Lesson 12: Equation of a Line (Slope Intercept Form)

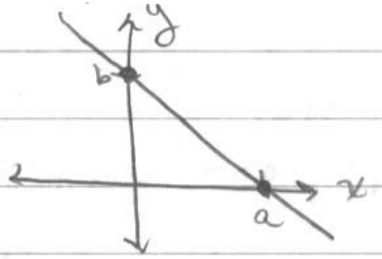
Recall the point slope form:

$y - y_1 = m(x - x_1)$ where x_1 and y_1 are the coordinates of a point on the line and m is the slope of the line.

Recall what an intercept is:

x-intercept = the point when the line crosses the x-axis

y-intercept = the point where the line crosses the y-axis



To be on the y-axis, the x-coordinate of the point must be 0 (why?)

To be on the x-axis, the y-coordinate of the point must be 0 (why?)

Then the y-intercept has coordinates $(0, b)$, where b can be any number and the x-intercept has coordinates $(a, 0)$ where a can be any number.

Recall that knowing any one point on the line and the slope of the line is enough to graph the line and find its equation.

Task: Find the equation of the line with slope m which goes through the point $(0, b)$.

(x_1, y_1)

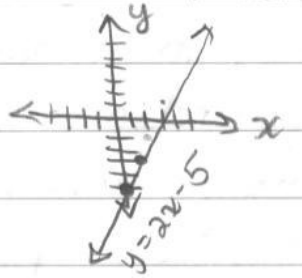
$$y - y_1 = m(x - x_1)$$
$$y - b = m(x - 0)$$
$$y - b = mx$$
$$+b \quad +b$$
$$y = mx + b$$

* Whenever we have an equation of that form ($y = \#x + \#$), we know that the number in front of x is the slope and the number without the x is b, which is the y-coordinate of the y-intercept, and we can graph.

* $y = mx + b$ is called the slope intercept form (m = slope, b = intercept)

ex: Graph $y = 2x - 5$

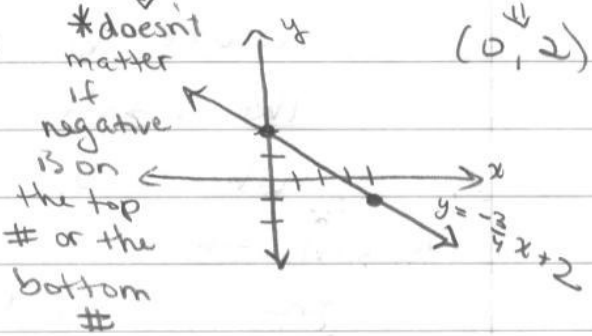
$m = \frac{2}{1}$ $\frac{\text{rise}}{\text{run}}$ $b = -5$ $(0, -5)$



ex Graph $y = -\frac{3}{4}x + 2$

$m = -\frac{3}{4}$ $b = 2$ $(0, 2)$

* doesn't matter if negative is on the top # or the bottom #



* if the equation is not in $y = mx + b$ form, just solve for y.

ex: $3x - 2y = 5$

* since it is easy to graph using $y=mx+b$, if the equation is not in $y=mx+b$ form, just solve for y and graph.

ex: $3x-2y=5$

$+2y \quad +2y$

$3x = 5+2y$

$-5 \quad -5$

$\frac{3x-5}{2} = \frac{2y}{2}$

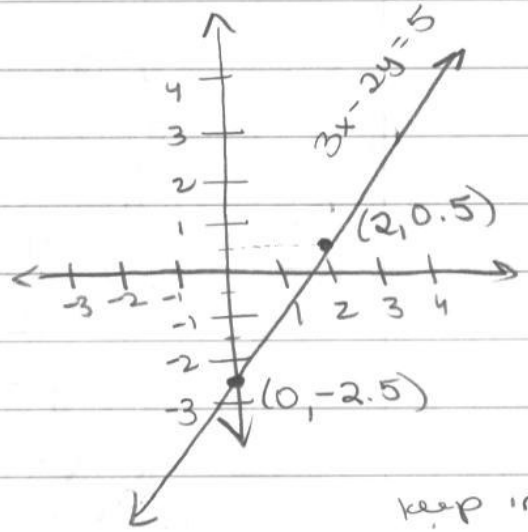
$\frac{3}{2}x - \frac{5}{2} = y$

so $m = \frac{3}{2}$

$b = -\frac{5}{2} = -2.5$

$(0, -2.5)$

make sure to distribute, it is $y=mx+b$, not $y=\frac{mx+b}{a}$



keep in mind

$-2.5 + 3 = 0.5$

ex: $-y+x-1=0$

$+1 \quad +1$

$-y+x=1$

$-x \quad -x$

$\frac{-y}{-1} = \frac{1-x}{-1}$

$y = -1 + x$

$m = \frac{1}{1}$

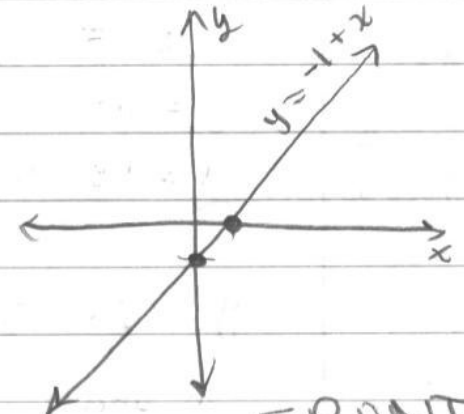
$b = -1 \Rightarrow (0, -1)$

(could have also done:

$-y+x-1=0$

$+y \quad +y$

$x-1=y$)



* notice order doesn't matter since $mx+b = b+mx$

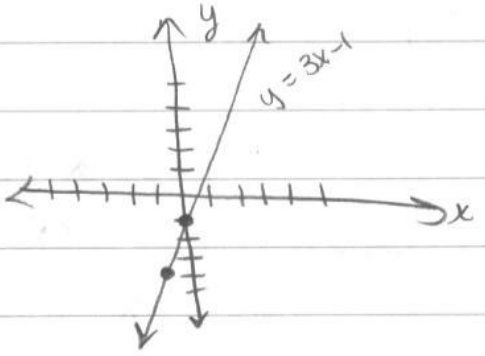
SLOPE IS ALWAYS IN FRONT OF X

★ Point-Slope form always simplifies to slope-intercept form:

ex: give the equation of the line with slope 3 and which goes through $(-1, -4)$

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = 3(x - (-1))$$
$$y + 4 = 3(x + 1)$$
$$y + 4 = 3x + 3$$
$$-4 \qquad -4$$
$$y = 3x - 1 \quad \leftarrow \text{so} \quad m = 3,\ b = -1 \Rightarrow (0, -1)$$

check: graph the line using initial conditions and see if its y-int is $(0, -1)$



yes! $(-1, -4)$
$+1 \quad +3$
$(0, -1)$ ☺

$y = mx + b$ form in disguise:

vertical lines

ex: $x = 5$

$y = mx + b$

$0y = 1x - 5$

$\frac{0y}{0} = \frac{1x}{0} - \frac{5}{0}$

$y = \frac{1}{0}x - \frac{5}{0}$

↑ m undefined

we have no y's, one x, and one loose #

$0y = 1x - 5$

$0 = x - 5$

$+5 \quad +5$

$5 = x$ ☺

horizontal lines

ex: $y = 3$

$y = mx + b$

one y, no x's, one loose #

$1y = 0x + 3$

$y = 0x + 3$

↑ $m = 0$

lines through the origin (0,0)

ex: $y = 2x$

$y = 2x + 0$

one y, 2 x's, no loose #'s

↑ $b = 0$

(makes sense since our y-int is 0)