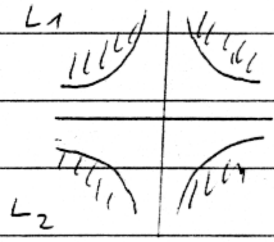
2. Serie (ab 28.10.02)

1a) $L = (-\infty; \frac{1}{2}]$

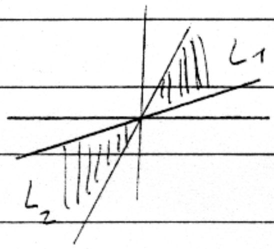
b) $L = (-3; 2)$

c) $L = \left[-7; \frac{-3-\sqrt{37}}{2}\right) \cup \left[-2; \frac{\sqrt{37}-3}{2}\right)$

d) $y > 0 \rightarrow L_1 = \{(x,y) \mid y > \frac{1}{|x|}\}$; $y < 0$ $\{y < -\frac{1}{|x|}\} = L_2$



e) $\underbrace{\{y \leq 2x \wedge y \geq \frac{x}{2}\}}_{L_1}, \underbrace{\{y \geq 2x \wedge y \leq \frac{x}{2}\}}_{L_2}$



f) $(x-3)^2 + (y-2)^2 < 9$ Kreisinneres!

3a) ! $z = x + yi$! ! $|z| = \sqrt{x^2+y^2}$! $|z-1| = |x+yi-1| = |(x-1)+yi| = \sqrt{(x-1)^2+y^2}$

damit $(x-1)^2+y^2<1$ bzw. $(x-1)^2+y^2>1$, Inneres bzw. Äußeres des Kreises

b) $(x-1)^2 + y^2 + (x+1)^2 + y^2 = 4 \rightarrow x^2+y^2 = 1$ (Kreis)

c) $\sqrt{(x-1)^2+y^2} + \sqrt{(x+1)^2+y^2} = 4$ (zweimal quadrieren) $\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$ Ellipse

d) $\sqrt{x^2+y^2} + x = 1 \rightarrow x = \frac{1}{2} - \frac{y^2}{2}$

e) $|z| \leq |\frac{z}{2}| + 1 \rightarrow |z| \leq \frac{1}{2}|z| + 1 \rightarrow |z| \leq 2$

f) $(z-z_1)(z-z_2) = z^2 - (1+2i)z + i - 3 \rightarrow z_1 = 2+i, \; z_2 = -1+i$

g) $k = 8 + 6i$

h) $\left|\frac{z-4}{z-8}\right| = 1$ liefert $z = \underline{\underline{6}} + yi$, in zweite Gleichung einsetzen

$\rightarrow z_1 = 6 + 17i$

$z_2 = 6 + 8i$

2a)

$$S_n = \sum_{k=1}^{n} \frac{1}{\sqrt{k}} \geq \sqrt{n} \quad ; \quad n \geq 1$$

$\boxed{\text{IA}}$ $n = 1: \quad 1 \geq 1$ ✓

$\boxed{\text{IV}}$ $n = p: S_p = \sum_{k=1}^{p} \frac{1}{\sqrt{k}} \geq \sqrt{p} \quad (p \geq 1)$

$\boxed{\text{IBeh.}}$ $n = p+1: S_{p+1} = \sum_{k=1}^{p+1} \frac{1}{\sqrt{k}} \geq \sqrt{p+1}$

$\boxed{\text{IBew.}}$! IV ! $\quad S_p \geq \sqrt{p} \quad \| + \frac{1}{\sqrt{p+1}}$

$$\underline{S_{p+1}} = S_p + \frac{1}{\sqrt{p+1}} \geq \sqrt{p} + \frac{1}{\sqrt{p+1}} \underline{> \sqrt{p+1}}$$

denn: $\sqrt{p} + \frac{1}{\sqrt{p+1}} = \frac{\sqrt{p}\sqrt{p+1} + 1}{\sqrt{p+1}} \overset{?}{>} \sqrt{p+1} \quad | \cdot \sqrt{p+1}$

$$\sqrt{p} \cdot \sqrt{p+1} + 1 \overset{?}{>} p+1$$

Rückschluß $\quad$ R.S. $\uparrow$ $\quad \sqrt{p}\,\underline{\sqrt{p+1}} > p$ offensichtlich!

2b) $\quad S_n = \sum_{\mu=1}^{n} \frac{1}{n+\mu} > \frac{13}{24} \quad ; \quad n \geq 2 \quad \Big| \quad S_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+(n-1)} + \frac{1}{(n+n)}$

$\boxed{\text{IA}}$ $n = 2: \quad \frac{1}{2+1} + \frac{1}{2+2} = \frac{7}{12} > \frac{13}{24}$ ✓

$\boxed{\text{IV}}$ $n = k: \quad S_k = \frac{1}{k+1} + \frac{1}{k+2} + \ldots + \frac{1}{k+(k-1)} + \frac{1}{k+k} > \frac{13}{24}$

$\boxed{\text{IBeh}}$ $n = k+1: \quad S_{k+1} = \frac{1}{(k+1)+1} + \frac{1}{(k+1)+2} + \ldots + \frac{1}{(k+1)+k-1} + \underline{\frac{1}{(k+1)+k}} + \underline{\frac{1}{(k+1)+k+1}} > \frac{13}{24}$

$\boxed{\text{IBew.}}$ $\quad S_{k+1} = S_k - \frac{1}{k+1} + \underline{\frac{1}{2k+1}} + \underline{\frac{1}{2k+2}} = S_k + \underbrace{\frac{1}{2k+1} - \frac{1}{2k+2}}_{>0} > S_k > \frac{13}{24}$

! IV !

also $\quad S_{k+1} > S_k > \frac{13}{24}$

$$S_{k+1} > \frac{13}{24}$$

2c) $\prod_{\nu=1}^{n} \frac{2\nu-1}{2\nu} < \frac{1}{\sqrt{2n+1}}$; $n>1$ | $\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\ldots\cdot\frac{2n-1}{2n} < \frac{1}{\sqrt{2n+1}}$

$\boxed{\text{IA}}$ $n=1$: $\frac{1}{2} < \frac{1}{\sqrt{3}}$ ✓

$\boxed{\text{IV}}$ $n=k$: $p_k = \frac{1}{2}\cdot\frac{3}{4}\cdots\frac{(2k-1)}{2k} < \frac{1}{\sqrt{2k+1}}$

$\boxed{\text{IBeh.}}$ $n=k+1$: $p_{k+1} = \frac{1}{2}\cdot\frac{3}{4}\cdots\frac{(2k-1)}{2k}\cdot\frac{(2k+1)}{2(k+1)} < \frac{1}{\sqrt{2k+3}}$

$\boxed{\text{IBew.}}$ $p_{k+1} = p_k \cdot \frac{2k+1}{2(k+1)} \overset{!\,\text{IV}\,!}{<} \frac{1}{\sqrt{2k+1}}\cdot\frac{2k+1}{2(k+1)} < \frac{1}{\sqrt{2k+3}}$,

denn: $\frac{1}{\sqrt{2k+1}}\cdot\frac{2k+1}{(2k+2)} \overset{?}{<} \frac{1}{\sqrt{2k+3}} \rightarrow \sqrt{2k+3}\,(2k+1) \overset{?}{<} \sqrt{2k+1}\,(2k+2)$

$\rightarrow \sqrt{2k+3}\sqrt{2k+1} < 2k+2 \rightarrow \underline{4k^2+8k+3 < 4k^2+8k+4}$ RS.

2d) $p_n = \prod_{k=1}^{n}(2k)! > [(n+1)!]^n$, $n \geq 2$ | $(2!)\cdot(4!)\cdot\ldots\cdot((2n)!) \cdot [(n+1)!]^n$

$\boxed{\text{IA}}$ $n=2$: $2\cdot 24 > (3!)^2$; $48 > 6^2$; $48 > 36$ ✓

$\boxed{\text{IV}}$ $n=t$: $p_t = \prod_{k=1}^{t}(2k)! > [(t+1)!]^t$ $(t \geq 2)$

$\boxed{\text{IBeh.}}$ $n=t+1$: $p_{t+1} = \prod_{k=1}^{t+1}(2k)! > [(t+2)!]^{t+1}$

$\boxed{\text{IBew.}}$ $\underline{p_{t+1}} = \underline{p_t}\cdot(2t+2)! \overset{!\,\text{IV}\,!}{>} \underline{[(t+1)!]^t}\cdot(2t+2)! > \underline{[(t+2)!]^{t+1}}$

NR, denn: $\underline{[(t+1)!]^t}(2t+2)! \overset{?}{>} [(t+2)!]^{t+1} = [(t+2)!]^t(t+2)!$

$(2t+2)! \overset{?}{>} (t+2)^t\cdot(t+2)!$

$\underline{1\cdot 2\cdots(t+2)}(t+3)\cdots(2t+2) \overset{?}{>} (t+2)^t\cdot\underline{1\cdot 2\cdots(t+2)}$

$\underbrace{(t+3)\cdot\ldots\cdot(2t+2)}_{t \text{ Stück}} > (t+2)^t$

t Stück, damit ist letzte Ungleichung offensichtlich richtig → Rückschluß