Differential equations and spatial relations for oscillating and radiating systems

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Differential equations and spatial relations for oscillating and radiating systems

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Summary. Traditionally, indoor heating is connected to power in W, but translation into room temperature is not direct. Here, we will consider formats related to differential equations and qualitative experiments. Modeling within the framework of thermodynamics, Poisson's equation and homology are combined. The results are functional relations for temperature and visualisation of shapes and paths where heat flows, spreads, propagates and flux.
Keywords: Temperature, Functions, Lamp, Experiments, Heat, Boundary Conditions (BC), Maxima-online, thermal efficiency

1. Introduction

Within fluid dynamics, various thermal devices, such as heaters, boilers, engines, pulsating combustion applications and dryers, are modelled with an input heat source (fuel-input or direct temperature), and differential equations for structural response in e.g. pipes, cover and valves. The systems are sometimes characterised as hyperbolic and elliptic for the main variables temperature and pressure (velocities, etc), and the behaviour changes between those states and its solutions.

The present paper concerns a thermal system where input heat is from an electric lamp inside a coverage of glass, c.f. Figure 1, left.

In modeling, we consider a linear second order system. At an harmonic oscillation input, the hyperbolic system has a solution that corresponds to heating of adjacent materials that, in turn, radiates.

To describe the temperature in shields around a lamp, two nonlinear functions are deduced. Related qualitative behaviour is ealuated in conjunction with models. Thereafter, aspects of the room will be compared to, in terms of rotations, elasticity, friction and Einstein relativity.

2. Material and methods

In many lamps, the functional format of a Sombrero, Figure 1, right, is descriptive for the heated air inside. The shape radiates upwards and also to the sides, since that area is relatively large.

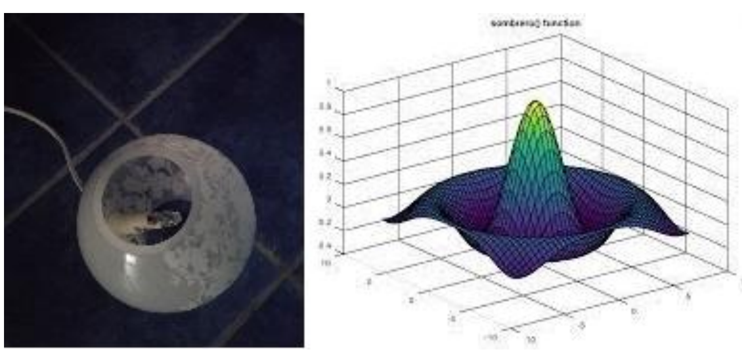

Figure 1. Left: A lamp with a spherical glass shield, thickness <1cm. Right. Temperature distributions around a more common lamp, radiating some heat.

The lamp where heat properties will be analysed and enhanced is shown in Figure 1, left. For that purpose, two actions are proposed, (Figure 2):

- to extend (and close) the cover with a cylindrical metal can
- to put a distance foot, allowing air from below, (into a possible circulation)

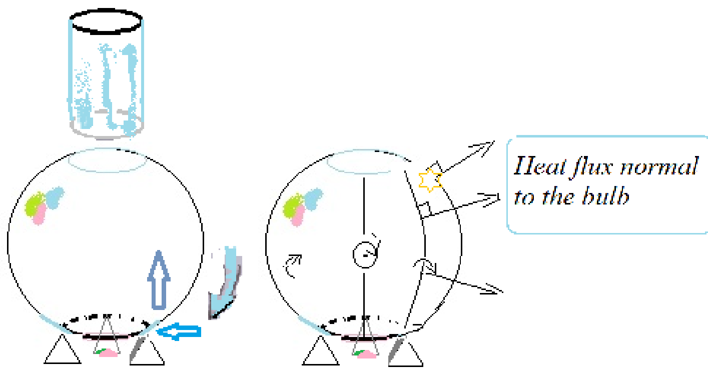

Figure 2. Left: The lamp with a cover on top and a 3-foot distance below. The arrows show how cold air goes downwards, (causing a rotational field meso-scale, perpendicular). Right: Rotation at the surface on a minor scale. Outward heat flux at meso and macro scale

In general, for a medium warm interior, other flows are likely to happen. Ruled by gravity, buoyancy and dynamics, cold air might go downwards, enter and cause a rotational field, perpendicular, as sketched in Figure 2.
If large scale, and dominating, the resulting flow could be an obstacle, not beneficial for heat flux. Instead, a special sheet leading heat to the glass bulb might be favourable, i.e. produce more heat in vicinity of the lamp. The original design is without air inlets from down under and a Dirac-distributed [1] temperature interior, Figure 1. When a macro-scale metal is attached directly on the lamp, high heat is spread and occasionally chemical reactions [2], into smoke.
However, also a composed flow might organise into spreading heat. This will be proposed in modeling below.

3. Results

The metal can becomes warm. The glass surface remains at constant temperature. Probably the distance is too large, to capture into heat radiation. This will be modelled below in Section 4.3-4.5.

4. Models

In the present section, physical models for the lamp-light and heat will be proposed. Both homologies (in terms of formations by space matter) and mathematical formulas will be used

4.1 Spatial modeling

A model for the flow is given by v=wxr, where w is angular velocity [3], x is the vector product and r is a radial coordinate. On a smaller scale, such an alternating flow(axial vector drifting/precession), can make a path on the surface, c.f. Figure 2, right. Partly, light matter may follow curved paths. This together with a perpendicular outward heat flux, have similarities with the composed motions derived and analysed in [4].

4.2 Model for EM, light, pressure and heat

AC realisations into lamps are surrounded by a periodic field due to the nature of electro-dynamics and light. For certain materials and geometries at meso-scale (e.g. airtight sealed bulbs, layers close to loaded boundaries & in wakes), this results in an oscillating pressure. In some materials and layer media, an increased pressure raises the temperature. When the delay time, for drop in temperature, is longer than the periods in the pressure oscillation or/& temperature gradients are higher towards colder areas, the body part region becomes 'steadily warm' and may radiate outwards, with its own quasi-static physics. This is observed in e.g. bulb lamps. A more simplified explanation, from micro scale to macro, is that the lamp radiates heat. That reveals no information about possibilities to magnify at a meso scale.

4.3 Amplification of oscillation in a hyperbolic system

A model with linear differential equations will be scrutinised.
Consider a time-dependent system (t, f,finp)that fulfils the differential equation
$f'' + Df=f_{inp}$

For D>0, it's elliptic and for D<0 hyperbolic. Since the solutions to the homogenous hyperbolic equation are not bounded, the change into such is sometimes known as unstable. Here, we will consider D<O, at an oscillating harmonic input and find how these interact when an additional time scale; 1/w and internal variables; C, B, D.

With f_{inp}=Ccos wt +BD

a solution is given by $f=C/(D-w^2)\cos wt+B(1-\exp(-(-D)^{0.5} t))$ .....(1)

Remark. Increased response with superimposed oscillation is obtained (but not the same with more amplification close to the eigen frequency as in an elliptic system.)

For certain values of parameters, the function is given in Figure 3.

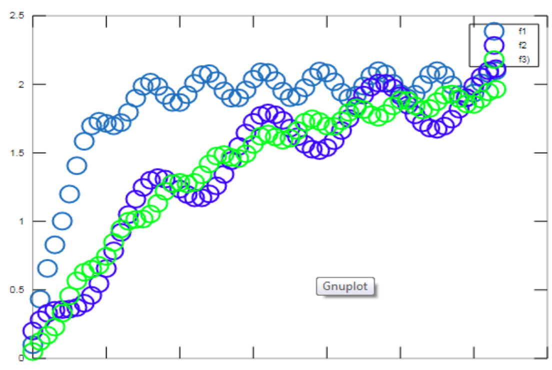

Figure 3. The function f(t) from (1) for the parameters given in the Script; Octaveonline [5]
t = 0:0.1:6.3; plot (t, 0.1*cos(8*t)+2*(1-exp(-2*t)), "o;f1;", t, 0.2*cos(4*t)+2*(1-exp(-0.5*t)), "ob;f2 ;",t, 0.05*cos(10*t)+2*(1-exp(-0.5*t)), "og;f3);");

For a subsystem e.g. a part of a shield: When starting to radiate, it consumes less power, and the input is not affected. The internal variables correspond to energy and interaction with the surrounding.

4.4 Functional expression from a MacLaurin expansion

A Maclaurin expansion of temperature T, at a spatial point 0, gives a functional expression T(r), where r is the radial coordinate for distance. Assuming a variable gradient given by so-called boundary flux [6], T(r) becomes nonlinear ; reading

$T(r)=(T_0-rhT_{ext})/(1-hr)$ .... (2)

where T_0 is the temperature at r=0, h is the factor in the rule for boundary flux $h(T-T_{ext})$ and T_{ext} is a constant external temperature.

Exercise 1. Plot the graph of T(r) for various values of parameters.

Exercise 2. Show that there are parameters such that T'(r)<0.
Solution: For example with numerical values of the parameters, or by deriving bounds for different behaviour (i.e. increasing and decreasing with r).

Remark. A Taylor series, emphasizes the relative distance for the coordinate, and contains more information about space.

Here, the notations above will be used for brevity, and additional reductions will be considered for an example in the next section:

Proposition: For certain values of parameters, the function (2) is proportional to 1/r ... (3)

Temperature at two locations

Close to a source, another matter(shield) with a temperature distribution given by $T_m=C-D(r-r_1)^2$ ... (4)

is assumed. This is a solution to Poisson's equation; the energy balance when a heat source and static.

The temperature functions given by (3) and (4), both valid/put together, gives two locations in space; Figure 4.

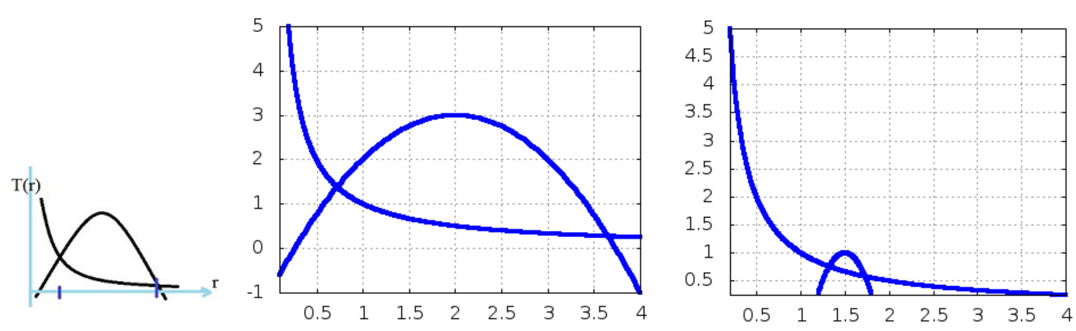

Figure 4. Intersection points of T(r) from (3) andT_m(r) given by (4), for two geometries. Left:

Qualitative. Quantitative, with Maxima online scripts. Center: draw2d (grid= true, color = blue,line_width = 4,explicit(1/x, x, 0.2, 4),explicit(3-(x-2)^2, x, 0.1, 4))$ Rightmost: draw2d (explicit(1/x, x, 0.2, 4),explicit(1-2*(2*x-3)^2, x, 1.2, 1.8)) $

The quadratic dependency is valid close to $r=r_1$. An interpretation is that shields, or a shield with a thickness will obtain those temperature. Then, if on a larger area it might radiate more efficiently. Without the shield, there is not so much contact with the surrounding. That is also found in the formula expression (2) alone: When T_{ext} is small, temperature may drop faster.

5. Concluding Remarks

Entropy

In statistical mechanics [7], the definition of entropy; S, is proportionality to kln Ω, where Ω is the number of available states for the molecules, and k is Boltzmann constant. Then, temperature is proposed, and could be considered proportional to entropy in intervals, which means that it increases when more dynamics, e.g. changes between states and by that friction.

In thermodynamics, entropy, is the energy conjugate to temperature. The change between various energies is given by e.g. adding the product TS, where T is temperature and S is entropy.

Intuitively, one may expect that the rotations of air around the sphere cause a decrease in temperature outside the shell/shield, since radiation outwards is obstructed. This agrees with increasing entropy when more states, thermodynamics when the product TS remains constant, and that the static pressure (assuming an ideal gas) drops when the air is moving.

Homology

With a spatial analogy; on a finer scale, we find organised motions that might promote outward heating: Radiation could be composed such that the normal flux begins after a rotational path on the surface, Section 4.1.

Matter formation with elastic energy

- The frictional motion on the surface, is connected to mechanical heat.
- According to [8], electric power may subdivide into rate of potential work for a viscous fluid, and also a varying temperature field contribute.
- In analogy with mechanics; while making a turn or part of laps, the air matter could mimic a torsional spring, providing energy for the perpendicular heat flux.

Einstein relativity

The torsion spring could be connected to length contraction in SR, e.g. as interpreted in [9].

Motion on a curved space relates to FLWR universe in GR, however we considered heat derived from light, (not same as moving light).

Applications would require experiments, e.g. 3D-printed [10] material that reach close to the lamp. Also elastic energy [11], could be invoked, adding into a motion between the lamp and the glass shield.

6. Concluding summary

- Magnification (and localisation) of heat radiation from lamp designs were analysed.
- Results for heat flux were deduced, and a measurment might give time harmonics [12].
- Temperature with two dependencies, gives locations in space.

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