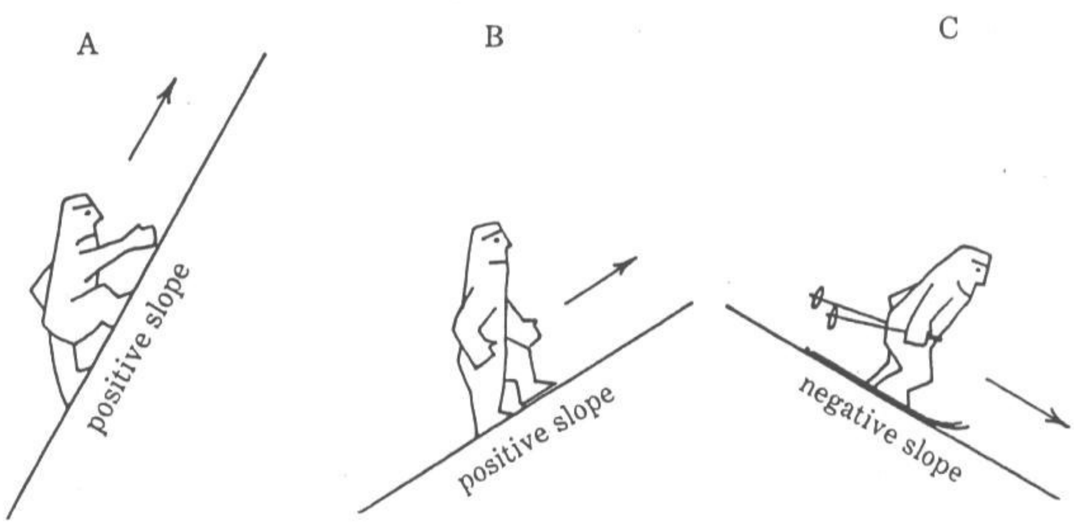
Lesson 10

Slope

In this lesson we build a way of measuring the steepness of a line. **Slope** is the word we use to describe steepness.

Man A is walking up a steeper line than man B. Since Line A is steeper than Line B, we say Line A has a greater slope than Line B. We also say that the slope is positive if the line goes up as you walk from left to right. Lines A and B have positive slope. If the line goes down as you walk from left to right, the slope of the line is negative. So Man C is on a line with negative slope. Be careful! You must always think "left to right."



To measure the slope of the line in Figure 1 notice that as the man goes from P to Q he goes up 3 units while going to the right one unit. We say the slope of this line is $\frac{3}{1}$ which is +3.

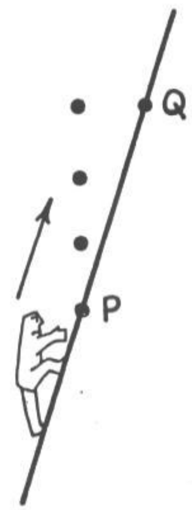


Figure 1

To measure the slope of the line in Figure 2 notice that as the man goes from A to B he goes up 2 units while going to the right one unit. The slope is $\frac{2}{1}$ which is +2. Suppose the man walks from A to C. Then he goes up 4 units while going 2 units to the right. The slope of the line is $\frac{4}{2}$ which is +2. Notice we get the same slope no matter which two points on the line we pick.

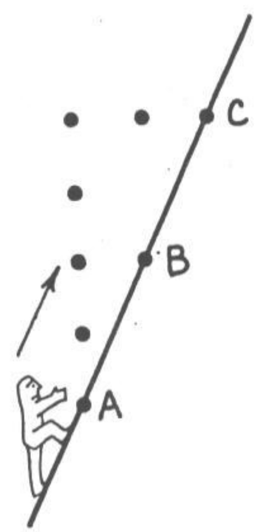


Figure 2

Let's measure the slope of the line in Figure 3 in three different ways.

(1) From P to R. Two units up, two units to the right. Slope is $\frac{2}{2}$ which is 1.
(2) From P to S. Three units up, three units to the right. Slope is $\frac{3}{3}$ or 1.
(3) From Q to R. One unit up, one unit to the right. Slope is $\frac{1}{1}$ or 1.

Note again that no matter what two points on this line we pick, we get the same slope, +1.

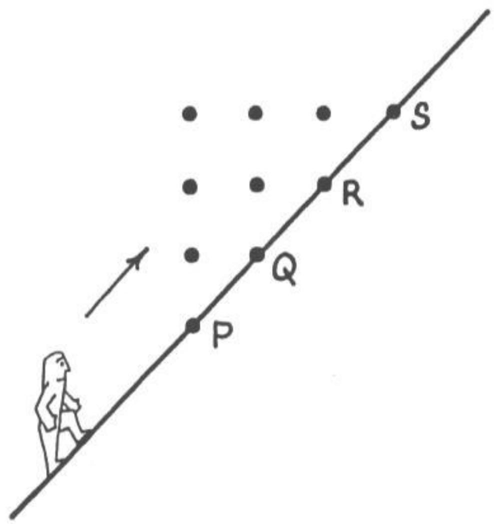


Figure 3

Measure the slope of the line in Figure 4. From D to E we go up two units and to the right three units. The slope is $+\frac{2}{3}$.

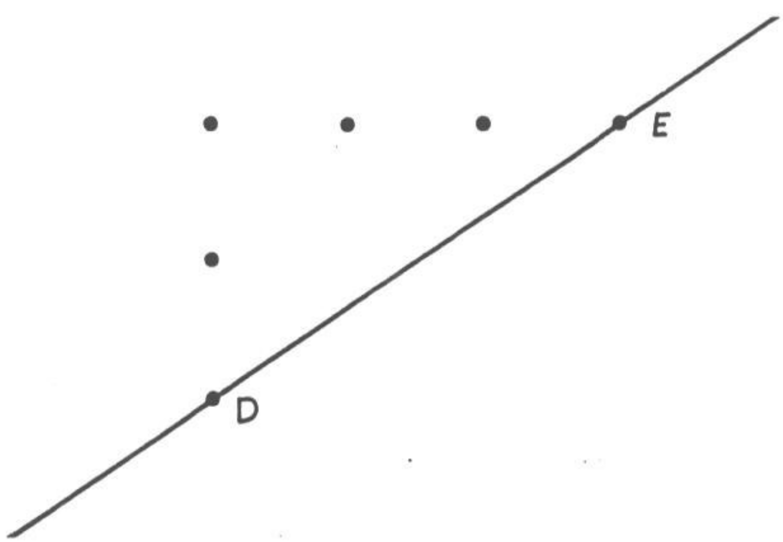


Figure 4

To measure the slope of the line from P to Q in Figure 5, we go down five units and to the right two units. So the slope is $\frac{-5}{2}$. Notice that since we went down five units, we use negative five. Since $\frac{-5}{2}$ is the same as $-\frac{5}{2}$, the slope of this line is a negative number. Just as in graphing, we use positive numbers for Up ↑ and Right → and negative numbers for Down ↓ and Left ←.

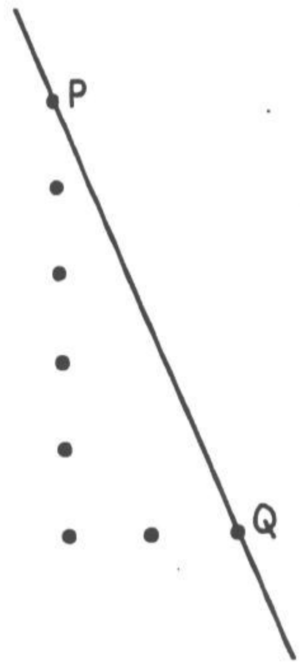


Figure 5

For each of the lines in Figures 1 thru 5 the slope is the change in height divided by the change in level distance.

$$\text{Slope} = \frac{\text{Change in height}}{\text{Change in level distance}}$$

Shorthand for the words "change in height" is Δy (pronounced delta y). Shorthand for the words "change in level distance" is Δx (pronounced delta x).

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$

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EXAMPLE 1 Find the slope of the line in Figure 6.

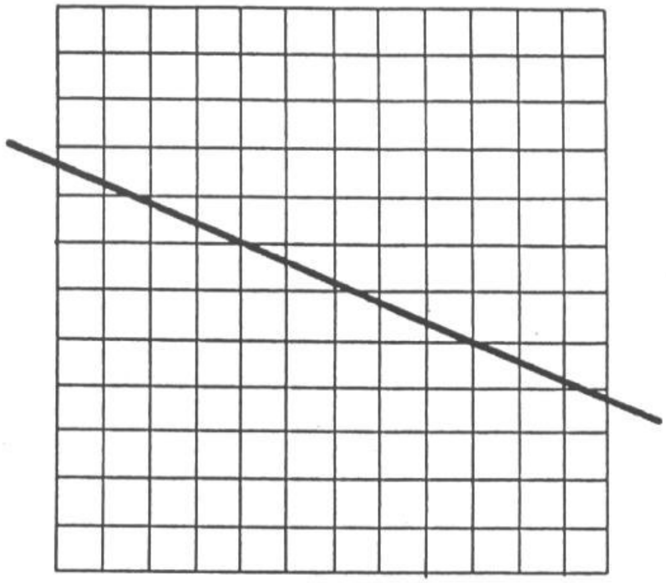

Figure 6

First, select any two points on the line. In Figure 7, we selected points A and B.

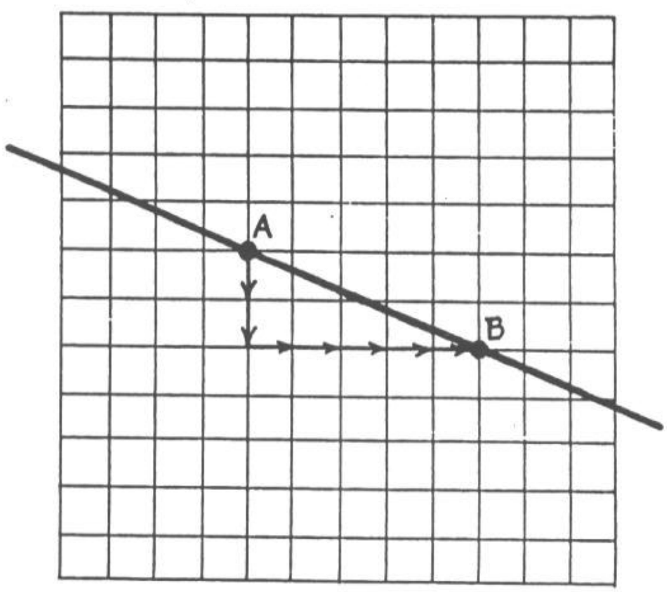

Figure 7

From A to B we go 2 units down (−2) and 5 units to the right (+5). Thus $\Delta y = -2$; $\Delta x = +5$.

So the slope of the line $= \dfrac{\Delta y}{\Delta x} = \dfrac{-2}{+5} = -\dfrac{2}{5}$.

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EXAMPLE 2 Find the slope of the line in Figure 8.

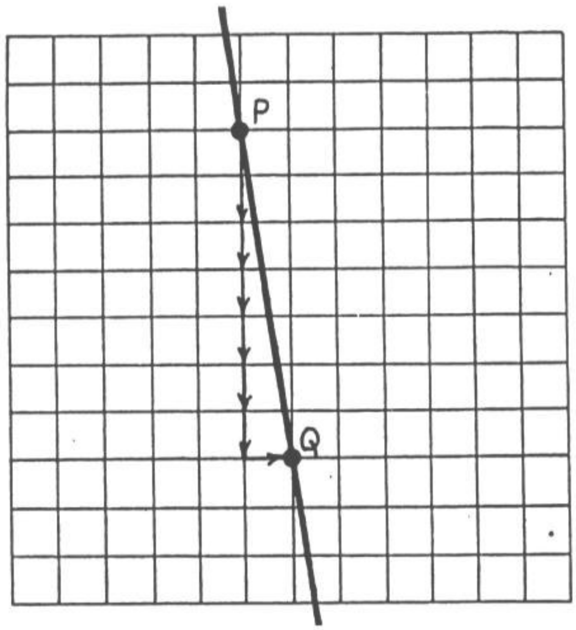
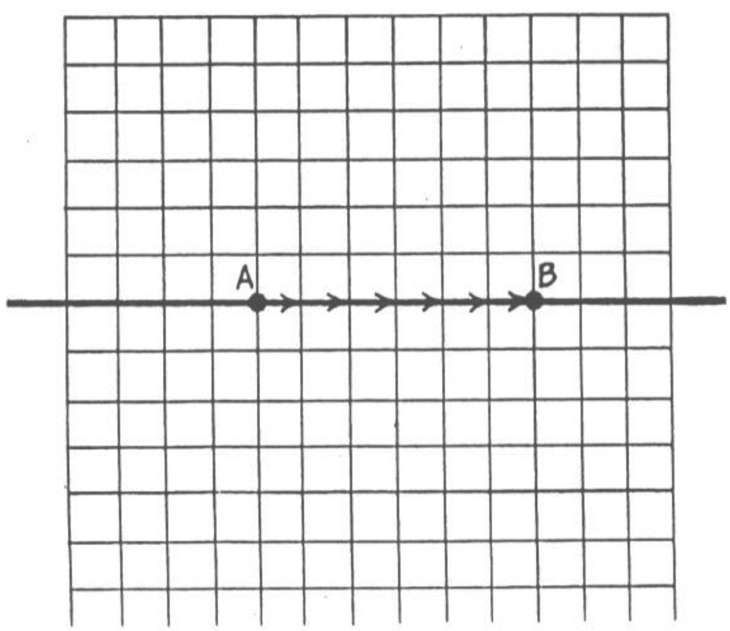


Figure 8

To find the slope of the line in Figure 8, select two points, say P and Q. From P to Q, we go 7 units down (−7) and 1 unit to the right (+1). Thus $\Delta y = -7$ and $\Delta x = +1$.

So the slope of the line $= \dfrac{\Delta y}{\Delta x} = \dfrac{-7}{+1} = -7.$

EXAMPLE 3 Find the slope of the line in Figure 9.



To find the slope of the line in Figure 9, select two points on the line, say A and B. Since A and B are at the same height, Δy is zero (you don't go up or down). Δx is 6. So the slope $= \frac{\Delta y}{\Delta x} = \frac{0}{6} = 0.$

The slope of every horizontal line is zero, and every line whose slope is zero is horizontal.

EXAMPLE 4 Find the slope of the line in Figure 10.

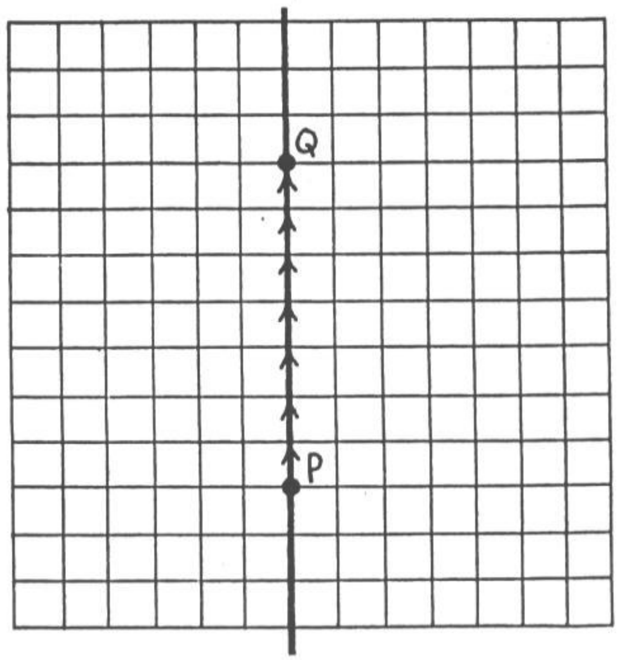


Figure 10

To find the slope of the line in Figure 10, select two points on the line, say P and Q. To go from P to Q you go up 7 units and over 0 units. So $\Delta y = +7$ and $\Delta x = 0$.

But slope $= \frac{\Delta y}{\Delta x} = \frac{+7}{0}$. Since $\Delta x = 0$ and division by zero has no meaning, we say that the slope of this line is **undefined**; that is, the line doesn't have any slope. The slope of every vertical line is undefined, and every line with undefined slope (that is, without a slope) is vertical.

In all the examples above we were given a line, and the job was to find the slope. Now we will be given a slope, and the job will be to draw a line with that slope.

EXAMPLE 5 Draw a line with a slope of $\frac{5}{7}$.

To draw a line with a slope of $\frac{5}{7}$ you write

$$\text{Slope} = \frac{5}{7} = \frac{\Delta y}{\Delta x}$$

So we can let $\Delta y = +5$ and $\Delta x = +7$.

Start from any point in Figure 11, say P. Go up 5 units (since $\Delta y = +5$), go to the right 7 units (since $\Delta x = +7$), and make a dot; call it Q. Draw a line through P and Q. This line has the slope of $\frac{5}{7}$.

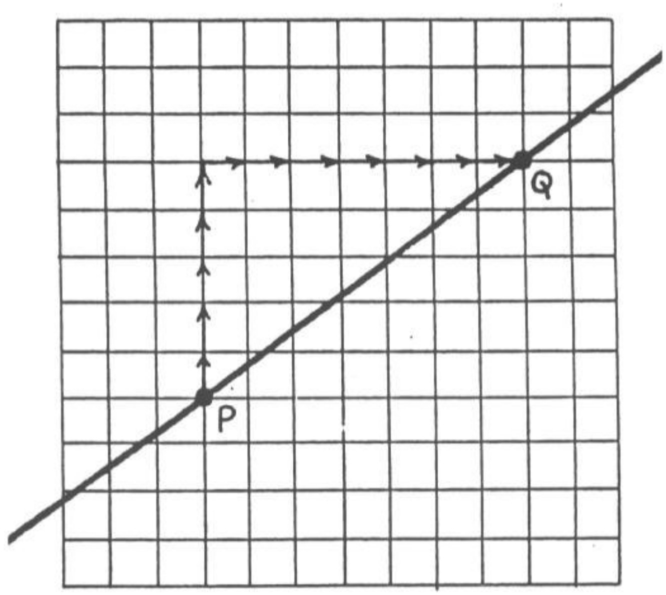


Figure 11

Note: The line in Figure 11 goes up as we go from left to right. This is what we expected since its slope, $\frac{5}{7}$, is positive.

EXAMPLE 6 Draw a line with slope $-\frac{2}{3}$.

To draw a line with slope $-\frac{2}{3}$ (since $-\frac{2}{3} = \frac{-2}{3}$), we write

$$\text{Slope} = \frac{-2}{3} = \frac{\Delta y}{\Delta x}$$

So we can let $\Delta y = -2$ and $\Delta x = +3$.

Start with any point in Figure 12, say A. Go down two units (since $\Delta y = -2$), go 3 units to the right (since $\Delta x = +3$), make a dot; call it B. To be sure we did not make any mistakes we can repeat the process starting from B, and obtain another point C. Draw a straight line through A, B and C. This line has the slope of $-\frac{2}{3}$.

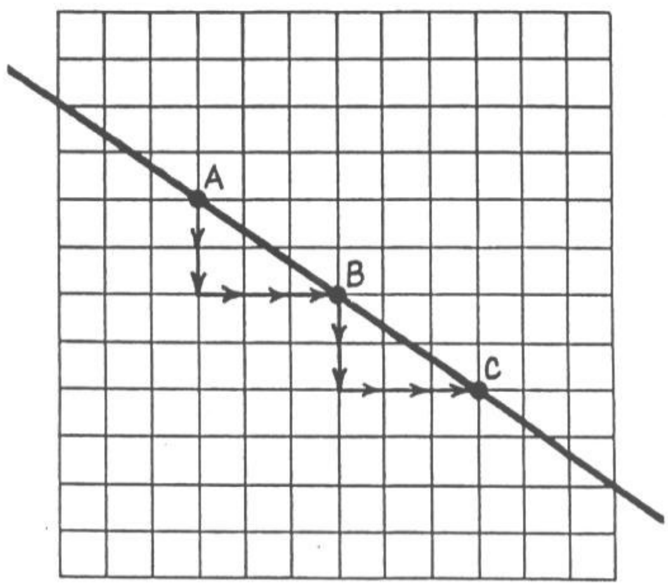


Figure 12

Note: The line in Figure 12 goes down as we go from left to right. This is what we expected since its slope, $-\frac{2}{3}$, is negative.

EXAMPLE 7 Draw a line with slope 3.

To draw a line with slope 3, we write

$$\text{Slope} = 3 = \frac{\Delta y}{\Delta x}$$

It would be nice to write 3 as a fraction, say $\frac{3}{1}$ or $\frac{6}{2}$ or $\frac{9}{3}$ or something else equal to 3. Let's choose $\frac{3}{1}$.

$$\text{Slope} = \frac{3}{1} = \frac{\Delta y}{\Delta x}$$

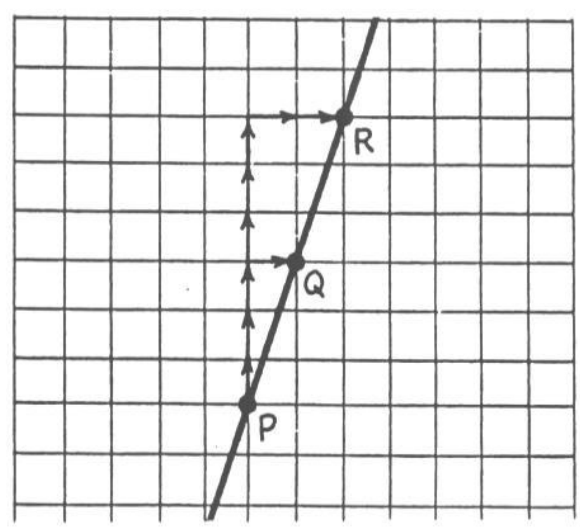
So we can let $\Delta y = +3$ and $\Delta x = +1$.

Start with any point in Figure 13, say P. Go up 3 units (since $\Delta y = +3$), go 1 unit to the right (since $\Delta x = +1$), and make a dot; call it Q. Draw a line through P and Q. This line has the slope of 3.

Suppose we had chosen $3 = \frac{6}{2}$.

$$\text{Slope} = \frac{6}{2} = \frac{\Delta y}{\Delta x}$$

And again, starting at P, go up 6 units (since $\Delta y = +6$), go 2 units to the right (since $\Delta x = +2$), and make a dot; call it R. Notice that the point R is on the line that we drew through P and Q.



EXAMPLE 8 Draw a line with slope 0.6.

To draw a line with slope 0.6, we change the decimal number to a fraction. 0.6 can be written as $\frac{6}{10}$ or $\frac{3}{5}$.

$$\text{Slope} = 0.6 = \frac{3}{5} = \frac{\Delta y}{\Delta x}$$

So we can let $\Delta y = +3$ and $\Delta x = +5$.

Start with any point in Figure 14, say P. Go up 3 units and then go 5 units to the right and make a dot. We labeled this dot Q. A line through P and Q has slope 0.6.

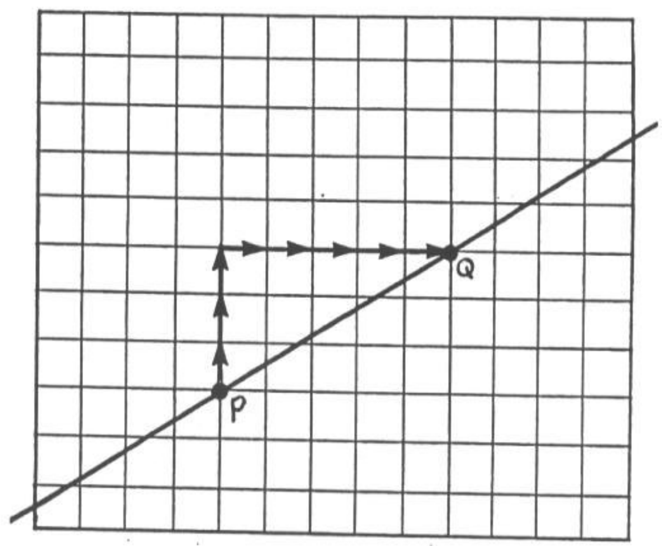


Figure 14

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Exercises

1. Calculate the slope of the line in Figures 15 and 16.

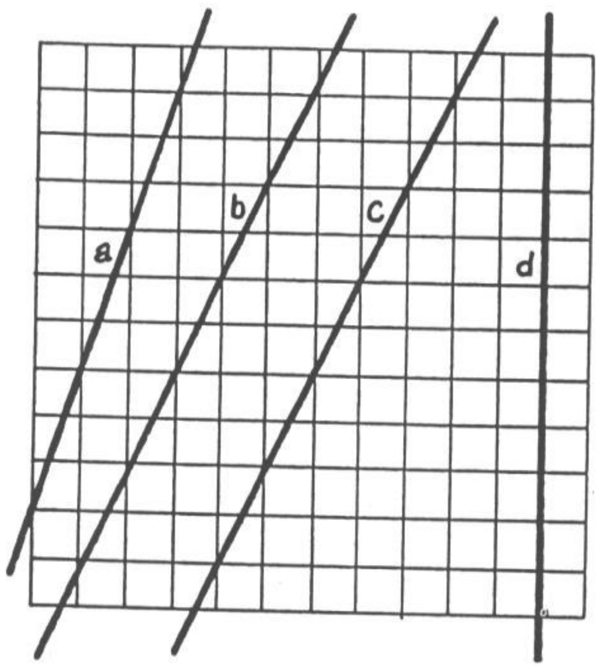

Figure 15

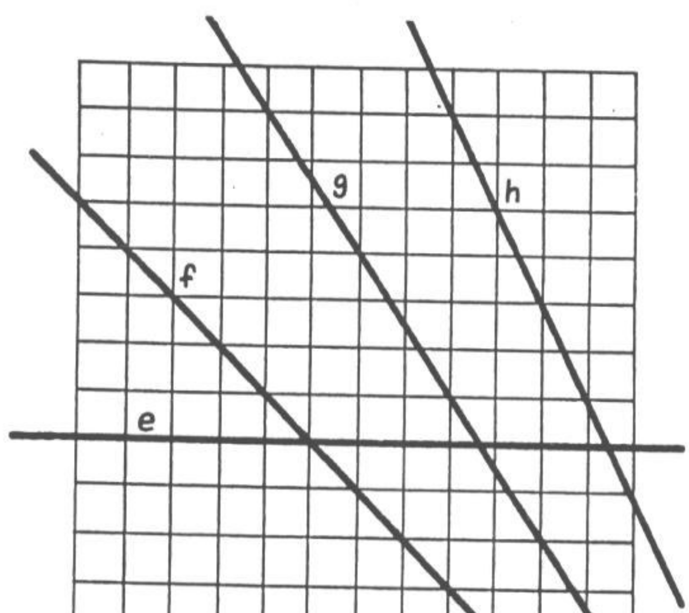

Figure 16

2. Match each line in Figures 17 and 18 with one of the given slopes.

<u>Slope</u>

$\frac{2}{3}$

-3

0

$\frac{3}{4}$

undefined

-1

$-\frac{2}{5}$

1

$\frac{1}{2}$

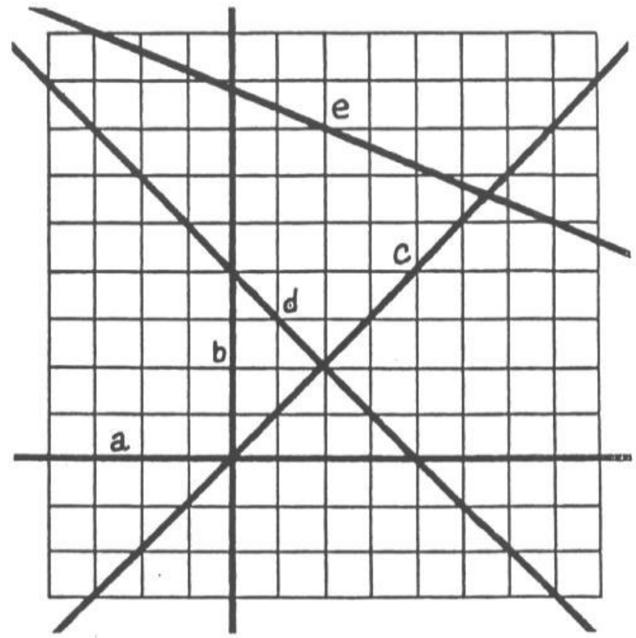

Figure 17

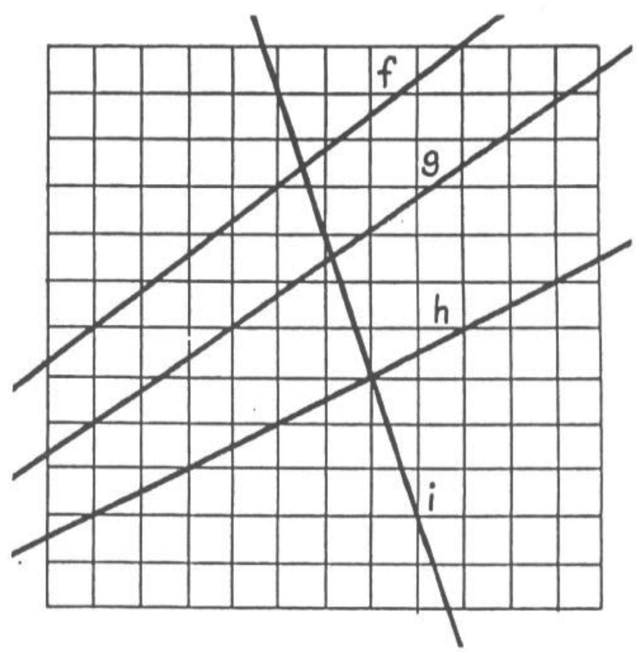

Figure 18

3. From the point P in Figure 19, draw lines a, b, c and d with the given slopes. Identify each line.

Line	Slope
a	1
b	−1
c	2
d	−2

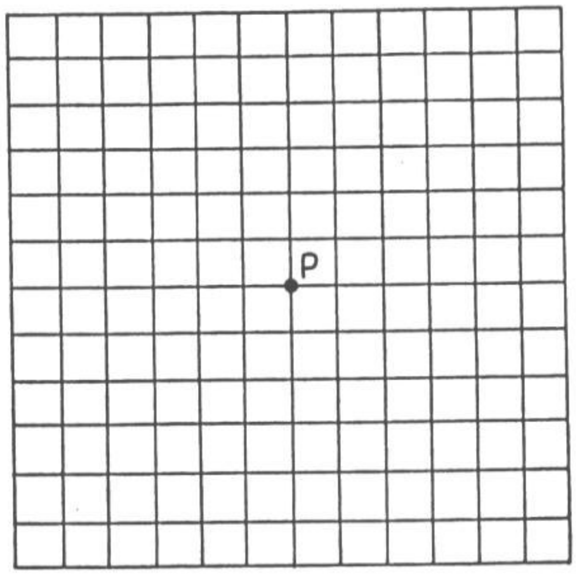

Figure 19

4. From the point P in Figure 20, draw lines e, f, g and h with the given slopes. Identify each line.

Line	Slope
e	$\frac{1}{3}$
f	$-\frac{1}{3}$
g	$\frac{2}{5}$
h	$-\frac{2}{5}$

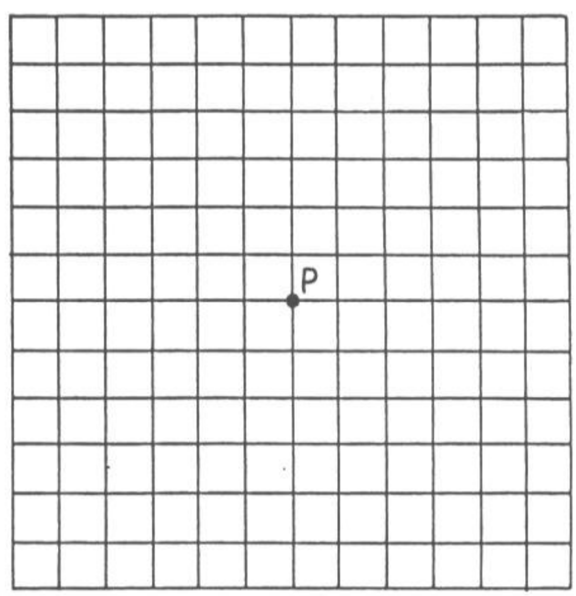

Figure 20

5. From the point P in Figure 21, draw lines i, j, k, l and m with the given slopes. Identify each line.

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Line	Slope
i	0
j	$\frac{3}{2}$
k	$-\frac{3}{2}$
l	$\frac{4}{5}$
m	0.5

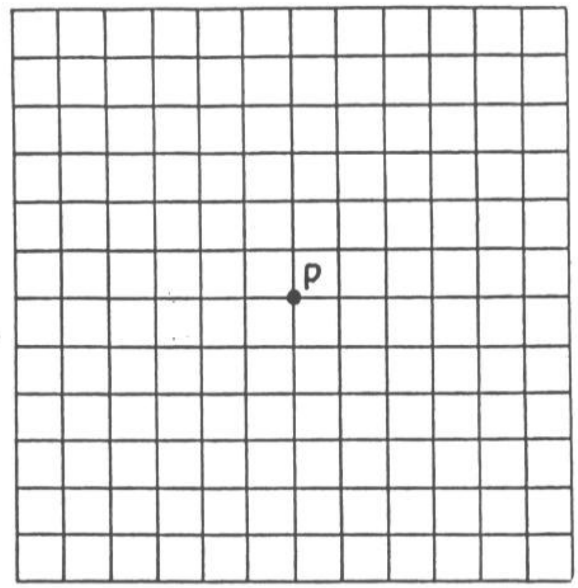

Figure 21