A Two Phase Technique for Optimal Tessellation of Complex Geometric Models

Dinesh Shikhare, S. Gopalsamy and S. P. Mudur
Graphics and CAD Division,
National Centre for Software Technology,
Gulmohar Cross Road 9, Juhu, Mumbai 400049, India.
e-mail: {dinesh, gopal, mudur}@ncst.ernet.in

Abstract

This paper presents a two phase technique for grid generation over trimmed parametric surface patches that model components of the surface of a geometrically complex object like an aircraft or an automobile. In the first phase, the parametric domain of a trimmed surface patch is reparameterized to a computationally convenient parametric domain – a square. In the second phase, an initial surface grid generated using the new parametric domain is optimized with respect to an objective function. The objective function is non-linear and is formulated for three desirable properties – smoothness, orthogonality, and adaptivity. Simulated annealing method is used for obtaining solution to the non-linear optimization problem. The use of a simple reparameterized domain enables us to derive simple and efficient methods for choosing a valid initial grid and also for ensuring grid validity during the optimization phase.

Keywords: trimmed surface patch, reparameterization, grid generation, grid optimization, and simulated annealing.

1 Introduction

Most engineering analysis and visualization processes require tessellated representations of CAD system generated geometric models in the form of meshes made up of planar quadrilateral or triangular elements. Automatic tessellation of such trimmed surface patches has been a problem of significant interest in the graphics and CAGD fields for quite some time now. The right tessellation can tremendously improve the efficiency of the engineering process using the tessellated model. It is well known that mesh quality affects both efficiency and accuracy of CFD solutions [6].

One of the primary difficulties arises out of the fact that the surfaces are defined using higher order non-linear functions, typically, rational polynomial functions. Further, providing control over the tessellation in a fashion that can make the engineering process efficient makes it even more complex. The desired tessellation is one which satisfies a number of properties like control over shape of elements, mesh size optimality at the desired level of detail, distribution of elements in proportion to the variation in the property and smoothness in element size variation.

A large number of techniques have been proposed for automatic tessellation of trimmed surface patches, each with its own performance properties and its own application domain. Until recently most methods were simple parameter subdivision based tessellation techniques [1, 11], that are based on the efficient evaluation of the surface geometry and surface geometric properties, like tangents, normals, curvature etc. [13]. In [12], implementation of structured and unstructured grid generation procedures are described for trimmed parametric surface patches. For an unstructured grid, the trimmed parametric domain is used as such, and the quality of the triangles over the surface is controlled by computing them directly over the surface. Both advancing front and Delaunay methods have been extended to work directly on the physical domain. For a structured grid, the trimmed parametric domain is reparameterized to a square domain using Coons' patch [7]. Even though this works for many cases, there is always the possibility that the Coons' patch will extend beyond the original parametric domain particularly when it is concave, leading to invalid grids.

The aim of the work reported here is twofold: first, we need a simple method of ensuring valid grids at any time in the grid generation process, and second, we must achieve control over the quality of the grid specified in terms of smoothness, orthogonality and adaptivity in distribution of points. We describe a two phase technique, with the first phase called as reparameterization, serving the purpose of obtaining a suitable intermediate mapping method that ensures valid grids. The second phase in our technique is grid optimization for optimally positioning mesh points on the trimmed surface patches, once again ensuring grid validity when grid points are moved.

1.1 The Two-phase Optimal Grid Generation Technique

The surface patches presented to the tessellator may be defined over bi-parametric polynomial surfaces (e.g. NURBS) or over triangulated surfaces.

Reparameterization: In case of parametric surfaces, the pre-image of the trimmed patch may take any arbitrary shape. This pre-image forms a very inconvenient computational domain to work with. A convenient computational domain for placement of points on the surface patch boundary and the interior region is first obtained. This is done by deriving a new parameterization for the trimmed patch that gives a square parametric domain. Then onwards, the mapping of points on the surface patch is done via the new parameterization. In case of triangulated representation of the surface patch, the patch is parameterized by a method suggested by Floater [5]. Floater discusses parameterization of a surface defined by a triangulation. Even though the purpose is surface fitting for shape fidelity, his work contains interesting techniques which we have adapted and used in surface grid generation.

Grid Optimization: Phase two treats the different surface patch representations uniformly. In this phase, an objective function is formulated for optimizing the grid point positions satisfying the given boundary constraints and also maximizing the desired qualities. The function is formulated in terms of metrics derived from the surface patch and the topological relationships which are known on the computational domain. We have used simulated annealing algorithm [2] for this optimization.

2 Grid Generation Problem

Without any loss of generality, we discuss the specific formulation we have used for our problem of *structured quadrilateral grid generation on four-sided trimmed surface patch* both for ease of explanation and for the relevant examples.

Commonly the computer models of geometric entities are composed of multiple surface patches. To generate a grid on the entire surface of the entity, it is first decomposed into multiple topological regions [10]. Each such region is a trimmed surface patch. A trimmed surface patch is typically obtained by intersection of surfaces. The region on the parametric domain corresponding to the trimmed surface patch is further modeled topologically as being four-sided, for structured grid generation.

As of now, this topological decomposition and four-sided region specification must be manually done in most CAD systems. Grid points are first distributed on the boundaries of four-sided patches. The task is now to generate a suitable grid on each patch.

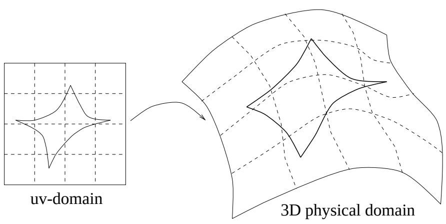

Figure 1: Patch boundaries are concave and not aligned with isoparametric curves

We note the following aspects of this problem:

- The boundary curves of the trimmed patch may not be aligned with the isoparametric curves of the surface on which the patch is identified. (see Figure 1)
- The shape of the boundary curves on the trimmed patch as seen on the parametric domain may lead to a concave shape.

These observations imply that firstly, it is non-trivial to obtain a grid such that all the grid points lie inside the domain and the elements are valid; secondly improvement of surface grid to achieve properties like smoothness, orthogonality while maintaining a valid grid is difficult.

In the following sections, we present our technique for generation of structured grid on an arbitrary four-sided patch and its optimization and adaptation for desired properties.

3 Reparameterization of Patch

We look for a parameterization such that the square domain is topologically identical to the trimmed region in the uv-space. This implies one-to-one correspondence between boundary points and interior points. A further desirable quality of a parameterization is near-linearity in the mapping between the square domain and the four-sided patch on the surface. As already mentioned earlier, the traditional method of transfinite interpolation (Coons' patch) suffers from the fact that it does not guarantee this one-to-one correspondence. Various other reparameterization schemes have therefore been reported in the literature. Eck *et al* [3] use an approximation of harmonic maps for reparameterizing triangulated surfaces. Floater [5] discusses length-preserving and shape-preserving parameterizations of triangulated surface for generating smooth approximations.

In this section, we describe the reparameterization scheme used by us for mapping of grid points from a convenient parametric domain to the surface patch to be sampled.

3.1 Algorithm for Reparameterization

Given:

1. Biparametric surface of the form $F : [0,1] \times [0,1] \rightarrow R^3$.

2. A four-sided trimmed patch, represented as a sequence of four bounding curves intersecting and trimmed at four points on the parametric domain, (u_1, v_1), (u_2, v_2), (u_3, v_3), and (u_4, v_4), in counter-clockwise sense. Let us define the four trimmed sides as e_1: from (u_1, v_1) to (u_2, v_2), e_2: from (u_2, v_2) to (u_3, v_3), e_3: from (u_3, v_3) to (u_4, v_4), and e_4: from (u_4, v_4) to (u_1, v_1).

3. Distribution of points on the four sides of the patch, such that, the number of points on the opposite sides is same (requirement for structured grids).

Using the following steps we derive the reparameterization for the domain described above:

Step 1: Construct a new square domain and let the parameters in this domain be $(\xi, \eta) \in [0,1] \times [0,1]$. We assume a one-to-one correspondence between the sides $(\xi = 0) \leftrightarrow e_1$, $(\eta = 0) \leftrightarrow e_2$, $(\xi = 1) \leftrightarrow e_3$, $(\eta = 1) \leftrightarrow e_4$,

Step 2: Distribute points on the boundary of the $\xi\eta$-domain according to the arc-length distributions of points on the corresponding sides in uv-domain.

Step 3: Consider the points of distribution on the sides in uv-domain in counter-clockwise sense; this gives us a simple polygon. We triangulate this domain using Delaunay triangulation algorithm [8], suitably extended to ensure that no triangle is formed with all three points on a single edge. This is necessary to ensure that we do not end up with degenerate triangles in the $\xi\eta$-domain in Step 4 below. The extension to Delaunay triangulation is in the form of using edge swapping to avoid such degeneracy, and if required, inserting interior points.

Step 4: We use the connectivity information from the triangulation in trimmed region of uv-domain, and the boundary distribution of $\xi\eta$-domain to achieve a corresponding triangulation in $\xi\eta$-domain by reparameterization procedure of [5] The procedure involves forming a system of linear equations and solving them for the interior points in the new parametric domain.

The above process will give triangles on the $\xi\eta$-domain which are equal in number as those on uv-domain; and there is a one-to-one correspondence between the triangles, edges and vertices across the domains. Now, any point (ξ, η) in $\xi\eta$-domain can be mapped onto the trimmed patch on uv-domain taking the following steps (see Figure 2):

Step a: Determine the triangle in $\xi\eta$-domain in which the given point (ξ, η) lies. Let the vertices of the triangle be ν_1, ν_2 and ν_3.

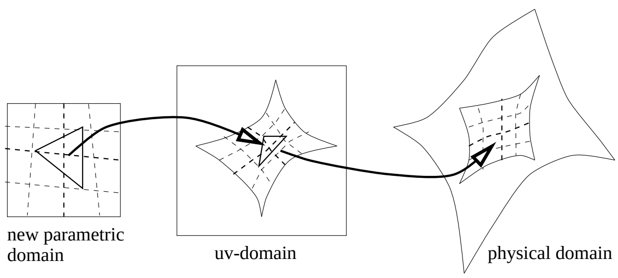

Figure 2: Two stage mapping of points from $\xi\eta$-domain to the trimmed surface patch

Step b: Determine the barycentric coordinates of the point (ξ, η) with respect to ν_1, ν_2 and ν_3; as α_1, α_2 and α_3.

Step c: Map the point onto the uv-domain by using the barycentric coordinates and the points of the corresponding triangle in uv-domain.

Here we have determined a parameterization for the trimmed region on the uv-domain, and effectively, for the patch on the surface.

4 Initial Distribution of Points

We first distribute the points on the trimmed patch ensuring a valid grid, but without worrying about the quality of the grid. The parametric domain for this process will be the newly derived $\xi\eta$-domain. Valid elements on $\xi\eta$-domain are guaranteed to be valid elements on the trimmed patch. We generate the initial distribution in this domain and improvement of the grid will be done by moving points in this domain.

In case of structured grids, the opposite sides of the four-sided domain have equal number of grid points, and the connectivity of the grid points is known in advance. The initial distribution of points is achieved by joining grid lines between the corresponding points on the opposite sides (see Figure 2). The grid points are then taken to be the intersections of the grid lines so drawn.

Note that these grid points are mapped to the surface via uv-domain by making use of the reparameterization described in the previous section. This distribution may start us off with a poor quality grid on the trimmed patch in R^3, but it is improved in the second phase.

5 Optimization of Surface Grid

In this section we describe the modeling of objective function used for grid optimization and the solution method for obtaining the optimal values for grid point positions. But first, we briefly identify the optimization objectives and analyze the existing methods for grid optimization.

5.1 Earlier Grid Optimization Methods

The qualities we optimize in the surface grids are:

Smoothness: the distance between the adjacent grid-lines should not change suddenly.

Orthogonality: the angle made by grid-lines meeting at all vertices should be close to 90^o.

Adaptivity: the process of grid generation should be controllable by specifying the desired density function at different parts in the physical domain.

In the literature we find a few methods for optimizing surface grids using iterative methods. The Laplacian smoothing [4] method moves each grid point to centroid of its neighboring points. Achieving high quality grids while ensuring validity of grid elements has been a known problem with this method. Also, unless the initial grid is reasonable, the iterative procedure of Laplacian smoothing tends to get trapped in a local minimum that most often does not correspond to a desirable grid. The grid smoothing algorithm discussed by TzuYi Yu and Bharat Soni in [15] does not address issues of grid generation over arbitrary trimmed patches or of element validity problems and does not model the objectives mentioned above. The method reported by Li [16] minimizes the sum of the squares of the chord-lengths between the connected grid points. A limitation of this method is that the chords across highly curved regions will completely ignore the curved features of the surface of interest. The method works only when (a) the surface is flat enough, or (b) the initial sampling is performed densely. The above methods either do not have safeguards to ensure valid grids or have not been designed to handle trimmed surface patches. The CFD community has been traditionally addressing smoothing of meshes using the Elliptic Grid Generation method that formulates the above objectives in the form of Euler-Lagrange equations [14]. These are then solved as elliptic differential equations. The literature mostly discusses use of elliptic solvers for smoothing planar grids. However, the formulation gets very complex for arbitrary four-sided patches on parametric surfaces and workable methods for solving them have yet to be derived.

5.2 Formulation of the Objective Function

The design variables in the grid optimization problem are the grid points on $\xi\eta$-domain. Note that there are twice as many variables in the system as the number of interior grid points. We formulate an energy function that models the properties of smoothness, orthogonality and adaptivity, such that the energy function with minimum value provides most desirable surface grid.

We model smoothness in distance between grid points along grid-lines by formulating a term that captures the sum of the squares of the arc-lengths of grid-lines on the surface patch. The smoothness term is then stated as:

$$\mathfrak{E}_{smooth} = \sum_{e \in E} w_e (arclength(e))^2 \qquad (1)$$

where, E is the set of all edges of elements of the surface grid and the function $arclength(e)$ returns the arclength of the edge computed along the surface. The factor w_e indicates the density of the grid point distribution on the surface patch. A large value for w_e would shorten the edge in the grid for the minimum value of $\mathfrak{E}_{smooth}$, and a small value of the factor will stretch the edge on the surface. This factor is specified for adaptive point distribution.

The orthogonality term ($\mathfrak{E}_{ortho}$) in the energy function is formulated as the sum of the squares of dot products of the vectors obtained at the intersection of the grid lines at the grid points.

The two terms mentioned above have been weighted and combined to form the objective function for the optimization procedure.

$$\mathfrak{E} = c_s \mathfrak{E}_{smooth} + c_o \mathfrak{E}_{ortho} \qquad (2)$$

5.3 Our Solution Method

We have used Goffe's implementation (called SIMANN) [9] of Simulated Annealing algorithm described in [2] for global optimization of multi-modal non-linear objective functions.

The algorithm takes a vector **x** (with $x_1, \ldots, x_n$ as its components) to be optimized for the minimum value of the objective function $f(\mathbf{x})$. One specifies the upper and lower bounds for ranges of the variables x_i to localize and accelerate the search. This stochastic algorithm does extensive search of the solution space in order to find the optimum values for **x**.

One has to provide an initial approximate solution, bounds for the variables and an objective function which is called by SIMANN internally. We have tuned the parameters of this code for our application.

6 Results

Figure 3 shows an example of improvement in smoothness and orthogonality at intersections of grid lines on trimmed patch over a bi-parametric surface.

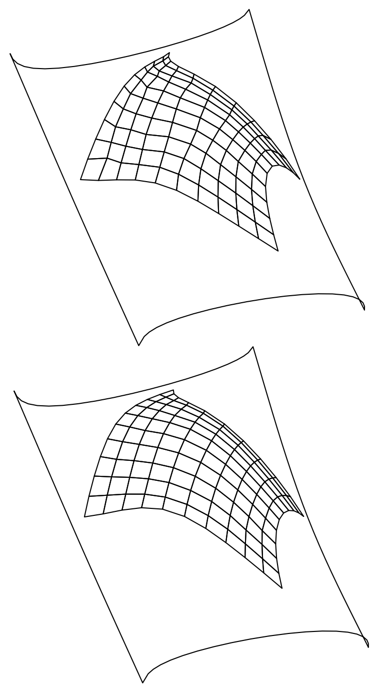

Figure 3: Structured grid on a trimmed four-sided patch before and after smoothing.

7 Conclusions and Future Work

The generation of high quality grids on complex surface components of engineering bodies has been a problem of interest in the fields of computer graphics, CAGD and the different engineering analysis areas. The earlier proposed solutions work for a number of cases, but fail to generate high quality grids in many cases, since they work on local optimization of grids to ensure validity.

This paper has presented a two phase technique consisting of derivation of a reparameterized domain which is used for the mapping of grid points and the use of simulated annealing for optimizing the grid to obtain desired properties. The two phase method has been implemented and results of one example have been presented. Because of the use of a simple square domain, ensuring validity of the grid in any stage of the grid generation process is very simple and does not impose any additional computational burden when using global optimization technique.

We have an efficient implementation of the first phase of the algorithm. While the quality of grid is as desired and control over the properties is explicitly modeled, the method needs acceleration in the second phase. Simulated annealing requires computation of objective function to be very efficient. Our immediate efforts are towards design of a simple objective function that adequately models the objectives and is still computationally efficient.

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About the Authors

Dinesh Shikhare, MS, is a scientist with the Graphics & CAD division of National Centre for Software Technology (NCST), Mumbai. His current activities include development of a CAD package for aircraft surface modeling, surface/volume grid generation and visualization.

S. Gopalsamy, PhD, is a research scientist with the Graphics & CAD division of NCST. His research interests include geometric modeling, grid generation, FEM and computer graphics. Currently he is a visiting scholar at Hong Kong University of Science and Technology (gopal@cs.ust.hk).

S. P. Mudur, PhD, is associate director of NCST and head of Graphics & CAD division. His research interests include geometric modeling, global illumination and image synthesis, physically based modeling and visualization, multimedia virtual reality environments, multi-lingual text processing.

The authors can be reached at the address mentioned on the title page of this paper.