COURSE OUTLINE

MATH 7

2013-14

Note: Optional topics will not be on Midterms, Finals or Proficiency examinations

I. **SETS** (~10 days)

- Finite and infinite sets; partition of a set; special infinite sets (natural and whole numbers, integers, primes and composites, evens and odds)
- Subsets and proper subsets of a set
- Power set; proof that the number of subsets of a set with n elements is 2^n
- Union and intersection of sets; Venn diagrams
- Universal set; complement of a set
- Set-builder notation
- **Optional** topics: Using a one-to-one correspondence between sets to demonstrate that two infinite sets have the same cardinality, $\aleph_0$; $\aleph_1$.

II. **DIVISIBILITY** (~9 days)

- Definition of divisor, proper divisor, prime number, composite number
- Euclid's proof that the number of primes is infinite
- Divisibility tests, including justifications
- Exponents; prime factorization; Fundamental Theorem of Arithmetic
- Finding the number of positive divisors of a given whole number
- Greatest common factor; least common multiple; relatively prime

Optional:

- Bases of numeration (converting a number in a given base to base 10 and vice-versa)
- Perfect, abundant, and deficient numbers
- Goldbach's Conjecture
- **Optional** topics: Mersenne primes; Four square theorem; game of Nim

III. **NUMERICAL AND ALGEBRAIC EXPRESSIONS** (~13 days)

Numerical:

- Order of operations; evaluating a numerical expression (including fractions and decimals)
- Mental arithmetic
- Factorials
- Absolute value and opposite (additive inverse)
- Adding, subtracting, multiplying, and dividing signed numbers
- Properties of integers (closure, commutative, associative, distributive, identities, inverses)
- Estimation problems

Algebraic:

- Solving open sentences by inspection

- Evaluating algebraic expressions using signed numbers; include special expressions, e.g., circumference and area of a circle; $n^2 - n + 41$; Fermat primes: $2^{2^n} + 1$; $F = \frac{9}{5}C + 32$; $s = 16t^2$; $T_n = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$
- Adding and subtracting like terms; distributing through parentheses
- **Optional** topics: Brocard's conjecture; special problems using factorials; magic squares

IV. **SOLVING ALGEBRAIC EQUATIONS AND INEQUALITIES** (~14 days)

Algebraic Equations:

- Solving linear equations using the addition and multiplication properties of equality
- Adding and subtracting like terms
- Solving linear equations with variables on both sides
- Solving equations with parentheses
- Solving equations of the form $|ax + b| = c$
- Translating verbal expressions into algebraic expressions
- Solving verbal problems: number, consecutive integer, age, geometric (may include area, perimeter, supplementary and complementary angles, sum of the measures of the angles of a triangle, problems involving properties of isosceles and equilateral triangles, triangle inequality)

Algebraic Inequalities:

- Solving linear inequalities and graphing the solution sets
- Union and intersection of inequalities
- Solving absolute value inequalities of the form $|x - b| \begin{matrix} < \\ > \\ \leq \\ \geq \end{matrix} c$, $c \geq 0$ and graphing the solution sets
- **Optional**: Solving equations of the form $a + b|cx + d| = e$

V. **INVESTIGATION OF RATIONAL NUMBERS** (~24 days)

- Definition of the set of rational numbers
- Determining which of two rational numbers is larger
- Decimal equivalents of rational numbers; terminating vs. non-terminating decimals
- Determining when $\frac{1}{n}$ is equivalent to a terminating or a repeating decimal
- Repetends, periods, perfect repetends, cyclic decimals
- Adding, subtracting, and multiplying with repeating decimals
- Converting a repeating decimal to fractional form
- Laws of exponents, including negative exponents
- Scientific notation: multiplying, dividing, adding and subtracting numbers in scientific notation
- Ratio, proportion and percent
- Solving verbal problems involving proportions
- Solving verbal problems involving percent (sales tax and discount problems; increase and decrease; investment problems)

VI. **IRRATIONAL NUMBERS** (~9 days)

- Definition of irrational numbers; approximating a square root to the nearest tenth
- Proof that $\sqrt{2}$ is irrational
- Simplifying radicals with index ≥ 2
- Adding, subtracting, multiplying, and dividing monomial radicals
- Pythagorean Theorem; Pythagorean triples; applications of the Pythagorean Theorem (e,g., finding the area of a square given the length of its diagonal, finding the length of the diagonal of a rectangular prism)
- Solving simple quadratic equations of the form $ax^2 \pm b = c$ and $\left(x \pm a\right)^2 = b$
- **Optional** topic: Formulas for generating primitive Pythagorean triples

VII. **ELEMENTARY ALGEBRAIC OPERATIONS** (~11 days)

- Adding, subtracting, multiplying and dividing monomials
- Addition and subtraction of polynomials
- Multiplication and division of a polynomial by a monomial (using a geometric interpretation where possible)