simulation enables one to construct a frequency distribution from which VaR can be picked off from the left tail.

These descriptions provide the general idea behind the historical method but the implementation is not as straightforward as it might seem. Consider that many firms will hold quite diverse portfolios of assets and liabilities. Suppose a portfolio consists of various positions in LIBOR based instruments such as FRAs, caps, floors, and swaps. It also holds some treasury notes, some common stocks, some short positions in stock index futures and some currency options. The returns on these assets are not generated by the same source of uncertainty. To produce a portfolio variance

it is necessary to estimate the correlations among the different classes of instruments. Though this is not a difficult task, it is certainly subject to estimation error. In addition, the number of correlations that one must estimate is N(N-1)/2 where N is the number of distinct instruments, which can be quite large. For a Monte Carlo simulation, one must generate random numbers representing prices, interest rates, and exchange rates according to the relative frequency with which these numbers would appear in practice. Thus, the simulator must take into account the correlations among the prices and rates.

When estimating VaR using the analytical method, one must determine the means, variances and covariances of the existing assets and liabilities based on information currently available, in contrast to historical information. Some of this information can be obtained from the term structure, and models used in capturing its evolution. For example, one might be able to obtain good estimates of deltas and gammas for swaps, caps, etc. Since deltas and gammas tell us how a derivative or asset price will change if the underlying changes, we can obtain an estimate of the appropriate volatility of the derivative or asset. Obtaining these estimates for other instruments such as stocks is more difficult. In any case, one must obtain estimates of expected changes, variances and covariances based on current and expected future data. From this one can identify the VaR as represented by a point on the left tail end of the distribution. Alternatively, given estimates of means, variances and covariances, simulation can also be conducted so as to generate a frequency distribution from which the left tail end can be examined for the VaR.

There are many variations and nuances associated with each of these methods. In some cases the values of certain instruments, such as options, are estimated based on the use of a delta or delta-plus-gamma, given an appropriate change in the underlying. In this manner, changes in the underlying that occur 1- or 5-percent of the time can be converted into changes in option values so as to determine how the options will perform when the underlying makes an extreme move. Often associated with VaR is the concept of stress testing, which is simulating extremely unlikely events and observing how the value of the portfolio changes. Most stress testing, however, focuses on bad events and places insufficient emphasis on good events. In fact VaR in general has been criticized for its focus only on the downside risk. Portfolios with considerably attractive upside features could be avoided because of an arbitrarily unacceptable VaR.

Some VaR models are factor-based. They attempt to identify a small number of common factors that drive all assets. If one can then determine the sensitivity of each asset to the factor, then it is much easier to compute VaR for large portfolios.

Studies have shown that VaR estimated using different methods with data extracted over different length time periods can be quite different. Clearly VaR is not a perfect measure but no statistic is perfect. Users should be aware that VaR provides only an estimate, is potentially subject to large error, and should not be relied upon as the sole source of information when managing risk.

References

Since 1995, there is probably no subject more written about than VaR. The following items are particularly notable.

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