

Lesson 10: Slope Formula

Graphing a line given point and slope

What is a slope? An incline with some steepness.
 What affects how steep it is? Height and length.
 What does height do? What does length do?

To test, keep length same, alter height

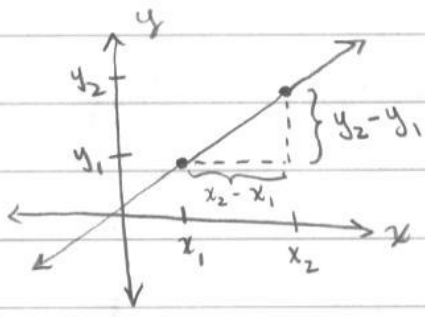


the higher, the steeper
varies directly



the longer, the less steep
varies inversely

The slope is the ratio (fraction) of height to length. The letter representing slope is m .



keep in mind, the subscripts stand for the point, i.e. x_1 is the x-coord. of the first point, y_2 is the y-coord of the second point, etc.

Consider two points, (x_1, y_1) and (x_2, y_2) and a line through them. Then the "height" of the slope is

the difference in y-coordinates $(y_2 - y_1)$ and the "length" is the difference in x-coordinates $(x_2 - x_1)$. Why? 😊

Slope is the ratio of height to length.
Thus:

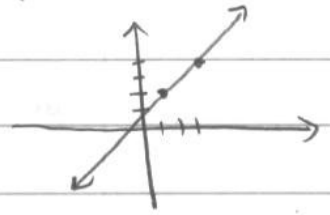
$$m = \frac{\text{diff in } y}{\text{diff in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

★ make sure it's $\frac{y_2 - y_1}{x_2 - x_1}$, NOT $\frac{y_2 - y_1}{x_1 - x_2}$

25

ex: find the slope of the line that passes through (x_1, y_1) and (x_2, y_2) .

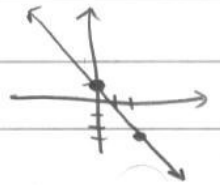
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{3 - 1} = \frac{3}{2}$$



* line going up from left to right has positive slope

ex: find the slope of the line that passes through $(0, 1)$ and $(2, -3)$

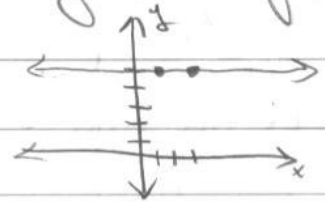
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 1}{2 - 0} = \frac{-4}{2} = -2$$



* line going down from left to right has negative slope

ex: find the slope of the line passing through (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 5}{3 - 1} = \frac{0}{2} = 0$$



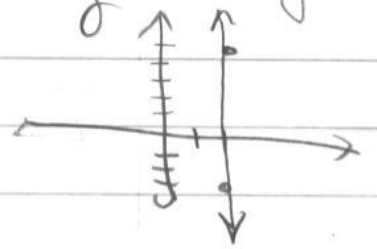
* horizontal line is flat (not steep) s. slope is 0

ex: find the slope of the line passing through (2, 5) and (2, -3)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 5}{2 - 2} = \frac{-8}{0}$$

undefined!

can't divide by 0



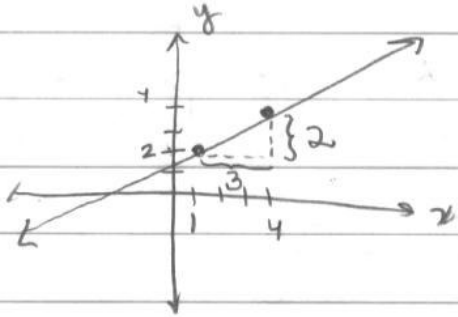
* a vertical line has enormous steepness! (death drop 😊), so its slope is undefined because we can't divide by 0.

What can a slope tell us?

Recall that a slope contains information about the difference in y and x values between 2 points. Thus, if we only know one point, we can use the slope to get to the next. Thus A SLOPE TELLS HOW TO GET FROM ONE POINT TO ANOTHER ON A LINE!

ex: A line has slope $\frac{2}{3}$ and goes through the point (1, 2). Graph the line.

⇒ (1, 2) is our starting point. The slope, $m = \frac{2}{3}$ tells that the difference in y from one point to the next is 2 and the difference in x is 3. Thus, from (1, 2) go up 2 and to the right 3 to get the next point.



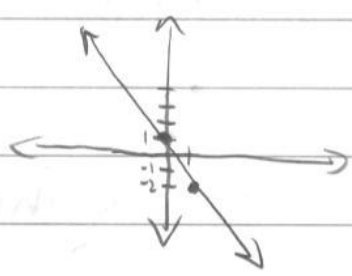
So $m = \frac{2}{3} \rightarrow \Delta y$
 $\rightarrow \Delta x$
 and the next point
 from (1, 2) is (4, 4)

Why?
 $m = \frac{y_2 - y_1}{x_2 - x_1}$
 $\frac{2}{3} = \frac{y_2 - 2}{x_2 - 1}$

$\Rightarrow 2 = y_2 - 2 \quad - \quad 3 = x_2 - 1$
 $\quad \quad \quad +2 \quad \quad +2 \quad \quad +1 \quad \quad +1$
 $\quad \quad \quad 4 = y_2 \quad \quad 4 = x_2 \quad \text{So } (x_2, y_2) = (4, 4)$

ex: Line has $m = -3$ and point (0, 1). Graph line.
 \Rightarrow we know $m = \frac{\Delta y}{\Delta x}$ so change m to
 a fraction

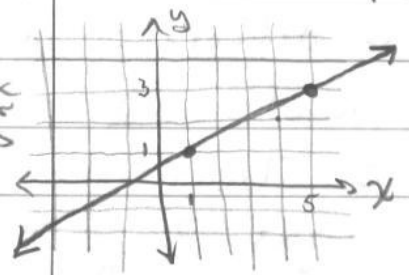
Thus $m = \frac{-3}{1} \leftarrow \Delta x$ So start from (0, 1),
 go down 3, and to the right 1.



★★ A slope cannot be
 graphed. Slope just
 tells how to get from
 one point to another on
 a line !! It is a relationship

Slope always
 determined going
 from left
 to right

ex: Tell the slope of the following line:



Count boxes up and to the right
 from one point to the next:

$m = \frac{2}{4} = \frac{1}{2}$

★ Or just get coordinates of points
 (1, 1) and (5, 3) and plug into $m = \frac{y_2 - y_1}{x_2 - x_1}$