

Lesson 1: Solving Linear Inequalities and Word Problems

A. What is a linear equation?

- linear = line, as seen later, variable x is to the 1st degree
- equation \Rightarrow equal sign

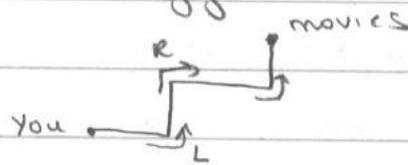
Ex: Are these linear equations?

- | | | | |
|-----------------|------------|-------------------|--|
| a) $3x + 5 = 8$ | Yes, $x =$ | c) $5x^2 + 2 = 7$ | No, $x^2 \leftarrow 2^{\text{nd}}$ deg |
| b) $4x - 3$ | No, no = | d) $3x - 19 = 2x$ | Yes, $x =$ |

B. What is solving? \Rightarrow finding what the unknown variable is equal to (ie: $x = ?$)

C. How do we solve?

- analogy:



going to movies to go back

- | | |
|------|------|
| 1) L | 1) R |
| 2) R | 2) L |
| 3) L | 3) R |

- to go back, we undo the forward operations backwards (undoing last, second to last, ... first)

what operations undo each other (inverses)

- a) left, right b) up, down c) $+$, $-$ d) \times , \div e) x^2 , \sqrt{x}

- in an equation, imagine you are x , the operations being done are like left and right, and the movies is what it all equals. Thus, you undo the mathematical operations to get back to x .

* because $=$ says what's on one side is the same as what's on the other, we must perform operations on both sides of the $=$ to balance the changes

(2)

ex: $3x = 6$ ← what is being done to x ? It is being mult. by 3, thus to cancel that, we divide by 3 (on both sides!)

$$\frac{3x}{3} = \frac{6}{3}$$

$$\underline{x} = \underline{2}$$

ex: $x + 2 = 8$ ← 2 is being added to x , so to undo, we subtract 2 from both sides!

$$\underline{x} = \underline{6}$$

ex: $2x + 5 = 3x + 4$ ← Remember our solution is $x = \#$. x 's on one side, $\#$'s on the other.

$$\begin{array}{r} -2x \\ \hline -2x \end{array}$$

canceling x 's here, so I want $\#$'s to be on this side

$$\begin{array}{r} 5 = x + 4 \\ -4 \quad -4 \\ \hline 1 = x \end{array}$$

Pick a side for each and cancel the ones that shouldn't be there

ex: $3x + 2 - 5x = 9$ ← Here x 's are on one side.

$$\begin{array}{r} \downarrow \\ -2x + 2 = 9 \\ -2 \quad -2 \\ \hline -2x = 7 \\ -2 \quad -2 \\ \hline \end{array}$$

canceling brings them to other side, which we don't want. Here, we just combine like terms

$$\Rightarrow x = -\frac{7}{2}$$

ex: $3(x+2) = 9$ ← x is not free to move. Two ways

a) distribute and solve

$$3(\overbrace{x+2}) = 9$$

$$3x + 6 = 9$$

$$\begin{array}{r} -6 \quad -6 \\ \hline \end{array}$$

$$\begin{array}{r} 3x = 3 \\ \hline 3 \quad 3 \end{array}$$

$$x = 1$$

b) undo $\times 3$ first

$$\begin{array}{r} 3(x+2) = 9 \\ \hline 3 \quad 3 \end{array}$$

$$x + 2 = 3$$

$$\begin{array}{r} -2 \quad -2 \\ \hline \end{array}$$

$$x = 1$$

* Sometimes quicker, sometimes not

word Problems * read slowly !!

Phrases to consider:

a less than b $\Rightarrow b-a$ (a is subtracted from b)

quotient of a and b $\Rightarrow \frac{a}{b} \Rightarrow b\sqrt{a}$

difference between a and b $\Rightarrow a-b$

product of a and b $\Rightarrow (a)(b)$ (multiplication)

sum of a and b $\Rightarrow a+b$ (addition)

Ex. five times a number is three less than seven

* easier \rightarrow times that number. What is the number?

to
solve
using
symbols

$$\rightarrow 5x = 7x - 3$$

$$\begin{array}{rcl} \text{Solve: } 5x = 7x - 3 & \xrightarrow{-7x} & -2x = -3 \\ -7x & & \xrightarrow{-2} \end{array} \quad \begin{array}{l} x = \frac{3}{2} \end{array}$$

Ex. The difference between sixteen and nine times a number is three less than the quotient of that number and two.

$$16 - 9x = \frac{x}{2} - 3$$

$$2(16 - 9x) = 2\left(\frac{x}{2} - 3\right)$$

$$32 - 18x = x - 6$$

$$+18x +18x$$

$$32 = 19x - 6$$

$$+6 +6$$

can multiply both sides

by 2 to cancel x's

denominator (since x

is \div by 2, $\times 2$ will

cancel the $\div 2$, leaving just x)

$$\frac{38}{19} = \frac{19x}{19} \Rightarrow 2 = x$$

Consecutive Integers: (consecutive = one after the other)

a) 1, 2, 3, 4, etc \Rightarrow to get from one to the next, add 1

If we don't know 1st one, make that x , so

next is $x+1$, $x+2$, $x+3$, etc.

b) consecutive even (and odd) integers are 2 apart, so if x is the 1st, next is $x+2$, $x+4$, $x+6$, etc.

(4)

Ex. The sum of three consecutive even numbers is eighteen. Find the numbers.

Let $x = 1^{\text{st}} \#$

$x+2 = 2^{\text{nd}} \#$

$x+4 = 3^{\text{rd}} \#$

$$\text{sum: } x + \underline{x+2} + \underline{x+4} = 18$$

$$3x + 6 = 18$$

$$\underline{-6} \quad \underline{-6}$$

$$\frac{3x}{3} = \frac{12}{3}$$

need all

$$x = 4 \quad * \text{NOT DONE YET!}$$

$$3 \# \rightarrow x = 4, x+2 = 6, x+4 = 8$$