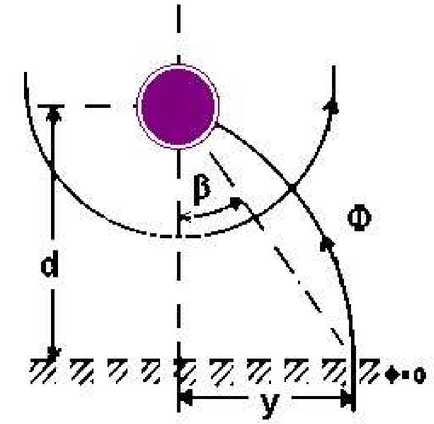
Full analysis & design solutions for saturated corona current condition
Wire to plane EHD thrusters (Category: Ionocrafts)

Ing. Saviour – Blaze Labs 2004



Wire to plane ionocraft
Corona current distribution
Blaze Labs 2004

This paper deals with analysis & design solutions for the wire to grid ionocrafts. The charge flow is simplified as multiple paths of unipolar ions drifting all together in the form of an ion cloud with mobility μ and negligible diffusion effects. A voltage source of voltage V is applied between the corona wire and collector grid. The corona is a discharge where ionisation is non thermal.

In the above diagram, the top conductor is the corona wire (not to scale), d is the vertical distance from wire to plane grid, and y is the effective lateral distance that the cloud travels during its journey to the collector. β is called the distribution angle.

It can be assumed that, all charge transport through the gap is carried by charged particles having the same polarity as the corona. The ion flow lines coincide with the electric field lines, but, the electric field distribution is strongly dependent on the ion space charge. At high currents or corona saturation currents, the current distribution $j(\beta)$ was earlier found by Warburg in 1899, that it closely follows the so called Warburg $\cos^5$ distribution, given by:

$j(B) = j(0) \cos^m(\beta)$ …… m = 4.82 for positive corona and m = 4.65 for Negative corona

If one plots the Warburg distribution it can be clearly seen that for angles of 60^o and over, the current density falls sharply from 4% towards 0%, indicating that the field lines further out than point y, at which $\beta >= 60^o$ are not acting upon the ion cloud. Due to the small difference in m for different polarities, the angle β for positive ion clouds at which this threshold occurs is slightly less, calculated to be just one degree less , that is 59^o.

This first rule, clearly indicates that implementing collector grids, which laterally exceed $2d*\tan(\beta)$ will not have any beneficial effect upon the resulting thrust, and result only in additional 'dead' weight. Knowing β, we can now conveniently express **y**, half the width of the ion cloud in terms of the height **d** of the wire above the plane, since

$\tan(\beta) = y/d$ (1)

$\tan(60^o) = y/d$

$y = 1.73 * d$ ….. for negative emitter corona wire (2)

for positive ions

$\tan(59^o) = y/d$

$y = 1.66 * d$ ….. for positive emitter corona wire (3)

Effective ion cloud cross sectional area

Thus, the effective area of the negative or positive ion clouds at the grid per unit length ℓ of the grid becomes:

$A_n = 2 * y * \ell = 3.46 * d * \ell$ ... for negative corona (4)
$A_p = 2 * y * \ell = 3.32 * d * \ell$ ... for positive corona (5)

So, effective area is independent of actual grid lateral width (given that the width 2y >= 3.46*d or 3.32* d respectively)

Derivations for maximum pressure, force, air flow, air velocity & corona saturation current

Now, the ion flow is driven through the air gap by the electric field and braked by the friction with the neutral gas molecules. Ion acceleration in the wire to plane geometry is negligible, and thus all the electric energy from the field is ultimately transferred to the neutral molecules. We can thus integrate the force qE along all field lines, and take the effective force projected over the plane grid. This has been worked out by Sigmond et all, and the total force exerted by the corona current i over a gap distance d is given by:

$F = i d / \mu$ …….. F is force in Newtons, I current in Amps, d air gap in metres, μ = ion mobility in air

This shows that the vertical component of the force is independent of the actual ion path and electric field.
Again we have a small difference between negative and positive ions, due to a small variation in mobility for different polarities, μ_n= 2.7 E-4 m^2/Vs , μ_p= 2.0 E-4 m^2/Vs , thus we have:

F = i d / 2.7 E- 4 ….. for negative corona (6)

F = i d / 2.0 E- 4 …… for positive corona (7)

This force, is equal to the momentum transfer rate, and can be used to calculate gas flow and velocity at the lower side of the grid. If we assume a free flowing air stream through the collector grid effective area A, with average air flow S and velocity v (m/s), we have:

$F = i d / \mu = \rho S v = \rho S^2/A$, …………. ρ is the air density in kg/m3 , S= flow rate in litres/sec

$S = (i A d / \mu\rho)^{1/2}$ (8)
$v = S/A = (id / \mu\rho A)^{1/2}$ (9)

Now, the total maximum pressure acting over the active grid area A (having dimensions: length ℓ, width 2y), is equal to :

$P = F/A$
$P = id /(A\mu)$
$P = id /(2y \ell \mu)$ …. writing y in terms of d, y= 1.73d or y=1.66d for the respective corona polarities we have:

$P_{max} = id / (3.46 d \ell \mu) = i / (3.46 \mu \ell) = j / (3.46 \mu)$ … for negati ve coronas (10)

$P_{max} = id / (3.32 d \ell \mu) = i / (3.32 \mu \ell) = j / (3.32 \mu)$ … for positive coronas (11)

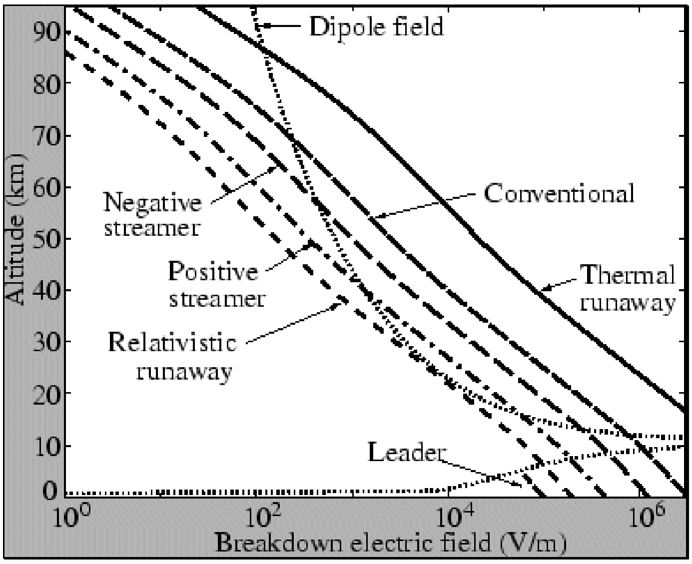
This shows that pressure generated does not depend on the length ℓ ,but only on saturation current density j.

Now the pressure gradient is equal to coulombs force acting on each ion.

From Gauss law, we have $\delta p/\delta z = \varepsilon_o * E * (\delta E/\delta z)$ …. where E varies from zero to the streamer breakdown voltage

Integrating for total pressure increase **$\Delta P_{max} = ½ \varepsilon_o E_{max}^2$ or $½ \varepsilon_o (V/d)^2$ (12)**

The breakdown in the ionised air gap occurs by means of a totally different mechanism than the conventional electron avalanche, namely by corona streamers. Streamers are ionisation waves which can propagate as narrow channels through regions where the electric field is less than the conventional breakdown voltage for air (30kV/cm). In the plot below, you can see the conventional Townsend avalanche breakdown voltage at sea level is approximately 3E6 V/m, but once we have corona streamers in action, this drops to either 1E6 V/m for negative streamers, or even worse, about 0.8E6 V/m for positive streamers. This self-propagation as we know, is due to highly nonuniform electric fields which result from significant gradient in current density, or space charge.



Note that again, we have a small variation between streamer breakdown voltage (V/d) for negative and positive coronas, where E_n ~ -11kV/cm or -1.1E6 V/m and E_p ~ +8kV/cm or +0.8E6 V/m. However this breakdown voltage varies with pressure, temperature, humidity and lateral wind speed. So, it is recommended to use actual streamer breakdown V/d values from experimental data.

We can now equate the above pressure equations:

$P_{max} = j_{max} / (3.46\,\mu) = ½ * \varepsilon_o E_n^2$ for negative coronas (13)

$P_{max} = j_{max} / (3.32\,\mu) = ½ * \varepsilon_o E_p^2$ for positive coronas (14)

Thus the saturation corona current per unit length (per metre) j_{max} in each case is given by:

$j_{max} = 1.73 * \mu\varepsilon_o E_n^2$ for negative coronas $E_n = -V/d$ (15)

$j_{max} = 1.66 * \mu\varepsilon_o E_p^2$ for positive coronas $E_p = +V/d$ (16)

This shows that an upper limit for current exists which is independent of actual ionocraft size, given it is operated at max V/d (= E_n or E_p). The same applies for pressure generated.

Substituting for **j_{max}** in $F = j \ell d / \mu$, we get

F_{max} per unit length = $1.73 * \varepsilon_o V^2 / d$ for negative coronas (17)

F_{max} per unit length = $1.66 * \varepsilon_o V^2 / d$ for positive coronas (18)

Using equations (17) & (15), the thrust to power ratio in g/Watt assuming $g=9.8m/s^2$:

T/P = (1000 / 9.8)* Fmax / (V*Imax) = 102 / (μE) where E= Ep or En depending upon polarity (19)

Machine efficiency = Mechanical energy output / Electrical energy input , assuming thruster reaches air velocity

Conversion efficiency % = 100 * F_{max} * v / (i V) (20)

Practical worked estimates for an Ionocraft working at +40kV (positive corona), length 1m.

From E_p ~ +0.8MV/m, we know that the gap distance d must be 40kV/(8kV/cm) = 5cm or 0.05m

Eqn(1)….tan(59^o) = y/d, y = 1.66 * d = 0.083cm

Eqn(5) ….Ion cloud area reaching grid = A_p = 2 * y * ℓ = 2 * 0.083 * 1 = 0.166m^2

Eqn(16)…. Saturated corona current per metre j_{max} = 1.66 * $\mu\varepsilon_o E_p^2$ = 1.66 * 2E-4 * 8.8542E-12 * $0.8E6^2$ = 1.88 mA/m or 18.8uA/cm

Eqn(8) …. air flow rate S= $(i\ A\ d\ /\ \mu\rho)^{½}$ = $(1.88E-3 * 0.166 * 0.05/ (2E-4 * 1.2))^{½}$ = 0.25 litres/sec

Eqn(9) …..air velocity v = S/A = $(id\ /\ \mu\rho A)^{½}$ = $(1.88E-3 * 0.05/(2E-4* 1.2 * 0.166))^{½}$ = 1.53 m/s

Eqn(18)….. F_{max} per unit length = 1.66 * $\varepsilon_o V^2 / d$ = 1.66 * 8.8542E-12 * $40E3^2$ / 0.05 = 0.47 N/m = 46gF /m

Eqn(14) ….. P_{max} = j / (3.32 μ) = 1.88E-3 / (3.32 * 2E-4) = 2.83 Pa (or P_{max} = ½ $\varepsilon_o E_{max}^2$ = 2.83 Pa) (or $P_{max} = F_{max}/ A_p$ = 2.83Pa)

Total power consumption = IV = 1.88mA * 40kV = 75.2 Watts

Eqn(19)…… T/P = 102 / (μE_p) = 102/ (2E-4 * 0.8E6) = 0.64 g / Watt

Eqn(20)……. Conversion efficiency % = 100 * F_{max} * v / (i V) = 100* 0.47 * 1.53/ 75.2 = 0.96%

So, this ionocraft will produce a 46gF thrust, and run at 1.88mA, 75.2Watts.

It can also be used as an EHD hovercraft generating a total pressure of 2.83Pa.

As an EHD air pump it will flow about 900 litres of air per hour

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Quick reference:

μ_n = 2.7 E-4 m^2/Vs
μ_p = 2.0 E-4 m^2/Vs

E_n ~ -11kV/cm or -1.1E6 V/m
E_p ~ +8kV/cm or +0.8E6 V/m

Warburg current distribution j(B) = j(0) $\cos^m(\beta)$
m = 4.82 for positive corona
m = 4.65 for negative corona

Distribution angle β = 60^o for negative corona
Distribution angle β = 59^o for positive corona

ε_o = 8.8542E-12 F/m
air density ρ = 1.2kg/m^3
g= 9.8 m/s^2