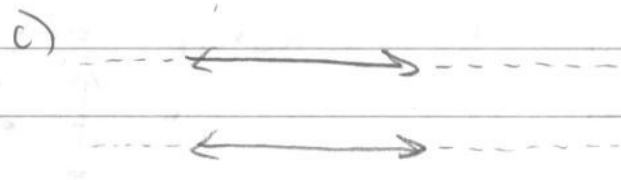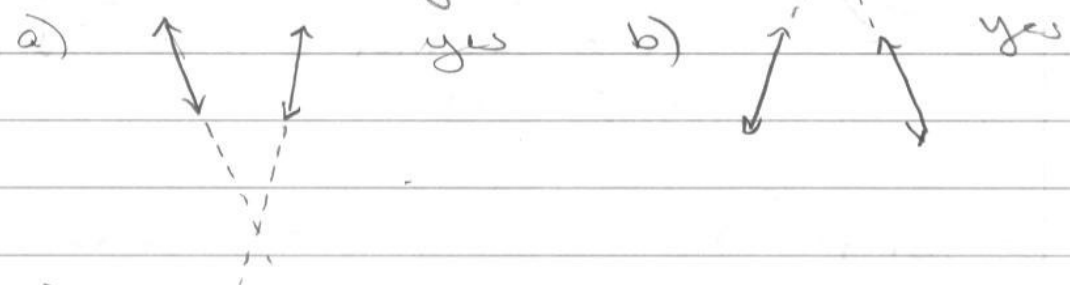Lesson 13: Parallel and Perpendicular Lines

What are parallel lines?
- Lines that never intersect
- Symbol for parallel: ||
Do the following intersect?

a)   yes   b)   yes

c)   No. Parallel lines
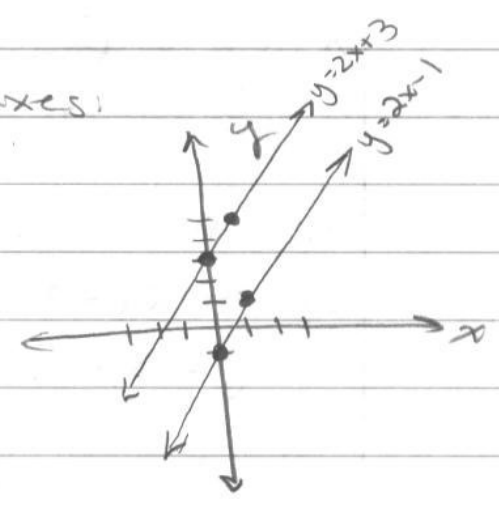   are like railroad
                             tracks, both in
                             the same direction,
                             same angle

What do parallel lines have in common?
→ lines are described by points and slopes
→ they don't have points in common since
    they don't intersect
☆ → slopes must be the same (same direction)

ex: graph on the same axes:
$$y = 2x + 3 \quad m = 2 \\ b = 3$$
$$y = 2x - 1 \quad m = 2 \\ b = -1$$

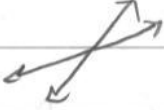☆ So parallel lines
have equal slopes

What are perpendicular lines?
- lines that intersect at a right (90°) angle
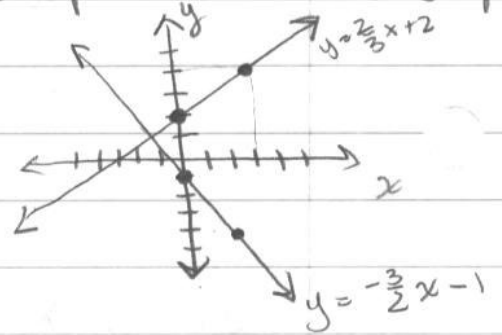- symbol for perpendicular: ⊥

Are the following perpendicular?

a) no, angle of intersection is not 90°

b) yes

ex. Graph the following on the same axes to determine the relationship between slopes of ⊥ lines.

$y = \frac{2}{3}x + 2$ $m = \frac{2}{3}$ $b = 2$

$y = -\frac{3}{2}x - 1$ $m = -\frac{3}{2}$ $b = -1$

$y = \frac{2}{3}x + 2$

$y = -\frac{3}{2}x - 1$

★. The slope of one is $\frac{2}{3}$, and of the other is $-\frac{3}{2}$. So the fraction is flipped, and negated. This is called a negative reciprocal.

★ The slopes of ⊥ lines are negative reciprocals of each other

Are the following parallel, perpendicular, or neither?

a) $y = 2x - 3$
   $y = 5 + 2x$
   ⇓
   parallel, $m = 2$ for both

b) $y = \frac{1}{2}x - 1$
   $y = 2x + 1$
   ⇓
   neither
   $m = \frac{1}{2}$ for one
   $m = 2$ for the other, not neg recip!

c) $y = \frac{2}{3}x - 5 \quad \Leftarrow m = \frac{2}{3}$

$2y = -3x - 8 \quad \Leftarrow \quad \frac{2y}{2} = \frac{-3x - 8}{2}$

$y = -\frac{3}{2}x - \frac{8}{2} = -\frac{3}{2}x - 4$

$m = -\frac{3}{2}$

not in $y = mx + b$, so can't just take the # in front of x, must put it into $y = mx + b$ form first

So $\perp$, since $\frac{2}{3}$ and $-\frac{3}{2}$ are neg. recip.

ex: Give the equation of the line that is parallel to $2x + y = 8$ and goes through $(1, 2)$

$\Rightarrow$ To give equation of line, need to use point slope formula, so we need a point (we have $(1,2)$) and a slope

$\Rightarrow$ Since our line is $\parallel$ to $2x + y = 8$, then the slope of OUR line is equal to the slope of $2x + y = 8$. What is the slope of $2x + y = 8$?

$2x + y = 8$

$\underline{-2x \qquad -2x}$

$y = -2x + 8 \Rightarrow m = -2$

$\Rightarrow$ So for our line, $m = -2$, $(1,2)$ (all we need)

$y - y_1 = m(x - x_1)$

$y - 2 = -2(x - 1)$

$y - 2 = -2x + 2$

$\underline{+2 \qquad\qquad +2}$

$y = -2x + 4$

ex: Give the equation of the line that goes through $(2,-3)$ and is perpendicular to $y = 5x + 1$.

$\Rightarrow$ Need to use $y - y_1 = m(x - x_1)$ so need point (which we have, $(2,-3)$) and slope.

$\Rightarrow$ Since our line is $\perp$ to $y = 5x + 1$, OUR slope is the negative reciprocal of the slope of $y = 5x + 1$. What is the slope of $y = 5x + 1$? $m = \frac{5}{1}$. So OUR slope is $-\frac{1}{5}$.

$\Rightarrow$ OUR line $\Rightarrow m = -\frac{1}{5}$, $(2,-3)$

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = -\frac{1}{5}(x - 2)$$
$$y + 3 = -\frac{1}{5}x + \frac{2}{5}$$
$$\underline{-3} \qquad\qquad \underline{-3\left(\frac{5}{5}\right)}$$
$$y = -\frac{1}{5}x + \frac{1}{5} - \frac{15}{5} \Rightarrow y = -\frac{1}{5}x - \frac{14}{5}$$

what is the $y = mx + b$ form of horizontal and vertical lines?

a) vertical $\Rightarrow x = 2$

$$y = mx + b$$
$$\text{why?} \leftarrow \quad 0y = 1x - 2$$
$$\frac{}{0} \qquad \frac{}{0}$$
$$y = \frac{1}{0}x - \frac{2}{0}$$
$$\underset{\uparrow}{} \quad m \text{ undefined}$$

$0y = 1x - 2$
$0 = x - 2$
$\underline{+2} \quad \underline{+2}$
$2 = x$

$\bigstar$ a line $\parallel$ to vertical line is also vertical $(x = \#)$

$\bigstar$ a line $\perp$ to vertical is horizontal $(y = \#)$

b) horizontal: $y = 2$

$$y = mx + b$$
$$y = 0x + 2$$
$$\underset{\uparrow}{} \quad m = 0$$

$\bigstar$ a line $\parallel$ to horizontal is horizontal $(y = \#)$

$\bigstar$ a line $\perp$ to horizontal is vertical $(x = \#)$